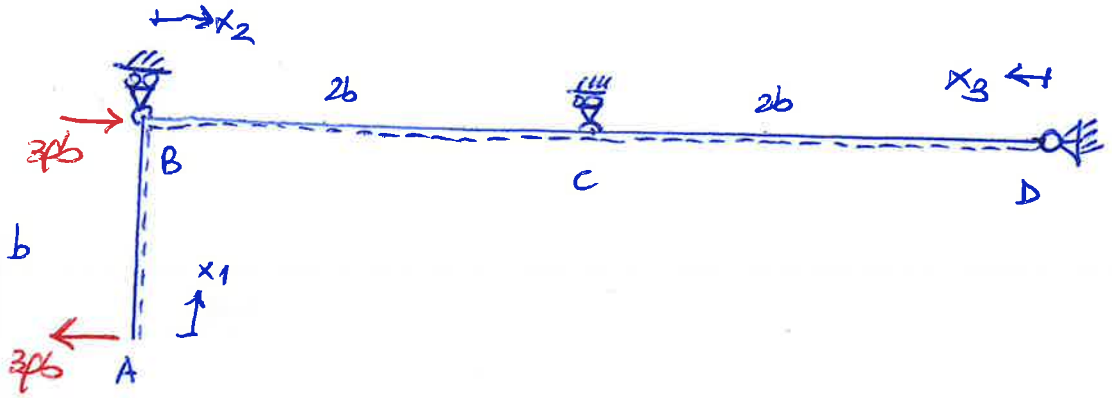
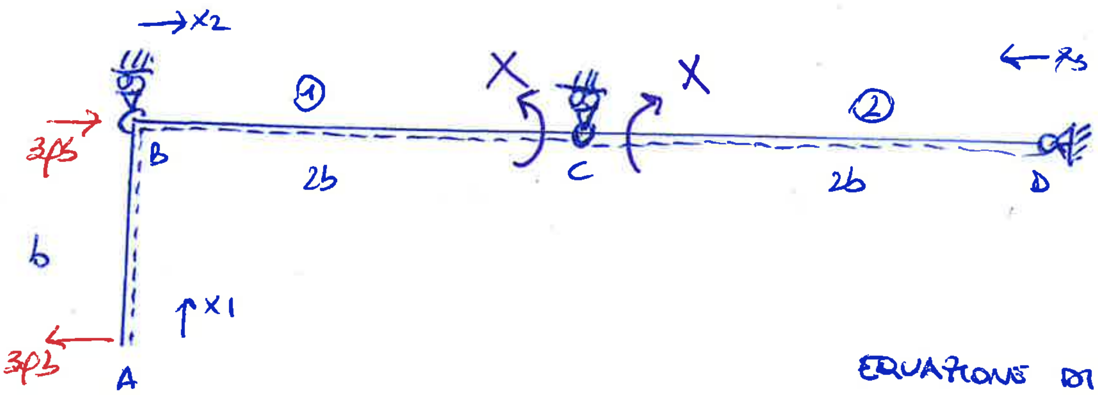
SdC SdA P2 27/01/2026 TESTO 2

