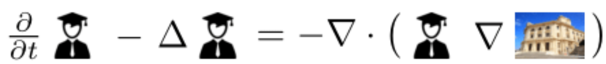
THE 8TH INTERNATIONAL WORKSHOP ON MATHEMATICAL ANALYSIS OF CHEMOTAXIS (iWMAC8)

Cagliari, May 12–16, 2025



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1 Introduction and acknowledgments

We are honored and delighted to host *The 8th International Workshop on Mathematical Analysis of Chemotaxis* (iWMAC8) at the University of Cagliari, Italy. This workshop represents an important opportunity to convene scholars and researchers from around the world who are engaged in the study of chemotaxis, particularly from the perspective of mathematical analysis.

Our focus lies in uniting experts in the field of Analysis, as well as specialists working on the application of Partial Differential Equations (PDEs) and related analytical methods to problems arising in mathematical biology. The primary objective of this event is to foster scientific exchange by providing a collaborative environment in which participants can present their latest research findings, engage in in-depth discussions, and explore open questions that remain at the frontier of the discipline.

The iWMAC series has historically served as a vital platform for advancing the theoretical understanding of chemotaxis and its applications, and we are confident that this year's edition will continue in that spirit.

This meeting has been made possible largely due to the generous availability and commitment of all the invited speakers, whose contributions form the scientific core of the workshop. We are sincerely grateful for their willingness to share their expertise and for their active participation in this event.

We would also like to extend our warm thanks to all our colleagues who, directly or indirectly, contributed their time and effort to the organization and success of iWMAC8. A special expression of gratitude is due to Professor Tomomi Yokota and Professor Michael Winkler, whose scientific guidance and ongoing support have been instrumental in shaping the identity and continuity of this workshop series. Their dedication to the advancement of mathematical biology and to the chemotaxis community has been a source of inspiration and a cornerstone of this initiative.

We are equally indebted to the *Fondazione Banco di Sardegna* and the Department of Mathematics and Computer Science (*Dipartimento di Matematica e Informatica*) at the University of Cagliari. Their financial and institutional support has been indispensable in bringing this event to fruition. Without their generous sponsorship, the organization of this conference would not have been possible.

With these acknowledgments, we warmly welcome all participants to Cagliari and wish everyone a stimulating and fruitful workshop.

Cagliari, May 12, 2025

The Scientific Committee

Sachiko Ishida (*Chiba University*)
Johannes Lankeit (*Leibniz University Hannover*)
Monica Marras (*University of Cagliari*)
Giuseppe Viglialoro (*University of Cagliari*)
Silvia Frassu (*University of Cagliari*)

2 INVITED SPEAKERS

1. JAEWOOK AHN (DONGGUK UNIVERSITY)
2. PIOTR BILER (UNIVERSITY OF WROCŁAW)
3. TOBIAS BLACK (PADERBORN UNIVERSITY)
4. XINRU CAO (DONGHUA UNIVERSITY)
5. YUTARO CHIYO (TOKYO UNIVERSITY OF SCIENCE)
6. ALESSANDRO COLUMBU (UNIVERSITY OF CAGLIARI)
7. RAFAEL DÍAZ FUENTES (UNIVERSITY OF CAGLIARI)
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20. POONAM RANI (INDIAN INSTITUTE OF TECHNOLOGY GANDHINAGAR)
21. TAKASI SENBA (KANAGAWA UNIVERSITY)
22. YUYA TANAKA (KWANSEI GAKUIN UNIVERSITY)
23. STELLA VERNIER-PIRO (UNIVERSITY OF CAGLIARI)
24. DUAN WU (PADERBORN UNIVERSITY)

3 Abstracts of the contributions

Boundedness in a 2D chemotaxis-Navier-Stokes system with tensor-valued sensitivity

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Abstract

Boundedness of smooth solutions to a two-dimensional chemotaxis-consumption system with tensor-valued sensitivity has been established by A.-Kang-Lee [1]. In this talk, we extend this result to the case where the chemotaxis system is coupled with the incompressible Navier-Stokes equations. The main challenge stems from the presence of the convection term $u \cdot \nabla c$ in the chemical concentration equation, which significantly complicates the analysis. To overcome this difficulty, we employ a Trudinger-Moser type inequality and derive a series of spatially localized estimates.

References

[1] J. Ahn, K. Kang and J. Lee, *Regular solutions of chemotaxis-consumption systems involving tensor-valued sensitivities and Robin type boundary conditions*, Math. Models Methods Appl. Sci. **33** (2023), pp. 2337–2360.

Radial solutions of the minimal chemotaxis model in $\mathbb{R}^d$

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Abstract

We discuss existence of radially symmetric solutions (evolution and self-similar cases) of the minimal Keller-Segel system in $\mathbb{R}^d$:

$$u_t - \Delta u + \nabla \cdot (u \nabla \varphi) = 0,$$

$$\Delta \varphi + u = 0,$$

under optimal assumptions on the initial data $u_0 = u(0,.)$. We are interested, in particular, in minimal regularity assumptions imposed on the initial data in order to a local-in-time solution exists, and size conditions for (approximate) dichotomy: global-in-time existence versus finite time blowup of solutions. The results are essentially those reported in [1] and [2].

