Versione A

1) $f(x) = \ln\left(\frac{x^2}{4-x^2}\right)$

• Dominio: $\frac{x^2}{4-x^2} > 0$

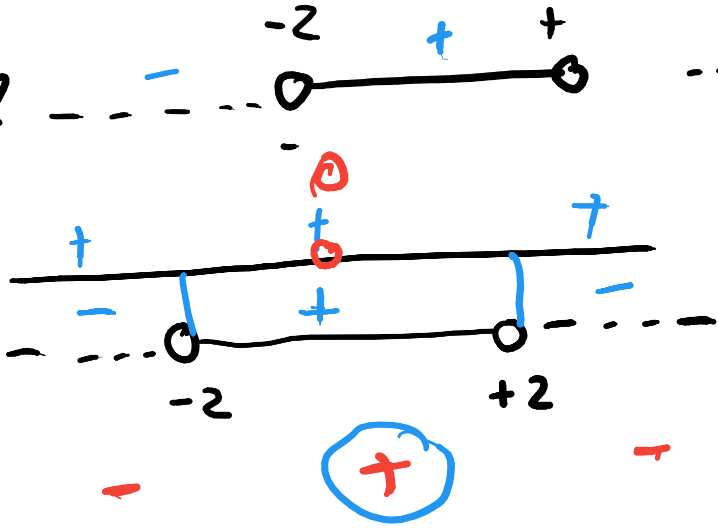
1) $x^2 > 0$ $\quad \forall x \in \mathbb{R} \setminus \{0\}$

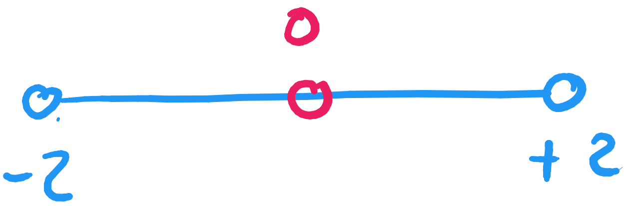
2) $4 - x^2 > 0$

$x \quad 4 \Rightarrow x < \pm 2$

N → $x^2 > 0$

D → $x^2 < 4$





$f(x)$ è definita $]-2;0[\cup]0;+2[$

oppure $(-2;0) \vee (0;+2)$

• **Simmetria:**

- $f(-x) = \ln\left(\frac{(-x)^2}{4-(-x)^2}\right) = \ln\left(\frac{x^2}{4-x^2}\right)$

Allora $f(x) = f(-x)$

$f(x)$ è Pari

• **Intersezione:**

- Con l'asse x: $\begin{cases} y = 0 \\ \ln\left(\frac{\;}{4-x^2}\right. \end{cases}$

$$\cancel{e}^{\cancel{\ln}\left(\frac{x^2}{4-x^2}\right)} = e^0 \Rightarrow \frac{x^2}{4-x^2} = 1$$

