# Esercitazione: goniometria

# Esercizi

**Traccia per punti il grafico delle seguenti funzioni:**

$$y = 2\cos(2x)$$

$$y = \frac{1}{2}\cos(4x)$$

$$y = 2\sin(x/2) - 1$$

$$y = 1 + \tan x$$

$$y = \cot x - 3$$

$$y = 1 + \sin x$$

$$y = |\sin x| + 1$$

$$y = \begin{cases} \sin(2x) & x \geq 0 \\ 3x^2 & x < 0 \end{cases}$$

$$y = \begin{cases} 2\cos(x/2) & x \geq 0 \\ 3x + 2 & x < 0 \end{cases}$$

$$y = \begin{cases} \cos x & x \leq 0 \\ e^x & x > 0 \end{cases}$$

# Esercizi

**Risolvi le seguenti equazioni in $\mathbb{R}$.**

$2\sin x = -\sqrt{2}$ $\left[\frac{5}{4}\pi + 2k\pi; \frac{7}{4}\pi + 2k\pi\right]$

$\sin x - 1 = 0$ $\left[\frac{\pi}{2} + 2k\pi\right]$

$2 \sin x = 6$ [impossibile]

$\sin x = \cos\frac{\pi}{6}$ $\left[\frac{\pi}{3} + 2k\pi; \frac{2}{3}\pi + 2k\pi\right]$

$-\sin x = 2$ [impossibile]

$2 \sin 3x - 1 = 0$ $\left[\frac{\pi}{18} + k\frac{2}{3}\pi; \frac{5}{18}\pi + k\frac{2}{3}\pi\right]$

$2\sin\frac{x}{3} + \sqrt{3} = 0$ $[-\pi + 6k\pi; 4\pi + 6k\pi]$

$2\sin\left(x - \frac{\pi}{3}\right) - 1 = 0$ $\left[\frac{\pi}{2} + 2k\pi; \frac{7}{6}\pi + 2k\pi\right]$

$3\sin x + 1 = 2 - \cos\left(\frac{\pi}{2} + x\right)$ $\left[\frac{\pi}{6} + 2k\pi; \frac{5}{6}\pi + 2k\pi\right]$

$\sin(x + 1) = \tan\frac{\pi}{3}$ [impossibile]

$2\sin 5x - \sqrt{2} = 0$ $\left[\frac{\pi}{20} + k\frac{2}{5}\pi; \frac{3}{20}\pi + k\frac{2}{5}\pi\right]$

$3 \sin x - 10 = 2(\sin x - 1)$ [impossibile]

$2 (\sin 2x + 3) - 1 = 3(1 - \sin 2x) + 2$ $\left[k\frac{\pi}{2}\right]$

$8 \sin 8x = 8$ $\left[\frac{\pi}{16} + k\frac{\pi}{4}\right]$

$\sin x + 3 = -2[\sin(-x) - 1]$ $\left[\frac{\pi}{2} + 2k\pi\right]$

$\frac{3}{5}\sin x - 2 = -\frac{2}{5}\sin x - \sin\frac{\pi}{2}$ $\left[\frac{\pi}{2} + 2k\pi\right]$

# Esercizi

**Risolvi le seguenti equazioni in $\mathbb{R}$.**

$\cos x = -1$ $[\pi + 2k\pi]$

$2\cos x = \sqrt{2}$ $\left[\pm\frac{\pi}{4} + 2k\pi\right]$

$4\cos x = 8$ [impossibile]

$2\cos 5x + \sqrt{2} = 0$ $\left[\pm\frac{3}{20}\pi + k\frac{2}{5}\pi\right]$

$2\cos 6x - 1 = 0$ $\left[\pm\frac{\pi}{18} + k\frac{\pi}{3}\right]$

$8\cos x = 1$ $\left[\pm\arccos\frac{1}{8} + 2k\pi\right]$

$2\cos x + \sqrt{3} = 0$ $\left[\pm\frac{5}{6}\pi + 2k\pi\right]$

$4\cos x - 2\sin\frac{\pi}{3} = \sqrt{3}$ $\left[\pm\frac{\pi}{6} + 2k\pi\right]$