References

[1] P. Biler, *Singularities of solutions to chemotaxis systems*, De Gruyter Series in Mathematics and Life Sciences **6**, Walter de Gruyter, Berlin, 2020.

[2] P. Biler, G. Karch and H. Wakui, *Large self-similar solutions of the parabolic-elliptic Keller–Segel model*, Indiana Univ. Math. J. **72** (2023), pp. 1027–1054.

Dead-core behavior in degenerate chemotaxis systems

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Abstract

In an earlier work we considered whether dead-cores, i.e. zero-density zones, can appear in a bacterial population whose evolution is governed by a chemotaxis consumption system with degenerate diffusion of the form

$$\begin{cases} u_t = \nabla \cdot \big(D(u)\nabla u - uS(u)\nabla v\big) + f(u,v), \\ v_t = \Delta v - uv. \end{cases}$$

It was shown in [1], that in the initial-boundary value problem with Neumann data a dead-core formation cannot occur when the initial-datum for the bacterial population u is strictly positive throughout Ω. Now, we extend our findings to a setting, where the initial-datum for u is only assumed to be non-negative. We show that dead-cores are unable to appear suddenly inside the interior of the support of the bacterial population, but rather have to expand from the zero-density zones of the initial datum ([2]). This result not only applies to the mentioned consumption setting above, but also to the standard Keller–Segel model obtained by replacing $-uv$ with $-v+u$ in the second equation.

References

[1] T. Black, *Absence of dead-core formations in chemotaxis systems with degenerate diffusion*, Appl. Math. Lett. **161** (2025).

[2] T. Black, *Avoidance of caldera-type deadcores in a chemotaxis system with degenerate diffusion and compactly supported initial population density*, preprint.

Critical mass in quasilinear Keller–Segel systems

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Abstract

In this talk, we will discuss critical mass in quasilinear Keller-Segel model with critical exponent. We show that if the total mass of the cells is small, the system admits a global bounded solution; if the total mass is sufficiently large, we can find initial data such that the corresponding solution must be unbounded.

References

[1] X. Cao and X. Gao, *Critical mass in a quasilinear parabolic-elliptic Keller-Segel model*, J. Differential Equations **361** (2023), pp. 449–471.

[2] X. Cao and X. Zhao, *Small mass solution in a three-dimensional quasilinear Keller-Segel-Stokes system with critical exponent*, preprint (2024).

Can repulsive effect lead to boundedness in a one-dimensional quasilinear chemotaxis system even with flux limitation?

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Abstract

This talk is based on a joint work [1].

We consider the Neumann initial-boundary value problem for the one-dimensional quasilinear attraction-repulsion chemotaxis system with flux limitation,

$$\begin{cases} u_t = \left((u+1)^{m-1} u_x - \dfrac{\chi u (u+1)^{p-2}}{(1+|v_x|^2)^k} v_x + \dfrac{\xi u (u+1)^{q-2}}{(1+|w_x|^2)^\ell} w_x \right)_x & \text{in } \Omega \times (0,\infty), \\ 0 = v_{xx} + \alpha u - \beta v & \text{in } \Omega \times (0,\infty), \\ 0 = w_{xx} + \gamma u - \delta w & \text{in } \Omega \times (0,\infty), \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}$ is a bounded open interval, and $m, p, q \in \mathbb{R}$, $k, \ell \geq 0$, $\chi, \xi, \alpha, \beta, \gamma, \delta > 0$ are constants.

Our main results read as follows.

Theorem 1 (The case $p < q$)

Assume that $p < q$ and that $k \geq 0$ as well as $0 \leq \ell < \frac{1}{2}$. Suppose that $u_0 \in C(\overline{\Omega}) \setminus \{0\}$ is nonnegative and satisfies

$$\begin{cases} \|u_0\|_{L^1(\Omega)} < \infty & \text{if } \ell = 0, \\ \|u_0\|_{L^1(\Omega)} < \dfrac{1}{\gamma} \sqrt{\left(\dfrac{1}{2\ell}\right)^{\frac{1}{\ell}} - 1} & \text{if } 0 < \ell < \dfrac{1}{2}. \end{cases} \tag{2}$$

Then there exists a unique global classical solution of (1). Moreover, u is bounded in the sense that

$$\sup_{t>0} \|u(\cdot,t)\|_{L^\infty(\Omega)} < \infty.$$

Theorem 2 (The case $p = q$)

Assume that $p = q$ and that $k \geq 0$ as well as $0 \leq \ell < \frac{1}{2}$. Suppose that $u_0 \in C(\overline{\Omega}) \setminus \{0\}$ is nonnegative and satisfies (2) and

$$\chi\alpha - \xi\gamma \left\{ \left(1 + \gamma^2 \|u_0\|_{L^1(\Omega)}^2\right)^{-\ell} - 2\ell \right\} < 0. \tag{3}$$

Then the same conclusion as in Theorem 1 holds.

