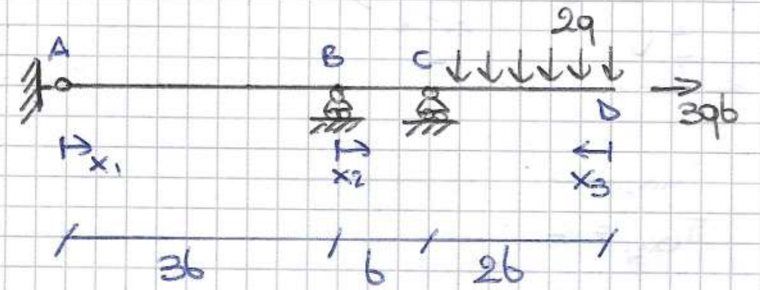
Esame Scienza delle Costruzioni – 11.06.2025

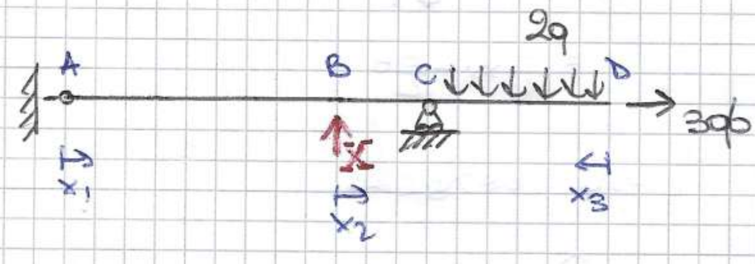


Struttura Iperstatica

GDL = 3

GDV = 2(A) + 1(B) + 1(C) = 4

GDL < GDV



Struttura isostatica

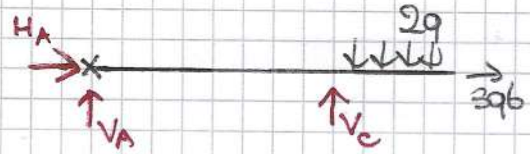
GDL = 3

GDV = 2(A) + 1(C) = 3

GDL = GDV

$v_C(P; X) = 0$

S_0 – Sistema Reale

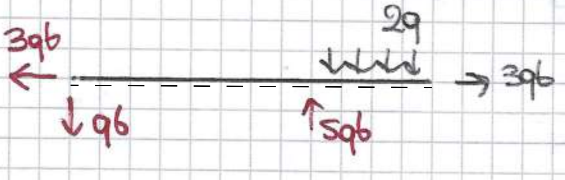


$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(A)} = 0 \end{cases} \quad \begin{cases} H_A + 3qb = 0 \ [1] \\ V_A + V_C - 2q(2b) = 0 \ [2] \\ V_C(4b) - 2q(2b)(5b) = 0 \ [3] \end{cases}$$

$H_A = -3qb$ [1]

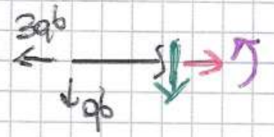
[3] $V_C = 5qb$

[2] $V_A = -qb$



Azioni Interne

A → B $0 \le x_1 \le 3b$

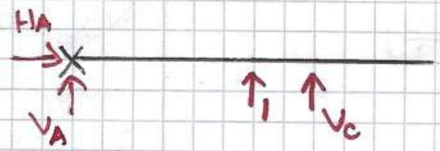


$N(x_1) = 3qb$

$T(x_1) = -qb$

$M(x_1) = -qb\,x_1$

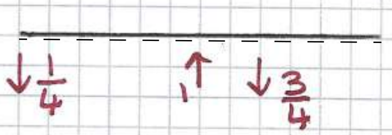
S_1 – Sistema Equilibrato



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(A)} = 0 \end{cases} \quad \begin{cases} H_A = 0 \\ V_A + V_C + 1 = 0 \\ 1(3b) + V_C(4b) = 0 \ [3] \end{cases}$$

