INTRODUZIONE AL CONCETTO DI SFORZO

Emanuele RECCIA

_ Azioni interne

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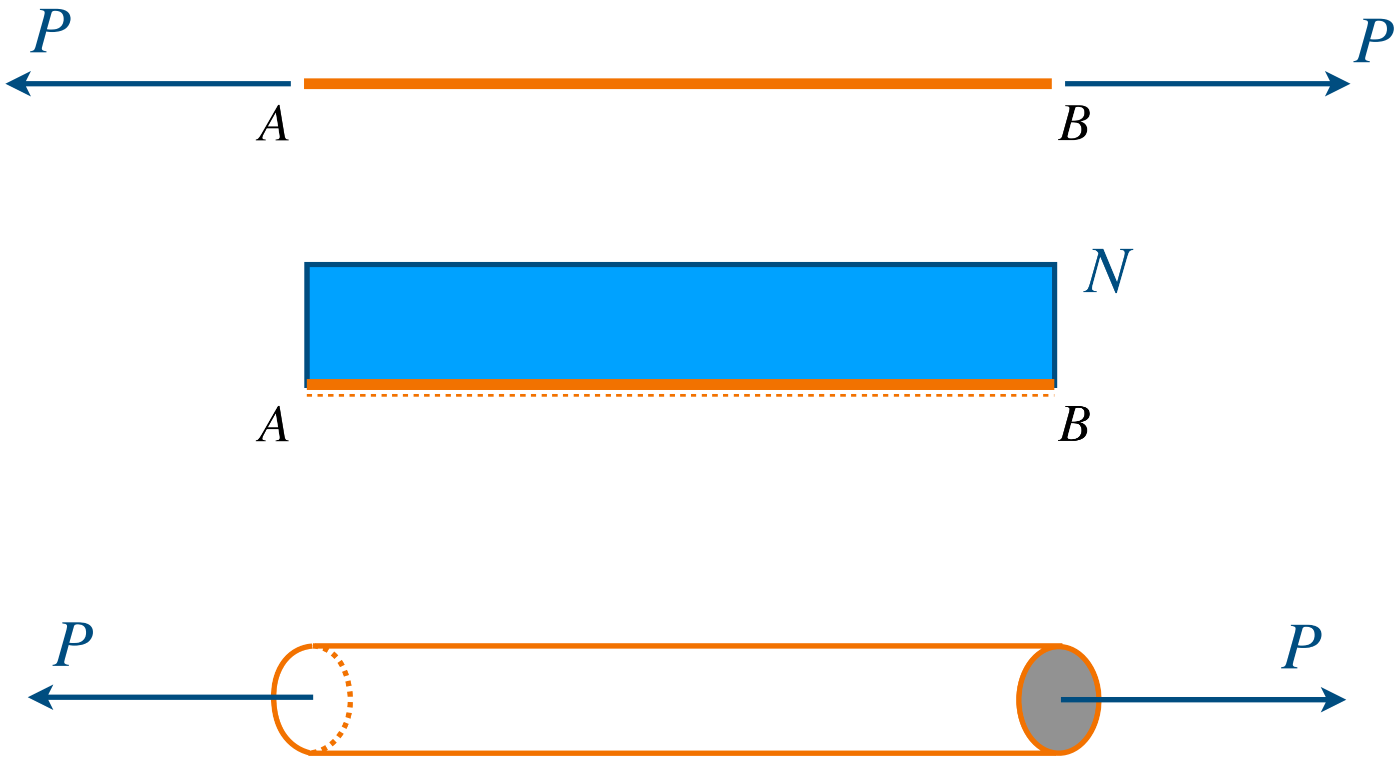

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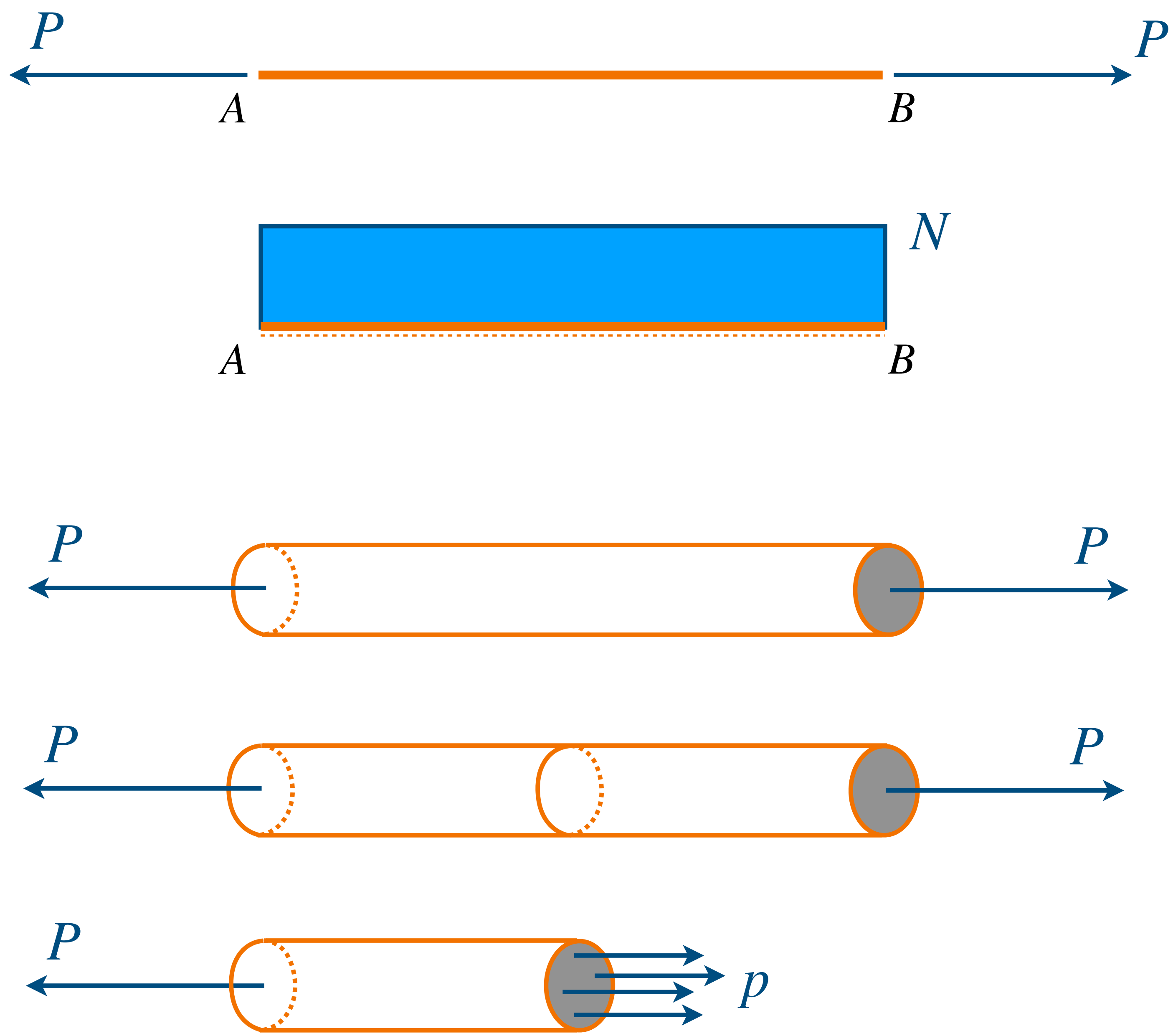

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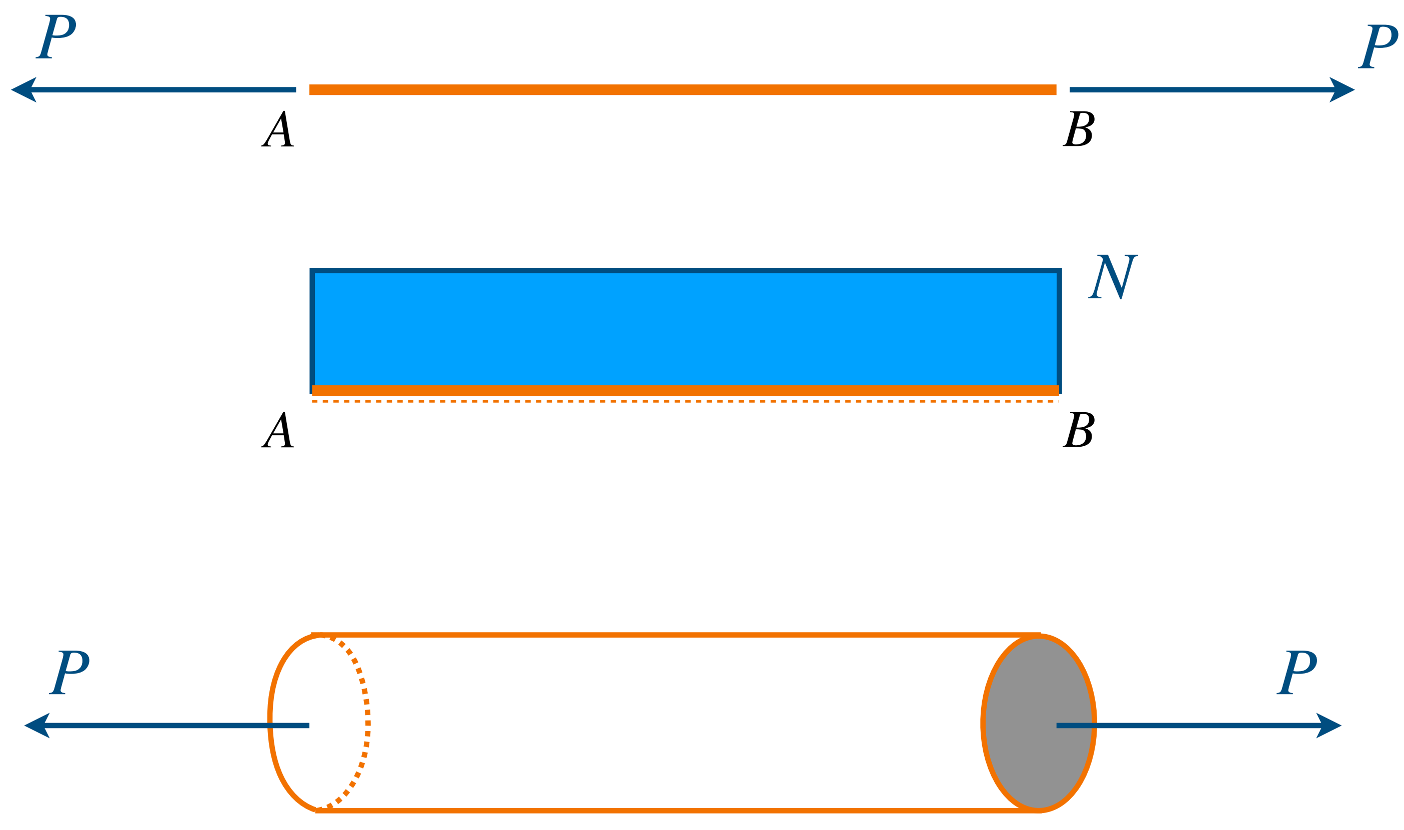