References

[1] Y. Chiyo, K. Hasegawa, S. Kohatsu and T. Yokota, *Boundedness in a one-dimensional quasilinear attraction-repulsion chemotaxis system with flux limitation*, submitted.

Dampening gradient terms in a consumption model

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Abstract

In this talk, we will discuss this consumption chemotaxis problem

$$\begin{cases} u_t = \Delta u - \chi \nabla \cdot u \nabla v + \lambda u - \mu u^2 - c|\nabla u|^\gamma, & \text{in } \Omega \times (0, T_{\max}), \\ v_t = \Delta v - uv, & \text{in } \Omega \times (0, T_{\max}), \end{cases}$$

in a bounded and smooth domain $\Omega \subset \mathbb{R}^n$, $n \geq 3$, under homogeneous Neumann boundary conditions, for $\chi, \lambda, \mu, c > 0$, $T_{\max} \in (0, \infty]$ and for u_0, v_0 positive initial data with a certain regularity. By taking inspiration from [1] and [2], we will show that the problem has a unique and uniformly bounded classical solution for $\gamma \in \left(\frac{2n}{n+1}, 2\right]$. Moreover, we have the same result for $\gamma = \frac{2n}{n+1}$ and a condition that involves the parameters c, μ, n, χ and the initial data.

References

[1] T. Li, D. Acosta Soba, A. Columbu, and G. Viglialoro, *Dissipative gradient nonlinearities prevent δ-formations in local and nonlocal attraction-repulsion chemotaxis models*, Stud. Appl. Math. **154** (2025).

[2] J. Lankeit and Y. Wang, *Global existence, boundedness and stabilization in a high-dimensional chemotaxis system with consumption*, Discrete Contin. Dyn. Syst. **37** (2017), pp. 6099–6121.

Dissipation Through Combinations of Nonlocal and Gradient Nonlinearities in Chemotaxis Models

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Abstract

In this presentation, we introduce a class of chemotaxis systems in which external sources influence the motion of a cell density attracted by a chemical signal [3]. The studied source terms comprise two scenarios: nonlocal and gradient-dependent damping reactions. For each case, we provide sufficient conditions on the parameters of the models to ensure that any nonnegative classical solution to the studied system is global and uniformly bounded in time. The introduction of dissipative effects, as expressed in both proposed source terms, constitutes the main novelty of this investigation. Most of the results in the literature on chemotaxis models with external sources deal with classical logistics. In this sense, we extend those works and others in which similar sources are examined [1, 2]. This is a joint research with Silvia Frassu and Giuseppe Viglialoro.

References

[1] S. Bian, L. Chen and E. A. Latos, *Nonlocal nonlinear reaction preventing blow-up in supercritical case of chemotaxis system*, Nonlinear Analysis, **176** (2018), pp. 178–191.

[2] Y. Chiyo, F.G. Düzgün, S. Frassu and G. Viglialoro, *Boundedness Through Nonlocal Dampening Effects in a Fully Parabolic Chemotaxis Model with Sub and Superquadratic Growth*. Appl. Math. Optim. **89** (2024).

[3] R. Díaz Fuentes, S. Frassu and G. Viglialoro, *Dissipation Through Combinations of Nonlocal and Gradient Nonlinearities in Chemotaxis Models*. Acta Appl. Math. **195** (2025).

A study of the bifurcation theory for Keller–Segel systems

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Abstract

We investigate the stationary system associated with the classical Keller–Segel model on a cylindrical domain. Using the Crandall–Rabinowitz bifurcation theorem, we establish the existence of non-constant solutions for this system and extend these results to other systems. Furthermore, by analyzing the spectral properties, we discuss criteria for stability and instability of the corresponding linearized system.

Formation of Dirac singularities in the parabolic-elliptic Keller-Segel system

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Abstract

We consider radially symmetric solutions to the parabolic-elliptic simplified Keller-Segel system of the form

$$\begin{cases} u_t = \Delta u - \nabla \cdot \big(u \nabla v\big), \\ 0 = \Delta v + u - \mu, \qquad \mu = \frac{1}{|\Omega|}\int_\Omega u_0, \end{cases} \tag{1}$$

with suitably regular initial data u_0, subjected to homogeneous Neumann boundary conditions and posed in the domain $\Omega = B_R(0) \subset \mathbb{R}^n$ for some $R > 0$ and $n \geq 3$.
In two dimensions, it is well-known that solutions blowing up in finite time converge to a Dirac profile in the vague topology. In contrast, in dimensions greater or equal to three, blow-up solutions with finite existence time T exhibit no such behavior; in particular, Souplet and Winkler ([2]) showed that for radially decreasing initial data, the blow-up profile $u(\cdot,T)$, which is attained as the $L^1(B_R(0))$-limit of $u(\cdot,t)$ as $t \nearrow T$, is bounded via