[3] $V_C = -\frac{3}{4}$

$V_A = \frac{3}{4} - 1 \Rightarrow V_A = -\frac{1}{4}$



Azioni Interne

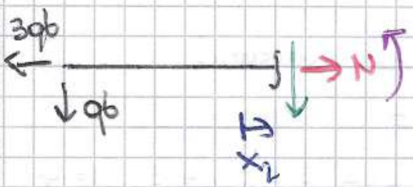
A → B $0 \le x_1 \le 3b$

$N(x_1) = 0$

$T(x_1) = -\frac{1}{4}$

$M(x_1) = -\frac{1}{4}x_1$

B → C $\quad 0 \le x_2 \le b$

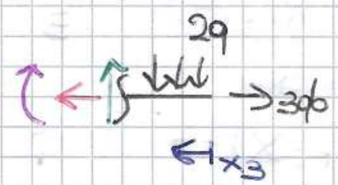


$N(x_2) = 3qb$

$T(x_2) = -qb$

$M(x_2) = -qb(3b + x_2)$
$= -3qb^2 - qbx_2$

C → D $\quad 0 \le x_3 \le 2b$



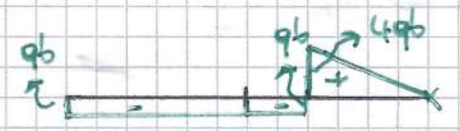
$N(x_3) = 3qb$

$T(x_3) = 2qx_3$

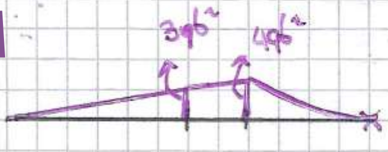
$M(x_3) = -2q(x_3)\left(\frac{x_3}{2}\right)$
$= -qx_3^2$

N

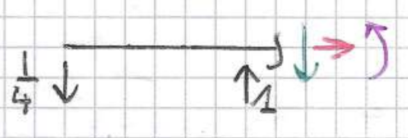
T



M



B → C $\quad 0 \le x_2 \le b$

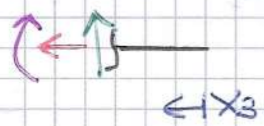


$N(x_2) = 0$

$T(x_2) = 1 - \frac{1}{4}$
$= \frac{3}{4}$

$M(x_2) = x_2 - \frac{1}{4}(3b + x_2)$
$= \frac{3}{4}x_2 - \frac{3b}{4}$

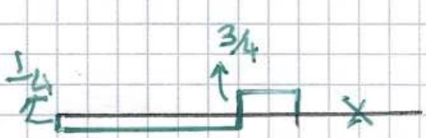
C → D $\quad 0 \le x_3 \le 2b$



$N(x_3) = 0$

$T(x_3) = 0$

$M(x_3) = 0$



P.L.V.

$\mathcal{L}V_e = \mathcal{L}V_i$

$\mathcal{L}V_e = 1 \cdot v_B = 0$

$\mathcal{L}V_i = \int_s m_1 \cdot \chi_i$

$\mathcal{L}V_i = \int_s m_1 \left(\frac{m_0 + X m_1}{EI}\right) ds \Rightarrow \int_s \left(\frac{m_1 m_0}{EI} + X \frac{m_1^2}{EI}\right) ds$

A → B

$m_0 = -qbx_1$

$m_1 = -\frac{1}{4}x_1$

$m_1 m_0 = \frac{1}{4} qb x_1^2$

$m_1^2 = \frac{1}{16} x_1^2$

B → C

$m_0 = -3qb^2 - qbx_2$

$m_1 = \frac{3}{4}x_2 - \frac{3}{4}b$

$m_1 m_0 = -\frac{9}{4}qb^2x_2 + \frac{9}{4}qb^3 - \frac{3}{4}qbx_2^2 + \frac{3}{4}qb^2x_2$