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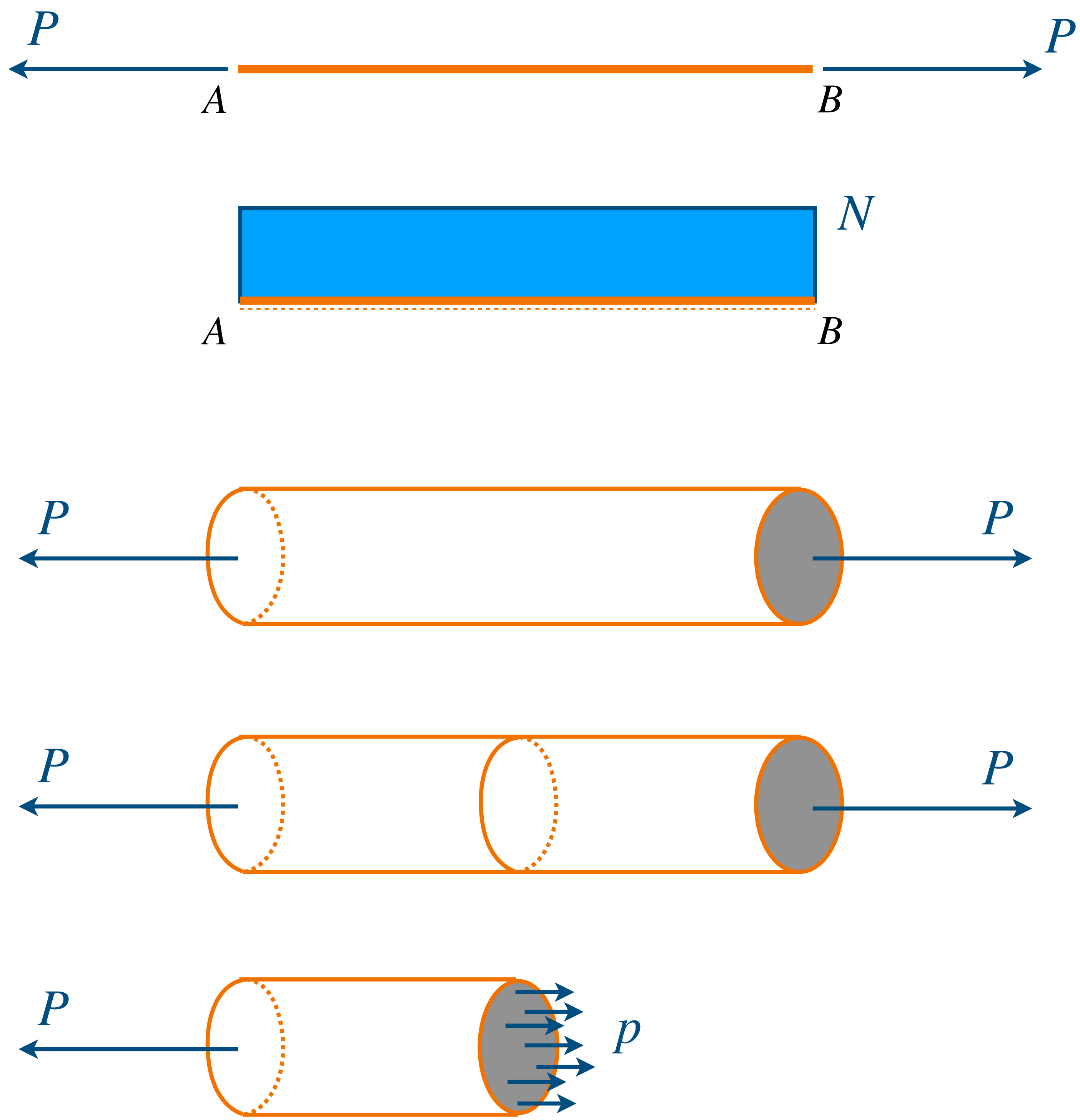

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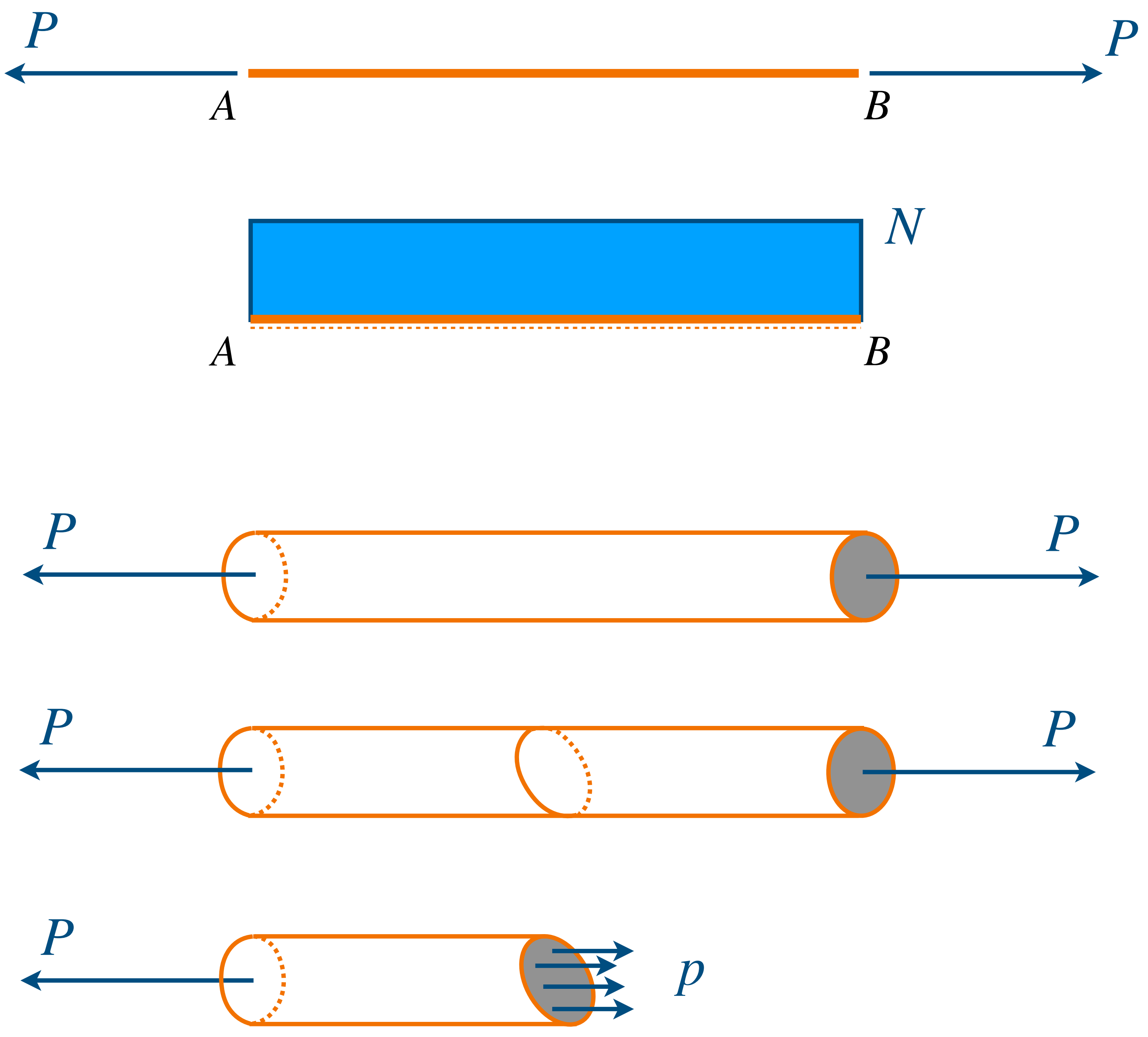

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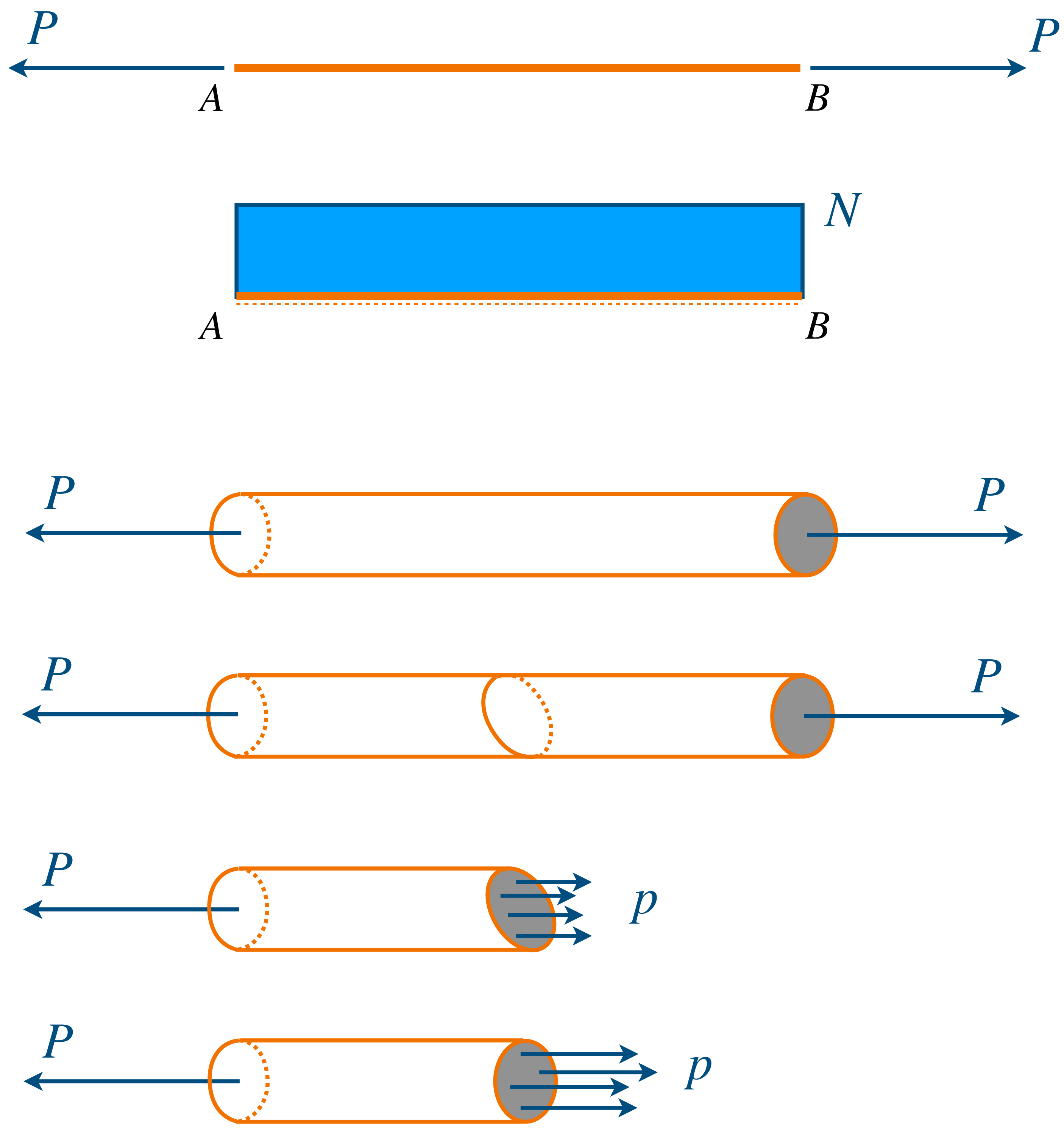

_ Azioni interne

La descrizione della distribuzione delle forze interne in un corpo viene fatta introducendo il concetto di sforzo, o tensione.

Lo sviluppo di questo concetto è iniziato con Galilei, James Bernoulli, Eulero, Newton…

Molti studiosi hanno contribuito in maniera significativa: Hooke, Young, Poisson…

_ Azioni interne

Un contributo fondamentale è stato dato da Navier

Claude-Louis-Marie-Henri Navier
(Digione 1785, Parigi 1836)

Navier, Claude-Louis-Marie-Henri (1827) Mémoire sur les lois de l'équilibre et du mouvement des corps solides élastiques, *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, vol.7, pp. 375-393.