ES 1



STRUTTURA REALE
1 VOLTA IPERSTATICA

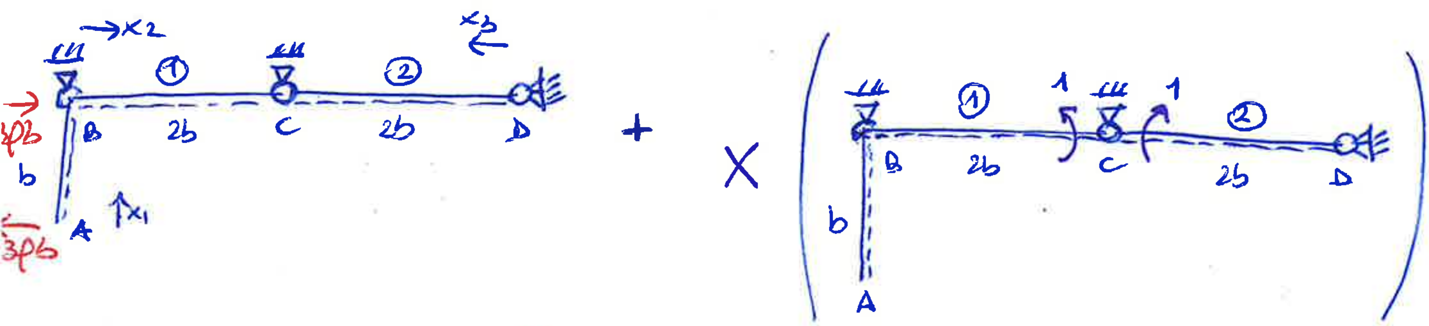


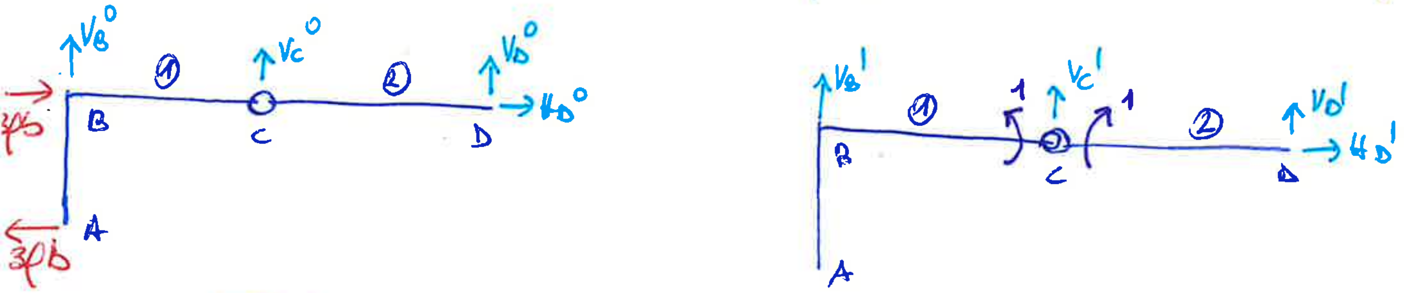
STRUTTURA ISOSTATICA CONGRUENTE

EQUAZIONE DI CONGRUENZA $\boxed{\Delta\varphi_C = 0}$

SISTEMA PRINCIPALE SP_0

SISTEMA AUSILIARIO SA_1





$\rightarrow R_x = 0 \qquad \boxed{H_D^0 = 0}$

$\uparrow R_y = 0 \qquad V_B^0 + V_C^0 + \cancel{V_D^0} = 0$

$\circlearrowleft M_{z(B)} = 0 \qquad -3pb^2 + V_C^0\, 2b + \cancel{V_D^0}\, 4b = 0$

$\circlearrowleft M_{z(C)}^{(2)} = 0 \qquad \boxed{V_D^0 = 0}$

$\rightarrow R_x = 0 \qquad H_D^1 = 0$

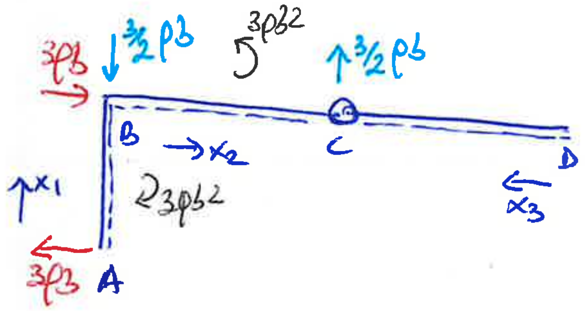
$\uparrow R_y = 0 \qquad V_B^1 + V_C^1 + V_D^1 = 0$

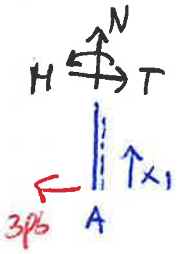
$\circlearrowleft M_{z(B)} = 0 \qquad V_C^1\, 2b + V_D^1\, 4b = 0$