$$u(x,T) \leq C|x|^{-2}$$

with $0 < |x| \leq R$ and some $C > 0$.
Examining the system

$$\begin{cases} w_t = n^2 s^{2-\frac{2}{n}} w_{ss} + nww_s - \mu s w_s, & s \in (0,R^n), t > 0, \\ w(R^n,t) = \frac{\mu R^n}{n}, & t > 0, \end{cases}$$

emerging from a transformation of (1), specifically $w(s,t) = \int_0^{s^{\frac{1}{n}}} \rho^{n-1} u(\rho,t) d\rho$, we may trace the evolution of solutions further than the blow-up time of (1). We shall establish that for certain initial data, there exists $t > 0$ such that $w(0,t) > 0$ ([1]), effectively corresponding to the formation of a Dirac mass at the origin in (1).

References

[1] G. Flüchter, *Formation of Dirac singularities in the parabolic-elliptic Keller-Segel system*, Preprint.

[2] P. Souplet and M. Winkler, *Blow-up Profiles for the Parabolic-Elliptic Keller-Segel System*, Comm. Math. Phys. **367** (2019), pp. 665–681.

Shrinking vs. expanding: the evolution of spatial support in degenerate Keller–Segel systems

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Abstract

We consider radially symmetric solutions to a degenerate parabolic–elliptic Keller–Segel system in bounded balls with initial data having compact support. Our main result shows that the initial evolution of the positivity set is essentially completely determined by the flatness/steepness of the initial data near a boundary point x_0 of the support. If they are sufficiently flat (respectively, steep), the support shrinks (respectively, expands) near x_0. We give concrete conditions for both behaviors and in particular show that there is a critical exponent and a critical parameter distinguishing between these cases.

This talk is based on a joint work with Frederic Heihoff.

References

[1] M. Fuest and F. Heihoff, *Shrinking vs. expanding: the evolution of spatial support in degenerate Keller–Segel systems*, Preprint (2025), arXiv:2501.19119.

Refined asymptotics near blow-up points for the planar Keller–Segel system

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Abstract

For the Keller–Segel system

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla v), \\ v_t = \Delta v - v + u, \end{cases} \tag{$\star$}$$

posed in a planar domain Ω with Neumann boundary conditions, the existence of classical solutions blowing up at some finite time T has long been established. In fact, it has been shown that for every blow-up point x the quantity $\int_{B_R(x)\cap\Omega} u(\cdot,t)\ln u(\cdot,t)$ is unbounded as $t \nearrow T$ for all $R > 0$ even though the global mass of u is always conserved ([3]). Since then, refining these still somewhat coarse blow-up asymptoptics has proven rather challenging. Indeed, successful efforts in this direction thus far seem to be restricted to tailor-made solutions to ($\star$) with an explicitly constructed blow-up mechanism ([2]). Using the concrete shape of these solutions, the blow-up asymptotics can then be more precisely quantified for this specific case as follows:

$$\limsup_{t \nearrow T} \frac{1}{\ln \frac{T}{T-t}} \int_{B_R(x)\cap\Omega} u(\cdot,t)\ln u(\cdot,t) > \delta_0 \tag{$\star\star$}$$

for all $R > 0$ with some $\delta_0 > 0$.

Expanding upon the knowledge in this area, we show that the blow-up asymptotics expressed in ($\star\star$) (as well as appropriate counterparts for L^p norms) not only hold true for the aforementioned specific solutions but can be generalized to all solutions of ($\star$) blowing up in finite time with the constant δ_0 depending only on the domain Ω ([1]).

References

[1] F. Heihoff and M. Winkler, *Refined asymptotics near blow-up points for the planar Keller–Segel system*, preprint.

[2] M. A. Herrero and J. J. L. Velázquez, *A blow-up mechanism for a chemotaxis model,* Ann. Scu. Norm. Sup. Pisa Cl. Sci. **24** (1997), pp. 633–683.

[3] T. Nagai, T. Senba and T. Suzuki, *Chemotactic collapse in a parabolic system of mathematical biology,* Hiroshima Math. J. **30** (2000), pp. 463–497.

