



UNIVERSITY OF CAGLIARI

DIEE - Department of Electrical and Electronic Engineering

STOCHASTIC MODELS (IA/0134/EN)

[MS Teams code: b3dlww4](#)

Alessandro Pilloni

Course	Curriculum	CFU	Length(h)
[70/91] INTERNET ENGINEERING	[91/00 - Ord. 2018] INGEGNERIA DELLE TECNOLOGIE PER INTERNET	5	50
[70/90] COMPUTER ENGINEERING, CYBERSECURITY AND ARTIFICIAL INTELLIGENCE	[90/00 - Ord. 2018] PERCORSO COMUNE	5	50



Lecturer



Alessandro Pilloni

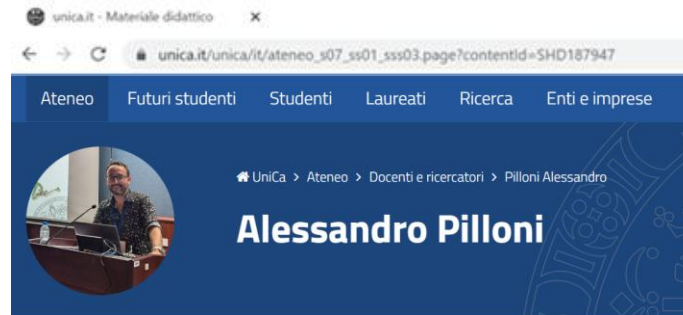
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SSD: ING-INF/04

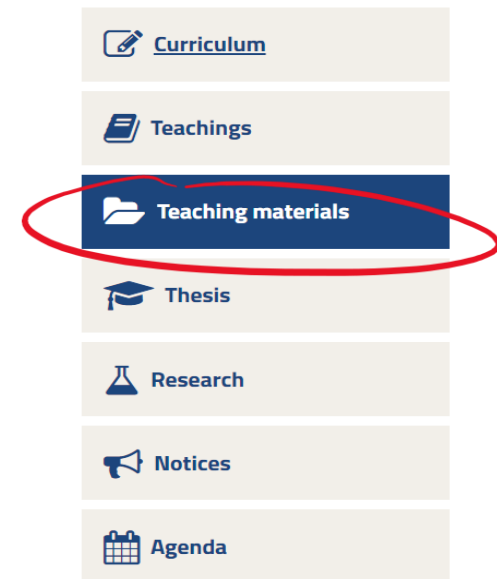
Office: Building M, 3° Floor

Reception: Appointment (Office/MS Teams)



STOCHASTIC MODELS

Tot. 50h



The course aims to...