$\circlearrowleft M_{z(C)}^{(2)} = 0 \qquad -1 + V_D^1\, 2b = 0 \rightarrow \boxed{V_D^1 = \frac{1}{2b}}$

$V_C^0\, 2b = 3\rho b^2$ $\qquad \boxed{V_C^0 = \frac{3}{2}\rho b}$

$V_B^0 = -V_C^0$ $\qquad \boxed{V_B^0 = -\frac{3}{2}\rho b}$

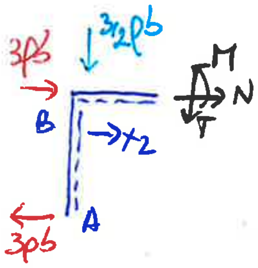




$N^0_{AB} = 0$

$T^0_{AB} = 3\rho b$

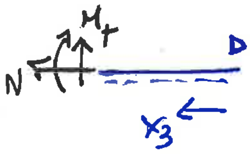
$M^0_{AB} = 3\rho b x_1$



$N^0_{BC} = 0$

$T^0_{BC} = -\frac{3}{2}\rho b$

$M^0_{BC} = 3\rho b^2 - \frac{3}{2}\rho b x_2$

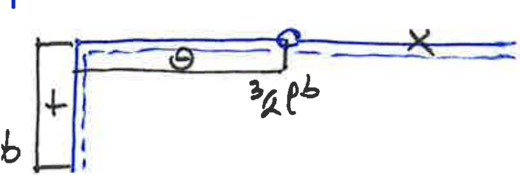


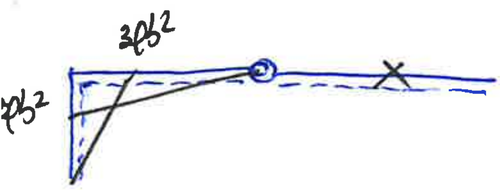
$N^0_{DC} = 0$

$T^0_{DC} = 0$

$M^0_{DC} = 0$

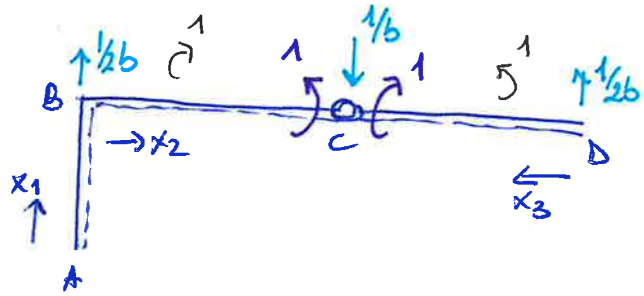
T^0

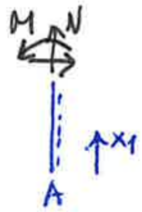




$V_C^1\, 2b = -V_D^1\, 4b$ $\qquad \boxed{V_C^1 = -\frac{1}{2b\cdot 2b}\cdot 4b = -\frac{1}{b}}$

$V_B^1 = -V_C^1 - V_D^1$ $\qquad \boxed{V_B^1 = -\frac{1}{2b} - \left(-\frac{1}{b}\right) = \frac{1}{2b}}$

