Es 1:

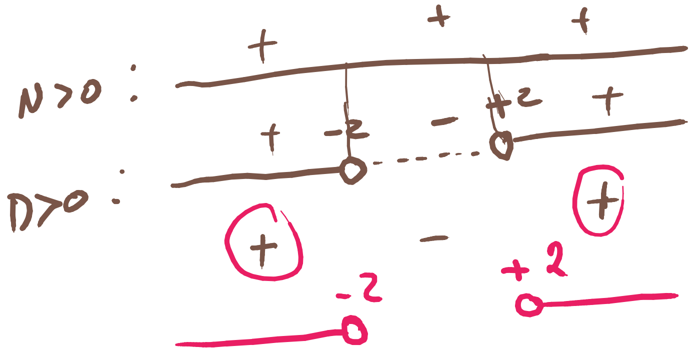
$$y = \ln\left(\frac{x^2}{x^2-4}\right)$$

• Dominio: $\frac{x^2}{x^2-4} > 0$

1) $x^2 > 0 \quad \forall x \in \mathbb{R}$

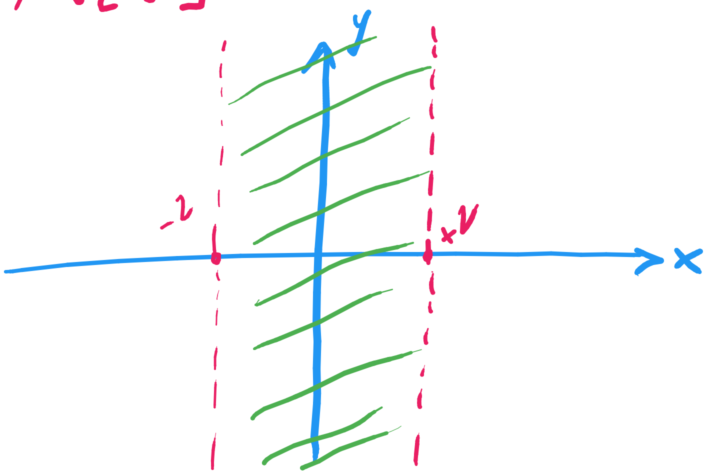
2) $x^2-4>0 \to x^2>4$

$x > \pm 2$



$$\begin{cases} x<-2 \ ; \quad x>+2 \\ (-\infty;-2) \ ; \ (+2;+\infty) \\ ]-\infty;-2[\ \cup \ ]+2;+\infty[\end{cases}$$

Dominio



• Simmetria:

$$f(x) = \ln\left(\frac{x^2}{x^2-4}\right)$$

$f(x)$ è pari?

$$f(-x) = \ln\left(\frac{(-x)^2}{(-x)^2-4}\right) = \ln\left(\frac{x^2}{x^2-4}\right) \overset{!}{=} f(x)$$

Allora $f(x)$ è pari.

Inters:

Con l'asse x $\begin{cases} y=0 \\ \ln\left(\frac{x^2}{x^2-4}\right) = 0 \end{cases} \to \begin{cases} y=0 \\ \cancel{\ln}\left(\frac{x^2}{x^2-4}\right) = \cancel{\ln}(1) \end{cases}$

$0 = \ln(1)$

$$\begin{cases} y=0 \\ \frac{x^2}{x^2-4} = 1 \end{cases} \to \begin{cases} y=0 \\ \frac{x^2}{x^2-4} - 1 = 0 \end{cases} \to \begin{cases} y=0 \\ \frac{\cancel{x^2} - \cancel{x^2} + 4}{x^2-4} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} y=0 \\ \frac{4}{x^2-4} = 0 \end{cases} \to \begin{cases} y=0 \\ 4 = 0 \quad \text{Mai} \end{cases}$$

Allora No Int. Con l'asse y.

Int. Con l'asse y:

$\begin{cases} x = 0 \end{cases} \rightarrow$ Però $x=0 \nexists$ Dom

Allora, No Int.

