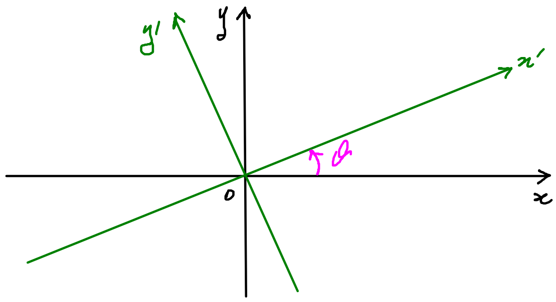
MA 21 OTT 2025

INDIPENDENZA DEL GRADIENTE DAL SISTEMA DI RIFERIMENTO



IN UN SISTEMA RUOTATO DI θ I VERSORI DEGLI ASSI SONO $\hat{\imath}' = \hat{\imath}\cos\theta + \hat{\jmath}\operatorname{sen}\theta$ E $\hat{\jmath}' =$

$$= \hat{\imath}\cos\left(\theta+\frac{\pi}{2}\right) + \hat{\jmath}\operatorname{sen}\left(\theta+\frac{\pi}{2}\right)$$

$$= -\hat{\imath}\operatorname{sen}\theta + \hat{\jmath}\cos\theta.$$

QUINDI SE UN VETTORE $\vec{E}$ HA COMPONENTI (x', y') NEL SISTEMA RUOTATO CIOÈ $\vec{E} = x'\hat{\imath}' + y'\hat{\jmath}'$,

ALLORA
$$\vec{E} = x'\hat{\imath}\cos\theta + x'\hat{\jmath}\operatorname{sen}\theta - y'\hat{\imath}\operatorname{sen}\theta + y'\hat{\jmath}\cos\theta$$

DUNQUE LE COMPONENTI DI $\vec{E}$ NEL SISTEMA xy SONO:

$$\begin{cases} x = x'\cos\theta - y'\operatorname{sen}\theta \\ y = x'\operatorname{sen}\theta + y'\cos\theta \end{cases}$$

QUINDI

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix}.$$

ORA SUPPONIAMO CHE $f(x,y)$ SIA DIFFERENZIABILE IN $(0,0)$ E QUINDI VALGA LA FORMULA

$$\frac{\partial f}{\partial \hat{v}}(0,0) = \hat{v}\cdot\nabla f(0,0).$$

CHI LAVORA NEL SISTEMA RUOTATO TROVA

$$\frac{\partial f}{\partial \hat{\imath}'}(0,0) = \frac{\partial f}{\partial x'}(0,0) = \hat{\imath}'\cdot\nabla f(0,0)$$
$$= \frac{\partial f}{\partial x}(0,0)\cos\theta + \frac{\partial f}{\partial y}\operatorname{sen}\theta$$

E ANCHE

$$\frac{\partial f}{\partial \hat{\jmath}'}(0,0) = \frac{\partial f}{\partial y'}(0,0) = \hat{\jmath}'\cdot\nabla f(0,0)$$
$$= -\frac{\partial f}{\partial x}(0,0)\operatorname{sen}\theta + \frac{\partial f}{\partial y}(0,0)\cos\theta$$

OVVERO

$$\begin{pmatrix} \frac{\partial f}{\partial \hat{\imath}'} \\ \frac{\partial f}{\partial \hat{\jmath}'} \end{pmatrix} = \begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}.$$

IL VETTORE AL PRIMO MEMBRO HA, NEL SISTEMA xy, COMPONENTI DATE DA

$$\begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$$