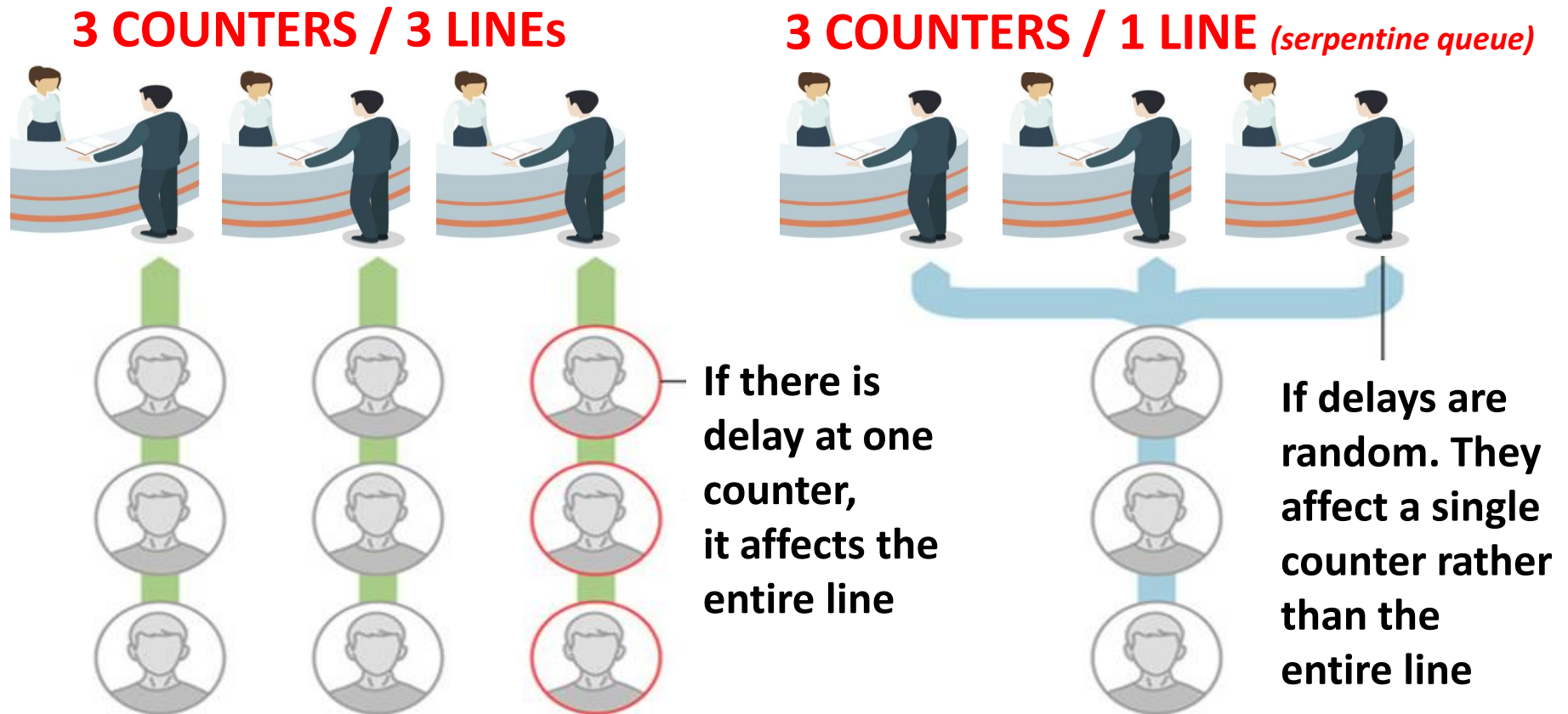
... provide an introduction to the “**analysis tools**” used in the ICT field for the **modeling, analysis and design** of applications operating in **random environments and over shared resources**

It will provide to **YOU** the “**math skills**” to

- **understand the stochasticity of “man-made systems”**
- **right-model prediction algorithms based on their randomness**
- **evaluate performance and QoS of Client-Server Systems** under random requests and to avoid congestion
- **understand how randomized distributed algorithms work**



An example... there was a time when...



“Stochastic models and queueing theory helps businesses, computer networks, and others to figure out how to keep things moving...

...while saving money” ...

From “The Wall Street Journal”

Real-world applications



ICT systems

- Communication link dimensioning
- Multiplexing techniques
- QoS evaluation in Client-Server systems
- Information spreading over graphs & networks
- Distributed estimation and optimization algorithms
- etc...

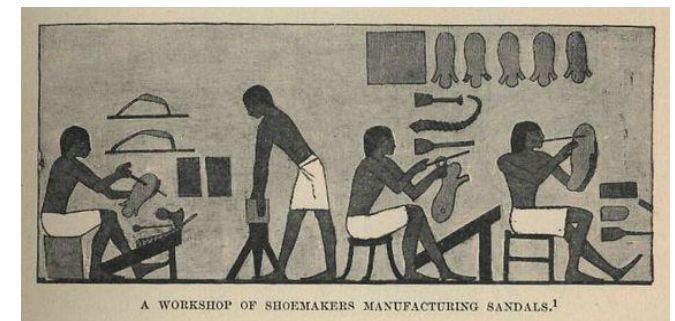
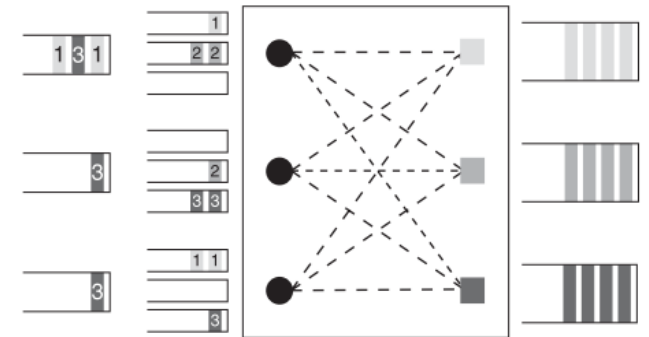
Manufacturing Systems and Logistics

