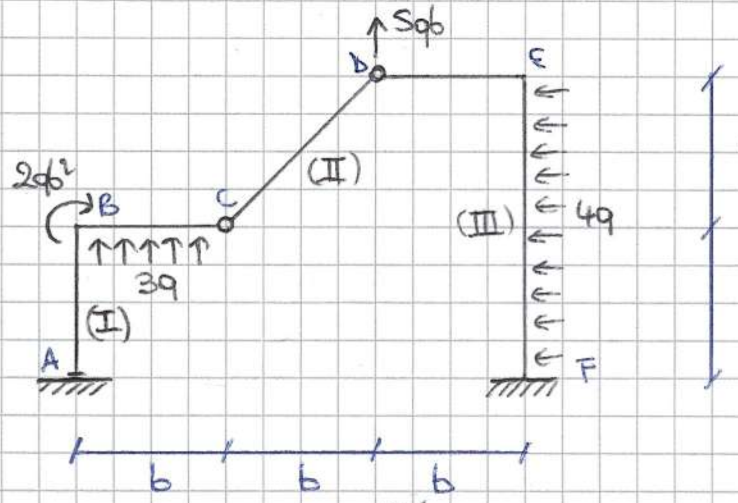
Esame statica_11.06.2025_esercizio 1_traccia II



STRUTTURA ISOSTATICA

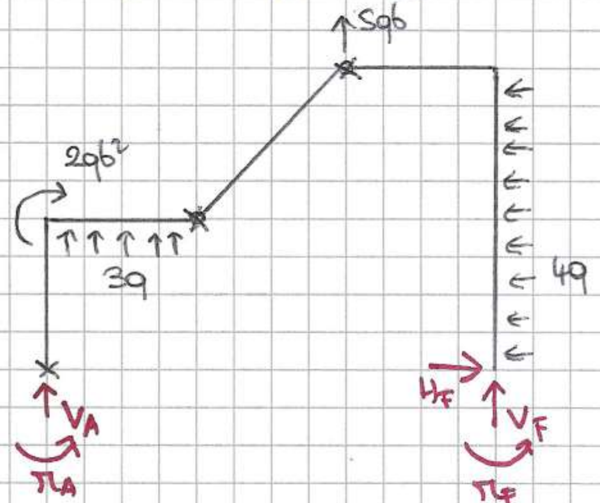
$GDL = 3(I) + 3(II) + 3(III)$

$GDL = 9$

$GDV = 2(A) + 2(C) + 2(D) + 3(F)$

$GDV = 9$

$GDL = GDV$



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(A)} = 0 \end{cases} \quad \begin{cases} H_F - 4q(2b) = 0 \Rightarrow \boxed{H_F = 8qb} \\ V_A + 3qb + 5qb + V_F = 0 \Rightarrow V_A + 8qb + V_F = 0 \Rightarrow V_A = -V_F - 8qb \\ M_A - 2qb^2 + 3qb\left(\frac{b}{2}\right) + 5qb(2b) + 4q(2b)(b) + V_F(3b) + M_F = 0 \quad [3] \end{cases}$$

$$\begin{cases} M^{I}_{z(C)} \\ M^{III}_{z(D)} = 0 \end{cases} \quad \begin{cases} M_A - V_A b - 3qb\left(\frac{b}{2}\right) - 2qb^2 = 0 \quad [4] \\ M_F + H_F(2b) + V_F b - 4q(2b)(b) = 0 \quad [5] \end{cases}$$

$[4]\quad M_A + V_F b + 8qb^2 - \frac{3}{2}qb^2 - 2qb^2 = 0 \Rightarrow M_A + V_F b + 6qb^2 - \frac{3}{2}qb^2$