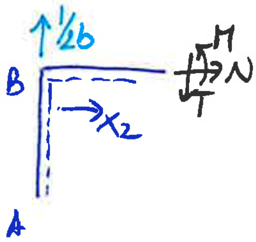




$N^1_{AB} = 0$

$T^1_{AB} = 0$

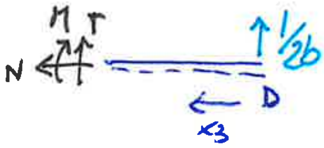
$M^1_{AB} = 0$



$N^1_{BC} = 0$

$T^1_{BC} = \frac{1}{2b}$

$M^1_{BC} = \frac{1}{2b} x_2$

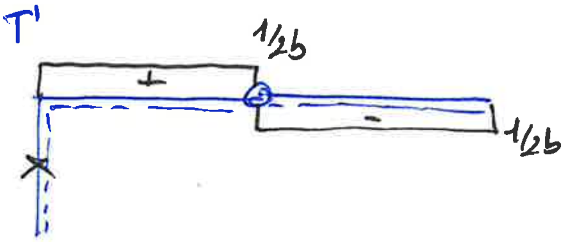


$N^1_{DC} = 0$

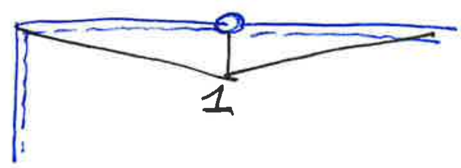
$T^1_{DC} = -1/2b$

$M^1_{DC} = \frac{1}{2b} x_3$

T^1



M^1



$\circled{3}$

$\delta \mathcal{L}_{EST} = \delta \mathcal{L}_{INT}$ $\qquad$ $\Delta\varphi_c = 0 \rightarrow \delta \mathcal{L}_{EST} = 0$ $\qquad$ $1 \cdot \Delta\varphi_c = 0$

$$\delta \mathcal{L}_{INT} = \int_{l_i} M' \frac{M^0 + XM'}{EJ} dx \qquad \text{N.B.} \quad \chi = \frac{M}{EJ} \rightarrow \chi = \frac{M^0 + XM'}{EJ}$$

$$\delta \mathcal{L}_{INT} = \underbrace{\int_{l_i} \frac{M'M^0}{EJ} dx}_{\eta_{10}} + X \underbrace{\int_{l_i} \frac{M'^2}{EJ} dx}_{\eta_{11}} = \eta_{10} + X\eta_{11} \rightarrow X = -\frac{\eta_{10}}{\eta_{11}}$$

TRATTO	L	M^0	M'	M^0M'	M'^2
AB	b	$3\rho b x_1$	//	//	//
BC	$2b$	$3\rho b^2 - 3/2\, \rho b x_2$	$\frac{1}{2b} x_2$	$\frac{3}{2}\rho b x_2 - \frac{3}{4} \rho x_2^2$	$\frac{1}{4b^2} x_2^2$
DC	$2b$	//	$\frac{1}{2b} x_3$	//	$\frac{1}{4b^2} x_3^2$

$$\eta_{10} = \frac{1}{EJ}\int^{2b}\left(\frac{3}{2}\rho b x_2 - \frac{3}{4}\rho x_2^2\right)dx = \frac{1}{EJ}\left[\frac{3}{2}\rho b \frac{x^2}{2} - \frac{3}{4}\rho\frac{x^3}{3}\right]_0^{2b} = \frac{1}{EJ}\left[\frac{3}{2}\rho b \frac{4b^2}{2} - \frac{3}{4}\rho \frac{8b^3}{3}\right]_0^{2b} =$$

$$= \frac{1}{EJ}\left(3\rho b^3 - 2\rho b^3\right) = \frac{\rho b^3}{EJ}$$

$$\eta_{11} = \frac{1}{EJ}\int^{2b}\frac{1}{4b^2}x_2^2\,dx + \frac{1}{EJ}\int^{2b}\frac{1}{4b^2}x_3^2\,dx = \frac{1}{EJ}\left[\frac{1}{4b^2}\frac{x^3}{3} + \frac{1}{4b^2}\frac{x^3}{3}\right]_0^{2b} = \frac{1}{EJ}\left[\frac{1}{4b^2}\frac{8b^3}{3} + \frac{1}{4b^2}\frac{8b^3}{3}\right]_0^{2b} =$$

$$= \frac{1}{EJ}\left(\frac{2}{3}b + \frac{2}{3}b\right) = \frac{4b}{3EJ}$$

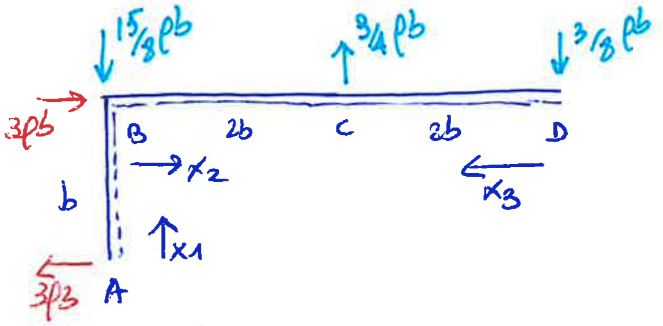
$$X = -\eta_{10}/\eta_{11} = -\frac{\rho b^3}{\cancel{EJ}} \cdot \frac{3\cancel{EJ}}{4b} = -\frac{3\rho b^2}{4}$$