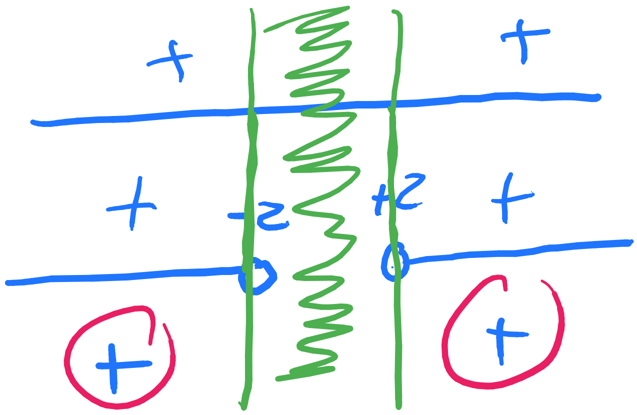
Conc: Non ci sono Int. con gli Assi

Segno: $y > 0 \rightarrow \cancel{\ln}\left(\frac{x^2}{x^2-4}\right) = \boxed{0}$ $\rightarrow \cancel{\ln}(1)$

$$\frac{x^2}{x^2-4} = 1 \text{ ---------} \rightarrow \frac{4}{x^2-4} > 0$$

$N > 0 \rightarrow 4 > 0 \quad \forall x \in \mathbb{R}$

$D > 0 \rightarrow x^2 - 4 > 0$ per



f non esiste

La funzione è sempre positiva $\forall x \in \text{Dom}(f)$

I limiti:

Dominio è $]-\infty; -2[\cup]+2; +\infty[$

4 estremi

I limiti da calcolare sono $\boxed{\lim_{x\to-\infty} f(x) ; \lim_{x\to+\infty} f(x)}$ $\boxed{\lim_{x\to-2^-} f(x) ; \lim_{x\to+2^+} f(x)}$

Sapendo che $f(x)$ è pari

Allora

$$\lim_{x\to\pm\infty} f(x) \quad / \quad \lim_{x\to-2^-} f(x) = \lim_{x\to+2^+} f(x)$$

$$\lim_{x\to\pm\infty} \ln\left(\frac{x^2}{x^2-4}\right) = \ln\left(\frac{\cancel{x^2}}{\cancel{x^2}\left(1-\frac{4}{x^2}\right)}\right) \quad \frac{4}{x^2} \to 0$$

$$\lim_{x\to\pm\infty} \ln(1) = 0$$

$$\lim_{x\to \underbrace{+2^+}_{\cong 2,1}} \ln\left(\frac{(2^+)^2}{(2)^2-4}\right) = \ln\left(\frac{4^+}{0^+}\right) = \ln(+\infty) = +\infty$$

Derivata Prima: $f(x) = \ln\left(\dfrac{x^2}{x^2-4}\right)$

$$f'(x) = \frac{1}{\dfrac{x^2}{(x^2-4)}} \cdot \frac{2x(x^2-4)-(x^2)\cdot(2x)}{(x^2-4)^2}$$

$$= \frac{(x^2-4)}{x^2} \cdot \frac{\cancel{2x^3} - 8x - \cancel{2x^3}}{(x^2-4)^2} = \boxed{\frac{(x^2-4)\cdot(-8x)}{(x^2)\cdot(x^2-4)^2}}$$