- Productions Lines
- Logistic systems of cargo ships
- Vehicles at the toll station
- traffic systems, etc...

Commercial and social service systems

- Waiting lists for organ transplants
- market queues, etc...

Switch with 3 I/O ports



Take stock of the situation...

- **Deterministic systems** are theoretically predictable.

Given the **initial condition (ICs)** and a **model**, its state in any foreseeable future can be forecasted.

- **Chaotic systems** are also **deterministic** (at least in “theory”)

Since they **respond drastically to infinitesimal ICs changes** (see *butterfly effect*), they are also unpredictable and unstable in practice.

- A **stochastic system** is instead **just a random phenomenon** and conclusions on the system behavior can be drawn only in the mean sense (by exploiting the process “average”, its “variance”, etc...)

Ps. The term “process” stresses randomness occurs as time passes

Contents

- Main modules ($\approx 34h$):

- Probability Theory (10h)

- Stochastic processes (4h)

- Markov chains (8h)

- Hidden Markov models (3)

- Queuing theory (6h)

- Randomized protocols (3h)

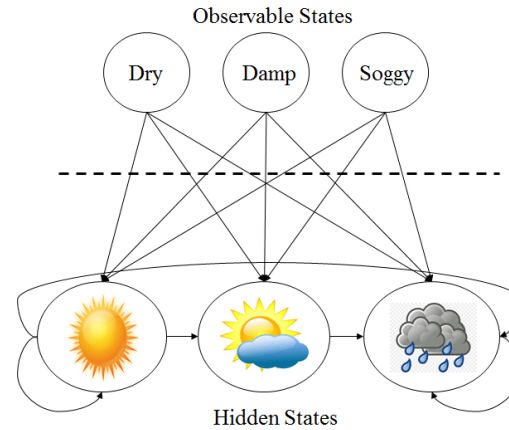
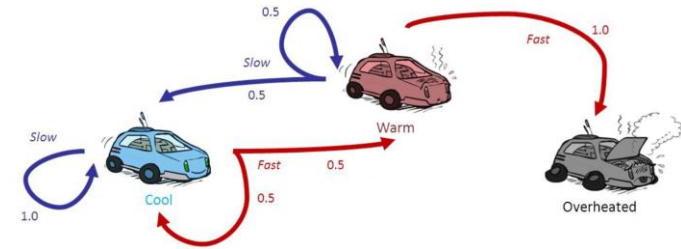
- Classworks (14/16h)

Tot. 50h



$$\{X_t, t = 0, 1, 2, \dots\} = \{X_0, X_1, X_2, X_3, \dots\}$$

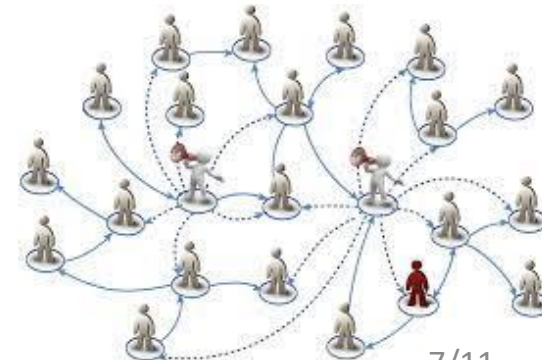
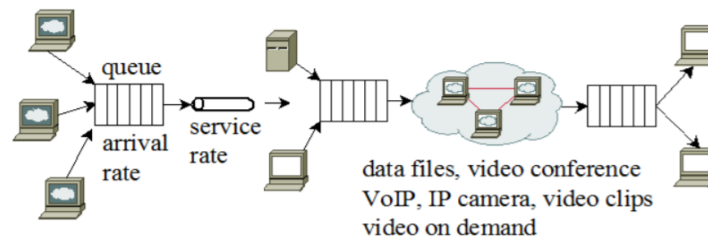
$$\pi(k+1) = \pi(k) \cdot P$$