_ Azioni interne

La trattazione completa si deve a Cauchy

Augustin-Louis Cauchy
(Parigi 1789, Sceaux 1857)

Cauchy, Augustin-Louis (1827) De la pression ou tension dans un corps solide, *Exercices de Mathématiques*, vol. 2, pp. 42-56
(Œuvres complètes, série 2, tome 7, pp. 60-78)

Cauchy, Augustin-Louis (1827) Addition à l'article précédent, *Exercices de Mathématiques*, vol. 2, pp. 57-59
(Œuvres complètes, série 2, tome 7, pp. 79-81)

Cauchy, Augustin-Louis (1828) Sur les équations qui expriment les conditions d'équilibre ou les lois du mouvement intérieur d'un corps solide, élastique, ou non élastiques, *Exercices de Mathématiques*, vol. 3, pp. 160-187.
(Œuvres complètes, série 2, tome 8, pp. 195-226)

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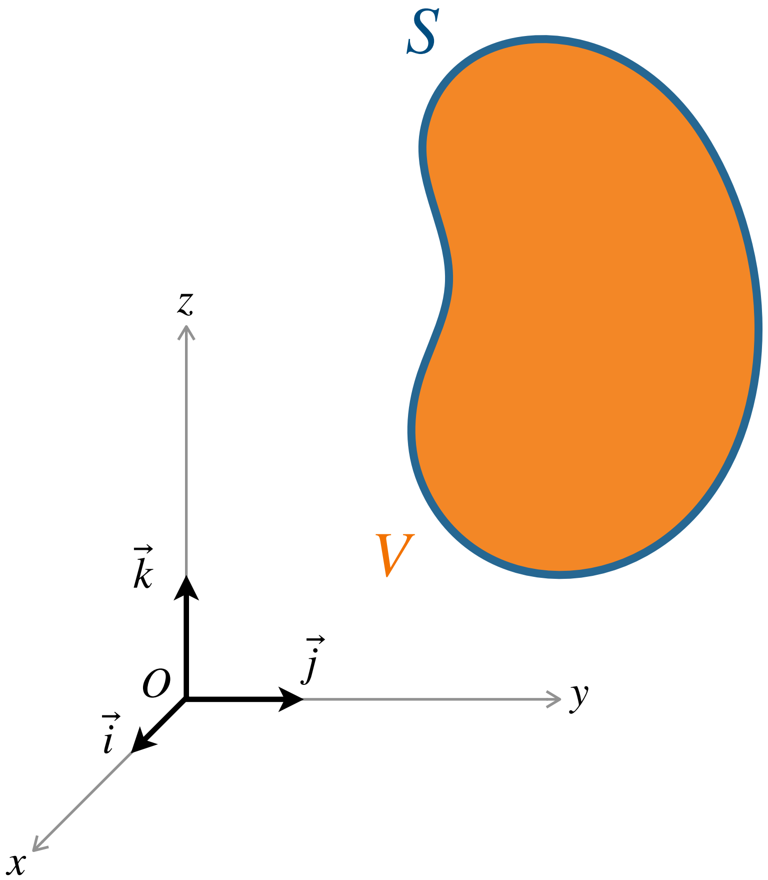

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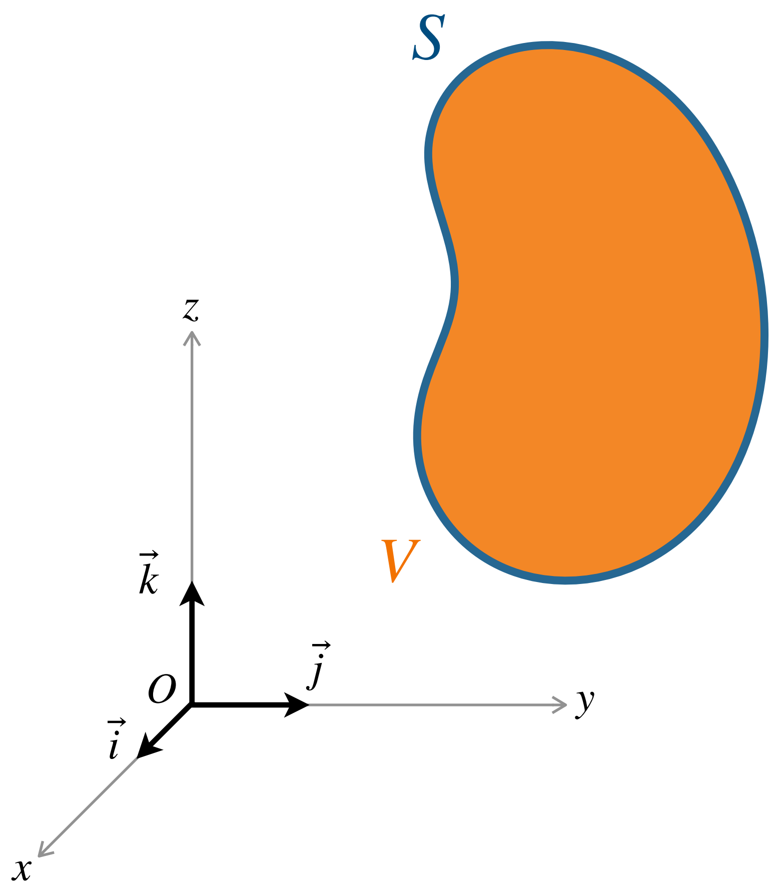

FORZE DI SUPERFICIE $\vec{p}$

FORZE DI VOLUME $\overrightarrow{F}$

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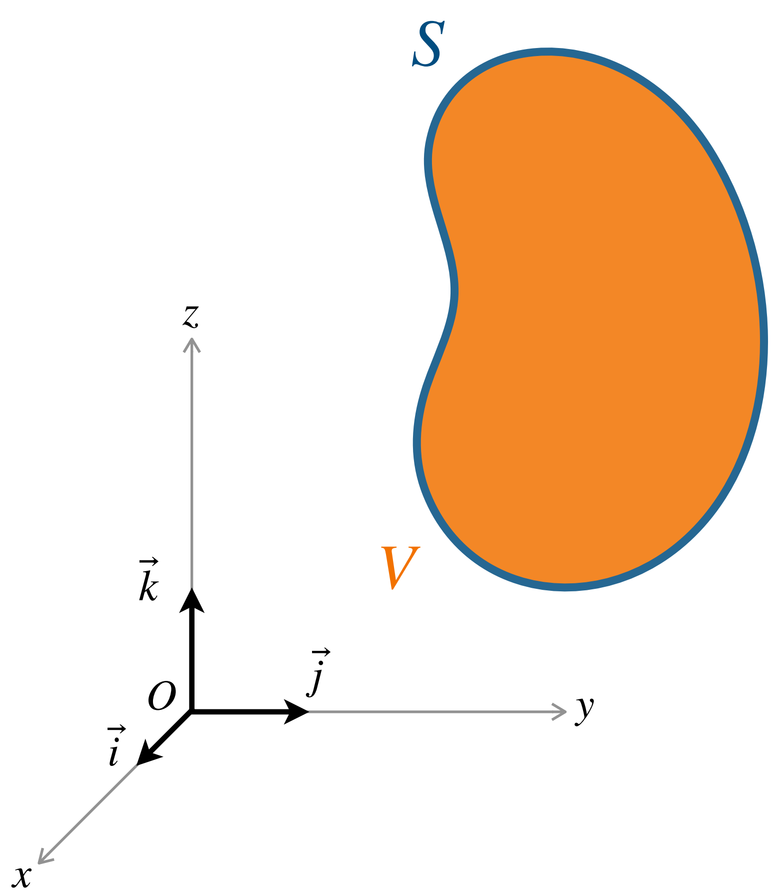

FORZE DI SUPERFICIE $\vec{p}$

$$\vec{p} = p_x\vec{i} + p_y\vec{j} + p_z\vec{k} \qquad \vec{p} = \left\{ p_x, p_y, p_z \right\}$$

$$p = \frac{[N]}{[L]^2}$$ PRESSIONE $Pa = N/m^2$

FORZE DI VOLUME $\overrightarrow{F}$

$$\overrightarrow{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k} \qquad \overrightarrow{F} = \left\{ F_x, F_y, F_z \right\}$$

$$F = \frac{[F]}{[L]^3}$$ PESO SPECIFICO $\rho = N/m^3$

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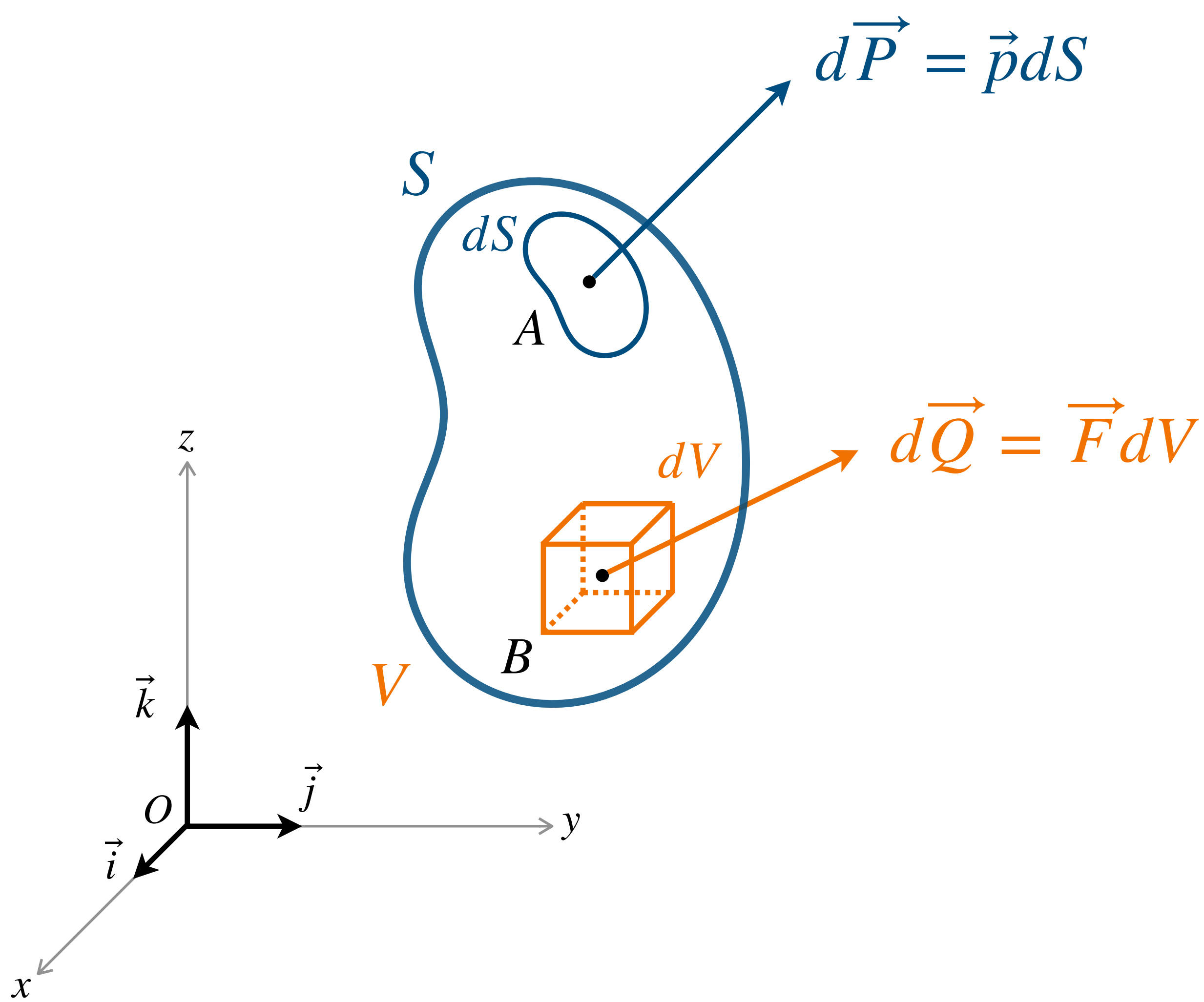

