

1. $\vdash_{ND} (\alpha \rightarrow \beta) \rightarrow ((\alpha \rightarrow \gamma) \rightarrow (\beta \rightarrow (\alpha \rightarrow \gamma)))$
2. $\vdash_{ND} (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow (\beta \rightarrow (\alpha \rightarrow \gamma))$
3. $\vdash_{ND} ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma))$
4. $\vdash_{ND} (\alpha \rightarrow \gamma) \rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma))$
5. $\vdash_{ND} ((\alpha \rightarrow \beta) \rightarrow (\beta \rightarrow \alpha)) \rightarrow (\beta \rightarrow \alpha)$
6. $\vdash_{ND} (\alpha \rightarrow \beta) \rightarrow (\alpha \wedge \gamma \rightarrow \beta \wedge \gamma)$
7. $\vdash_{ND} (\alpha \rightarrow \beta) \rightarrow (\alpha \vee \gamma \rightarrow \beta \vee \gamma)$
8. $\vdash_{ND} (\alpha \rightarrow \beta \wedge \gamma) \rightarrow (\alpha \rightarrow \beta) \wedge (\alpha \rightarrow \gamma)$
9. $\vdash_{ND} (\alpha \rightarrow \gamma) \vee (\beta \rightarrow \gamma) \rightarrow (\alpha \wedge \beta \rightarrow \gamma)$
10. $\vdash_{ND} (\alpha \rightarrow \beta) \vee (\alpha \rightarrow \gamma) \rightarrow (\alpha \rightarrow \beta \vee \gamma)$
11. $\vdash_{ND} \alpha \vee (\alpha \wedge \beta) \rightarrow \alpha$

Gli stessi esercizi possono essere svolti anche come esercizi su PK , relativi a sequenze con antecedente vuoto:

1. $\vdash_{PK} \Rightarrow (\alpha \rightarrow \beta) \rightarrow ((\alpha \rightarrow \gamma) \rightarrow (\beta \rightarrow (\alpha \rightarrow \gamma)))$
2. $\vdash_{PK} \Rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma)) \rightarrow (\beta \rightarrow (\alpha \rightarrow \gamma))$
3. $\vdash_{PK} \Rightarrow ((\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \gamma)) \rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma))$
4. $\vdash_{PK} \Rightarrow (\alpha \rightarrow \gamma) \rightarrow (\alpha \rightarrow (\beta \rightarrow \gamma))$
5. $\vdash_{PK} \Rightarrow ((\alpha \rightarrow \beta) \rightarrow (\beta \rightarrow \alpha)) \rightarrow (\beta \rightarrow \alpha)$
6. $\vdash_{PK} \Rightarrow (\alpha \rightarrow \beta) \rightarrow (\alpha \wedge \gamma \rightarrow \beta \wedge \gamma)$
7. $\vdash_{PK} \Rightarrow (\alpha \rightarrow \beta) \rightarrow (\alpha \vee \gamma \rightarrow \beta \vee \gamma)$
8. $\vdash_{PK} \Rightarrow (\alpha \rightarrow \beta \wedge \gamma) \rightarrow (\alpha \rightarrow \beta) \wedge (\alpha \rightarrow \gamma)$
9. $\vdash_{PK} \Rightarrow (\alpha \rightarrow \gamma) \vee (\beta \rightarrow \gamma) \rightarrow (\alpha \wedge \beta \rightarrow \gamma)$
10. $\vdash_{PK} \Rightarrow (\alpha \rightarrow \beta) \vee (\alpha \rightarrow \gamma) \rightarrow (\alpha \rightarrow \beta \vee \gamma)$
11. $\vdash_{PK} \Rightarrow \alpha \vee (\alpha \wedge \beta) \rightarrow \alpha$