VI Workshop on Trends in Nonlinear Analysis (WTNLA6)

September 3–5, 2026

Cagliari, Sardinia (Italy)

Fondazione di Sardegna

Contents

Program Overview

Time	**3/9 (Thu)**	**4/9 (Fri)**	**5/9 (Sat)**
09:30 – 10:15	N. Fusco	S. Polidoro	G. Porru
10:15 – 11:00	U. Gianazza	V. Ferone	V. Vespri
11:00 – 11:30	*Coffee Break*		
11:30 – 12:00	F. M. Cassanello	V. Iorio	A. Columbu
12:00 – 12:45	G. Ciraolo	T. Yokota	S. Vernier Piro
12:45 – 14:30	*Lunch*		*Closing*
14:30 – 15:15	Y. Liu	M. Squassina	
15:15 – 16:00	R. Filippucci	S. Fagioli	
16:00 – 16:30	A. Rebucci	M. Kamath Katapady	
16:30 – 17:00	*Discussion*		
20:30	**Social Dinner**		

A word from the organizers

"*Mathematics is best done with friends*" (P.M.)

Organizing scientific conferences in small/average Universities, especially when it comes with geographical hurdles, sometimes can be tricky, even when in a lively and competitive environment like Cagliari, not to mention creating a tradition of smart, high-quality scientific meetings in pure mathematics. It takes dedication, passion for the subject, and an ongoing commitment to find resources.

The *Workshop on Trends in Nonlinear Analysis* is the brainchild of Prof. Stella Vernier Piro, who, with some colleagues and students, promoted the first edition (simply named *Trends in Nonlinear Analysis*) in 2014. Then the Workshop went on with 2015, 2017 (celebrating Prof. Vernier Piro's 70th anniversary), 2022, and 2024 editions, mainly devoted to the study of existence, regularity, and qualitative properties of solutions to nonlinear partial differential equations and their applications (especially to biological phenomena).

Meanwhile, an enrichment was brought in by the adding of thematic conferences *Geometric Properties of Solutions to Elliptic and Parabolic Problems* (Santa Margherita di Pula, 2016), *Research Meeting on Non-Local Operators* (2016), *Partial Differential Equations in Analysis and Mathematical Physics* (Santa Margherita di Pula, 2019), the *8th International Workshop on Mathematical Analysis of Chemotaxis* (2025).

Each of these conferences has represented a unique opportunity for the mathematical analysis community of Cagliari, whose main focus in research is on partial differential equations, to tie or renew scientific connections with renowned experts and emerging researchers in the field from both Italian and international Universities, as well as to discover new directions in study and share their results - and to do so as a group, beside individual collaborations that many of us entertain with mathematicians from outside the Island.

Special emphasis was always placed on contributions by Ph.D. students and young researchers.

But what makes the Cagliari workshop a meaningful experience is the aim to create a cheerful, familiar atmosphere, surrounding peer-to-peer discussions on mathematics (and more) in accordance with the traditional Sardinian hospitality, despite the mixed origins of organizers.

Either you are a long-time contributor of the Workshop or an appreciated new guest, we hope you enjoy this 2026 edition!

III Workshop on Trends in Nonlinear Analysis, Cagliari, September 7-9 2017

Preliminaries

Organizing Committee

- **Rafael Díaz Fuentes** - University of Cagliari
- **Fatma Gamze Düzgün** - University of Cagliari
- **Antonio Iannizzotto** - University of Cagliari
- **Monica Marras** - University of Cagliari

Locations

- **Main Venue:** Palazzo delle Scienze, Via Ospedale 72, Cagliari
  Dipartimento di Matematica e Informatica, Università degli Studi di Cagliari.
- **Social Dinner:** To be announced.

Sponsors

We are deeply thankful to the agencies and institutions that support this event:

- **Fondazione di Sardegna** (Grant M. Marras)
- **AISAC** (A. Fanti, M. Marras)
- **Department of Mathematics and Computer Science**, University of Cagliari

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Abstracts

Filippo M. Cassanello

Title: *Hölder regularity for the fractional p-Laplacian*

Abstract: We present an alternative proof for local Hölder regularity of the solutions of the fractional p-Laplace equations based on clustering and expansion (more precisely, recentering) of solutions.