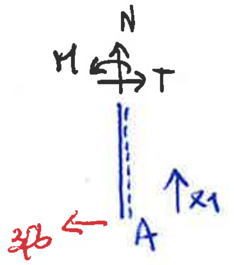
$$\boxed{V_B = V_B^0 + X V_B' = -\frac{3}{2}\rho b - \frac{3}{4}\rho b^2\left(\frac{1}{2b}\right) = -\frac{3}{2}\rho b - \frac{3}{8}\rho b = \frac{(-12-3)}{8}\rho b = -\frac{15\rho b}{8}}$$

$$\boxed{V_C = V_C^0 + X V_C' = \frac{3}{2}\rho b - \frac{3}{4}\rho b^2\left(-\frac{1}{b}\right) = \frac{3}{2}\rho b + \frac{3}{4}\rho b = \frac{(6+3)\rho b}{4} = \frac{9\rho b}{4}}$$

$$\boxed{V_D = V_D^0 + X V_D' = 0 - \frac{3}{4}\rho b^2\left(\frac{1}{2b}\right) = -\frac{3\rho b}{8}}$$

$$\boxed{H_B = H_B^0 + X H_B' = 0 - \frac{3}{4}\rho b^2(0) = 0}$$



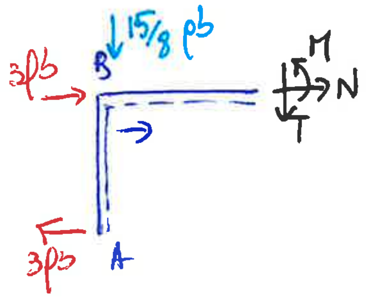


TRATTO AB È UGUALE AL TRATTO AB DEL SPo

$N_{AB} = 0$

$T_{AB} = 3\rho b$

$M_{AB} = 3\rho b x_1$



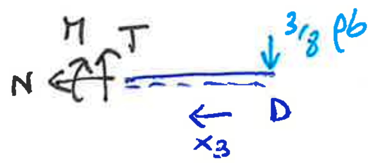
$N_{BC} = 0$

$T_{BC} = -\frac{15}{8}\rho b$

$M_{BC} = 3\rho b^2 - \frac{15}{8}\rho b x_2$

$x_2 = 0 \quad M = 3\rho b^2$

$x_2 = 2b \quad M = -3/4\,\rho b^2$



$N_{DC} = 0$

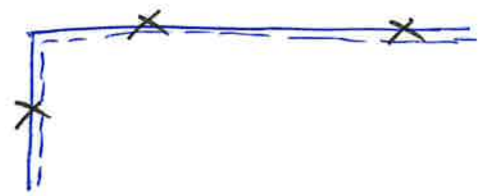
$T_{DC} = \frac{3}{8}\rho b$

$M_{DC} = -3/8\,\rho b x_3$