$= -\frac{3}{2}qb^2x_2 + \frac{9}{4}qb^3 - \frac{3}{4}qbx_2^2$

$m_1^2 = \frac{9}{16}x_2^2 + \frac{9}{16}b^2 - \frac{9}{8}x_2 b$

C → D

$m_0 = -qx_3^2$

$m_1 = 0$

$m_0 m_1 = 0$

$m_1^2 = 0$

$$\mathcal{L}V_i = \int_0^{3b} \frac{1}{EI}\left(\frac{1}{4}qbx_1^2\right) + \frac{X}{EI}\left(\frac{1}{16}x_1^2\right) dx_1 + \int_0^{b} \frac{1}{EI}\left(-\frac{3}{2}qb^2x_2 + \frac{9}{4}qb^3 - \frac{3}{4}qbx_2^2\right) +$$
$$+ \frac{X}{EI}\left(\frac{9}{16}x_2^2 + \frac{9}{16}b^2 - \frac{9}{8}x_2 b\right) dx_2$$

$$\mathcal{L}V_i = \left[\frac{1}{EI}\left(\frac{1}{12}qbx_1^3\right) + \frac{X}{EI}\left(\frac{1}{48}x_1^3\right)\right]_0^{3b} + \left[\frac{1}{EI}\left(-\frac{3}{4}qb^2x_2^2 + \frac{9}{4}qb^3x_2 - \frac{1}{4}qbx_2^3\right) +\right.$$
$$\left. + \frac{X}{EI}\left(\frac{3}{16}x_2^3 + \frac{9}{16}b^2x_2 - \frac{9}{16}x_2^2 b\right)\right]_0^{b}$$

$$\mathcal{L}V_i = \frac{1}{EI}\left(\frac{9}{4}qb^4 + X\frac{9}{16}b^3\right) + \frac{1}{EI}\left(-\frac{3}{4}qb^4 + \frac{9}{4}qb^4 - \frac{1}{4}qb^4 + X\frac{3}{16}b^3 + X\frac{9}{16}b^3 + \right.$$
$$\left. - X\frac{9}{16}b^3\right)$$

$\mathcal{L}V_i = \frac{1}{EI}\left(\frac{7}{2}qb^4 + \frac{12}{16}Xb^3\right)$; $\mathcal{L}V_i = 0 \Rightarrow \frac{1}{EI}\left(\frac{7}{2}qb^4 + \frac{3}{4}Xb^3\right) = 0$

$\frac{3}{4}Xb^3 = -\frac{7}{2}qb^4 \Rightarrow X = -\frac{7}{2}\cdot\frac{4}{3}qb \Rightarrow X = -\frac{14}{3}qb$

Reazioni Vincolari

$H_A = H_{A_0} + X H_{A_1}$

$H_A = -3qb$

$V_A = V_{A_0} + X V_{A_1}$

$V_A = -qb - \frac{14}{3}\left(-\frac{1}{4}\right)qb$

$= -qb + \frac{7}{6}qb$

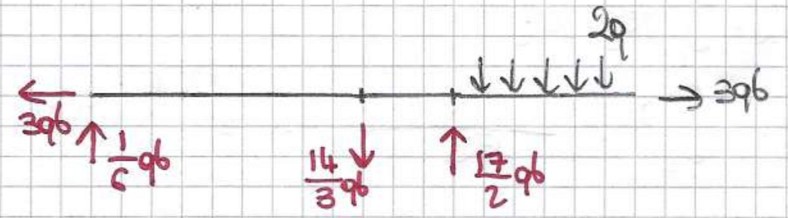
$V_A = \frac{1}{6}qb$

$V_B = -\frac{14}{3}qb$

$V_C = V_{C_0} + X V_{C_1}$

$= 5qb - \frac{14}{3}\left(-\frac{3}{4}\right)qb$

$V_C = \frac{17}{2}qb$



Azioni Interne

$N(x_1) = N(x_1)_0 + X N(x_1)_1$

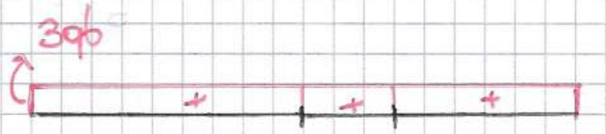
$N(x_1) = 3qb$

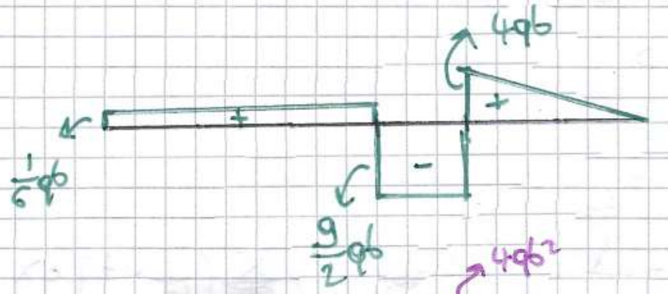
$T(x_1) = -qb - \frac{14}{3}\left(-\frac{1}{4}\right)qb$