Global existence for a tumor invasion model with small initial data

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Abstract

We consider the following initial-boundary value problem:

$$\begin{cases} u_t = \Delta u - \nabla \cdot (S(u)\nabla v), & x \in \Omega,\ t > 0, \\ v_t = \Delta v + wz, & x \in \Omega,\ t > 0, \\ w_t = -wz, & x \in \Omega,\ t > 0, \\ z_t = \Delta z - z + u, & x \in \Omega,\ t > 0, \\ (\nabla u - S(u)\nabla v) \cdot \nu = \nabla v \cdot \nu = \nabla z \cdot \nu = 0, & x \in \partial\Omega,\ t > 0, \\ (u, v, w, z)(x, 0) = (u_0, v_0, w_0, z_0)(x), & x \in \Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ be a bounded domain with smooth boundary $\partial\Omega$. Here, S is assumed to fulfill $0 \leq S(\sigma) \leq \sigma^\alpha$ $(\forall \sigma \geq 0)$ with some $\alpha \geq 0$, and the initial data satisfies $u_0, v_0, w_0, z_0 \geq 0$ and

$$(u_0, v_0, w_0, z_0) \in L^\infty(\Omega) \times W^{1,\infty}(\Omega) \times W^{1,\infty}(\Omega) \times W^{1,\infty}(\Omega).$$

The model (1) was proposed by [1] as a modified tumor invasion model with chemotaxis effect. As known results, the papers [2, 3] established global existence and boundedness of solutions to (1) under the condition $\alpha < \frac{4}{N}$. In this talk, we consider the case where $\frac{4}{N} \leq \alpha < \frac{4}{3}$ and show that if

$$\|u_0\|_{L^{\frac{\alpha N}{4}}(\Omega)}, \|z_0\|_{L^{\frac{\alpha N}{4-2\alpha}}(\Omega)} \text{ and } \|\nabla v_0\|_{L^{\frac{\alpha N}{4-3\alpha}}(\Omega)} \text{ are sufficiently small}$$

(depending on $\|w_0\|_{L^\infty(\Omega)}$), then there exists a global bounded solution of (1).

References

[1] K. Fujie, A. Ito and T. Yokota, *Existence and uniqueness of local classical solutions to modified tumor invasion models of Chaplain–Anderson type*, Adv. Math. Sci. Appl. **24** (2014), pp. 67–84.

[2] S. Ishida and T. Yokota, *Boundedness and weak stabilization in a degenerate chemotaxis model arising from tumor invasion*, J. Differential Equations, **371** (2023), pp. 450–480.

[3] H.Y. Jin, Z. Liu and S. Shi, *Global dynamics of a quasilinear chemotaxis model arising from tumor invasion*, Nonlinear Anal. Real World Appl. **44** (2018), pp. 18–39.

Global dynamics for a population model with repulsive chemotaxis

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Abstract

In this talk, we are concerned with the spatiotemporal population model with repulsive chemotaxis in a bounded domain. Based on the energy estimates, we first establish the global existence of classical solution. Moreover, depending on the input rate function, we also show the global stabilization of constant steady state, non-constant positive steady states and periodic solution.

Global Classical Solutions for a 3D Axisymmetric Chemotaxis-Fluid System Without Swirl

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Abstract

We study a chemotaxis-consumption system coupled with the 3D axisymmetric incompressible Navier–Stokes equations without swirl. The chemotactic sensitivity S is assumed to be axisymmetric and satisfies a decay condition of the form $|S(x,\rho,c)| \leq \chi(1+\rho)^{-\alpha}$ with some $\alpha > 0$. Under suitable compatibility conditions on the initial data, we establish the global existence of a classical solution to the system.

On steady states of certain chemotaxis-consumption system with an inflow of a nutrient

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Abstract

During the talk I will discuss recent results concerning existence and uniqueness of solutions (U,V) of the following system of equations

$$\begin{cases} 0 = \nabla \cdot (\nabla U - U\nabla V) & x \in \Omega \\ \partial_\nu U - U\partial_\nu V = 0 & x \in \partial\Omega \\ 0 = \Delta V - UV & x \in \Omega \\ \partial_\nu V = \bar{v} & x \in \partial\Omega \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^n$ and $\bar{v}$ is a given positive constant. The results for (1) were obtained jointly with Frederic Heihoff, [1].

References

[1] F. Heihoff and P. Knosalla, *A chemotaxis-consumption model with boundary inflow of a nutrient, steady states*, Preprint, 10 p.

Properties of solutions to flux-limited Keller–Segel systems with critical and supercritical exponents

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Abstract

This talk consists of two parts:

Part 1: Forward self-similar solutions and steady states

In this part we consider the flux-limited Keller–Segel system

$$\begin{cases} u_t = \Delta u - \nabla \cdot \left(\dfrac{u}{|\nabla v|^\alpha} \nabla v \right) & \text{in } \mathbb{R}^n \times (0,\infty), \\ 0 = \Delta v + u & \text{in } \mathbb{R}^n \times (0,\infty), \end{cases} \tag{1}$$

where $n \in \mathbb{N}$ and $\alpha \in \mathbb{R}$. The solution (u,v) of the system (1) is called a *forward self-similar solution* if there exist functions φ, ψ on $\mathbb{R}^n$ fulfilling

$$u(x,t) = \frac{1}{t^{\frac{2-\alpha}{2(1-\alpha)}}} \varphi\left(\frac{x}{\sqrt{t}}\right), \quad v(x,t) = \frac{1}{t^{\frac{\alpha}{2(1-\alpha)}}} \psi\left(\frac{x}{\sqrt{t}}\right) \quad (x \in \mathbb{R}^n,\ t > 0).$$

Our main results read as follows.