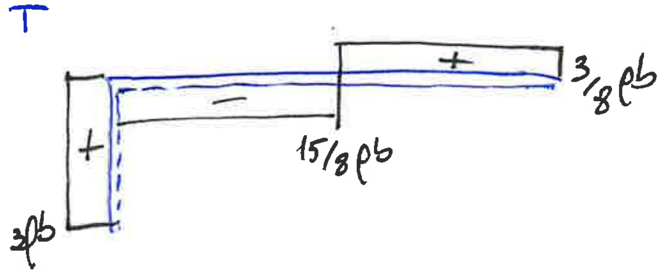
$x_3 = 0 \quad M = 0$

$x_3 = 2b \quad M = -\frac{3}{4}\rho b^2$

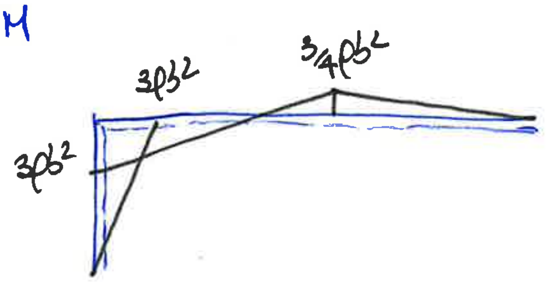
N



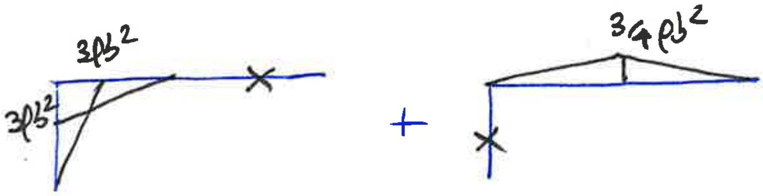
T



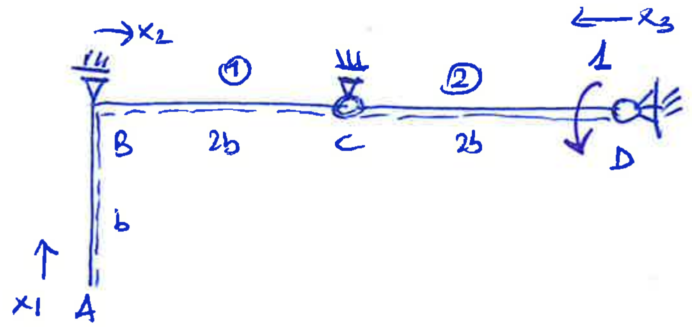
M

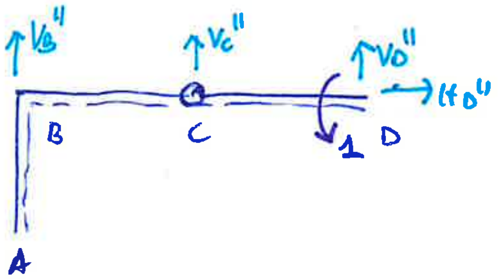


N.B.



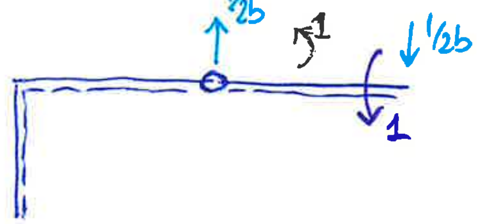
$\varphi_D = ?$ $\delta A2$





$$\begin{cases} R_x = 0 & \boxed{H_D'' = 0} \\ R_y = 0 & V_B'' + V_C'' + V_D'' = 0 \\ \circlearrowleft M_{t(B)} = 0 & V_C'' 2b + V_D'' 4b + 1 = 0 \end{cases}$$

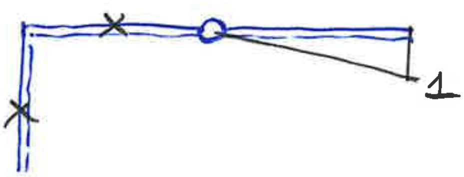
$\circlearrowleft M^{(2)}_{t(C)} = 0 \quad V_D'' 2b + 1 = 0 \quad \boxed{V_D'' = -\frac{1}{2b}}$



$V_C'' 2b = -V_D'' 4b - 1 \qquad V_C'' 2b = -\left(-\frac{1}{2b}\right)4b - 1 = 2 - 1 = 1$

$\boxed{V_C'' = \frac{1}{2b}}$

$V_B'' = V_C'' + V_D'' = \frac{1}{2b} - \frac{1}{2b} \qquad \boxed{V_B'' = 0}$



$M''_{AB} = 0$

$M''_{BC} = 0$

$M''_{DC} = 1 - \frac{1}{2b} x_3$

$\delta L_{ext} = \delta L_{int} \qquad \delta L_{ext} = 1 \cdot \varphi_D = \varphi_D \qquad \delta L_{int} = \int_{\ell_i} M'' \frac{M}{EJ} dx$