$\cos\frac{x}{4} - 1 = 0$ $[8k\pi]$

$\frac{\pi}{2}\cos\left(\frac{\pi}{9} - x\right) = 0$ $\left[\frac{11}{18}\pi + k\pi\right]$

$\cos 3x = -1 - \cos 3x$ $\left[\pm\frac{2}{9}\pi + k\frac{2}{3}\pi\right]$

$\cos 4x = \frac{1}{3}$ $\left[\pm\frac{1}{4}\arccos\frac{1}{3} + k\frac{\pi}{2}\right]$

$\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{4}$ $\left[\frac{\pi}{6} \pm \arccos\frac{1}{4} + 2k\pi\right]$

$\sqrt{3}\sec 2x = 2$ $\left[\pm\frac{\pi}{12} + k\pi\right]$

$2\cos\left(x + \frac{\pi}{6}\right) + 1 = 0$ $\left[\frac{\pi}{2} + 2k\pi; -\frac{5}{6}\pi + 2k\pi\right]$

$2\cos x + 2 = \cos x + 2\sin\frac{\pi}{2}$ $\left[\frac{\pi}{2} + k\pi\right]$

$2\sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{6} = \tan\pi$ $\left[\pm\frac{\pi}{3} + 2k\pi\right]$

$2(\cos 2x + 3) - 3 = 3(1 - \cos 2x)$ $\left[\pm\frac{\pi}{4} + k\pi\right]$

$2|\cos x| = 1$ $\left[\pm\frac{\pi}{3} + k\pi\right]$

$\left|\cos\left(x - \frac{\pi}{4}\right)\right| = \frac{\sqrt{2}}{2}$ $\left[\frac{\pi}{2} + k\frac{\pi}{2}\right]$

# Esercizi

**Risolvi le seguenti equazioni in $\mathbb{R}$.**

$\tan x - \sqrt{3} = 0$ $\left[\frac{\pi}{3} + k\pi\right]$

$\tan x = \sin\pi$ $[k\pi]$

$\tan x = 2$ $[\arctan 2 + k\pi]$

$\tan\frac{x}{4} + 1 = 0$ $[-\pi - 4k\pi]$

$3\tan\left(x + \frac{\pi}{9}\right) - \sqrt{3} = 0$ $\left[\frac{\pi}{18} + k\pi\right]$

$3\tan x = \tan x$ $[k\pi]$

$\tan\left(x + \frac{\pi}{6}\right) + 1 = 0$ $\left[-\frac{5}{12}\pi + k\pi\right]$

$2\sin\frac{\pi}{3}\tan x - \tan\frac{\pi}{3} = 0$ $\left[\frac{\pi}{4} + k\pi\right]$

$\cos\frac{3}{2}\pi + \tan\frac{x}{2} - \sin\frac{3}{2}\pi = 2$ $\left[\frac{\pi}{2} + 2k\pi\right]$

$\tan 3x = 3$ $\left[\frac{1}{3}\arctan 3 + k\frac{\pi}{3}\right]$

$\cot 2x = 1$ $\left[\frac{\pi}{8} + k\frac{\pi}{2}\right]$

$\tan(50° - x) = 0$ $[50° + k180°]$

$3\tan 6x + \sqrt{3} = 0$ $\left[-\frac{\pi}{36} + k\frac{\pi}{6}\right]$

$3\tan 3x = -1 + 2\tan(\pi + 3x)$ $\left[-\frac{\pi}{12} + k\frac{\pi}{3}\right]$

$\tan\left(x - \frac{\pi}{5}\right) + 7 = 0$ $\left[\frac{\pi}{5} + \arctan(-7) + k\pi\right]$

$\left|\tan\left(x - \frac{\pi}{3}\right)\right| = \sqrt{3}$ $\left[k\pi; \frac{2}{3}\pi + k\pi\right]$

$-2\sqrt{3}\tan(x + 100°) = 4\left[1 + \sqrt{3}\tan(x + 100°)\right] + 2$ $[-130° + k180°]$

$\tan\left(x + \frac{\pi}{4}\right) = 2\left(\sin\frac{2}{3}\pi + \cos\frac{\pi}{6}\right) - \tan\frac{\pi}{3}$ $\left[\frac{\pi}{12} + k\pi\right]$

