SECONDA PROVA PARZIALE del 16.06.2026 – A – (150 minuti)

Cognome	
Nome	
N.matricola	

- Svolgere il compito solo in questo foglio. Utilizzare altri fogli solamente se necessari e comunque, dopo averli chiesti al docente.
- Esercizio: **7.5** punti.
- L'esame sarà considerato superato se la somma dei punteggi totali ottenuti raggiunge almeno 18/30

1) Calcolare il seguente integrale:

$$\int \frac{4x-7}{x^2-3x+2}\,dx$$

2) Calcolare il seguente integrale doppio nel dominio D:

$$\int xy\ dxdy$$

$$D = \{(x,y) \in R^2: \quad 1 \le x^2+y^2 \le 4, \qquad x \ge 0, \qquad y \ge 0\}$$

Suggerimento: (usare le coordinate Polari)

3) Determinare l'integrale particolare del seguente problema di Cauchy:

$$\begin{cases} y''-2y'+4y = 0 \\ y(0)=1 \\ y'(0)=4 \end{cases}$$

4) Determinare i massimi e minimi relativi della seguente funzione:

$$f(x,y) = x^3 - y^3 + xy$$

Es 1)

$$\int \frac{4x-7}{\underbrace{x^2-3x+2}} \cdot dx \hookrightarrow \text{eq. } 2^\circ \text{ grado}$$

$$\Delta = 9-8 = 1 > 0 \Rightarrow x_{1,2} = \frac{3 \pm \sqrt{1}}{2} = \begin{cases} 2 \\ 1 \end{cases}$$

*Scomposizione:

$$\frac{4x-7}{x^2-3x+2} = \frac{A}{(x-2)} + \frac{B}{(x-1)}$$

$$= \frac{A(x-1)+B(x-2)}{(x-2)(x-1)} = \frac{Ax - A + Bx - 2B}{x^2-3x+2}$$

$$\begin{matrix} x \\ N \end{matrix} \begin{cases} A+B = 4 \\ -A-2B = -7 \end{cases} \Rightarrow \begin{cases} A = 4-B \\ -4+B-2B = -7 \end{cases}$$

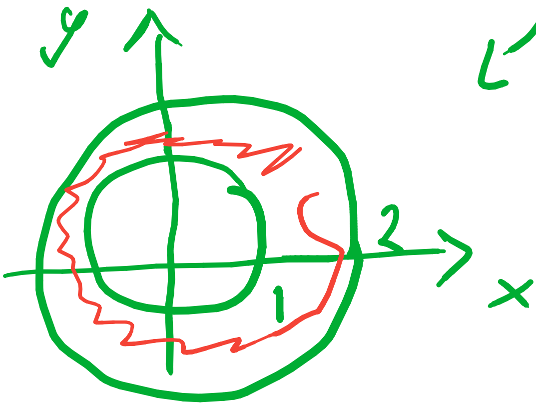
$$\begin{cases} A = 4-B \\ -4+7 = B \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = 3 \end{cases}$$

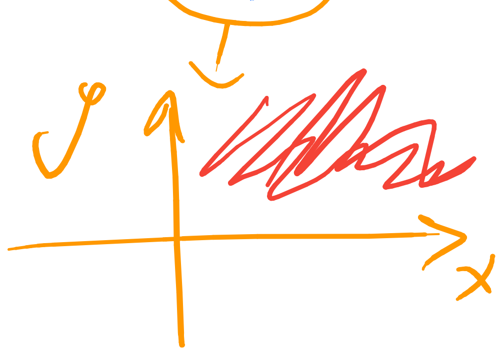
$$\Rightarrow \int \left(\frac{1}{x-2} + \frac{3}{x-1} \right) \cdot dx = \int \frac{1}{x-2} \cdot dx + 3 \int \frac{1}{x-1} \cdot dx$$