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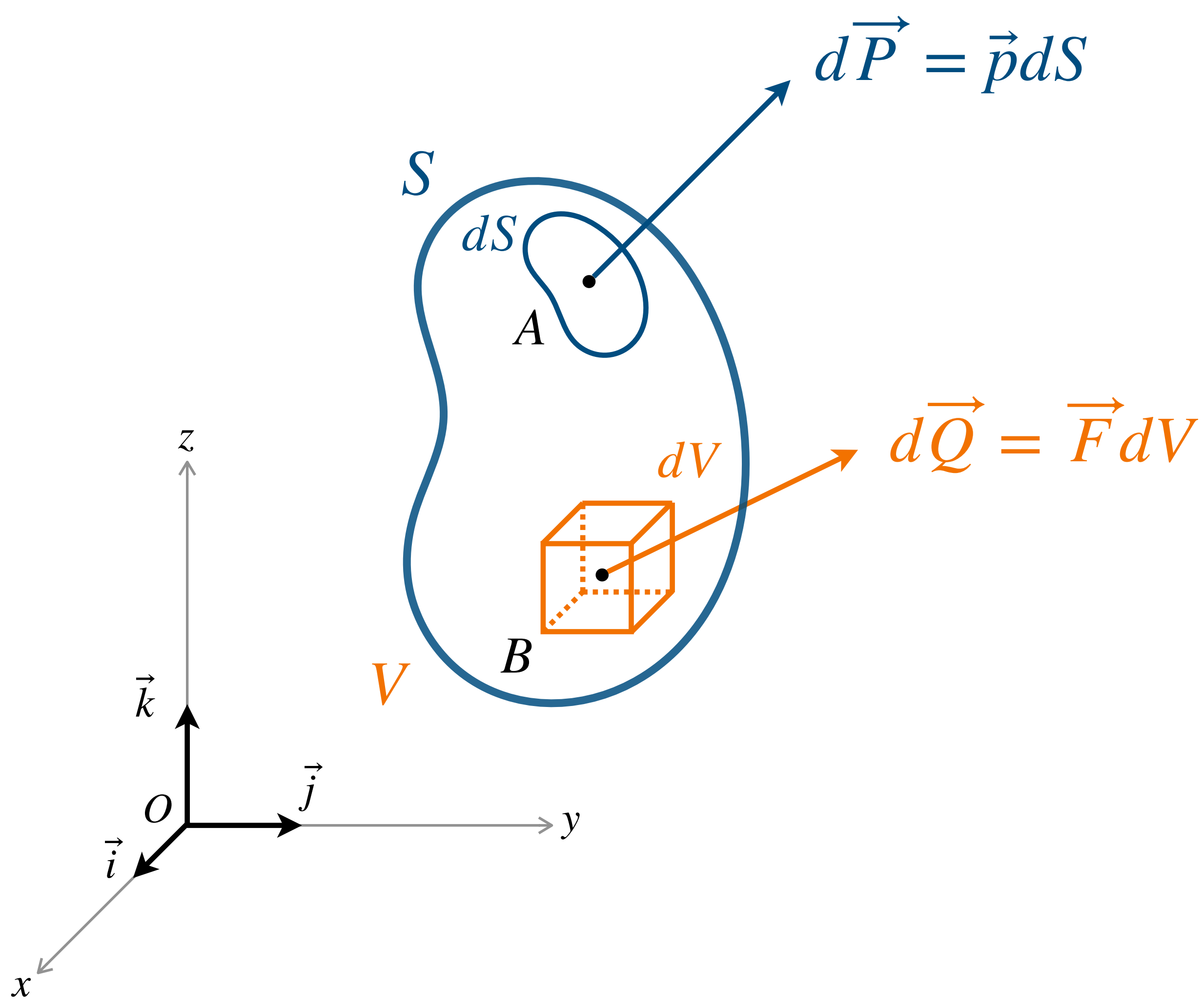

CONDIZIONI DI EQUILIBRIO

$$\overrightarrow{R} = \vec{0} \qquad \& \qquad \overrightarrow{M}_{(O)} = \vec{0}$$

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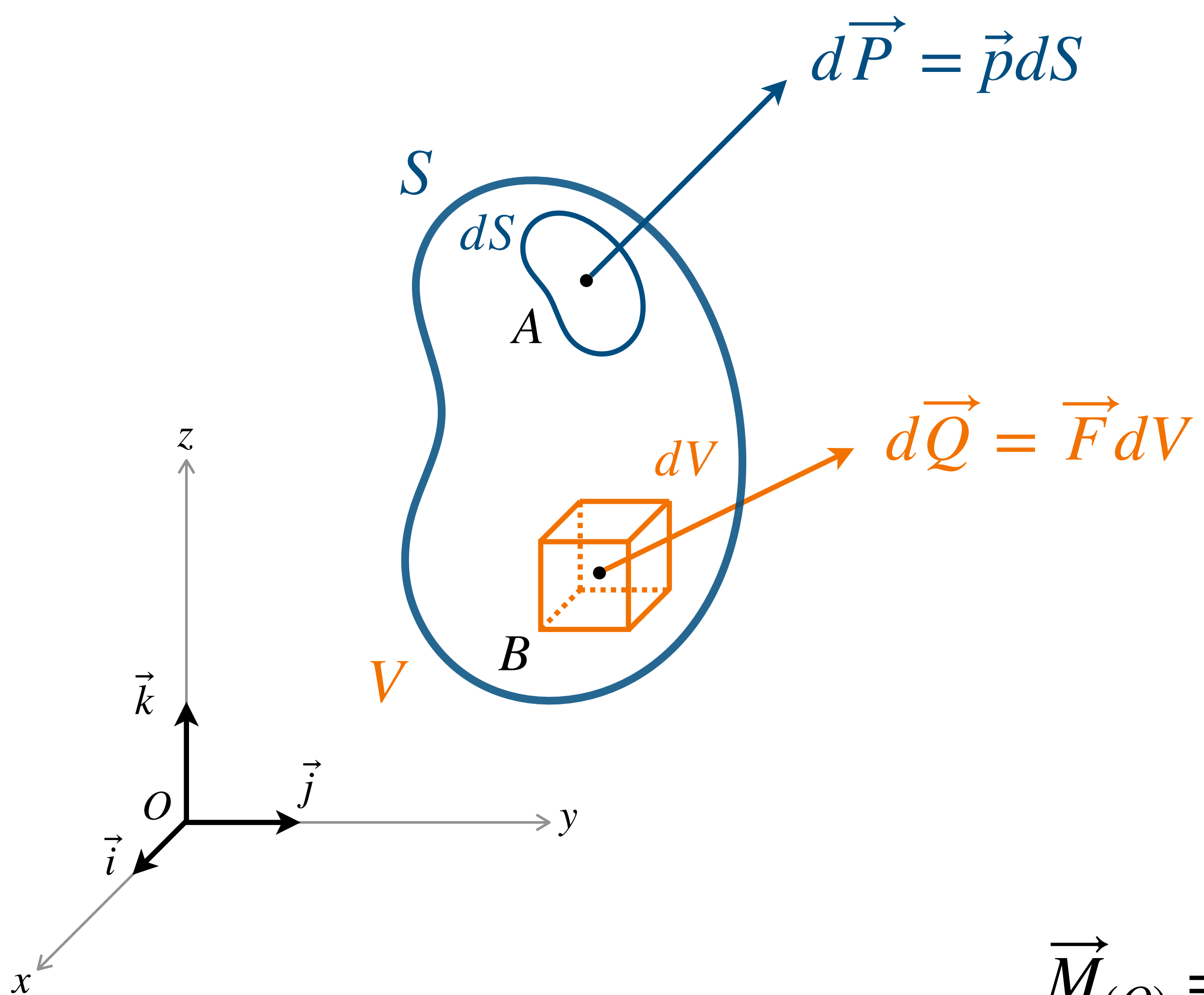

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$$\vec{M}_{(O)} = \vec{0}$$

$$\vec{M}_{(O)} = M_{(O)x}\vec{i} + M_{(O)y}\vec{j} + M_{(O)z}\vec{k} = \vec{O} \qquad \begin{cases} M_{(O)x} = 0 \\ M_{(O)y} = 0 \\ M_{(O)z} = 0 \end{cases}$$

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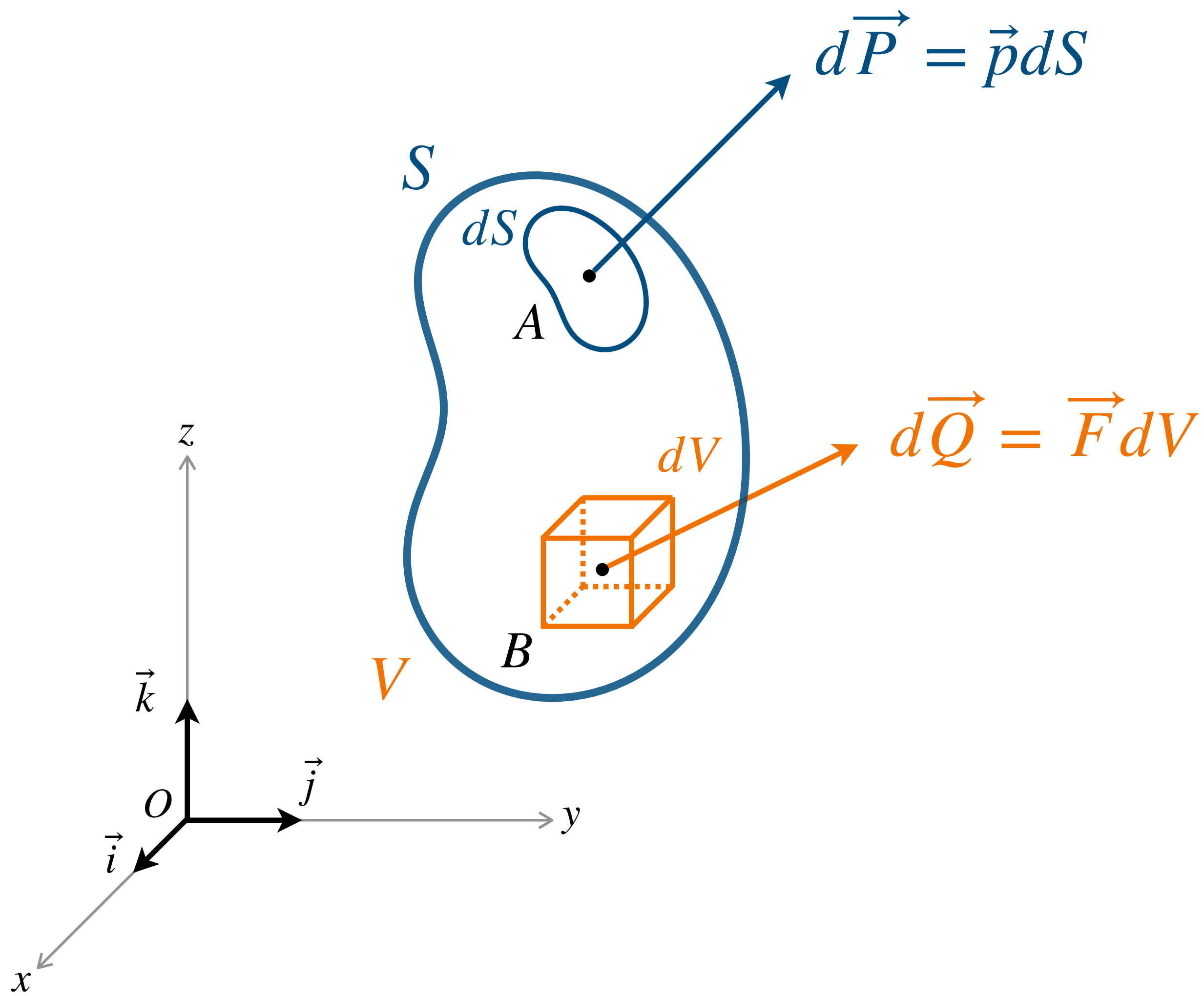

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$$R_x = 0 \qquad R_x = \int_S p_x dS + \int_V F_x dV = 0$$