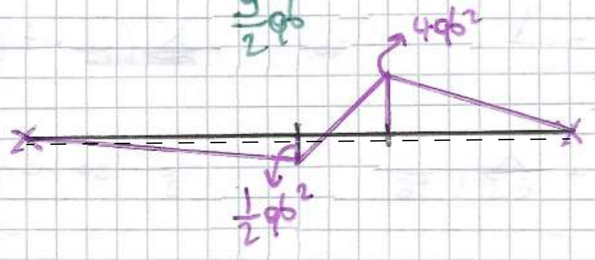
$T(x_1) = \frac{1}{6}qb$

$M(x_1) = -qbx_1 - \frac{14}{3}\left(-\frac{1}{4}x_1\right)qb$

$M(x_1) = +\frac{1}{6}qbx_1$







$N(x_2) = 3qb$

$T(x_2) = -qb - \frac{14}{3}qb\left(\frac{3}{4}\right)$

$T(x_2) = -\frac{9}{2}qb$

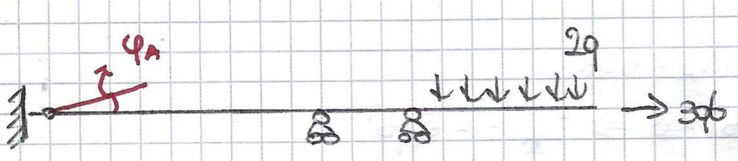
$M(x_2) = -3qb^2 - qbx_2 - \frac{14}{3}qb\left(\frac{3}{4}x_2 - \frac{3}{4}b\right)$

$= -3qb^2 - qbx_2 - \frac{7}{2}qbx_2 + \frac{7}{2}qb^2$

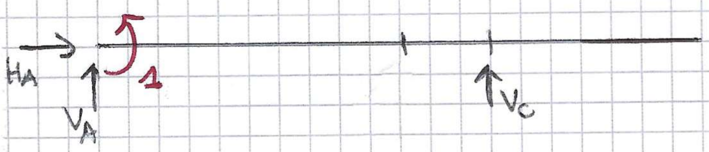
$M(x_2) = \frac{1}{2}qb^2 - \frac{9}{2}qbx_2$

$N(x_3) = 3qb$; $T(x_3) = 2qx_3$; $M(x_3) = -qx_3^2$

• Rotazione del punto A; φ_A



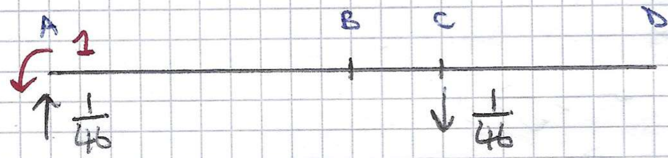
S_2



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(A)} = 0 \end{cases} \qquad \begin{cases} H_A = 0 \\ V_A + V_C = 0 \\ 1 + V_C(4b) = 0 \end{cases}$$

[3] $V_C = -\frac{1}{4b}$

$V_A = \frac{1}{4b}$



Azioni Interne (momento)