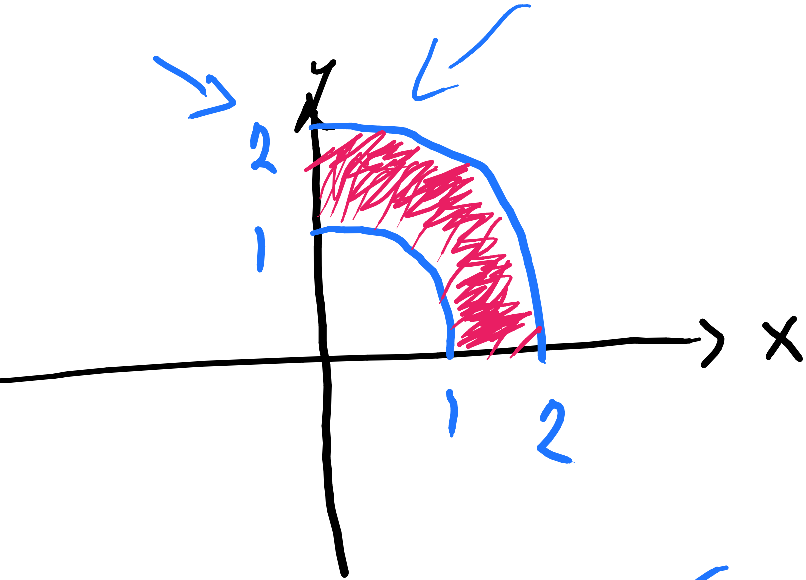
$$= \boxed{\ln|x-2| + 3 \ln|x-1| + C}$$

Es. 2) $\displaystyle\iint_D xy \; dx\,dy$

$$D = \left\{ (x,y) \in \mathbb{R}^2 : \; 1 \le x^2+y^2 \le 4 , \; \begin{matrix} x \ge 0 \\ y \ge 0 \end{matrix} \right\}$$







$$\begin{cases} x = \rho \cdot \cos(\theta) \\ y = \rho \cdot \operatorname{sen}(\theta) \end{cases}$$

$$1 \le \rho \le 2 \; , \quad 0 \le \theta \le \pi/2$$

$$dx\,dy = \rho \cdot d\rho \cdot d\theta$$

$$\Rightarrow \int_1^2 \int_0^{\pi/2} \rho \cos(\theta) \cdot \rho \cdot \operatorname{sen}(\theta) \cdot \rho \, d\rho \cdot d\theta$$

$$\hookrightarrow \int_1^2 \rho^3 \cdot d\rho \cdot \int \operatorname{sen}(\theta) \cdot \cos(\theta) \cdot d\theta$$

$$\int_1^2 \rho^3 \cdot d\rho = \frac{\rho^4}{4}\Big|_1^2 = \frac{1}{4}\left[(2)^4 - (1)^1\right]$$

$$= \frac{1}{4}(16 - 1) = \frac{15}{4}$$

$$\int_0^{\pi/2} \underbrace{\sin(\theta)}_{\hookrightarrow\, t} \cdot \cos(\theta) \cdot d\theta = \int t \cdot d\theta$$

$$\hookrightarrow \quad 1 \cdot dt = \cos(\theta) \cdot d\theta$$

$$= \frac{t^2}{2} = \frac{\sin^2(\theta)}{2}\Big|_0^{\pi/2} = \frac{1}{2}\left[\underbrace{\sin^2(\pi/2)}_{1} - \underbrace{\sin^2(0)}_{0}\right]$$

$$= \frac{1}{2}$$

$$\int \cdot \int = \frac{15}{4} \cdot \frac{1}{2} = \frac{15}{8}$$

Es. 3) $y'' - 2y' + 4y = 0$

$$\lambda^2 - 2\lambda + 4 = 0 \Rightarrow \Delta = 4 - 16 = -12 \quad < 0$$

$$\lambda_{1,2} = \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm \sqrt{12 i^2}}{2} = \frac{2}{2} \pm \frac{2\sqrt{3}\, i}{2}$$