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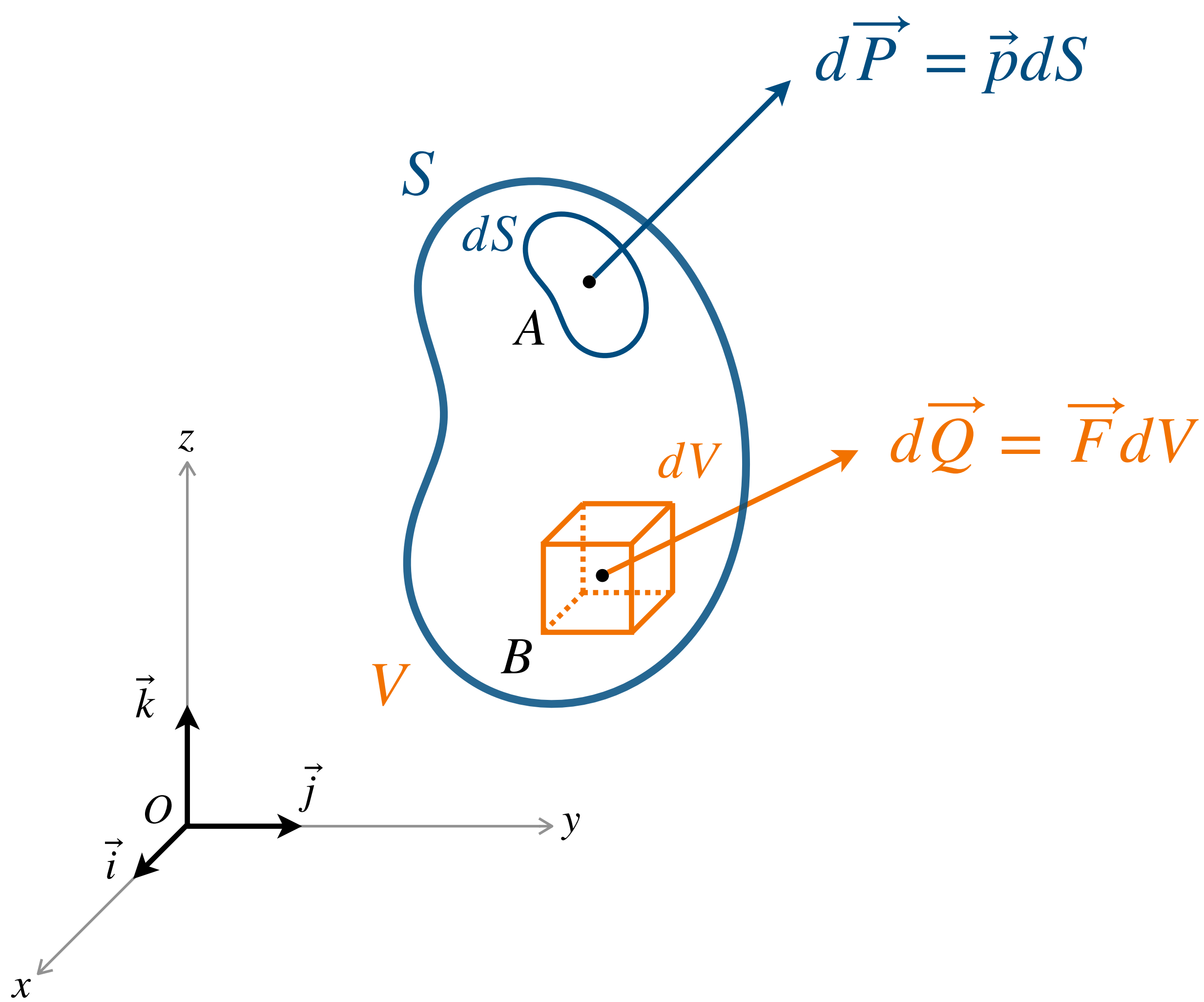

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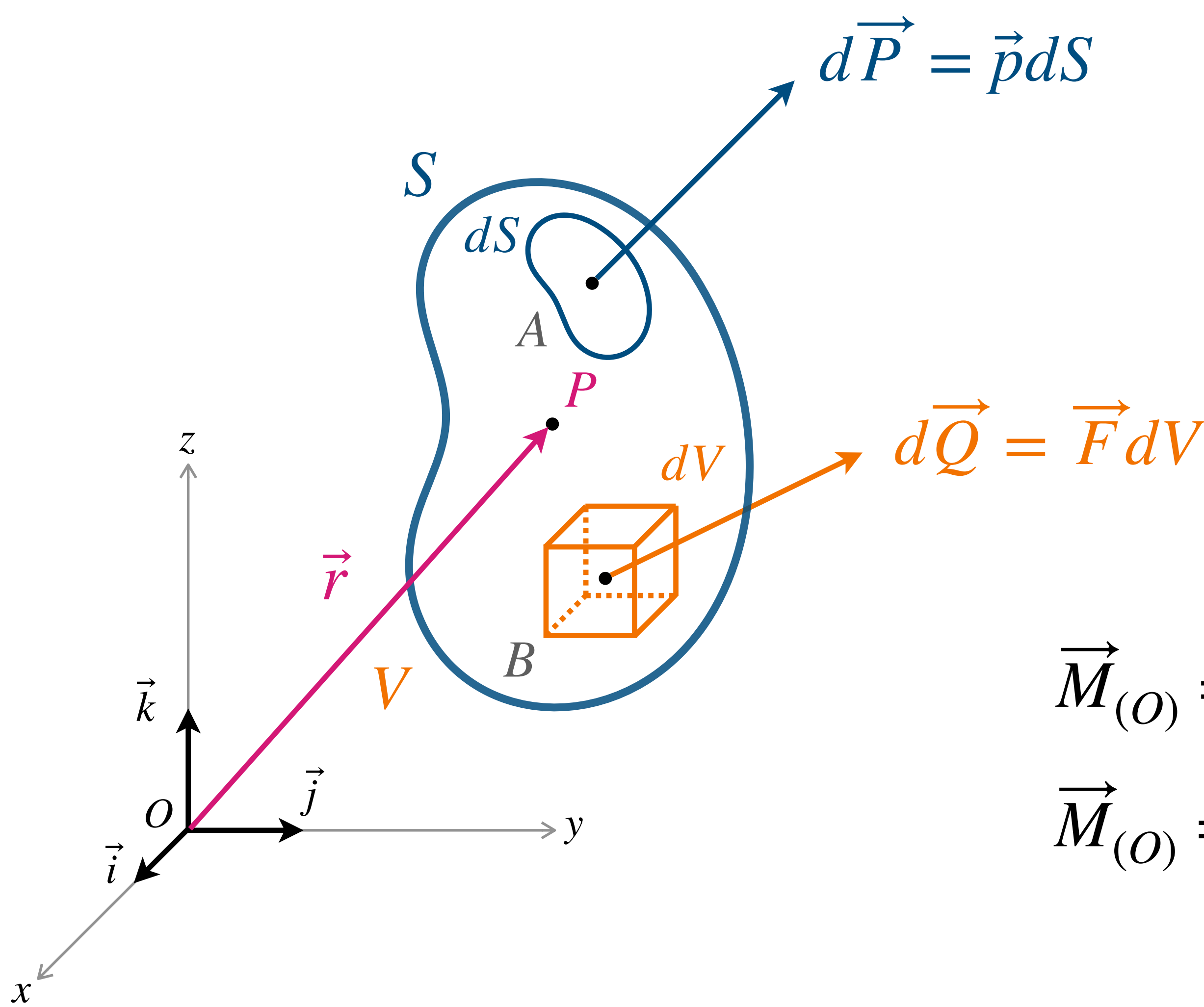

$$P = (x, y, z)$$

$$\overrightarrow{P - O} = \vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$\vec{r} = \{x, y, z\}$$

CONDIZIONI DI EQUILIBRIO

$$\overrightarrow{R} = \vec{0} \qquad \& \qquad \overrightarrow{M}_{(O)} = \vec{0}$$

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$$\overrightarrow{M}_{(O)} = \int_S \vec{r} \wedge d\overrightarrow{P} + \int_V \vec{r} \wedge d\overrightarrow{Q} = \vec{0}$$

$$\overrightarrow{M}_{(O)} = \int_S \vec{r} \wedge \vec{p}dS + \int_V \vec{r} \wedge \overrightarrow{F}dV = \vec{0}$$

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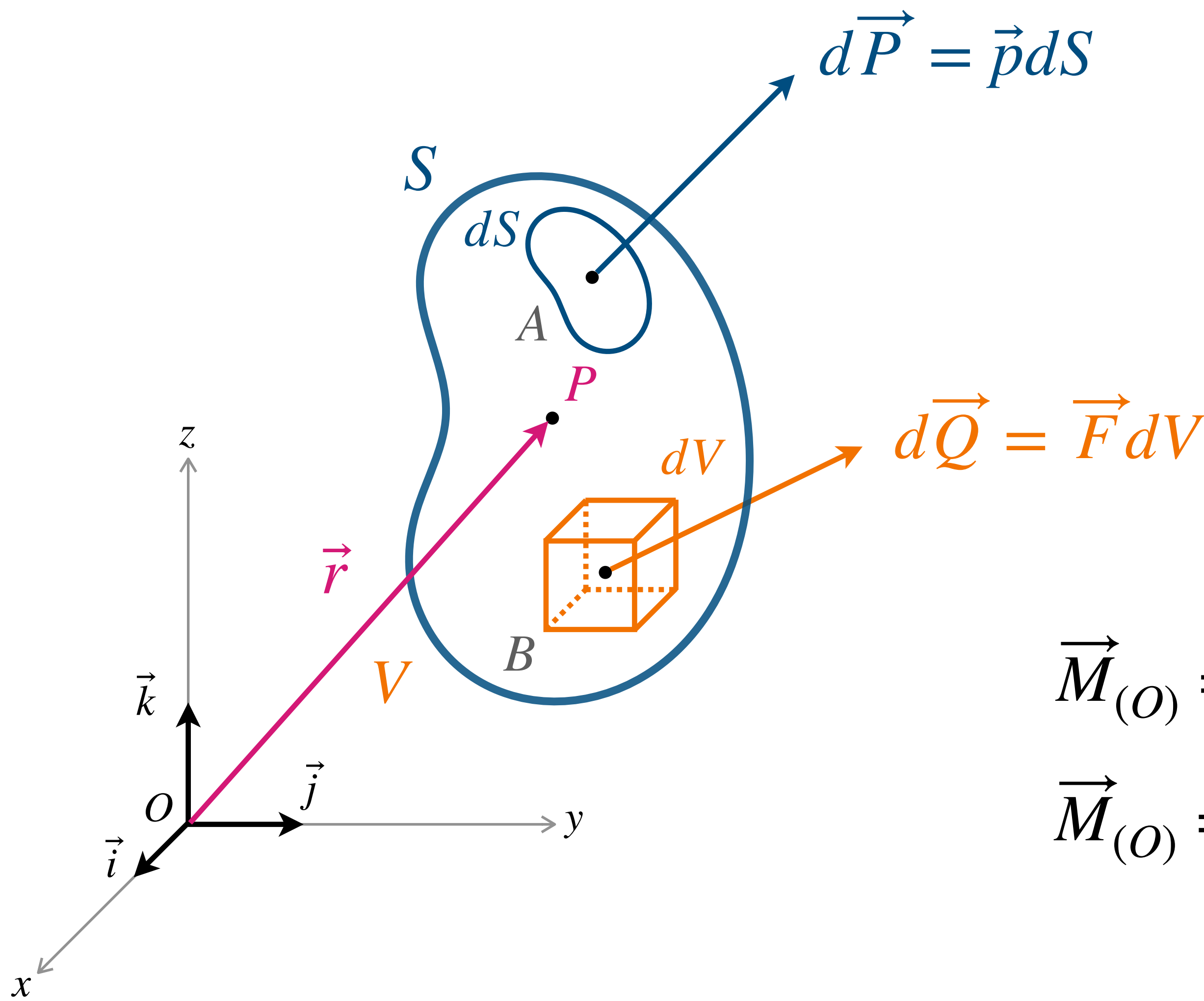