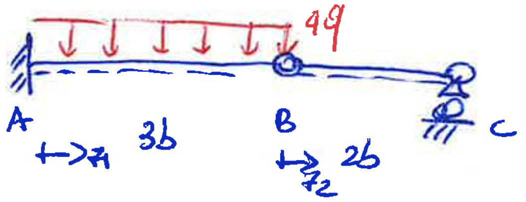
TRATTO	L	M	M''	MM''
AB	b	$3\rho b x_1$	//	//
BC	$2b$	$3\rho b^2 - \frac{15}{8}\rho b x_2$	//	//
DC	$2b$	$-\frac{3}{8}\rho b x_3$	$1 - \frac{1}{2b} x_3$	$-\frac{3}{8}\rho b x_3 + \frac{3}{16}\rho x_3^2$

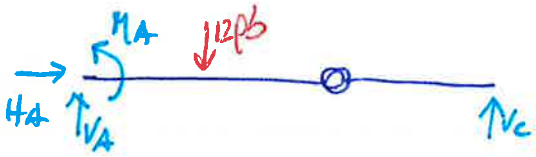
$$\frac{1}{EJ}\int_0^{2b}\left(-\frac{3}{8}\rho b x_3 + \frac{3}{16}\rho x_3^2\right)dx = \frac{1}{EJ}\left[-\frac{3}{8}\rho b \frac{x^2}{2} + \frac{3}{16}\rho \frac{x^3}{3}\right]_0^{2b} = \frac{1}{EJ}\left[-\frac{3}{8}\rho b \frac{4b^2}{2} + \frac{3}{16}\rho \frac{8b^3}{3}\right] =$$

$$= \frac{1}{EJ}\left(-\frac{12\rho b^3}{16} + \frac{8\rho b^3}{16}\right) = -\frac{\rho b^3}{4EJ}$$

$\boxed{\varphi_D = -\frac{\rho b^3}{4EJ}}$ $(\circlearrowright)$

ES 2



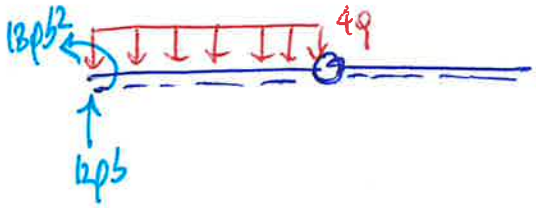


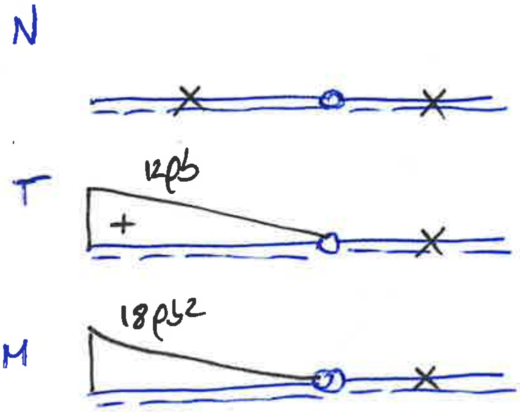
$$\begin{cases} \rightarrow R_x = 0 & \boxed{H_A = 0} \\ \uparrow R_y = 0 & V_A + V_c - 12pb = 0 \\ \circlearrowleft M_{z(A)} = 0 & M_A - 12pb \cdot \frac{3}{2}b + V_c \cdot 5b = 0 \end{cases}$$

$$\circlearrowleft M^{(2)}_{z(B)} = 0 \quad \boxed{V_c = 0}$$

$$\boxed{V_A = 12pb}$$

$$\boxed{M_A = 12pb \cdot \frac{3}{2}b = 18pb^2}$$





$N_{AB} = 0 \qquad N_{BC} = 0$

$T_{AB} = 12pb - 4pz_1 \qquad T_{BC} = 0$

$M_{AB} = -18pb^2 + 12pb\,z_1 - 2pz_1^2 \qquad M_{BC} = 0$

[N.B. BC È 1 BIELLA!]