$\Rightarrow M_A = -V_F b - \frac{9}{2}qb^2$

$[5]\quad M_F + 16qb^2 + V_F b - 8qb^2 = 0 \Rightarrow M_F = -8qb^2 - V_F b$

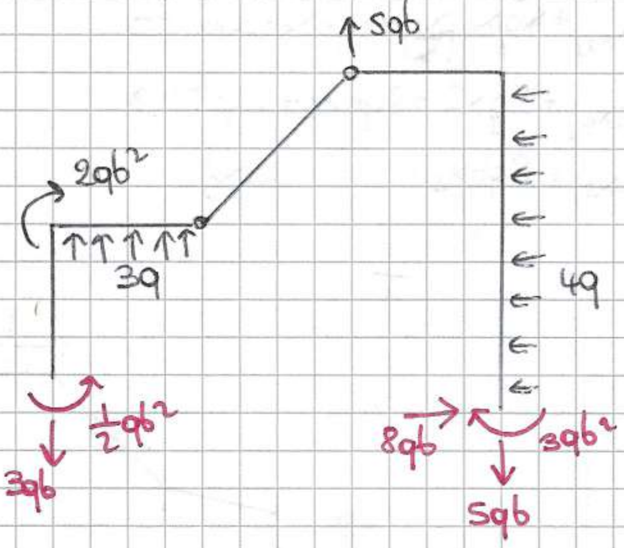
$[3]\quad -V_F b - \frac{9}{2}qb^2 - 2qb^2 + \frac{3}{2}qb^2 + 10qb^2 + \cancel{8qb^2} + 3V_F b - \cancel{8qb^2} - V_F b = 0$

[3] $V_F b - 3qb^2 + 8qb^2 = 0 \Rightarrow \boxed{V_F = -5qb}$

$V_A = -V_F - 8qb \Rightarrow \boxed{V_A = -3qb}$

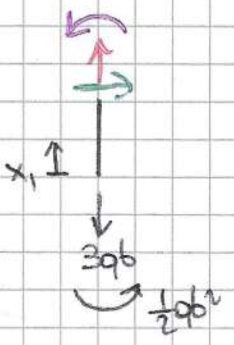
$M_A = -V_F b - \frac{9}{2}qb^2 \Rightarrow \boxed{M_A = \frac{1}{2}qb^2}$

$M_F = -8qb^2 - V_F b \Rightarrow \boxed{M_F = -3qb^2}$



AZIONI INTERNE

A → B $0 \le x_1 \le b$

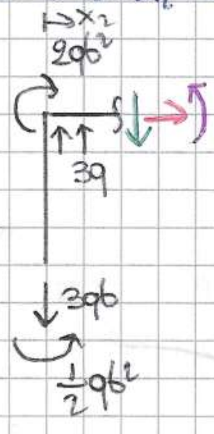


$N_{(x_1)} = 3qb$

$T_{(x_1)} = 0$

$M_{(x_1)} = -\frac{1}{2}qb^2$

B → C $0 \le x_2 \le b$



$N_{(x_2)} = 0$

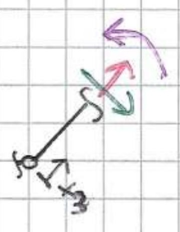
$T_{(x_2)} + 3qb - 3qx_2 = 0$

$M_{(x_2)} + \frac{1}{2}qb^2 + 3qbx_2 - 2qb^2 +$

$- 3qx_2\left(\frac{x_2}{2}\right) = 0$

$M_{(x_2)} = \frac{3}{2}qx_2^2 - 3qbx_2 + \frac{3}{2}qb^2$

C → D $0 \le x_3 \le b\sqrt{2}$

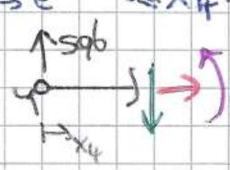


$N_{(x_3)} = 0$

$T_{(x_3)} = 0$

$M_{(x_3)} = 0$

D → E $0 \le x_4 \le b$

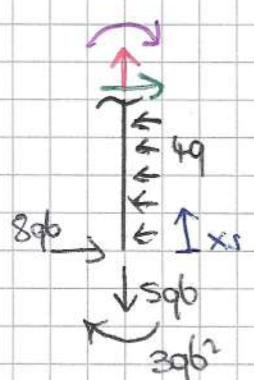


$N_{(x_4)} = 0$

$T_{(x_4)} = 5qb$

$M_{(x_4)} = 5qb\,x_4$

E → F



$N_{(x_s)} = 5qb$

$T_{(x_s)} + 8qb - 4q(x_s) = 0$

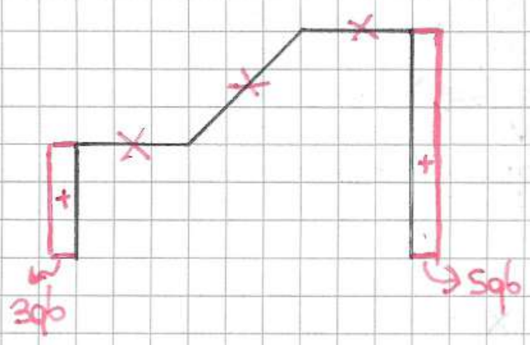
$T_{(x_s)} = 4qx_s - 8qb$

$M_{(x_s)} + \overset{2}{\cancel{4}}qx_s\left(\frac{x_s}{\cancel{2}}\right) - 8qbx_s + 3qb^2 = 0$