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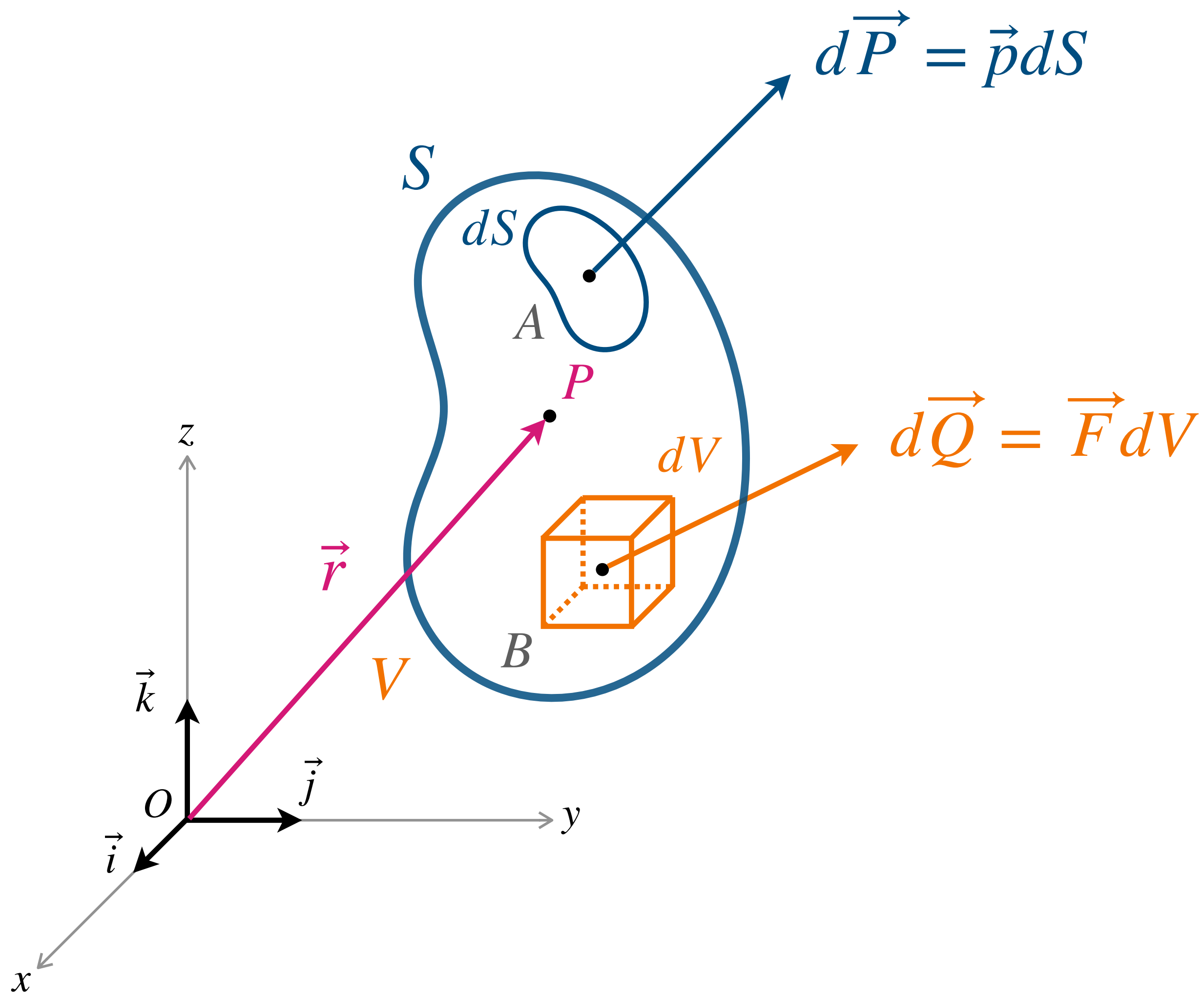

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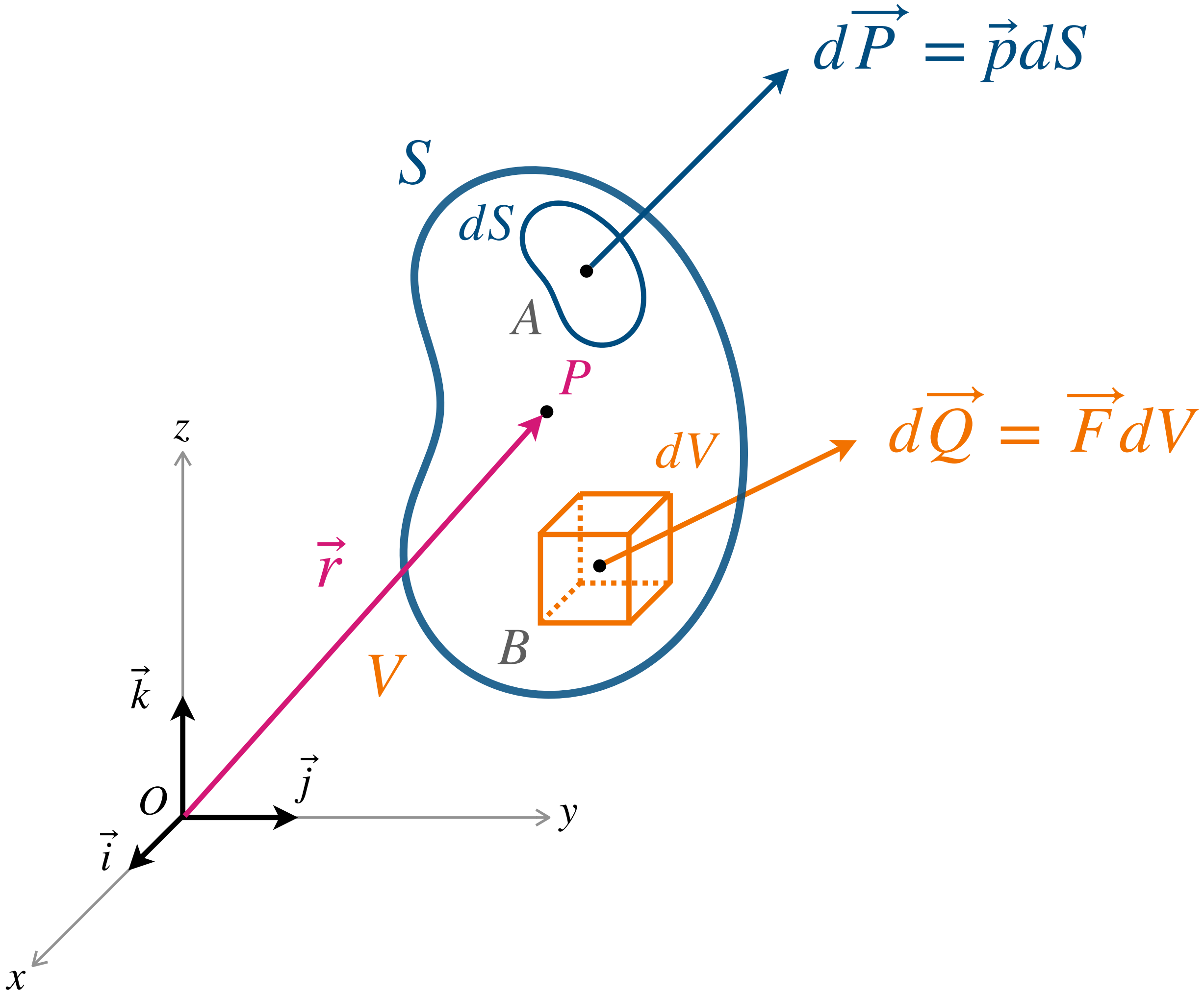

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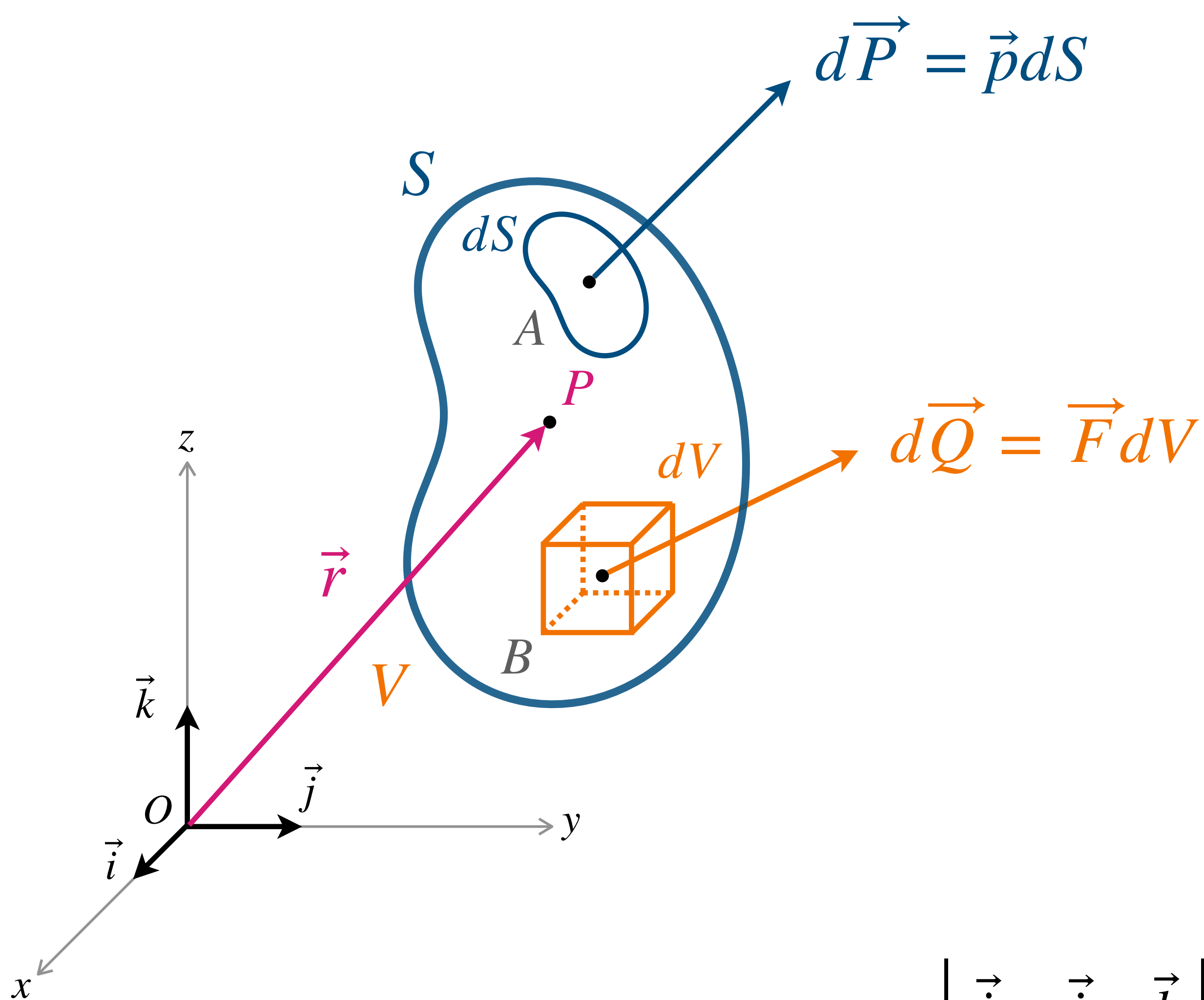

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$$\overrightarrow{R} = \vec{0} \qquad \& \qquad \overrightarrow{M}_{(O)} = \vec{0}$$

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$P = (x, y, z)$

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$$\boxed{\vec{r} \wedge \vec{p}} = \det \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ p_x & p_y & p_z \end{vmatrix} = + \vec{i} \begin{vmatrix} y & z \\ p_y & p_z \end{vmatrix} - \vec{j} \begin{vmatrix} x & z \\ p_x & p_z \end{vmatrix} + \vec{k} \begin{vmatrix} x & y \\ p_x & p_y \end{vmatrix} =$$

$$= \vec{i}\left(p_z y - p_y z\right) - \vec{j}\left(p_z x - p_x z\right) + \vec{k}\left(p_y x - p_x y\right)$$