$$1$$

$$\frac{x^2}{4-x^2} - 1 = 0 \Rightarrow \frac{x^2-4+x^2}{4-x^2} = 0$$

$$2x^2 - 4 = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$$

$A(-\sqrt{2};0)$; $B(+\sqrt{2};0)$

• Con y: $\{ x=0 \;\nexists\; D_{f(x)}$

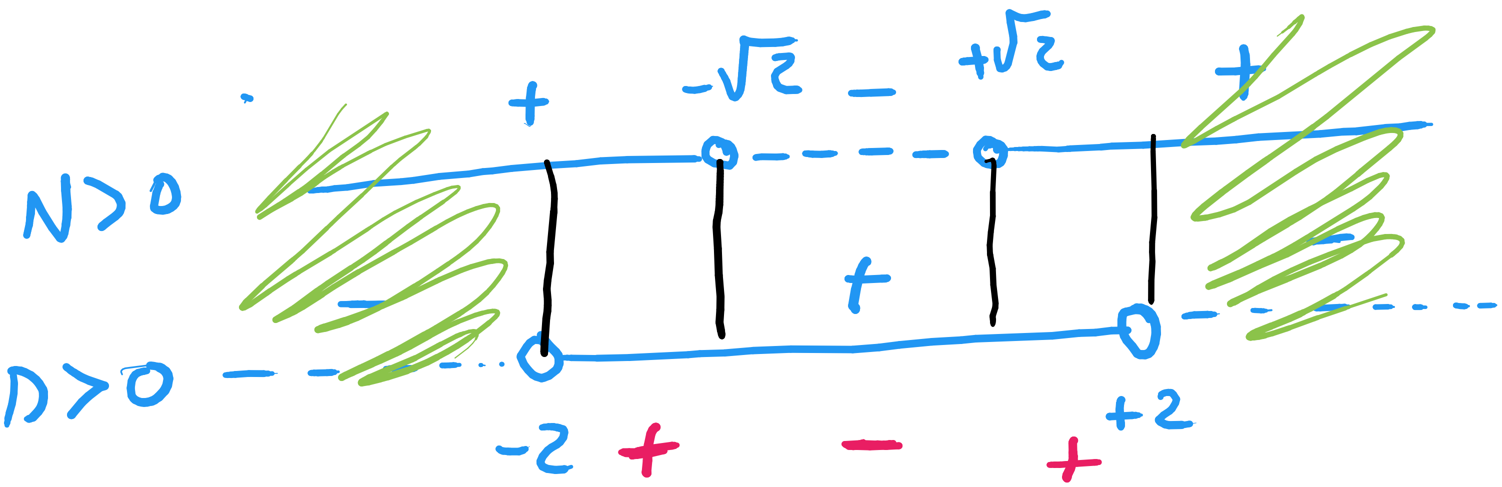
No Int. con gli assi

+ Segno: $y>0 \Rightarrow \left(\ln\left(\frac{x^2}{4-x^2}\right)\right)_e > (0)_c$

$$\frac{x^2}{4-x^2} > 1 \Rightarrow \frac{x^2}{4-x^2} - 1 > 0$$

$$\frac{+2x^2-4}{4-x^2} > 0 \rightarrow 2x^2-4>0 \rightarrow x^2>2 \rightarrow x > \pm\sqrt{2}$$

$$\hookrightarrow 4-x^2>0 \rightarrow x^2<4 \rightarrow x < \pm 2$$



$y>0$ per $-2<x<-\sqrt{2}$, $+\sqrt{2}<x<+2$

$y<0$ per $-\sqrt{2}<x<+\sqrt{2}$

I limiti:

f(x) è Pari

$$\lim_{x \to \underbrace{-2^+}_{\cong -1,9}} \ln\left(\frac{x^2}{4-x^2}\right) = \ln\left(\frac{(-2^+)^2}{4-(-2^+)^2}\right)$$

$$\lim_{x \to -2^+} \ln\left(\frac{+}{0^+}\right) = \ln(+\infty) \overset{!}{=} +\infty$$

$$\lim_{x \to -2^+} f(x) = \lim_{x \to +2^-} f(x)$$

$$\lim_{x \to \underbrace{0^-}_{\cong -0,1}} f(x) = \lim_{x \to 0^-} \ln\left(\frac{x^2}{4-x^2}\right) = \ln\left(\frac{0^+}{4-0}\right)$$

$$\ln\left(\frac{0^+}{4}\right) = \ln(0^+) = -\infty$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = -\infty$$

Derivata Prima:

$$f'(x) = \frac{1}{\frac{x^2}{4-x^2}} \cdot \frac{2x(4-x^2) - x^2 \cdot (-2x)}{(4-x^2)^2}$$