$$\lambda_{1,2} = \underbrace{1}_{\alpha} \pm \underbrace{\sqrt{3}}_{\beta}\, i$$

$$y = e^{x}\left(c_1 \operatorname{sen}(\sqrt{3}\,x) + c_2 \cos(\sqrt{3}\,x)\right)$$

$$y(0) = 1 \Rightarrow 1 = \underbrace{e^{(0)}}_{1}\left(c_1 \underbrace{\cancel{\operatorname{sen}(0)}}_{0} + c_2 \underbrace{\cos(0)}_{1}\right)$$

$$\boxed{1 = c_2}$$

$$y'(0) = 4$$

$$y' = e^{\overset{0}{x}}\left(c_1 \underbrace{\cancel{\operatorname{sen}(\sqrt{3}\,x)}}_{0} + c_2 \overbrace{\cos(\sqrt{3}\,x)}^{1}\right) + e^{x}\left(c_1 \underbrace{\cos(\sqrt{3}\,x)}_{1} \cdot \sqrt{3} + c_2 \cdot \left(-\underbrace{\cancel{\operatorname{sen}(\sqrt{3}\,x)}}_{0} \cdot \sqrt{3}\right)\right)$$

$$4 = c_2 + \sqrt{3}\, c_1$$

$$4 = c_2 + \sqrt{3}$$

$$c_2 = 4 - \sqrt{3}$$

$$\boxed{y = e^{x}\left(\underbrace{1}_{c_1}\operatorname{sen}(\sqrt{3}\,x) + \underbrace{(4-\sqrt{3})}_{c_2}\cos(\sqrt{3}\,x)\right)}$$

Integrale Particolare

Del Problema Di Cauchy

Es 4) $f(x,y) = x^3 - y^3 + xy$

$$\Delta f = 0 \begin{cases} f_x = 3x^2 + y = 0 \\ f_y = -3y^2 + x = 0 \end{cases} \Rightarrow$$

$$\begin{cases} y = -3x^2 \\ -3(-3x^2)^2 + x = 0 \end{cases} \Rightarrow -3(9x^4) + x = 0$$

$$\begin{cases} \\ -27x^4 + x = 0 \end{cases} \to \begin{cases} \\ x(-27x^3+1) = 0 \end{cases}$$

$$\begin{cases} \\ \Rightarrow \boxed{x = 0} \\ 2) -27x^3 + 1 = 0 \Rightarrow x^3 = \frac{1}{27} \Rightarrow x = \sqrt[3]{\frac{1}{27}} \end{cases}$$

$$\Rightarrow \boxed{x = \frac{1}{3}}$$

$$\begin{cases} y = -3x^2 \\ x = 0 \end{cases} \Rightarrow \begin{cases} y = 0 \\ x = 0 \end{cases} \quad A(0;0)$$

$$\begin{cases} y = -3x^2 \\ x = 1/3 \end{cases} \Rightarrow \begin{cases} y = -3\left(\frac{1}{3}\right)^2 = -3 \cdot \frac{1}{9} = \frac{-1}{3} \\ x = 1/3 \end{cases}$$

$B(1/3 ; -1/3)$

$$\begin{cases} f_x = 3x^2 + y \\ f_y = -3y^2 + x \end{cases}$$

$f_x \to f_{xx} = 6x$

$f_x \to f_{xy} = 1$

$f_y \to f_{yy} = -6y$

$f_y \to f_{yx} = 1$

$$H = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = \begin{vmatrix} 6x & 1 \\ 1 & -6y \end{vmatrix}$$

$$H(A) = H\underbrace{(0;0)}_{} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 0 - 1 = -1 < 0$$

P_to di sella

$$H(B) = H\underbrace{\left(\tfrac{1}{3}; -\tfrac{1}{3}\right)}_{} = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \qquad 4 - 1 = 3 > 0$$

$2 > 0$

↳ Mini—ssoluto.

$$\begin{cases} y = \\ x = 1/3 \end{cases}$$