$$\mathcal{O} = \{o_0, o_1, o_2, \dots, o_k\}$$

$$\mathcal{R} = \{x_0, x_1, x_2, \dots, x_k\}$$

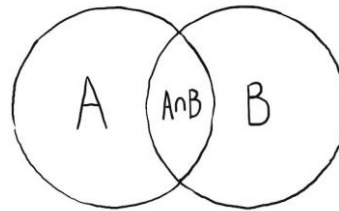
$$\bar{X} = \{X_0, X_1, X_2, \dots, X_k\}$$



Prerequisites

Basic concepts of mathematics:

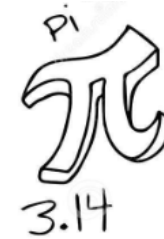
- Algebra and Set theory



- Differential and Integral calculus

$$\frac{a \text{ MILK}}{dx} = \text{CHEESE}$$

$$\int \text{MILK} dx = \text{COW}$$



Basics of System Theory and Stability

- LTI systems $\frac{d\pi(t)}{dt} = \pi(t) \cdot Q$

$$\pi_i(t) = \Pr(X_t = i)$$

Probability the experiment a time "t" takes value "i"

- Laplace's transformation

$$\Pi(s) = \mathcal{L}\{\pi(t)\} = \pi(0) (s \cdot I - Q)^{-1}$$

$$\pi(t) = \mathcal{L}^{-1}\{\Pi(s)\} = \sum_{i=1}^n \mathcal{L}^{-1}\left\{\frac{R_i}{(s - p_i)}\right\}$$

Exam's organization and mark

The final examination is **ORAL**, and lasts \approx **60/90 min.**

It **begins** by asking the candidate to present a **topic of his/her choice** among the many of the course (**\approx 30min of informal talk at the blackboard, no slides, no long proof, the focus must be on explaining concepts**)

Then...

During this discussion, it is allowed use an A4 sheet of paper where they write a few formulas... in order to not spend effort to memorize it...

ps: ...only formulas...no explanations....

Then it will be requested to ...

- **Answer to some related theoretical questions on his/her talk**
- **Solve 2 exercises like those seen in the course (often simpler 😊)**