$f' > 0$

$$\frac{(x^2-4)\cdot(-8x)}{x^2\cdot(x^2-4)^2} > 0$$

$\to (x^2-4)\cdot(-8x) > 0$

$\to x^2-4>0$

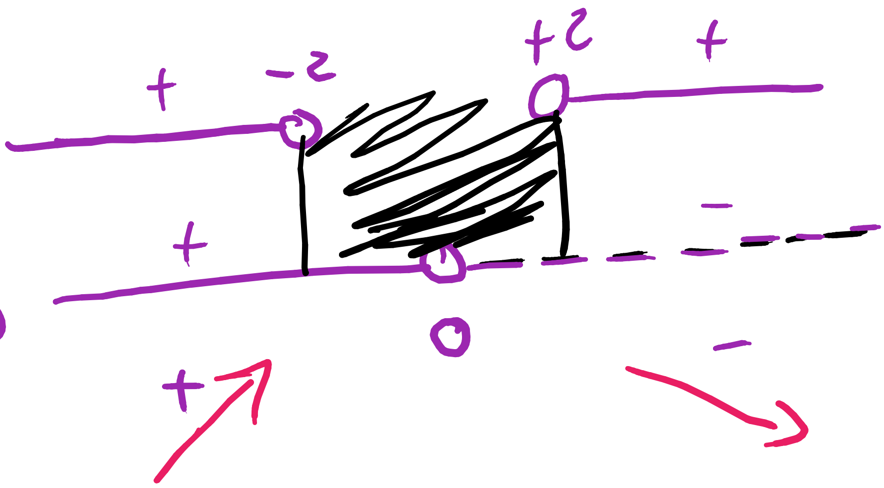
$\to -8x > 0$

$\to (x^2)\cdot(x^2-4)^2 > 0$

sempre verificata

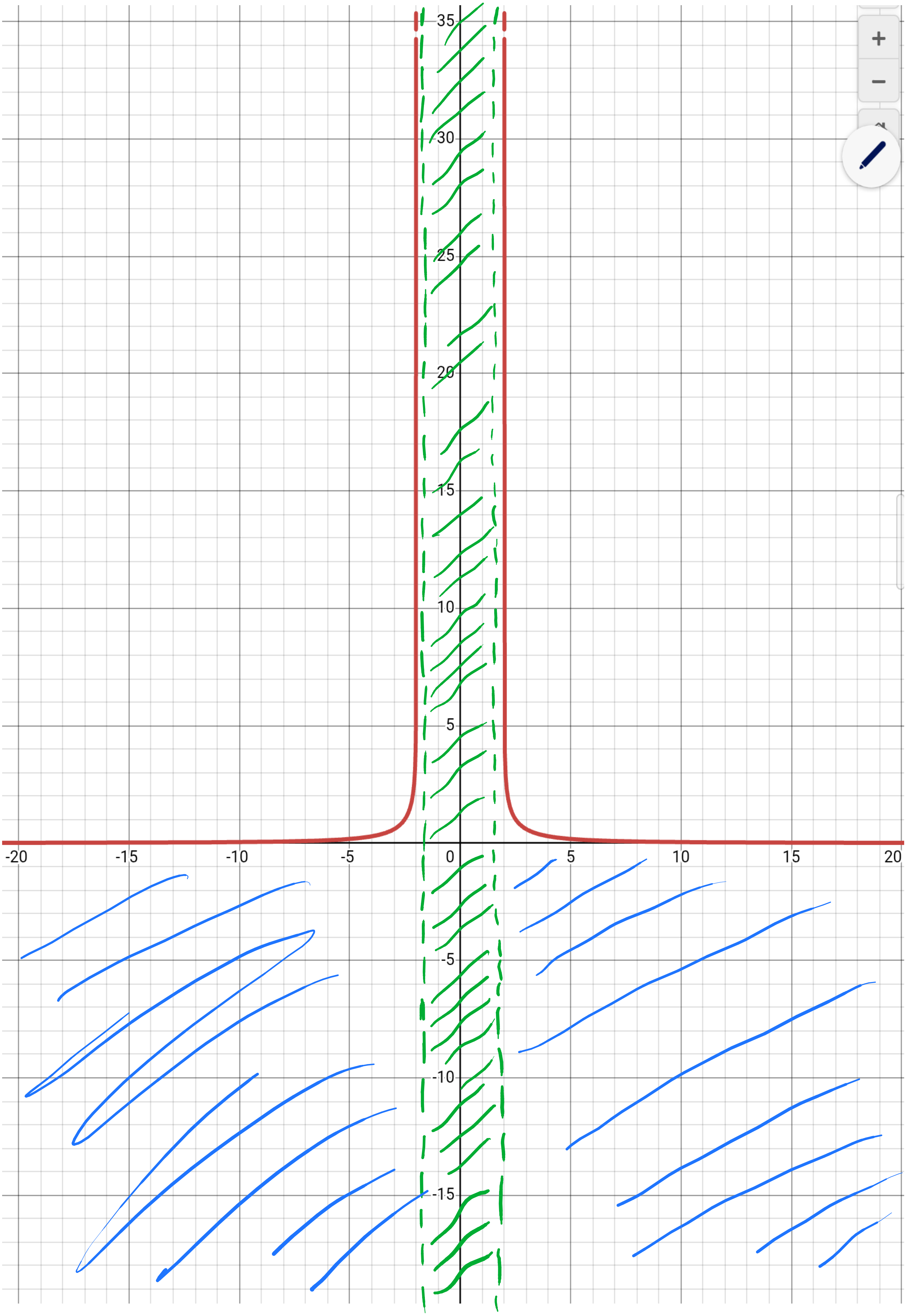
$x^2-4>0$ per

$-8x>0$ per $x<0$



f crescente per $x<-2$

f Decrescente per $x>+2$



35
30
25
20
15
10
5
0
-5
-10
-15
-20
-15
-10
-5
5
10
15
20

Es 2:

$$\lim_{x \to 0} \frac{3x^2 \cdot \operatorname{sen}(x^2)}{1-\cos^2(x^2)}$$

Utilizzando I limiti Notevoli:

$$\lim_{x \to 0} \frac{\operatorname{sen}(x)}{x} = 1 \implies \operatorname{sen}(x^2) \sim x^2$$

$$\lim_{x \to 0} \frac{1-\cos^2(x)}{x^2} = \frac{1}{2} \implies$$

$$\iff \lim_{x \to 0} \frac{1-\cos^2(x^2)}{(x^2)^2} = \frac{1-\cos^2(x^2)}{x^4} = \frac{1}{2}$$

$$\frac{1}{2}x^4$$

Sostituendo: $\dfrac{3\cancel{x^2}\cdot\cancel{x^2}}{\frac{1}{2}\cancel{x^4}} = \dfrac{3}{\frac{1}{2}} = 3 \cdot 2 = 6$

Es. 3)

$$y = x^2 \cdot \ln(x) \quad \text{in} \quad x_0 = 1$$

$$y = f'(x_0)(x - x_0) + f(x_0)$$

- $f(x_0) = (1)^2 \cdot \ln(1) = 1 \cdot 0 = 0$