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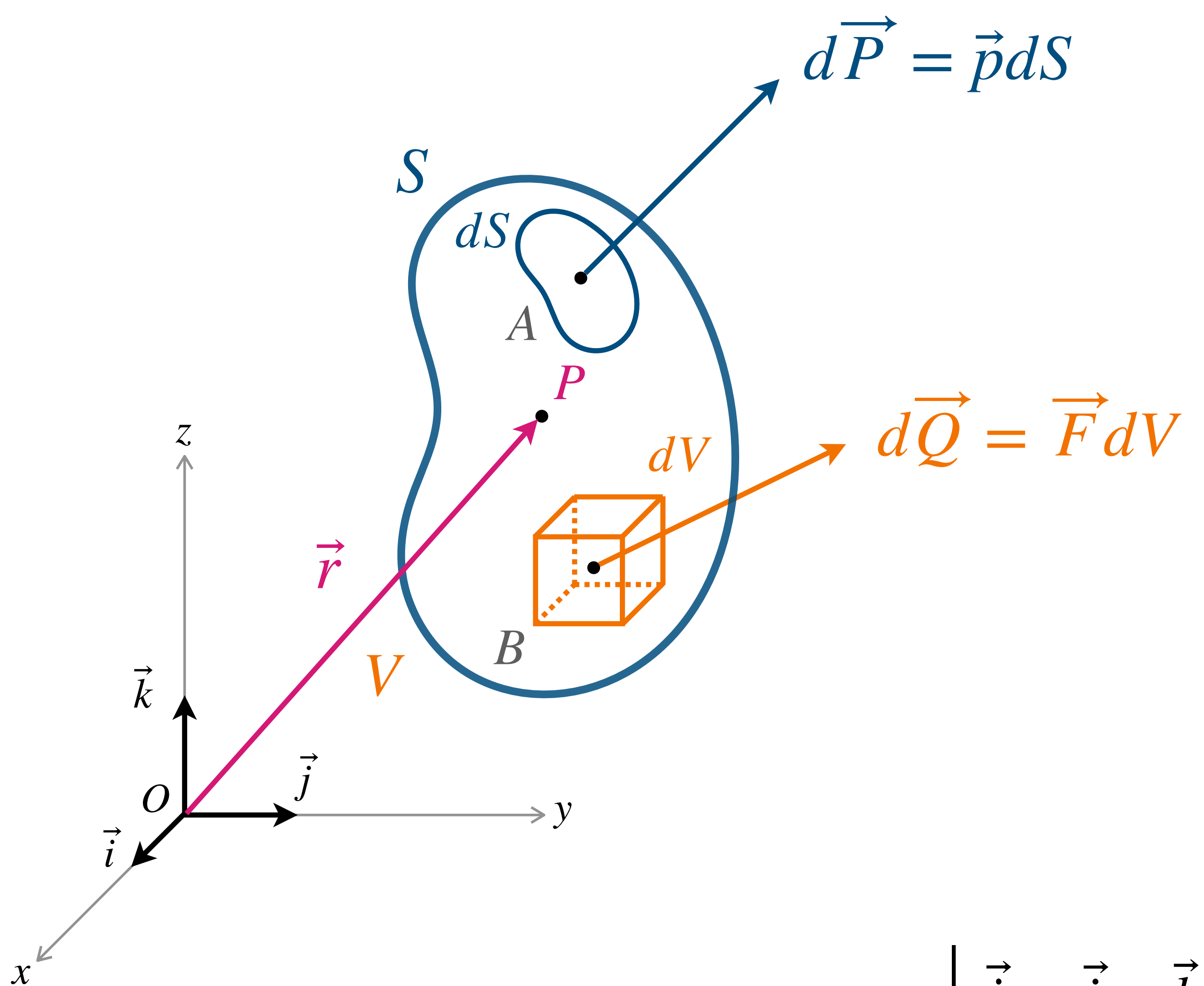

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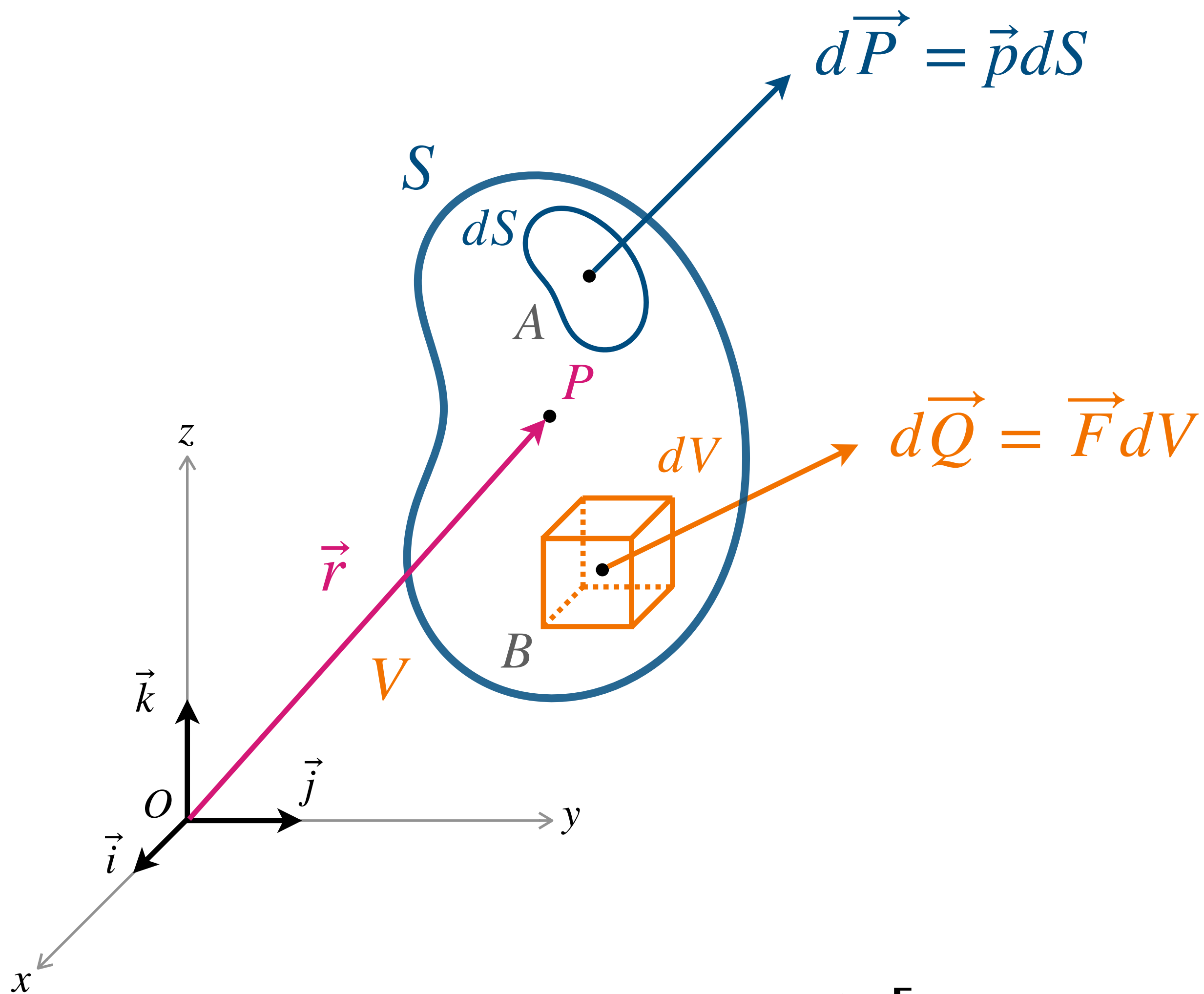

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$$\overrightarrow{M}_{(O)} = \int_S \left[\vec{i}\left(p_z y - p_y z\right) - \vec{j}\left(p_z x - p_x z\right) + \vec{k}\left(p_y x - p_x y\right) \right] dS +$$

$$+ \int_V \left[\vec{i}\left(F_z y - F_y z\right) - \vec{j}\left(F_z x - F_x z\right) + \vec{k}\left(F_y x - F_x y\right) \right] dV = \vec{0}$$

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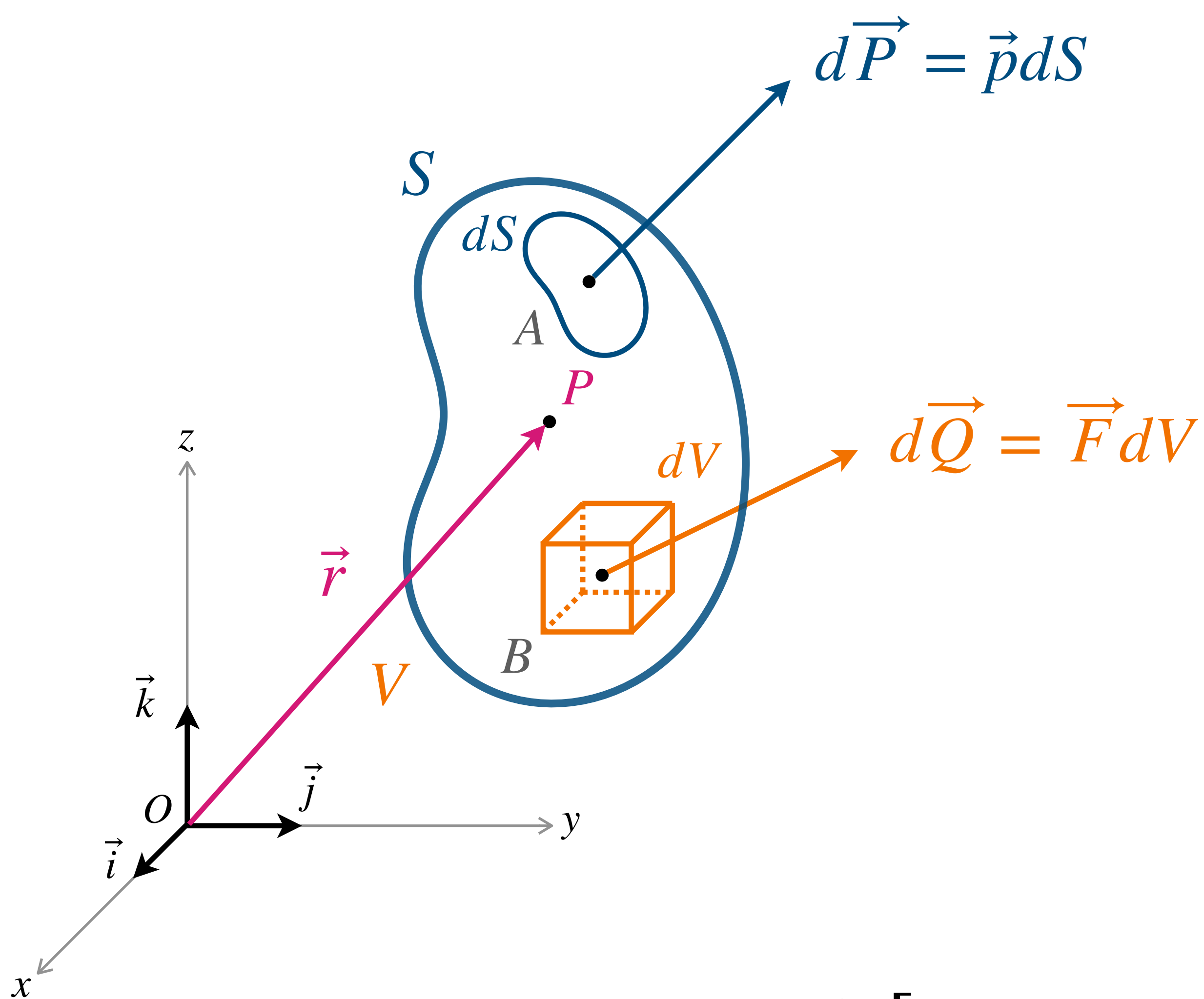