Theorem 1 (cf. [2, Theorems 1.1 and 1.2])

(i) Let $n \geq 3$ and $\alpha = \frac{n-2}{n-1}$. Then for any $m \ll 1$ there exists a radial forward self-similar solution $(u,v) \in \left(C^\infty(\mathbb{R}^n \times (0,\infty))\right)^2$ such that

$$u(\cdot,t) \overset{\star}{\rightharpoonup} m\delta_0 \quad \text{in } \mathscr{M}(\mathbb{R}^n) \quad \text{as } t \searrow 0,$$

where $\mathscr{M}(\mathbb{R}^n)$ is the space of Radon measures on $\mathbb{R}^n$.

(ii) Let $n \geq 2$ and $\alpha < \frac{n-2}{n-1}$. Then there exist $\gamma > 0$ and a radial forward self-similar solution $(u,v) \in \left(C^\infty(\mathbb{R}^n \times (0,\infty))\right)^2$ such that

$$u(\cdot,t) \overset{\star}{\rightharpoonup} \gamma |x|^{-\frac{2-\alpha}{1-\alpha}} \quad \text{in } \mathscr{M}(\mathbb{R}^n) \quad \text{as } t \searrow 0.$$

Theorem 2 (cf. [2, Theorem 1.3])

(i) Let $n \geq 3$ and $\alpha = \frac{n-2}{n-1}$. Then radial steady states of (1) are represented as

$$u(x) = u_\lambda(x) := \frac{n^{2n-1}}{(n-1)^{n-1}} \cdot \frac{\lambda^n}{(1+|\lambda x|^{\frac{n}{n-1}})^n} \qquad (x \in \mathbb{R}^n),$$

$$v(x) = v_\lambda(x) := D - \int_0^{|x|} \frac{1}{r^{n-1}} \int_{|y|<r} u_\lambda(y)\, \mathrm{d}y \mathrm{d}r \qquad (x \in \mathbb{R}^n),$$

where $\lambda > 0$ and $D \in \mathbb{R}$.

(ii) Let $n \geq 3$ and $\alpha \in [-1, \frac{2n-5-\sqrt{8n+1}}{2(n-1)}]$. Then the radial steady state (u,v) of (1) satisfies

$$\lim_{x\to\infty} \left(|x|^{\frac{2-\alpha}{1-\alpha}} u(x)\right) = \left(n - \frac{2-\alpha}{1-\alpha}\right) \left(\frac{2-\alpha}{1-\alpha}\right)^{\frac{1}{1-\alpha}}.$$

Remark. Theorem 1 implies that for suitable rough initial data the system still admits global solutions which immediately become smooth, noting that such a smoothing property for the fully parabolic variant was recently established when $\alpha > \frac{n-2}{n-1}$ ([1]).

Part 2: Critical mass and stability of steady states

In this part we consider the flux-limited Keller–Segel system with critical exponent,

$$\begin{cases} u_t = \Delta u - \nabla \cdot \left(\dfrac{u}{|\nabla v|^{\frac{n-2}{n-1}}} \nabla v \right), & x \in \mathbb{R}^n, \quad t > 0, \\ 0 = \Delta v + u, & x \in \mathbb{R}^n, \quad t > 0, \\ u(x,0) = u_0(x), & x \in \mathbb{R}^n, \end{cases} \tag{1}_{\mathrm{c}}$$

where $n \geq 3$, and where u_0 is a nonnegative and radial function on $\mathbb{R}^n$.

The first result establishes the exact critical mass.

Theorem 3

Let $n \geq 3$, and assume that u_0 is nonnegative, radial, Hölder continuous and

$$u_0 \in L^1(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n), \quad u_0(0) > 0, \quad \sup_{x \in \mathbb{R}^n} (1 + |x|^n) u_0(x) < \infty.$$

Then there exist $T_{\max} \in (0, \infty]$ and a unique radial mild solution (u, v) of $(1)_{\mathrm{c}}$ in $\mathbb{R}^n \times (0, T_{\max})$. Moreover,

(i) if $\|u_0\|_{L^1} < \left(\dfrac{n^2}{n-1} \right)^{n-1} \dfrac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}$, then $T_{\max} = \infty$ and $\sup_{t>0} \|u(\cdot,t)\|_{L^\infty} < \infty$;

(ii) if $\|u_0\|_{L^1} > \left(\dfrac{n^2}{n-1} \right)^{n-1} \dfrac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}$, then $T_{\max} < \infty$ and $\limsup_{t \nearrow T_{\max}} \|u(\cdot,t)\|_{L^\infty} = \infty$.

Our second result is concerned with stability of the steady states u_λ obtained in Theorem 2(i).