$$f'(x) = 2x \cdot \ln(x) + \cancel{x^2} \cdot \frac{1}{\cancel{x}}$$

$$= 2x \cdot \ln(x) + x$$

- $f'(x_0) = 2(1) \cdot \underbrace{\ln(\cancel{1})}_{0} + 1 = 1$

$$y = 1(x - 1) + 0 = x - 1$$

l'equa. Della retta Tangente a y in x_0;

$$\boxed{y = x - 1}$$

Es. 4)

$x_0 = 0$, $n = 3$, $f(x) = \ln(1+\cos(x))$

Con gli sviluppi di $\cos(x)$, $\ln(1+x)$

$$\boxed{\cos(x) = 1 - \frac{x^2}{2} +} \frac{x^4}{24}$$

$n=3$

Diventa

$\hookrightarrow f(x) = \ln\left[1+\left(1-\frac{x^2}{2}\right)\right]$

$n=3$

Sapendo che $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \Big| + \frac{x^4}{4} + \cdots$

N.B: l'argomento è $\left(1-\frac{x^2}{2}\right)$

allora

$\ln(1+\cos(x)) = \ln\left(1+\left(1-\frac{x^2}{2}\right)\right)$

$\rightarrow \left(1-\frac{x^2}{2}\right) - \frac{\left(1-\frac{x^2}{2}\right)^2}{2} + \frac{\left(\left(1-\frac{x^2}{2}\right)\right)^3}{3}$

Es. 5

$$y = \operatorname{arctg}\left(\frac{x+1}{x-1}\right)$$

$$y' = \frac{1}{1+\left(\frac{x+1}{x-1}\right)^2} \cdot \frac{(1)\cdot(x-1) - (x+1)\cdot(1)}{(x-1)^2}$$