Giulio Ciraolo

Title: *Quantitative stability for the critical p-Laplace equation*

Abstract: The critical p-Laplace equation arises in the context of determining the optimal constant in the Sobolev inequality and is also related to the Yamabe problem when $p = 2$. Since the seminal work of Gidas, Ni & Nirenberg, considerable effort has been devoted to the classification of positive solutions to this equation and, more broadly, to the study of symmetry and monotonicity properties of positive solutions to equations modeled on the critical one. Following Struwe's qualitative stability result, a number of works have addressed quantitative aspects of stability of the critical equation for $p = 2$ and functions in the energy space $\mathcal{D}^{1,2}(\mathbb{R}^n)$. In this seminar, we present a recent stability result for the general case $1 < p < n$.

Alessandro Columbu

Title: *Blow-up vs. Boundedness in a Chemotaxis System with Flux Limitation and Nonlinear Signal Production*

Abstract: This work, carried out in collaboration with Monica Marras and Shohei Kohatsu, is devoted to the analysis of a parabolic-elliptic cross-diffusion system with gradient-dependent sensitivity and logistic growth. We consider radially symmetric solutions in a smoothly bounded domain of $\mathbb{R}^N$, $N \geq 3$, under homogeneous Neumann boundary conditions. The cross-diffusion mechanism is modulated by a nonlinear sensitivity function of the gradient of the elliptic variable, while the reaction term includes logistic damping.

We investigate the interplay between aggregation effects, nonlinear diffusion, and logistic growth. Under suitable assumptions on the sensitivity function, the production term, and the initial data, we establish finite-time blow-up of solutions in the L^∞-norm. Moreover, we prove blow-up in suitable L^p-spaces and derive a lower bound for the blow-up time. On the other hand, we identify conditions ensuring global existence and uniform boundedness of solutions. The results reveal how gradient-dependent flux limitation, nonlinear production, and logistic effects interact in determining whether solutions remain globally bounded or develop singularities in finite time.

Simone Fagioli

Title: *Singular Limits and Well-Posedness for Multi-Species Nonlocal Interaction Models with Riesz-Type Potentials*

Abstract: We investigate various versions of systems involving many species, modeling aggregation phenomena through nonlocal interaction terms. We establish a rigorous connection between kinetic and macroscopic descriptions by considering the small-inertia limit at the kinetic level. The results are proven either under smoothness assumptions on all interaction kernels or

under singular assumptions for *self-interaction* potentials. Utilizing different techniques in the two cases, we demonstrate the existence of a solution to the kinetic system, provide uniform estimates with respect to the inertia parameter, and show convergence towards the corresponding macroscopic system as the inertia approaches zero, see [1]. Furthermore, by exploiting optimal transportation theory and the theory of gradient flows in Wasserstein spaces, we establish the existence of weak solutions for the macroscopic system under singularity assumptions on all interaction potentials, provided the cross-interaction ones satisfy a symmetry condition. Our analysis extends previous results available for either single-species equations or multi-species systems with smoother cross-interaction kernels, see [2].

References

[1] Y.-P. Choi, S. Fagioli, V. Iorio, Small inertia limit for coupled kinetic swarming models. *J Nonlinear Sci*, **35**, 39 (2025)

[2] S. Fagioli, V. Iorio: Two-species system with nonlocal interactions driven by Riesz potentials. *Submitted preprint*, (2026)

Vincenzo Ferone

Title: *First eigenvalue and torsional rigidity: isoperimetric inequalities for the fractional Laplacian*

Abstract: We present a fractional counterpart of a generalized Kohler-Jobin inequality, showing that, among all bounded, open sets $\Omega \subset \mathbb{R}^N$ with Lipschitz boundary, having the same fractional torsional rigidity, the first Dirichlet eigenvalue $\lambda_1(\Omega)$ of the fractional Laplacian attains its minimum on balls. With the same arguments we also establish a reverse Hölder inequality for an eigenfunction corresponding to $\lambda_1(\Omega)$. The results have been obtained in collaboration with B. Brandolini, I. de Bonis, G. Piscitelli and B. Volzone.

Roberta Filippucci

Title: *A priori estimates and Liouville type results for inhomogeneuos problems with convection*

Abstract: Starting from new connections between Liouville type theorems and local properties of nonegative solutions to superlinear elliptic problems investigated in [4] in light of previous results in [5], in this talk I willl consider the framework of inhomogeneuos problems involving the (p, q)-Laplacian operator and a rection taking different forms, in some cases including gradient terms, focusing on both supersolutions and solutions. In the first case, treated in [1], we propose an alternative approach to the celebrated capacity method of Mitidieri-Pohozaev [3], provided higher regularity of supersolutions is available. In particular, we prove a key lemma via an iteration method along the way we improve the lower bound for each solutions to each iteration step.