Theorem 4

Let $n \geq 3$, and assume that u_0 is nonnegative, radial, Hölder continuous,

$$u_0 \in L^1(\mathbb{R}^n) \cap L^\infty(\mathbb{R}^n), \quad u_0(0) > 0, \quad \sup_{x \in \mathbb{R}^n} (1 + |x|^n) u_0(x) < \infty,$$

$$\|u_0\|_{L^1} = \left(\frac{n^2}{n-1} \right)^{n-1} \frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}, \quad \int_{\mathbb{R}^n} (1 + |x|^2) |u_0(x) - u_\lambda(x)| \, \mathrm{d}x < \infty$$

with some $\lambda > 0$. Then the corresponding radial mild solution (u, v) of the system $(1)_{\mathrm{c}}$ satisfies

$$\lim_{t \to \infty} \|u(\cdot,t) - u_\lambda\|_{L^\infty} = 0.$$

Remark. If $n = 2$, then $\left(\dfrac{n^2}{n-1} \right)^{n-1} \dfrac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})} = 8\pi$.

To prove Theorem 4, we construct a new Lyapunov functional for the flux-limited Keller–Segel system $(1)_{\mathrm{c}}$ in a radially symmetric setting.

This is a joint work with Professor Takasi Senba (Kanagawa University, Japan).

References

[1] S. Kohatsu, *Instantaneous regularization of measure-valued population densities in a Keller–Segel system with flux limitation*, J. Differential Equations **408** (2024), pp. 201–229.

[2] S. Kohatsu and T. Senba, *Self-similar solutions to a flux-limited Keller–Segel system*, Nonlinear Anal. Real World Appl. **84** (2025), Paper No. 104308, 25 pp.

Updates on chemotaxis–consumption

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Abstract

In this talk, based on a joint work with Michael Winkler, we will consider the prototypical chemotaxis–consumption system and discuss eventual smoothness and stabilization, along with global existence, of weak solutions.
We will, in particular, highlight differences to the proofs of the previously known related theorems.

References

[1] J. Lankeit and M. Winkler, *Chemotaxis-consumption interaction: Solvability and asymptotics in general high-dimensional domains*, preprint.

Analysis of Taxis-Consumption Systems with Signal-Dependent Motilities

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Abstract

In this talk, we present recent advances in the analysis of taxis-consumption systems with signal-dependent motilities. We will explore basic solution theory and examine the qualitative behavior of solutions in both non-degenerate and degenerate motility cases.

References

[1] G. Li and Y. Lou *Roles of density-related diffusion and signal-dependent motilities in a chemotaxis-consumption system*, Calc. Var. Partial Differ. Equ. **63** (2025), 195.

Properties of blow-up points in chemotaxis systems with environmental dependent logistic source

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Abstract

This talk discusses possible points of blow-up in chemotaxis systems with spatially heterogeneous logistic term in two-dimensional smoothly bounded domains under the Neumann boundary conditions and initial conditions. About this problem, properties of possible blow-up points are recently studied; in the parabolic-elliptic setting, it was shown that finite-time blow-up of the classical solution can only occur in points where a coefficient is zero ([1]). In this talk we present a recent development regarding a property of blow-up points in the parabolic-parabolic setting of the system. This talk is based on a joint work with Dr. Mario Fuest (Leibniz University Hannover) and Professor Johannes Lankeit (Leibniz University Hannover).

References

[1] T. Black, M. Fuest, J. Lankeit and M. Mizukami, *Possible points of blow-up in chemotaxis systems with spatially heterogeneous logistic source*, Nonlinear Anal. Real World Appl. **73** (2023), Paper No. 103868, 14 pp.

A Quasilinear Chemotaxis-Haptotaxis System: Existence and Blow-Up Results

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Abstract

We consider the following chemotaxis-haptotaxis system:

$$\begin{cases} u_t = \nabla \cdot (D(u)\nabla u) - \chi \nabla \cdot (S(u)\nabla v) - \xi \nabla \cdot (u \nabla w), & x \in \Omega, t > 0, \\ v_t = \Delta v - v + u, & x \in \Omega, t > 0, \\ w_t = -vw, & x \in \Omega, t > 0, \end{cases}$$

under homogeneous Neumann boundary conditions in a bounded smooth domain $\Omega \subset \mathbb{R}^n, n \geq 3$. It is proved that for $\frac{S(s)}{D(s)} \leq A(s+1)^{\alpha}$ for $\alpha < \frac{2}{n}$ and under suitable growth conditions on D, there exists a uniform-in-time bounded classical solution. Also, we prove that for radial domains, when the opposite inequality holds, the corresponding solutions blow-up in finite or infinite-time. This is a joint work with my supervisor Prof. Jagmohan Tyagi.

References

[1] P. Rani and J. Tyagi, A quasilinear chemotaxis-haptotaxis system: existence and blow-up results, J. Differential Equations **402** (2024), pp. 180–217.