$2(\tan x + 1) + 3(1 - \tan x) = -2(\tan x - 1) + 4$ $\left[\frac{\pi}{4} + k\pi\right]$

# Esercizi

**Risolvi le seguenti disequazioni in $\mathbb{R}$.**

$\sin x < -\frac{1}{2}$ $\left[\frac{7\pi}{6}+2k\pi < x < \frac{11}{6}\pi+2k\pi\right]$

$3\tan x+\sqrt{3} < 0$ $\left[\frac{\pi}{2}+k\pi < x < \frac{5}{6}\pi+k\pi\right]$

$2\cos x \geq -1$ $\left[-\frac{2}{3}\pi+2k\pi \leq x \leq \frac{2}{3}\pi+2k\pi\right]$

$\tan x \geq \cos\pi$ $\left[-\frac{\pi}{4}+k\pi \leq x < \frac{\pi}{2}+k\pi\right]$

$2\sin x+\sqrt{2} > 0$ $\left[-\frac{\pi}{4}+2k\pi < x < \frac{5}{4}\pi+2k\pi\right]$

$2\cos x+\sqrt{3} \geq 0$ $\left[2k\pi \leq x \leq \frac{5}{6}\pi+2k\pi \vee \frac{7}{6}\pi+2k\pi \leq x \leq 2\pi+2k\pi\right]$

$\cos\left(x+\frac{\pi}{3}\right) \geq \frac{\sqrt{2}}{2}$ $\left[-\frac{7}{12}\pi+2k\pi \leq x \leq -\frac{\pi}{12}+2k\pi\right]$

$\sin x+3 \geq 2(\sin x+2)$ $\left[x=\frac{3}{2}\pi+2k\pi\right]$

$2(\sin x+3) < 3[2+\sin(-x)]$ $[\pi+2k\pi < x < 2\pi+2k\pi]$

$2(\tan x+1)+3(1-\tan x) < 2\tan(\pi-x)+6$ $\left[-\frac{\pi}{2}+k\pi < x < \frac{\pi}{4}+k\pi\right]$

$-4+2\sin\left(\frac{\pi}{2}+x\right)+3\cos x < 3(\cos x-1)$ $\left[\frac{\pi}{3}+2k\pi < x < \frac{5}{3}\pi+2k\pi\right]$

$2\cos^2 x-1 \leq 2(1-\sin x)(1+\sin x)+\cos x$ $[\forall x \in \mathbb{R}]$

$\tan 2x \leq 0$ $\left[\frac{\pi}{4}+k\frac{\pi}{2} < x \leq \frac{\pi}{2}+k\frac{\pi}{2}\right]$

$3\cot x \geq \sqrt{3}$ $\left[k\pi < x \leq \frac{\pi}{3}+k\pi\right]$

$\cot x < \sqrt{3}$ $\left[\frac{\pi}{6}+k\pi < x < \pi+k\pi\right]$

$2\sin x \leq -\sqrt{2}$ $\left[\frac{5}{4}\pi+2k\pi \leq x \leq \frac{7}{4}\pi+2k\pi\right]$

# Riferimenti bibliografici


[1] Abate, M. **Geometria**. McGraw-Hill

[2] Bergamini M., Barozzi G., Trifone A. **Matematica.blu 2.0 3**. Zanichelli

[3] Bergamini M., Barozzi G., Trifone A. **Matematica.blu 2.0 4**. Zanichelli

[4] Bergamini M., Barozzi G., Trifone A. **Matematica.verde 4A**. Zanichelli

[5] Bergamini M., Barozzi G., Trifone A. **Matematica.verde 4B**. Zanichelli

[6] Bertsch M., Dal Passo R., Giacomelli L. **Analisi matematica**. McGraw-Hill Education

[7] Conti M., Ferrario D. L., Terracini S., Verzini G. **Analisi matematica. Dal calcolo all'analisi, Vol 1**. Apogeo

[8] Marcellini P., Sbordone C. **Esercitazioni di Matematica. Primo volume, parte prima.** Liguori Editore