In case of solutions, we employ a nontrivial adaptation of the Bernstein method to the setting of nonhomogeneous operators. This adaptation requires careful handling of the (p, q)-structure, as it addresses the challenge of variable coefficients that may be unbounded from above. It allows us to derive a crucial a priori gradient estimate valid in any domain, which leads directly to the required Liouville properties.

References

[1] M. Bhakta, A. Biswas, and R. Filippucci, Liouville properties for differential inequalities with (p,q)-Laplacian operator, *J. Lond. Math. Soc.* **113e**, 3 (2026), e70490

[2] M. Bhakta, A. Biswas, and R. Filippucci, Liouville results for (p, q)-Laplacian elliptic equations with source terms involving gradient nonlinearities, *submitted* , 31 pp.

[3] E. Mitidieri, S. Pohozaev, The absence of global positive solutions to quasilinear elliptic inequalities. *Dokl. Akad. Nauk*, **359** (1998), 456-460; English translation in Dokl. Math., 57 (1998), 250-253.

[4] P. Poláčik, P. Quitter, and P. Souplet, Singularity and decay estimates in superlinear problems via Liouville-type theorems, I: Elliptic equations and systems, *Duke Math. J.*, **139** (2007), 555–579.

[5] J. Serrin, H. Zou, Cauchy-Liouville and universal boundedness theorems for quasilinear elliptic equations and inequalities. *Acta Math.* **189**, 1 (2002), 79-142.

Nicola Fusco

Title: *Consistency for the Surface diffusion flat flow in three dimensions*

Abstract: We will discuss the existence of a flat flow solution for the surface diffusion equation

$$V_t = \Delta_{\partial E_t} H_{E_t}$$

via the discrete minimizing movements scheme proposed by Cahn and Taylor in [1]. Here H_{E_t} is the mean curvature of the evolving set, $\Delta_{\partial E_t}$ is the Laplace-Beltrami operator on ∂E_t and V_t is the normal velocity at the boundary. We will show that in three dimensions, if the initial set is sufficiently regular, the scheme converges to the unique smooth solution of the equation, as long as the latter exists.

Joint paper with Marco Cicalese, Vesa Julin and Andrea Kubin, see [2].

References

[1] J.W. Cahn, J.E. Taylor, Surface motion by surface diffusion, *Acta Metall. Mater.* **42**, 4 (1994), 1045–1063

[2] M. Cicalese, N. Fusco, V. Julin, and A. Kubin, Consistency for the Surface diffusion flat flow in three dimensions, *arXiv:2502.13556* (2025), 45 pp.

Ugo Gianazza

Title: *Self-improvement phenomena in nonlinear diffusion equations*

Abstract: Self-improvement phenomena are a central topic in the analysis of partial differential equations. A classical example is the Gehring-type higher integrability of the gradient of weak solutions to certain partial differential equations and of minimizers of variational functionals. Even though the stationary case is by now well understood, the evolutionary setting poses additional challenges, particularly for nonlinear equations, and many interesting questions have received only partial answers.

In this talk, I will present recent advances in the local higher integrability of the spatial gradient of weak solutions to two classes of nonlinear diffusion equations and systems. I will also address the so-called *sub-critical* regime, where solutions may be unbounded. The proofs are based on intrinsic scaling, estimates for super-level sets, and supremum bounds obtained under mild integrability assumptions on the solution.

These results have been obtained in collaboration with V. Bögelein, F. Duzaar, N. Liao, and S. Schwarzacher.

Megha Kamath Katapady

Title: *Weak Solutions to a Constrained Aggregation-Diffusion-Reaction Model for Multiple Sclerosis*

Abstract: In this talk, we establish an existence result for weak solutions to an aggregation-diffusion-reaction equation with a constraint, arising in the modeling of multiple sclerosis. The model is derived from a general chemotaxis-type framework and describes the time evolution of the density of activated macrophages, which is subject to attraction by oligodendrocytes. The latter are governed by a constraint equation. The proof relies on a variational splitting scheme that isolates the transport (aggregation-diffusion) and reaction contributions. The structure of the constraint makes it possible to recover the density of oligodendrocytes as the limit of a sequence of characteristic functions.

References

[1] S. Fagioli, M. Kamath Katapady, Existence of Weak Solutions to a Constrained Aggregation-Diffusion-Reaction Model for Multiple Sclerosis, *arXiv:2601.19427* (2026)

Valeria Iorio

Title: *On deterministic particle approximations of continuity equations with Morse-type nonlocal interaction*

Abstract: This talk concerns deterministic particle methods for continuity equations with nonlocal interactions driven by Morse-type potentials. We work in the space $\mathcal{P}_2(\mathbb{R})$ of probability measures on $\mathbb{R}$ with finite second moment, equipped with the 2-Wasserstein distance. We first consider the one-species case with a repulsive Morse potential. We prove that the particle scheme converges to weak solutions of the nonlocal continuity equation as the number of particles tends to infinity. Moreover, after suitably rescaling the Morse potential so that it converges to a Dirac delta, we show that the nonlocal particle scheme approximates the quadratic porous medium equation.