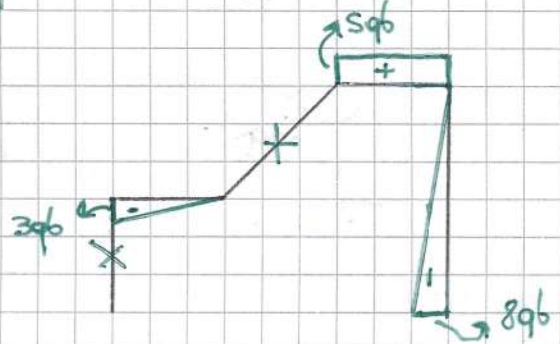
$M_{(x_s)} + 2qx_s^2 - 8qbx_s + 3qb^2 = 0$

$M_{(x_s)} = 8qbx_s - 3qb^2 - 2qx_s^2$

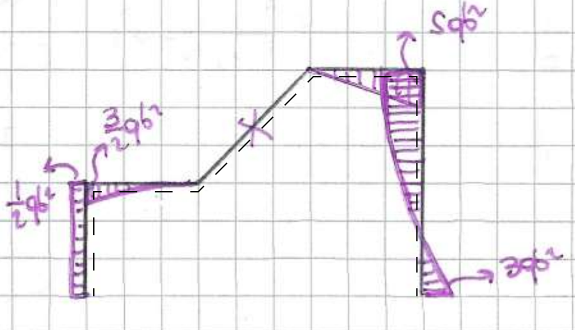
N



T

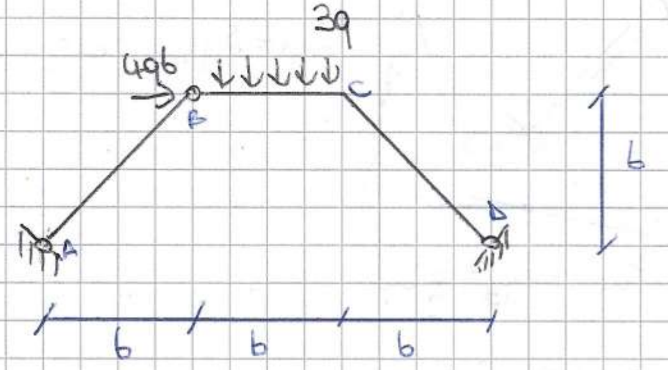


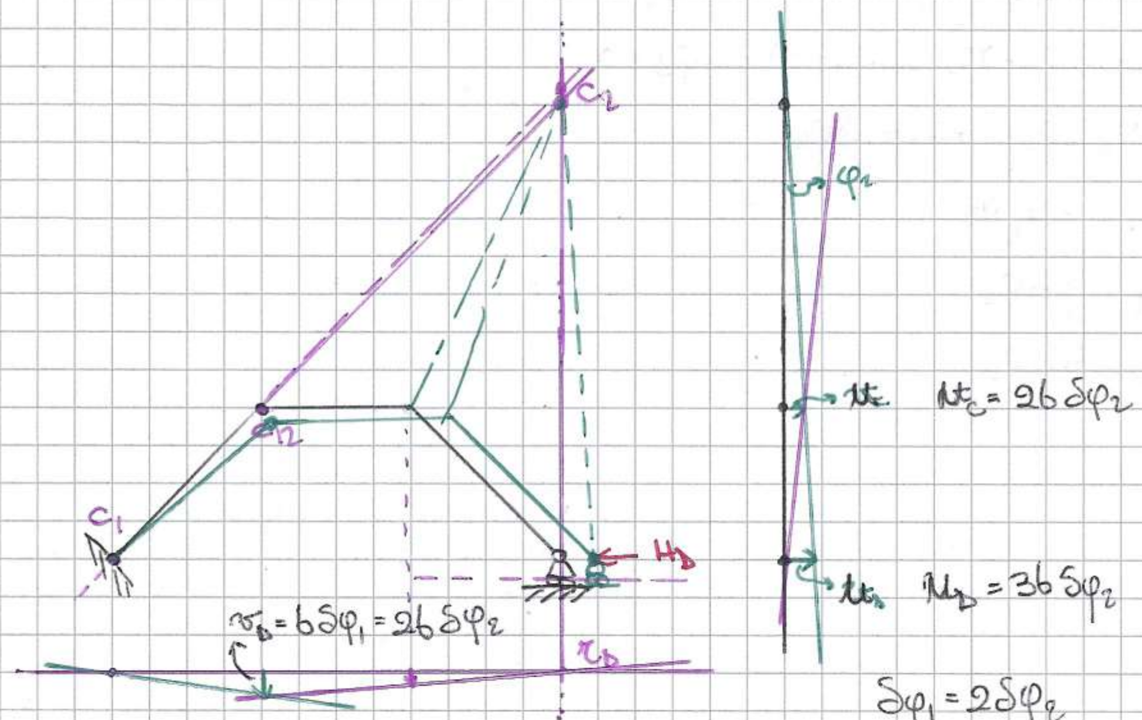
M



Esercizio 2._Traccia II

$H_D = ?$





$\delta\varphi_1 = 2\delta\varphi_2$

$C_1 = A = (0;0)$ $\qquad \delta\mathcal{L} = 0 \quad \forall \delta\varphi$

$C_{12} = B\,(b;b)$ $\qquad 4qb\,(2b\,\delta\varphi_2) + 3q(b)\left(b + \frac{b}{2}\right)\delta\varphi_2 - H_D\,(3b\,\delta\varphi_2) = 0$

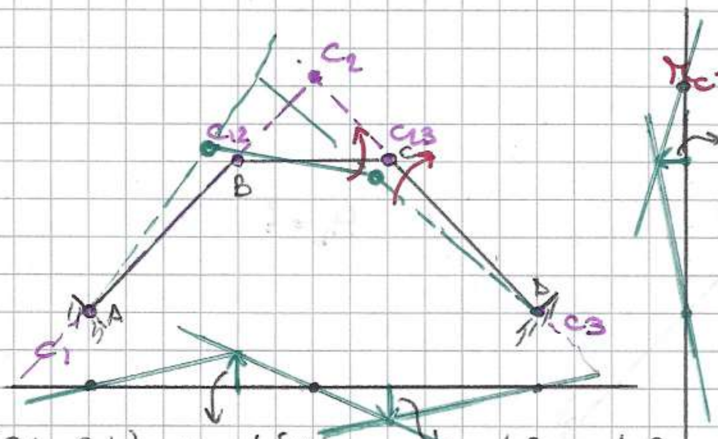
$C_2 \in r_D$ $\qquad 8qb^2\cancel{\delta\varphi_2} + \frac{9}{2}qb^2\cancel{\delta\varphi_2} - 3\cancel{b}\,H_D\,\cancel{\delta\varphi_2} = 0$