CONDIZIONI A CONTORNO:

A

INCASTRO $\quad v_A = 0 \rightarrow v_1(z_1 = 0) = 0$

$\varphi_A = 0 \rightarrow v_1'(z_1 = 0) = 0$

B

CERNIERA INTERNA

$v_B^{(1)} = v_B^{(2)} \rightarrow v_1(z_1 = 3b) = v_2(z_2 = 0)$

$\varphi^{(1)} \neq \varphi^{(2)}$

C

CARRELLO $\quad v_C = 0 \rightarrow v_2(z_2 = 2b) = 0$

$\varphi_C \neq 0$

$$v_1'' = -\frac{M}{EJ} = \frac{18pb^2 - 12pbz_1 + 2pz_1^2}{EJ}$$

$$v_1' = \frac{1}{EJ}\left(18pb^2 z_1 - \cancel{12}^{\,6}pb\frac{z_1^2}{\cancel{2}} + 2p\frac{z_1^3}{3}\right) + A_1$$

$$v_1 = \frac{1}{EJ}\left(18pb^2\frac{z_1^2}{2} - \cancel{6}^{\,2}pb\frac{z_1^3}{\cancel{3}} + \frac{\cancel{2}}{3}p\frac{z_1^4}{\cancel{4}_{2}}\right) + A_1 z_1 + A_2$$

$$= \frac{1}{EJ}\left(9pb^2 z_1^2 - 2pb z_1^3 + \frac{1}{6}p z_1^4\right) + A_1 z_1 + A_2$$

$$v_2'' = -\frac{M}{EJ} = 0$$

$$v_2' = B_1$$

$$v_2 = B_1 z_2 + B_2$$

$$v_1(z_1=0)=0 \rightarrow \boxed{A_2 = 0}$$

$$v_1'(z_1=0)=0 \rightarrow \boxed{A_1 = 0}$$

$$\boxed{v_1 = \frac{1}{EJ}\left(9pb^2 z_1^2 - 2pb z_1^3 + \frac{1}{6}p z_1^4\right)}$$

$$\boxed{v_1' = \frac{1}{EJ}\left(18pb^2 z_1 - 6pb z_1^2 + \frac{2}{3}p z_1^3\right)}$$

$$v_1(z_1=3b) = v_2(z_2=0) \rightarrow \frac{1}{EJ}\left[9pb^2(3b)^2 - 2pb(3b)^3 + \frac{1}{6}p(3b)^4\right] = B_2$$

$$\frac{1}{EJ}\left[9pb^2\cdot 9b^2 - 2pb\cdot 27b^3 + \frac{1}{6}p\,81b^4\right] = B_2$$

$$\frac{1}{EJ}\left[81pb^4 - 54pb^4 + \frac{81}{6}pb^4\right] = B_2$$

$$\frac{1}{EJ}\left[27pb^4 + \frac{81}{6}pb^4\right] = B_2$$

$$\frac{1}{EJ}\left[\frac{(162+81)pb^4}{6}\right] = B_2 \rightarrow \boxed{B_2 = \frac{\overset{243}{81}pb^4}{6EJ} = \frac{81pb^4}{2EJ}}$$

$$v_2(z_2=2b)=0 \rightarrow B_1\cdot 2b + \frac{81}{2EJ}pb^4 = 0 \quad \boxed{B_1 = -\frac{81pb^3}{4EJ}}$$

$$\boxed{v_2 = -\frac{81pb^3}{4EJ}z_2 + \frac{81pb^4}{2EJ}}$$

$$\boxed{v_2' = -\frac{81pb^3}{4EJ}}$$

$$v_B = v_2(z_2=0) = \frac{81pb^4}{2EJ} \quad (\downarrow)$$

$$\varphi_C = v_2'(z_2=2b) = -\frac{81pb^3}{4EJ} \quad (\circlearrowleft)$$