We then study a system of two continuity equations coupled through both attractive and repulsive Morse potentials. After establishing existence, uniqueness, and stability of gradient flow solutions in $\mathcal{P}_2(\mathbb{R})$ via the JKO scheme, we introduce a deterministic particle approximation and prove that the associated discrete densities converge to the corresponding gradient flow solution as the number of particles tends to infinity.

This talk is based on joint work with Marco Di Francesco and Markus Schmidtchen.

Yichen Liu

Title: *On the path-connectedness of the extremal sets with applications to control theory*

Abstract: This talk investigates the characterization of the image of a continuous operator over a nonempty compact set via the image of its extremal set. The path-connectedness property of the extremal set plays an essential role in the characterization. The main results are discussed in a general framework, and applications to control problems of several different types are provided, including control problems of linear and nonlinear differential equations. In each application, we examine the path-connectedness of the set of extremal controls. Moreover, we demonstrate that, depending on the application, either the reachable sets or the image of the cost functional can be entirely recovered from these extremal controls. This talk is based on a joint work with Zhiping Rao.

Sergio Polidoro

Title: *Mean value formulas for classical solutions to subelliptic partial differential equations in divergence form*

Abstract: This seminar provides a comprehensive overview of recent advances concerning mean value formulas for classical solutions to degenerate linear second-order partial differential equations in divergence form. We will first discuss the mean value theorems established in collaboration with D. Pallara [1], which heavily rely on De Giorgi's perimeter theory and its extension to Carnot groups. Building upon these geometric tools and classical PDE theory, we will then explore how these formulas lead to strong maximum principles and Harnack inequalities. Specifically, we will review the recent results obtained by G. Pecorella [2] for stationary subelliptic equations on Carnot groups, as well as the parallel developments by A. Rebucci [3] for degenerate Kolmogorov equations. Finally, we will outline some related open problems in the field.

References

[1] D. Pallara, S. Polidoro, Mean value formulas for classical solutions to subelliptic evolution equations in stratified Lie groups, *Calc. Var.* **64**, 208 (2025), 1–31

[2] G. Pecorella, Fundamental solution, maximum principle and Harnack inequality for second order subelliptic operators, *J Elliptic Parabol Equ* **11** (2025), 45–81

[3] A. Rebucci, Harnack inequality and maximum principle for degenerate Kolmogorov operators in divergence form, *J. Math. Anal. Appl.* **538**, 2 (2024), 128371

Giovanni Porru

Title: *Numerical Attractors*

Abstract: In the mathematical field of dynamical systems, an attractor is a set of states toward which a system tends to evolve for a wide variety of starting conditions of the system.

Geometrically, an attractor can be a point, a curve, a manifold, or even a complicated set with a fractal structure. Describing the attractors of chaotic dynamical systems has been one of the achievements of chaos theory.

I only consider elementary numerical attractors. No background is needed to follow this talk.

In 1949, the indian mathematician (autodidact) D. R. Kaprekar discovered the following fact. Pick a four digit number where at least 2 digits are different. Rearrange the digits in a decreasing order (larger number). Rearrange the digits in an increasing order (smaller number). Subtract the smaller number from the larger one. Repeat the process with the result until you find a number with the same digits.

After a maximum of 7 steps, you will find 6174. If you apply the process to 6174 you will find the same number, that is why it is called attractor.

Several mathematicians tried to extend Kaprekar's result to numbers with n digits. It was proved that this process works for 3 and for 4 digits only. In addition, for numbers with 4 digits, the process does not work if one adopts a base different from 10.

In this talk, I define variants of Kaprekar's process that work for numbers with n digits, also if we adopt a base different from 10.

Annalaura Rebucci

Title: *On the Harnack inequality for subelliptic evolution equations in stratified Lie groups*

Abstract: In this talk we will investigate the regularity properties of a large class of degenerate subelliptic evolution operators, building on the framework established in [1, 2]. Following a "descent method" due to Kuptsov, we will obtain mean value formulas with improved kernels. We will then exploit the mean value formulas to prove an invariant Harnack inequality and a strong maximum principle for classical solutions to the equations under study. Our analysis is carried out in the non-Euclidean framework of the associated Lie group and critically relies on the associated Carnot-Carathéodory geometry.