$$f' = \frac{(4-x^2)}{x^2} \cdot \frac{2x(4-\cancel{x^2}+\cancel{x^2})}{(4}$$

$$f' = \frac{(4-x^2)\cdot 8x}{x^2 \cdot (4-x^2)^2}$$

$\boxed{f' = 0}$ Ricerca Punti Critici

$f' = 0$ per $(4-x^2) \cdot 8x = 0$

- $4 - x^2 = 0$ per $x = \pm 2$
- $8x = 0$ per $x = 0$

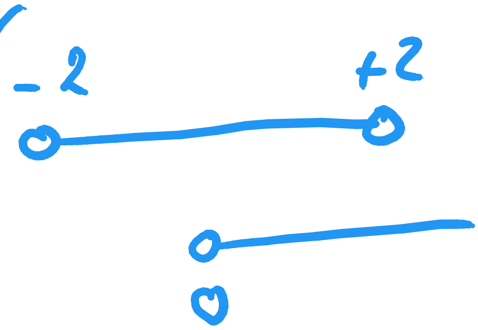
$\nexists$ Dom f

Allora Nessun Punto Critico

• $f' > 0$

$$\frac{(4-x^2)\cdot 8x}{x^2\cdot(4-x^2)^2} > 0$$

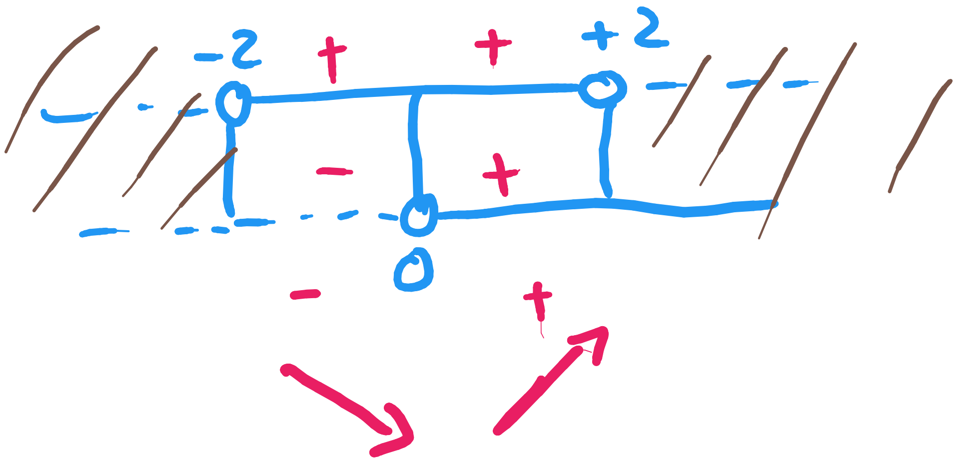
$N > 0$ per $4-x^2 > 0$ per



$8x > 0$ per $x > 0$

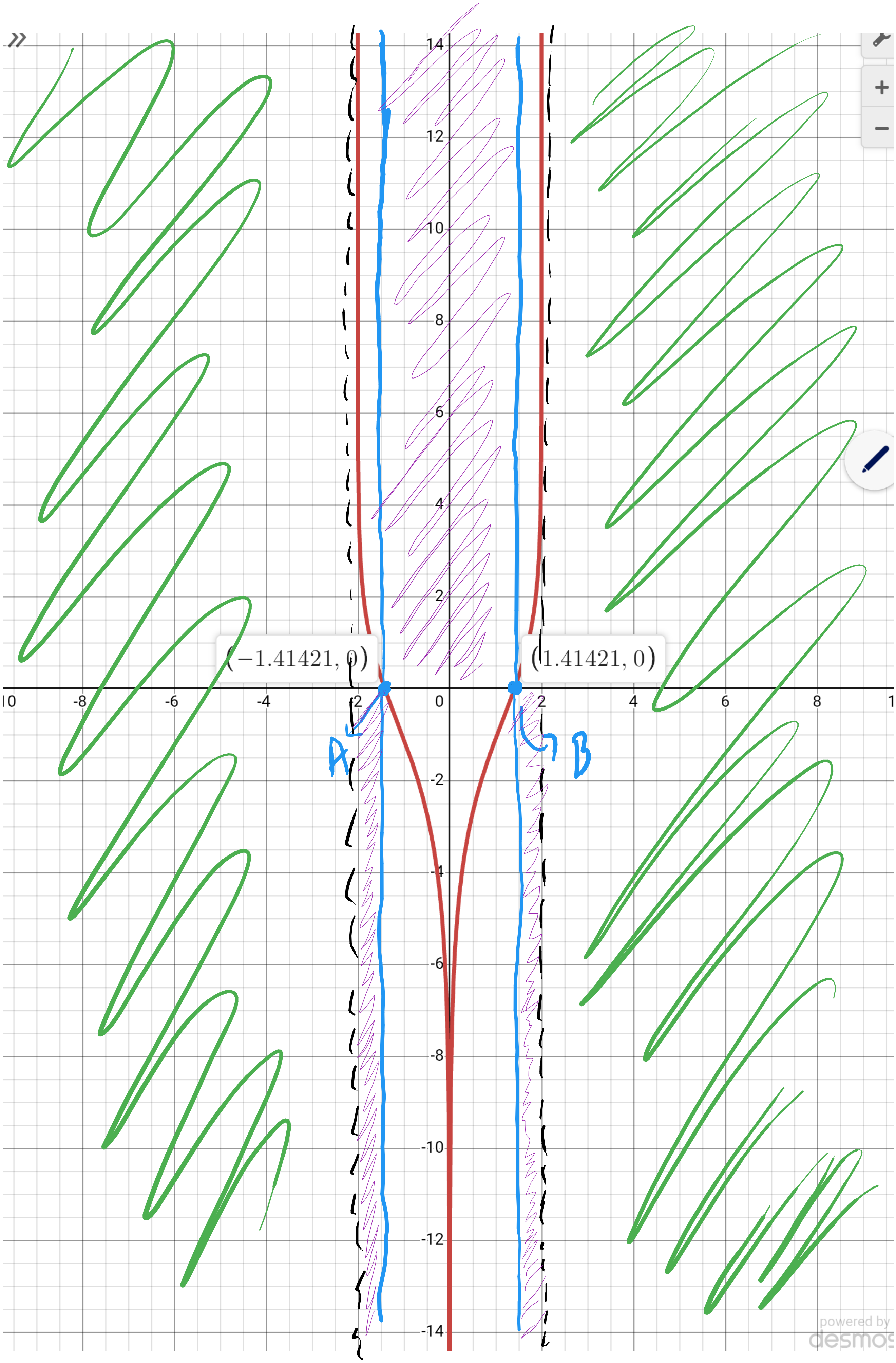
$D > 0$ per $x^2\cdot(4-x^2)^2 > 0$

sempre positiva



f è crescente per $x > 0$

" " Decrescente per $x < 0$



(−1.41421, 0)
(1.41421, 0)
A
B
powered by desmos

Es. 2)

$$\lim_{x \to 0} \frac{1-\cos(x^2)}{3x^2 \cdot \operatorname{sen}^2(x)} \quad \operatorname{sen}^2(x) \sim x^2$$

Con I limiti Notevoli:

- $\lim_{x \to 0} \frac{\operatorname{sen}(x)}{x} = 1$ Allora $\lim_{x \to 0} \frac{\operatorname{sen}^2(x)}{x^2} = 1$

Diventa:

- $\lim_{x \to 0} \frac{1-\cos(x^2)}{3x^2 \cdot x^2} = \frac{1-\cos(x^2)}{3x^4}$

$$\lim_{x \to 0} \frac{1-\cos(x)}{x^2} = 1/2 \Rightarrow \lim_{x \to 0} \frac{1-\cos(x^2)}{x^4} = 1/2$$

Allora $\lim_{x \to 0} \frac{1-\cos(x^2)}{3x^4} = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

$1/2$

Es.3)

$y = x^2 \cdot \ln(x)$ in $x_0 = 1$