$C_1 \Leftrightarrow C_{12} \Leftrightarrow C_2$ $\qquad +3H_D = +8qb + \frac{9}{2}qb$

$C_1 : (3b; 3b)$ $\qquad 3H_D = \frac{25}{2}qb \Rightarrow H_D = \frac{25}{6}qb$

$u_A = 0$

$v_C = -b\,\delta\varphi_2$



$v_B = v_C = b\delta\varphi_1 = b\delta\varphi_3$

$\delta\varphi_1 = \delta\varphi_3 \quad \delta\varphi_2 = 2\delta\varphi_3$

$c_2\left(\frac{3b}{2};\frac{3}{2}b\right) \quad u_B = b\delta\varphi_1 \quad u_C = b\delta\varphi_3 = \frac{b}{2}\delta\varphi_2$

$$-4qb(b\delta\varphi_1) - \pi_c\delta\varphi_2 - \pi_c\delta\varphi_3 = 0$$

$$-4qb(b\delta\varphi_1) - \pi_c(2\delta\varphi_3) - \pi_c\delta\varphi_3 = 0$$

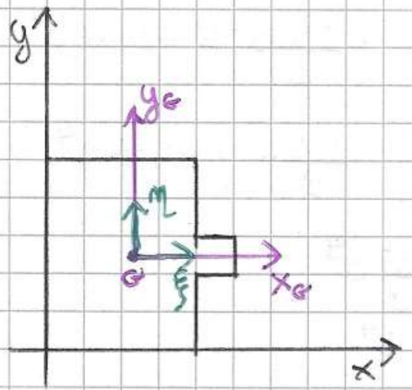
$$-3\pi_c\delta\varphi_3 = +4qb^2\delta\varphi_3$$