References

[1] G. Pecorella, A. Rebucci, Fundamental solution and Harnack inequality for subelliptic evolution operators on Carnot groups, *arXiv:2509.15982* (2025)

[2] A. Rebucci, Harnack inequality and maximum principle for degenerate Kolmogorov operators in divergence form, *J. Math. Anal. Appl.*, **538**, 2 (2024), 128371

Marco Squassina

Title: *Log-concave solutions of the log-Schrödinger equation in convex domains*

Abstract: The talk concerns concavity phenomena for positive solutions of elliptic equations in convex domains. After a brief overview of classical examples, such as the torsion problem and the first Dirichlet eigenfunction, I will discuss general notions of quasi-concavity and power-concavity, as well as some recent developments for quasilinear equations. The main focus will then be on log-concave solutions of the log-Schrödinger equation in convex domains. The aim is to present the subject from a broad perspective, highlighting both the underlying geometric ideas and some recent advances.

Stella Vernier -Piro

Title: *Hölder estimates of weak solutions to chemotaxis systems of fast diffusion type*

Abstract: I present some recent results on quasilinear chemotaxis systems of singular type, where the diffusion operator is given by $\mathrm{div}(\nabla u^m)$ with $0 < m < 1$, corresponding to the fast diffusion regime, and where the chemotactic drift is nonlinear. Since Hölder continuity constitutes the optimal regularity class for weak solutions to the porous medium equation, we establish analogous regularity results for bounded solutions of parabolic–parabolic chemotaxis systems in this setting. The proof is based on a refined De Giorgi–Di Benedetto iteration scheme adapted to the coupled structure of the system. These results advance the understanding of the fine regularity properties of chemotaxis models with nonlinear diffusion, and demonstrate that the interplay between singular diffusion and aggregation exhibits a regularizing mechanism consistent with the porous medium paradigm. This talk is mainly based on joint research with M.Marras, F. Ragnedda and V. Vespri.

References

[1] E. DiBenedetto, U. Gianazza, and V. Vespri, *Harnack's inequality for degenerate and singular parabolic equations*, Springer Monographs in Mathematics, Springer (2012)

[2] S. Ishida, T. Yokota, Global existence of weak solutions to quasilinear degenerate Keller-Segel systems of parabolic-parabolic type with small data, *J. Differential Equations*, **252** (2012), 2469–2491

[3] M. Marras, F. Ragnedda, S. Vernier-Piro, and V. Vespri, Hölder estimates of weak solutions to degenerate chemotaxis systems with a source term, *J. Differential Equations*, **366** (2023), 42–70

Vincenzo Vespri

Title: *A Clustering Theorem in Fractional Sobolev Spaces*

Abstract: We present a new clustering theorem for functions in the fractional Sobolev space $W^{s,p}$. The result shows that if the positivity set of a function occupies a sufficiently large portion of a cube and its fractional Sobolev seminorm is suitably controlled, then the function remains positive on a smaller cube with quantitatively large measure. The theorem extends the classical clustering results known for $W^{1,p}$ and BV spaces to the nonlocal fractional framework. Such estimates play a fundamental role in the regularity theory of elliptic and parabolic equations, particularly for problems driven by the fractional p-Laplacian. As a consequence, previously known clustering theorems in $W^{1,p}$ and BV are recovered through interpolation inequalities, providing a unified approach to clustering phenomena in local and nonlocal settings.

References

[1] F. G. Düzgün, A. Iannizzotto, and V. Vespri, A clustering theorem in fractional Sobolev spaces, *Ann. Fenn. Math.* **50** (2025), 243–252

Tomomi Yokota

Title: *Boundedness in a chemotaxis-consumption model with signal-dependent sensitivity*

Abstract: In this talk we consider a chemotaxis-consumption systems with signal-dependent sensitivity. In previous studies, boundedness of solutions to the system has been obtained under some restrictions on the dimension and the diffusion coefficient. The purpose of this talk is to remove these restrictions. This is a joint work [1] with Khadijeh Baghaei (Pasargad Institute for Advanced Innovative Solutions), Yutaro Chiyo (Tokyo University of Science) and Chihaya Machino (Tokyo University of Science).

References

[1] K. Baghaei, Y. Chiyo, C. Machino, and T. Yokota, Improvements of boundedness criteria for a class of indirect chemotaxis-consumption models with signal-dependent sensitivity, *Appl. Math. Lett.* **172** (2026), 109718