$$y = f'(x_0)(x - x_0) + f(x_0)$$

- $f(x_0) = (1)^2 \cdot \ln(1) = 1 \cdot 0 = 0$

$$f'(x) = 2x \cdot \ln(x) + \cancel{x^2} \cdot \frac{1}{\cancel{x}}$$

$$= 2x \cdot \ln(x) + x$$

+ $f'(x_0) = 2(1) \cdot \underbrace{\ln(\cancel{1})}_{0} + 1 = 1$

$$y = 1(x - 1) + 0 = x - 1$$

l'equa. Della retta Tangente a y in x_0;

$$\boxed{y = x - 1}$$

Es. 4)

$x_0 = 0$, $n = 3$, $f(x) = \ln(1+ \operatorname{sen}(x))$

Con gli sviluppi Di $\operatorname{sen}(x)$, $\ln(1+x)$

$$\boxed{\operatorname{sen}(x) = x - \frac{x^3}{3!}} + \frac{x^5}{5!} - \cdots$$

$n=3$

Diventa

$\Rightarrow f(x) = \ln\left[1+\left(x-\frac{x^3}{3!}\right)\right]$

$n=3$

Sapendo che $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

N.B: l'argomento è $\left(x-\frac{x^3}{3!}\right)$

Aora

$\ln(1+\operatorname{sen}(x)) = \ln\left(1+\left(x-\frac{x^3}{3!}\right)\right)$

$$= \boxed{\left(x-\frac{x^3}{3!}\right) - \frac{\left(x-\frac{x^3}{3!}\right)^2}{2} + \frac{\left(\left(x-\frac{x^3}{3!}\right)\right)^3}{3}}$$

Es.5) $y = \text{arctg}\left(\frac{x-1}{x+1}\right)$ $f' = ?$

y è composta da:

$\text{arctg}(\ ) \overset{D}{\Rightarrow} \frac{1}{1+(\ )^2}$

rapporto: $\frac{f}{g} \xrightarrow{D} \frac{f' \cdot g - f \cdot g'}{g^2}$

$$\Rightarrow y' = \frac{1}{1+\left(\frac{x-1}{x+1}\right)^2} \cdot \frac{(-1)\cdot(x+1) - (x-1)\cdot(1)}{(x+1)^2}$$