A → B

$M(x_1) = -1 + \frac{1}{4b} x_1$

B → C

$M(x_2) = -1 + \frac{1}{4b}(3b + x_2) = -1 + \frac{3}{4} + \frac{x_2}{4b}$

$M(x_2) = -\frac{1}{4} + \frac{x_2}{4b}$

C → D

$M(x_2) = 0$

PLV

$\mathcal{L}V_e = \mathcal{L}V_i$

$\mathcal{L}V_e = 1 \cdot \varphi_A = 0$

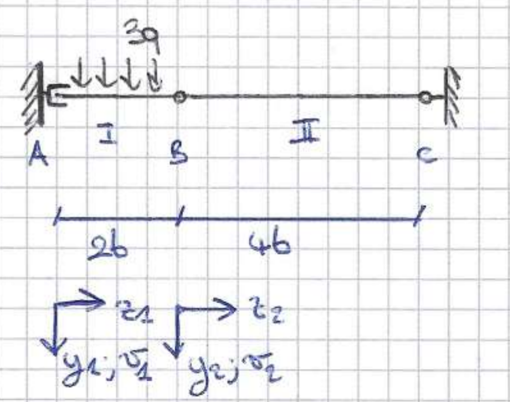
$$\mathcal{L}V_i = \int_0^{3b} \left(-1 + \frac{1}{4b} x_1\right)\left(\frac{1}{6} qb x_1\right)\left(\frac{1}{EI}\right) dx_1 + \int_0^{b} \left(\frac{x_2}{4b} - \frac{1}{4}\right)\left(\frac{1}{2} qb^2 - \frac{9}{2} qb x_2\right)\left(\frac{1}{EI}\right) dx_2$$

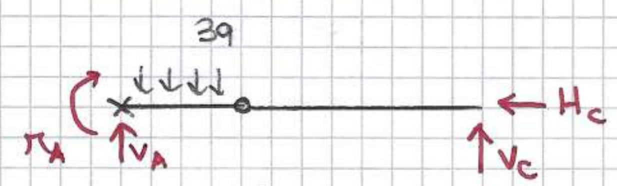
$$\mathcal{L}V_i = \frac{1}{EI}\left[-\frac{1}{12} qb x_1^2 + \frac{1}{72} q x_1^3\right]_0^{3b} + \left[\frac{1}{16} b x_2^2 q - \frac{9}{24} q x_2^3 - \frac{1}{8} qb^2 x_2 + \frac{9}{16} qb x_2^2\right]_0^{b}$$

$$= \frac{1}{EI}\left[-\frac{9}{12}qb^3 + \frac{27}{72}qb^3 + \frac{1}{16}qb^3 - \frac{9}{24}qb^3 - \frac{1}{8}qb^3 + \frac{9}{16}qb^3\right]$$

$$= \frac{1}{EI}\left[\frac{-12 + 1 - 2 + 9}{16} qb^3\right] \qquad \mathcal{L}V_i = -\frac{1}{4}\frac{qb^3}{EI} \circlearrowright$$

Esercizio 2.

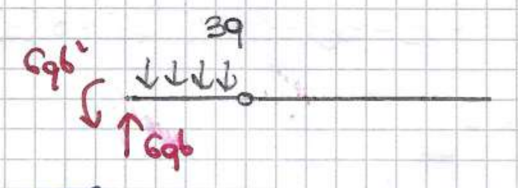




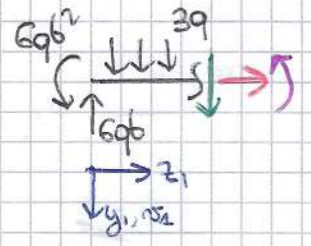
$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(A)} = 0 \end{cases} \quad \begin{cases} H_C = 0 \\ V_A + \cancel{V_C} - 3q(2b) = 0 \Rightarrow V_A = 6qb \\ M_A + 3q(2b)(b) - \cancel{V_C(6b)} = 0 \Rightarrow M_A = -6qb^2 \end{cases}$$

eq. aux

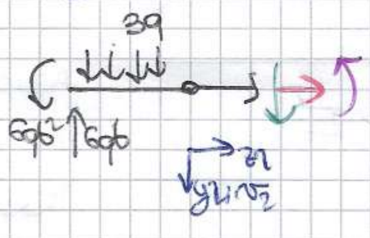
$$\left\{ M^{(II)}_{z(B)} \right. \quad \left\{ V_C(4b) = 0 \Rightarrow V_C = 0 \right.$$



