Updating Katz centrality in complex networks by counting walks

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Katz centrality is one of the most popular walk-based centrality indices in network science, with applications spanning from identification of potential disease genes in protein-interaction networks to the optimization of artificial neural networks.

Introduced in 1953, it builds on the simple fact that the powers of the adjacency matrix A of a graph can be used to count traversals in a network, and that longer walks should carry less importance then longer ones when defining a measure of centrality for nodes. More specifically, *Katz centrality of node i* is the ith entry of the vector

$$\sum_{r=0}^{\infty} \alpha^r A^r \mathbf{1}, \alpha > 0.$$

Whenever $\alpha\rho(A) < 1$, with $\rho(A)$ the spectral radius of A, the above converges to the matrix function $(I - \alpha A)^{-1}$ and computing Katz centrality reduces to solving a sparse linear system.

Since 1953, the size of networks for which computing Katz centrality is useful has grown exponentially. Moreover, the networks we interact with on a day-to-day basis change in time; roads get disrupted in road networks and users delete social media accounts.

In this talk we discuss the problem of approximately updating Katz centrality scores after nodes or edges are removed from the network. The approach we take in this talk is purely combinatoric and it is based on new results that allow to exactly count walks that visit a certain set of entities (nodes or edges) in the graph. We will describe two algorithms that update Katz centrality after node/edge removal using $O(m)$ operations, where m is the number of undirected edges in the original graph. Numerical experiments to support our theoretical findings will also be discussed.

If time allows, we will also discuss generalizations of these strategies to other centralities and to the case of directed and possibly weighted graphs.

The talk is based on [1] and on ongoing research with Mr Alessandro Filippo (Università degli Studi di Tor Vergata, Rome) and Prof. Vanni Noferini (Aalto University, Helsinki).

References

[1] FA, D. Bertaccini, and A. Filippo, *Updating Katz centrality by counting walks*, SIAM J. Matrix Anal. Appl, 46(2) (2025), pp. 2449–2474.