Properties of radial steady states to a flux-limited Keller-Segel system

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Abstract

This talk is based on a joint work with Shohei Kohatsu (Tokyo University of Science).
We consider radial solutions to the system

$$\begin{cases} U_t = \Delta U - \nabla \cdot \left(\dfrac{U}{|\nabla V|^{2\alpha}} \nabla V \right) & \text{in } \mathbf{R}^n \times (0,\infty), \\ 0 = \Delta V + U & \text{in } \mathbf{R}^n \times (0,\infty). \end{cases} \tag{1}$$

Here, α is a given constant.

From mathematical and biological points of view, flux-limited chemotaxis models such as (1) were introduced in the recent modeling literature (see e.g. [1, 2, 3]).

In particular, Winkler [5] considered the system

$$\begin{cases} U_t = \Delta U - \nabla \cdot \left(\dfrac{U}{(1+|\nabla V|^2)^{\alpha}} \nabla V \right), & x \in \Omega,\ t > 0, \\ 0 = \Delta V - \dfrac{1}{|\Omega|} \displaystyle\int_\Omega U + U, & x \in \Omega,\ t > 0, \\ \dfrac{\partial U}{\partial \nu} = \dfrac{\partial V}{\partial \nu} = 0, & x \in \partial\Omega,\ t > 0, \\ U(x,0) = U_0(x), & x \in \Omega, \end{cases} \tag{2}$$

in smoothly bounded domains $\Omega \subset \mathbf{R}^n$ $(n \geq 1)$, and identified a critical exponent $\alpha = (n-2)/(2n-2)$ for time-global existence and blow-up.

In the case where $\alpha = (n-2)/(2n-2)$, relation between steady states and behavior of solutions is becoming clear. Then, we consider that relation in the cases other than the critical case.

References

[1] N. Bellomo, A. Bellouquid, J. Nieto and J. Soler, *Multiscale biological tissue models and flux-limited chemotaxis for multicellular growing systems*, Math. Models Methods Appl. Sci. **20** (2010), pp. 1179–1207.

[2] A. Bianchi, K. J. Painter and J. A. Sherratt, *A mathematical model for lymphangiogenesis in normal and diabetic wounds*, J. Theoret. Biol. **383** (2015), pp. 61–86.

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[4] S. Kohatsu and T. Senba, *Self-similar solutions to a flux-limited Keller–Segel system*, Nonlinear Anal. RWA **84** (2025), 104308.

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Global existence and boundedness in Keller–Segel-type models with positive total flux

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Abstract

This talk is concerned with Keller–Segel-type models with positive total flux. In the majority of chemotaxis systems, it is assumed that the total flux is zero, which implies that a mass conservation property or an L^1 bound for the cell density can be obtained immediately. However, this is not the case when the total flux is positive. In this talk I will present results on L^1 bounds and global boundedness of solutions to Keller–Segel-type models involving strong logistic source with positive total flux. This is a joint work with Khadijeh Baghaei (Pasargad Institute for Advanced Innovative Solutions) and Silvia Frassu and Giuseppe Viglialoro (University of Cagliari).

Hölder continuity of weak solutions to chemotaxis systems

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Abstract

In this talk we consider in $\mathbb{R}^N \times [0,T]$ different classes of chemotaxis systems with porous media type diffusion, a drift term $div(u^{q-1}\nabla v),\ q > 1$, ([3]) and a source term depending on the cell density and their gradient. The Hölder regularity for bounded weak solutions is obtained by using De Giorgi-Di Benedetto approach [1], [2]. The key idea is to construct a family of nested cylinders with the same vertex, such that the essential oscillation ω_n of u in the cylinders is reduced. The results belong to a joint research with M. Marras, F. Ragnedda and V. Vespri [4], [5].

References

[1] E. DiBenedetto, *Degenerate parabolic equations*, Springer-Verlag, New York, 1993.

[2] E. DiBenedetto, U. Gianazza and V. Vespri, *Harnack's inequality for degenerate and singular parabolic equations*, Springer Monographs in Mathematics, Springer, 2012.

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The qualitative analysis to a doubly degenerate chemotaxis-consumption system with logistic source

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Abstract

In this talk, we focus on a doubly degenerate nutrient model

$$\begin{cases} u_t = \nabla \cdot \big(uv\nabla u\big) - \nabla \cdot \big(u^2 v\nabla v\big) + f(u), & x \in \Omega, t > 0, \\ v_t = \Delta v - uv, & x \in \Omega, t > 0, \end{cases}$$

in a smoothly bounded domain $\Omega \subset \mathbb{R}^N$, $N \geq 2$. The function f is assumed to satisfy

$$f(s) \leq \kappa s - \mu s^\gamma, \quad s > 0$$

with $\kappa \geq 0$, $\mu > 0$ and $\gamma > 1$.

Under suitable conditions on γ, we establish the existence of global bounded weak solutions for any arbitrary sufficiently regular initial data. In addition, we also investigate the large time behavior for these solutions.