A ⇒ B



B ⇒ C



$N_{z_1} = 0$

$T_{z_1} + 3qz_1 - 6qb = 0$

$T_{z_1} = 6qb - 3qz_1$

$M_{z_1} + 6qb^2 + 3qz_1\left(\frac{z_1}{2}\right) - 6qbz_1 = 0$

$M_{z_1} = 6qbz_1 - \frac{3}{2}qz_1^2 - 6qb^2$

$N_{z_2} = 0$

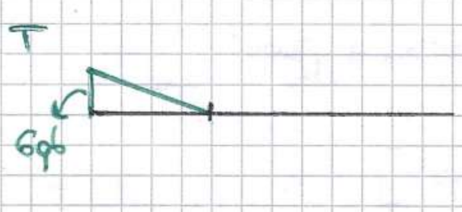
$T_{z_2} + 3q(2b) - 6qb = 0$

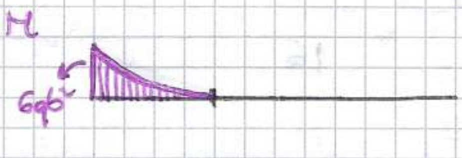
$T_{z_2} = 0$

$M_{z_2} + 3q(2b)(b + z_2) - 6qb(2b + z_2) + 6qb^2 = 0$

$M_{z_2} + \cancel{6qb^2} + \cancel{6qbz_2} - \cancel{12qb^2} - \cancel{6qbz_2} + \cancel{6qb^2} = 0$

$M_{z_2} = 0$





Eq. LINEA ELASTICA

A → B $0 \leq z_1 \leq 2b$

$$v''_{(z_1)} = -\frac{M}{EI} \Rightarrow v''_{z_1} = \left(-6qbz_1 + \frac{3}{2}qz_1^2 + 6qb^2\right)\left(\frac{1}{EI}\right)$$

$$v'_{(z_1)} = \frac{1}{EI}\left(-3qbz_1^2 + \frac{1}{2}qz_1^3 + 6qb^2z_1\right) + A_1$$

$$v_{(z_1)} = \frac{1}{EI}\left(-qbz_1^3 + \frac{1}{8}qz_1^4 + 3qb^2z_1^2\right) + A_1z_1 + A_2$$

B → C $0 \leq z_2 \leq 4b$

$$v''_{(z_2)} = -\frac{M}{EI} \Rightarrow v''_{(z_2)} = \frac{1}{EI}(0)$$

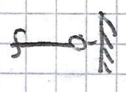
$$v'_{(z_2)} = B_1$$

$$v_{(z_2)} = B_1z_2 + B_2$$

COSTANTI A_1; A_2; B_1; B_2. CONDIZIONI A CONTORNO

A. 
MANICOTTO
{ IMPEDISCE
- SPOSTAMENTO ↕ ⇒ $v_{z_1}(A) = 0$
- ROTAZIONE ↻ ⇒ $v'_{z_1}(A) = 0$

B. 
CERNIERA
{ IMPONE
- UGUALE ABBASSAMENTO in B_1 e B_2 ⇒ $v_{(z_1)}(B) = v_{(z_2)}(B)$

C. 
CERNIERA
{ IMPEDISCE
- SPOSTAMENTO ↕ ⇒ $v_{z_2}(C) = 0$

IN A, CON $z_1 = 0$

$$v_{z_1}(z_1 = 0) = 0 \Rightarrow A_2 = 0$$

$$v'_{(z_1)}(z_1 = 0) = 0 \Rightarrow A_1 = 0$$

IN B, CON $z_1 = 2b$ e $z_2 = 0$

$$v_{(z_1)}(z_1 = 2b) = v_{(z_2)}(z_2 = 0)$$

$$\frac{1}{EI}\left(-8qb^4 + 2qb^4 + 12qb^4\right) = B_2$$

$$B_2 = \frac{6qb^4}{EI}$$

IN C CON $z_2 = 4b$

$$v_{z_2}(z_2 = 4b) = 0 \Rightarrow 4b B_1 + \frac{6qb^4}{EI} = 0 \Rightarrow B_1 = -\frac{3}{2}\frac{qb^3}{EI}$$

