Quantities of interest of Markovian Queueing Systems

queue type	M/M/1	M/M/m	M/M/∞	M/M/1/K
state space	$\mathbb{N}$	$\mathbb{N}$	$\mathbb{N}$	$\{0,1,\cdots K\}$
erogodicity conditions	$\varrho=\frac{\lambda}{\mu}<1$	$\varrho=\frac{\lambda}{m\mu}<1$	$\forall\varrho=\frac{\lambda}{\mu}$	$\forall\varrho=\frac{\lambda}{\mu}$
arrivals rate λ_i	$\lambda\ \ (\forall i)$	$\lambda\ \ (\forall i)$	$\lambda\ \ (\forall i)$	$\begin{cases}\lambda & (i<K)\\ 0 & (i\geq K)\end{cases}$
service rate μ_i	$\mu\ \ (\forall i)$	$\begin{cases}i\ \mu & (i<m)\\ m\mu & (i\geq m)\end{cases}$	$i\ \mu\ \ (\forall i)$	$\mu\ \ (\forall i)$
arrival rate within the system $\lambda_{\text{in}}=\sum_{i=0}^{\infty}\lambda_i\pi_i(\infty)$	λ	λ	λ	$\lambda\frac{1-\varrho^K}{1-\varrho^{K+1}}$
abandonement rate $\lambda_{\text{ab}}=\lambda-\lambda_{\text{in}}$	0	0	0	$\lambda\frac{\varrho^K(1-\varrho)}{1-\varrho^{K+1}}$
throughput (productivity) $\eta=\lambda_{\text{in}}$	λ	λ	λ	$\lambda\frac{1-\varrho^K}{1-\varrho^{K+1}}$
steady-state probability $\pi_i(\infty)=\Pr(x_i(\infty)=i)$	$(1-\varrho)\varrho^i\ \ (\forall i)$	$(*)$	$\frac{\varrho^i}{i!}e^{-\varrho}\ \ (\forall i)$	$\begin{cases}\varrho^i\frac{1-\varrho}{1-\varrho^{K+1}} & (i\leq K)\\ 0 & (i>K)\end{cases}$
utilization factor $\hat{U}=1-\pi_0$	ϱ	$1-\pi_0$	$1-e^{-\varrho}$	$\varrho\frac{1-\varrho^K}{1-\varrho^{K+1}}$
average number of costumer in the system $\overline{x}=\sum_{i=0}^{\infty}i\pi_i$	$\frac{\varrho}{1-\varrho}$	$m\varrho+\frac{m^m\varrho^{m+1}}{m!(1-\varrho)^2}\pi_0$	ϱ	$\frac{\varrho(1-(K+1)\varrho^K+K\varrho^{K+1})}{(1-\varrho^{K+1})\ (1-\varrho)}$
average time spent in the system $\overline{\vartheta}=\overline{x}/\lambda_{\text{in}}$	$\frac{1}{\mu\ (1-\varrho)}$	$\frac{\overline{x}}{\lambda}$	$\frac{1}{\mu}$	$\frac{(1-(K+1)\varrho^K+K\varrho^{K+1})}{\mu\ (1-\varrho^K)\ (1-\varrho)}$
average service time $\overline{\vartheta}_s=\frac{1}{\mu}$	$\frac{1}{\mu}$	$\frac{1}{\mu}$	$\frac{1}{\mu}$	$\frac{1}{\mu}$
average number of costumers in the buffer $\overline{x}_b=\overline{\vartheta}_b\lambda_{\text{in}}$	$\frac{\varrho^2}{1-\varrho}$	$\overline{x}-m\varrho$	0	$\frac{\varrho^2(1-K\varrho^{K-1}+(K-1)\varrho^K)}{(1-\varrho^{K+1})\ (1-\varrho)}$
average waiting time in the buffer $\overline{\vartheta}_b=\overline{\vartheta}-\overline{\vartheta}_s$	$\frac{\varrho}{\mu\ (1-\varrho)}$	$\frac{\overline{x}}{\lambda}-\frac{1}{\mu}$	0	$\frac{\varrho(1-K\varrho^{K-1}+(K-1)\varrho^K)}{\mu\ (1-\varrho^K)\ (1-\varrho)}$
average number of of busy servers $\overline{x}_s=\overline{x}-\overline{x}_b$	ϱ	$m\varrho$	ϱ	$\varrho\frac{1-\varrho^K}{1-\varrho^{K+1}}$
utilization factor of a single server $\tilde{\varrho}=\frac{\overline{x}_s}{m}$	ϱ	ϱ	0	$\varrho\frac{1-\varrho^K}{1-\varrho^{K+1}}$

(*) State probabilites of a M/M/m queue system:

$$\begin{cases}\pi_0=\dfrac{1}{\left(\displaystyle\sum_{i=0}^{m-1}\frac{m^i\varrho^i}{i!}\right)+\dfrac{m^m\varrho^m}{m!(1-\varrho)}}\\[2ex] \pi_i=\dfrac{m^i\varrho^i}{i!}\pi_0 & (i<m)\\[2ex] \pi_i=\dfrac{m^m}{m!}\varrho^i\pi_0 & (i\geq m)\end{cases}$$