$$\pi_c = -\frac{4}{3}qb^2$$

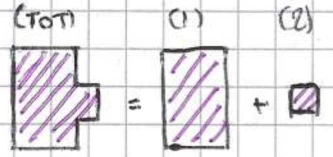
$$v_B = b\delta\varphi_1$$

$$v_C = b\delta\varphi_3$$

Esercizio 3 _ Traccia II



Momento statico



$S_x = A y_G \Rightarrow y_G = \frac{S_x}{A}$

$y_G = \frac{\cancel{105}^{\,5}}{2} a^3 \cdot \frac{1}{\cancel{21} a^2}$

$y_G = \frac{5}{2} a = 2{,}5a$

$S_y = A x_G \Rightarrow x_G = \frac{S_y}{A}$

$x_G = \frac{89}{2} a^3 \cdot \frac{1}{21 a^2}$

$x_G = \frac{89}{42} a \cong 2{,}119a$

$S_x^{(TOT)} = S_x^{(1)} + S_x^{(2)}$

$S_x^{(1)} = A_1 \cdot y_{G_1}$ con $A_1 = 20a^2$; $y_{G_1} = \frac{5}{2} a$

$S_x^{(1)} = \cancel{20}^{\,10} a^2 \cdot \frac{5}{\cancel{2}} a = 50 a^3$

$S_x^{(2)} = A_2 y_{G_2}$ con $A_2 = a^2$ $y_{G_2} = \frac{5}{2} a$

$S_x^{(2)} = a^2 \cdot \frac{5}{2} a = \frac{5}{2} a^3$

$S_x^{(TOT)} = 50a^3 + \frac{5}{2} a^3 = \frac{105}{2} a^3$

$S_y^{(TOT)} = S_y^{(1)} + S_y^{(2)}$

$S_y^{(1)} = A_1 x_{G_1}$ con $x_{G_1} = 2a$

$S_y^{(1)} = 20a^2 \cdot 2a = 40a^3$

$S_y^{(2)} = A_2 x_{G_2}$ con $x_{G_2} = \frac{9}{2} a$

$S_y^{(2)} = a^2 \cdot \frac{9}{2} a \Rightarrow S_y^{(2)} = \frac{9}{2} a^3$

$S_y^{(TOT)} = 40a^3 + \frac{9}{2} a^3 = \frac{89}{2} a^3$

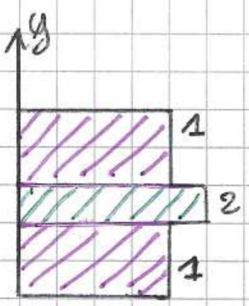
Momenti di inerzia

$I_{x_G} = \frac{b_1 h_1^3}{12} + \frac{b_2 h_2^3}{12}$ (1) (2)

$I_{x_G} = \frac{4a \cdot 125a^3}{12} + \frac{a \cdot a^3}{12} =$

$I_{x_G} = \frac{501}{12} a^4 = \frac{167}{4} a^4 \cong 41{,}75$

$I_{y_G} = I_y - A x_G^2$ $\qquad$ $I_y^{(TOT)} = 2 I_y^{(1)} + I_y^{(2)}$



$$I_y^{(1)} = \frac{h_1 \cdot b_1^3}{3} = \frac{2a \cdot 64a^3}{3} = \frac{128a^4}{3} \cong 42,666\,a^4$$

$$I_y^{(2)} = \frac{h_2 \cdot b_2^3}{3} = \frac{a \cdot 125a^3}{3} = \frac{125a^4}{3} \cong 41,666\,a^4$$

$$I_y^{(TOT)} = 2 \cdot \frac{128}{3}a^4 + \frac{125}{3}a^4$$

$$I_y^{(TOT)} = \frac{381}{3}a^4 = 127a^4$$

$$I_{y_G} = 127a^4 - \cancel{21}^{1}a^2\left(\frac{89a}{\cancel{42}_{2}}\right)^2$$

$$= 127a^4 - \cancel{21}^{1}a^2\left(\frac{7921}{\cancel{1764}_{84}}a^2\right)$$

$$= 127a^4 - \frac{7921a^4}{84}$$

$$= \frac{10668 - 7921}{84}a^4$$

$$I_{y_G} = \frac{2747}{84}a^4 \cong 32,702\,a^4$$

MOMENTO CENTRIFUGO

$I_{x_G y_G} = 0 \Rightarrow x_G$ = ASSE DI SIMMETRIA

$$tg\,2\vartheta = \frac{-2 I_{x_G y_G}}{I_{x_G} - I_{y_G}} = 0 \qquad tg\,2\vartheta = 0 \qquad I_{x_G} > I_{y_G} \Rightarrow \vartheta = 0$$

$$I_\xi = I_{max} = I_{x_G} = \frac{167}{4}a^4 \qquad I_\eta = I_{min} = I_{y_G} = \frac{2747}{84}a^4$$