DEFORMATA DELLA LINEA D'ASSE

$$v(z_1) = \frac{1}{EI}\left(-qbz_1^3 + \frac{1}{8}qz_1^4 + 3qb^2z_1^2\right)$$

$$v(z_2) = \frac{6qb^4}{EI} - \frac{3}{2}\frac{qb^3}{EI}z_2$$

DERIVATA PRIMA

$$v'(z_1) = \frac{1}{EI}\left(-3qbz_1^2 + \frac{1}{2}qz_1^3 + 6qb^2z_1\right)$$

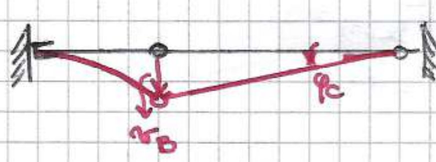
$$v'(z_2) = -\frac{3qb^3}{2EI}$$

ROTAZIONE NEL PUNTO C, φ_C

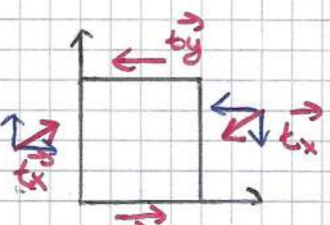
$$\varphi_C = -\frac{3qb^3}{2EI} \;\swarrow$$

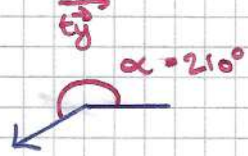
SPOSTAMENTO VERTICALE DEL PUNTO B, v_B.

$$v_B = \frac{6qb^4}{EI} \;\downarrow$$



Esercizio 3 _ Trazio 1





$|t_x| = 20\,MPa$

$\alpha = 210°$ $\quad \operatorname{sen}\alpha = -\frac{1}{2}$; $\cos\alpha = -\frac{\sqrt{3}}{2}$

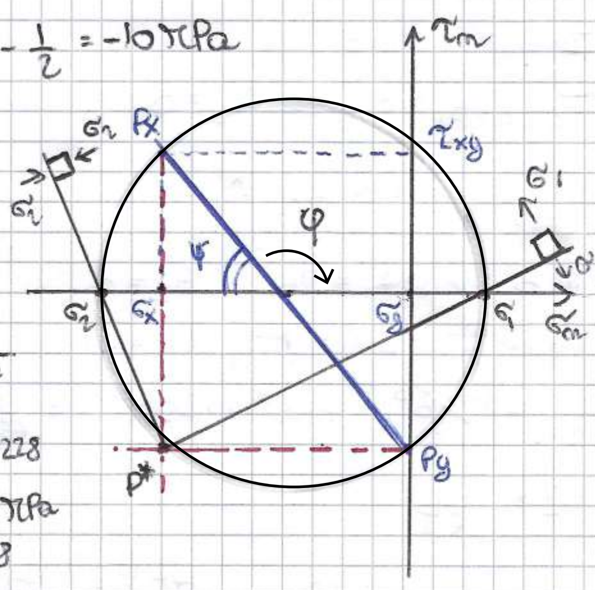
$\sigma_x = |t_x|\cos\alpha = 20\,MPa \cdot -\frac{\sqrt{3}}{2} = -10\sqrt{3}\,MPa \cong -17,320$

$\sigma_y = 0$

$\tau_{xy} = |t_x|\operatorname{sen}\alpha = 20\,MPa \cdot -\frac{1}{2} = -10\,MPa$

$$\underline{\underline{\sigma}} = \begin{pmatrix} -10\sqrt{3} & -10 & 0 \\ -10 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

CERCHIO DI MOHR - PIANO $(\sigma_n ; \tau_n)$



$P_x(\sigma_x ; \tau_{xy}) \Rightarrow \tau_{xy}$ ↻

$P_x(-10\sqrt{3} ; 10) \Rightarrow (-17,32 ; 10)$

$P_y(\sigma_y ; \tau_{yx}) \Rightarrow \tau_{yx}$ ↺

$P_y(0 ; -10)$

$C\left(\frac{\sigma_x + \sigma_y}{2} ; 0\right) = (-8,66 ; 0)$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{174,995} \cong 13,228$

$\tau_{max} = R = 13,228\,MPa$

$\sigma_1 = \sigma_C + R = 4,568$

$\sigma_2 = \sigma_C - R = -21,889$

$\varphi = ?$ $\quad tg\,2\varphi = \frac{\tau_{xy}}{\sigma_x - \sigma_C} = \frac{+10}{-8,66} = 1,154$ $\quad arctg(1,154) \cong 49,089$

$2\varphi = 180 - 49,089 = 130,911° \Rightarrow \varphi = -65,455°$