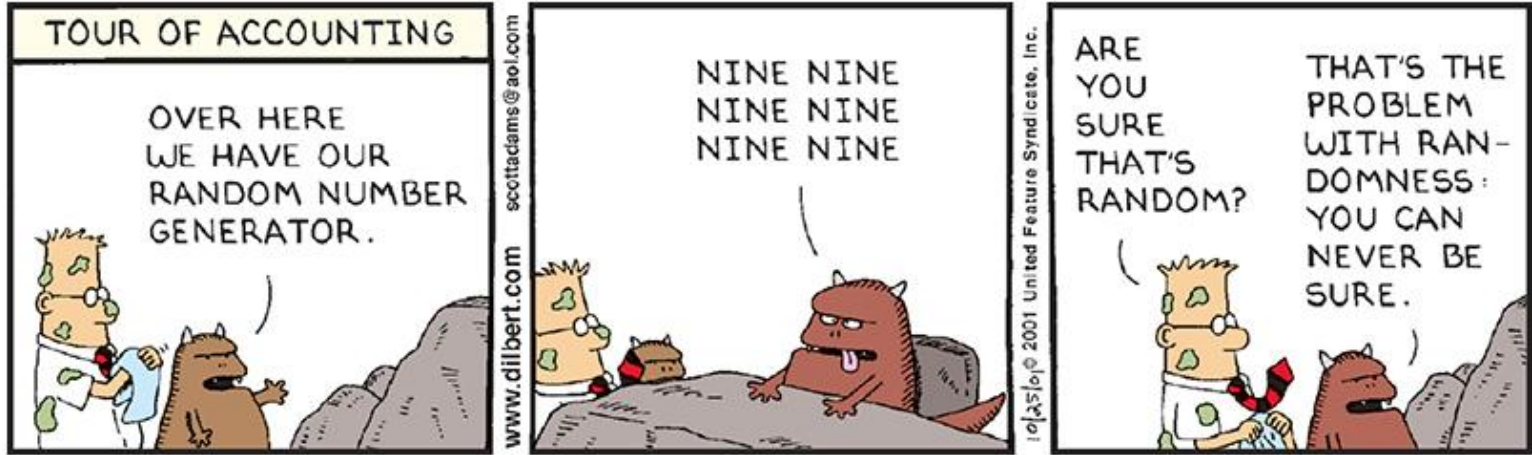
Exam's organization and mark

The **final marks** accounts:

- The **knowledge** of the **main topics** of the course (**30%**)
- The **consciousness** to approach the problems solutions (**30%**)
- The **autonomy in making judgments** (**30%**)
- The **use of technical language** (**10%**)

**Some possible
discussion topics...**

- *From random variables to stochastic processes up to Markov chains*
- *An overview from Markov chains to Queueing Theory*
- *Markov chains and connections the rumor spreading problem*
- *Multiplexing performances analysis through stochastic queues*
- *Communication Link's dimensioning*
- *Randomized distributed estimation through the consensus*
- *Etc....*

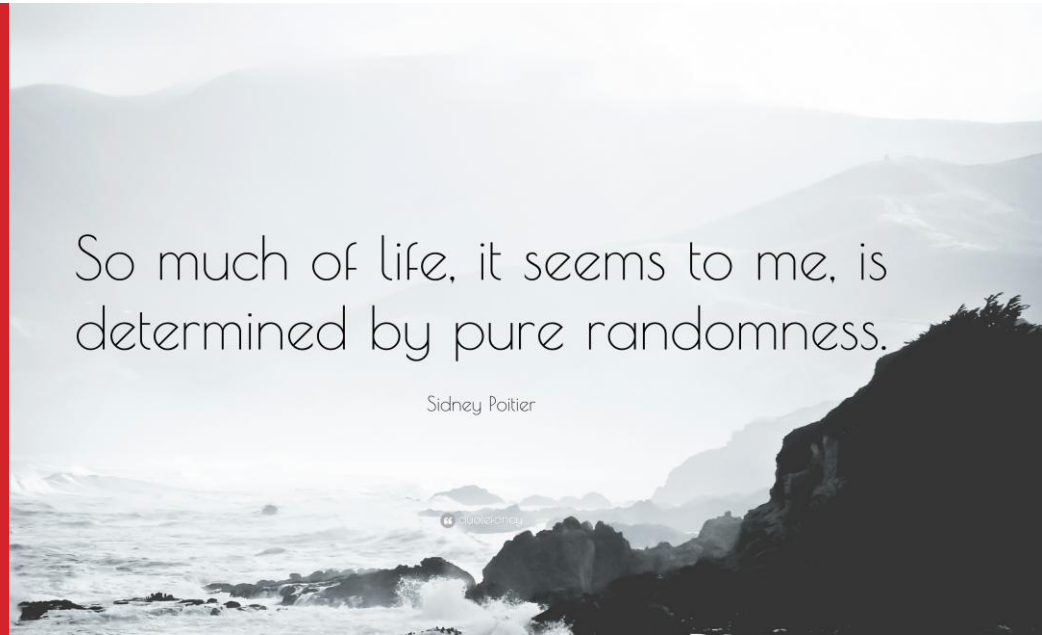


“Probability is not a mere computation of odds on the dice or more complicated variants; it is the acceptance of the lack of certainty in our knowledge and the development of methods for dealing with our ignorance.”

— Nassim Taleb

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So much of life, it seems to me, is determined by pure randomness.

Sidney Poitier

Thanks for the Attention!