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$$+ \int_V \left[\vec{i}\left(F_z y - F_y z\right) + \vec{j}\left(F_x z - F_z x\right) + \vec{k}\left(F_y x - F_x y\right)\right] dV = \vec{0}$$

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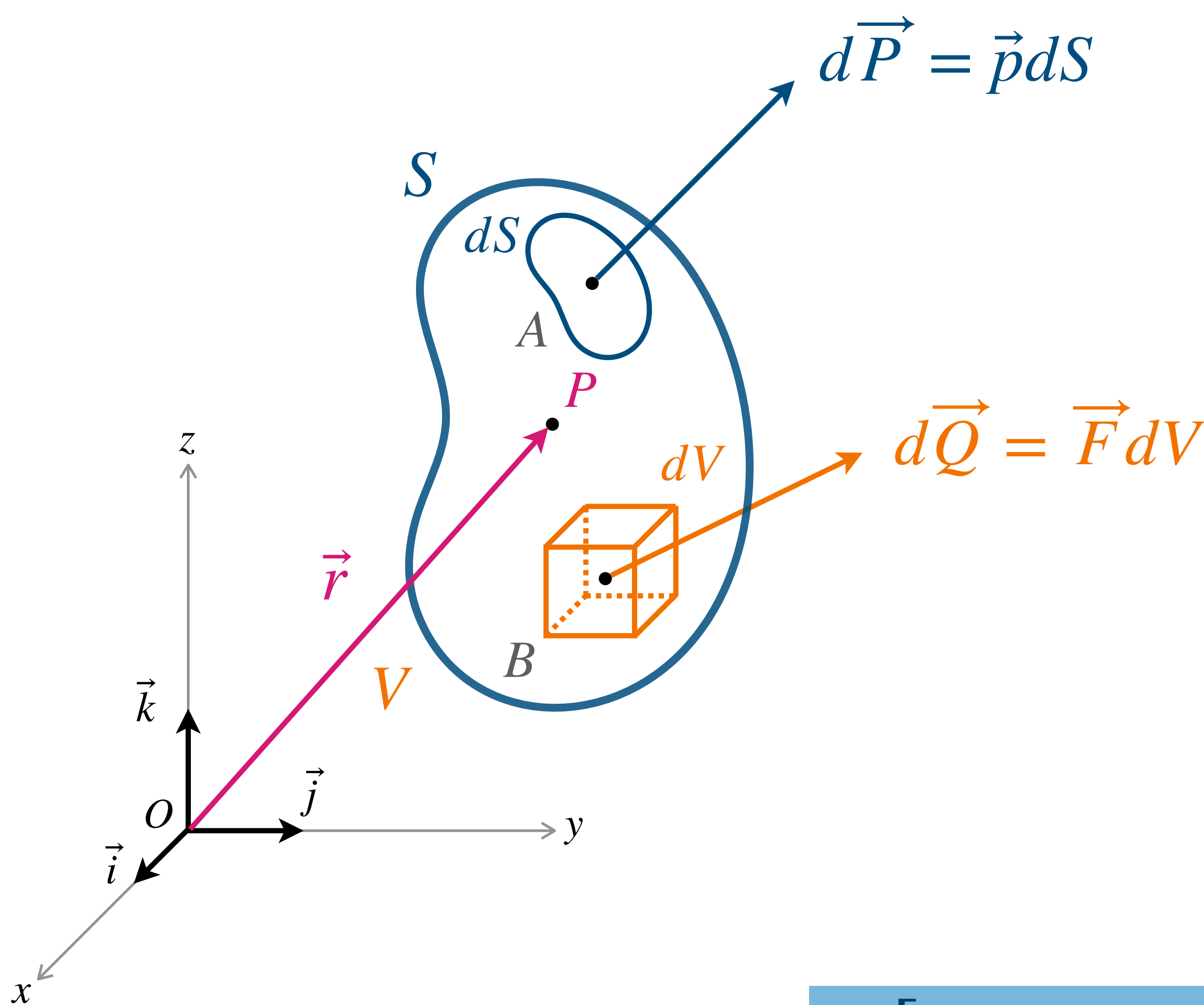

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$$+ \int_V \left[\vec{i}\left(F_z y - F_y z\right) + \vec{j}\left(F_x z - F_z x\right) + \vec{k}\left(F_y x - F_x y\right)\right] dV = \vec{0}$$

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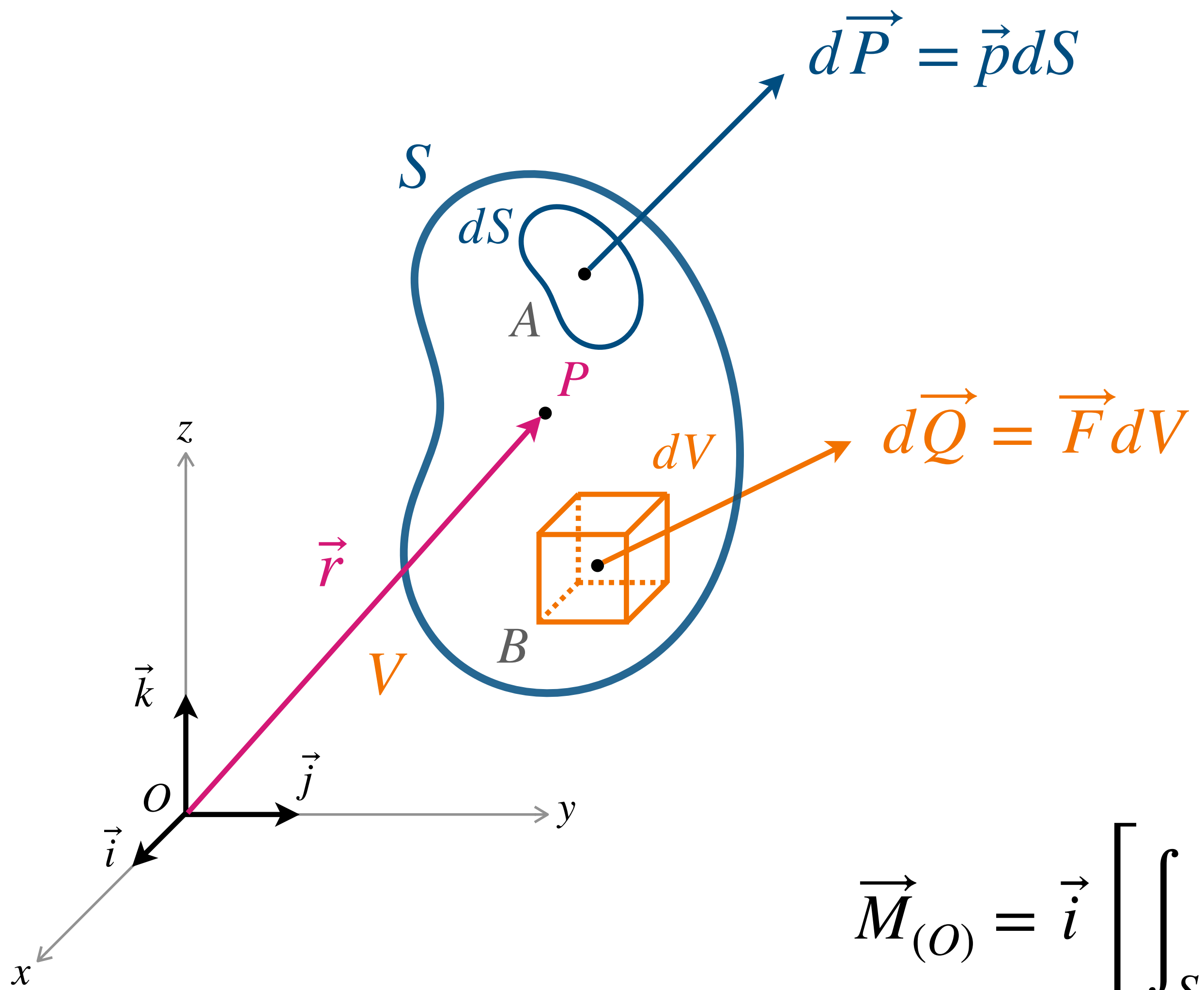

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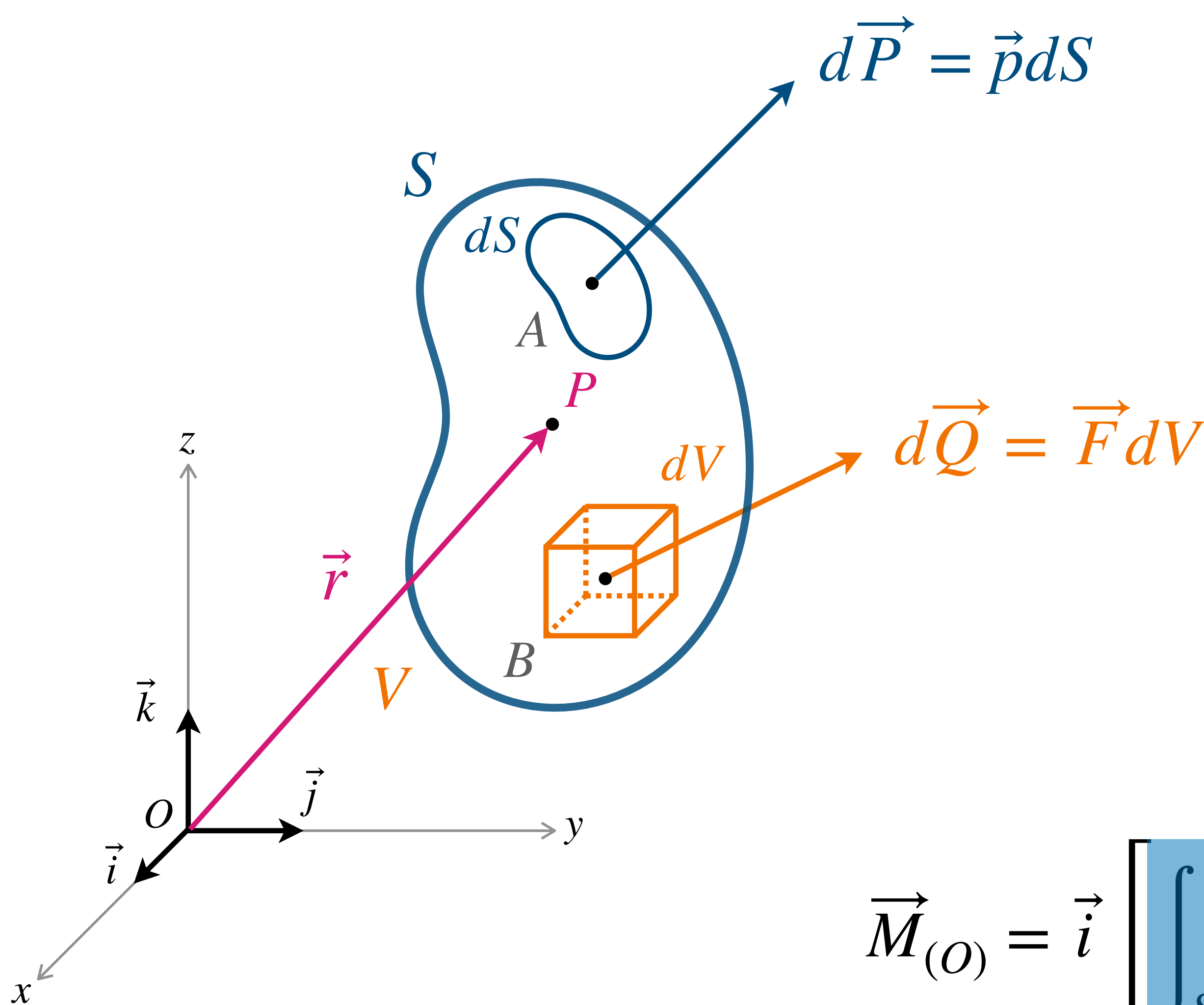

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$$+\vec{j}\left[\int_S \left(p_x z - p_z x\right) dS + \int_V \left(F_x z - F_z x\right) dV = 0\right] +$$

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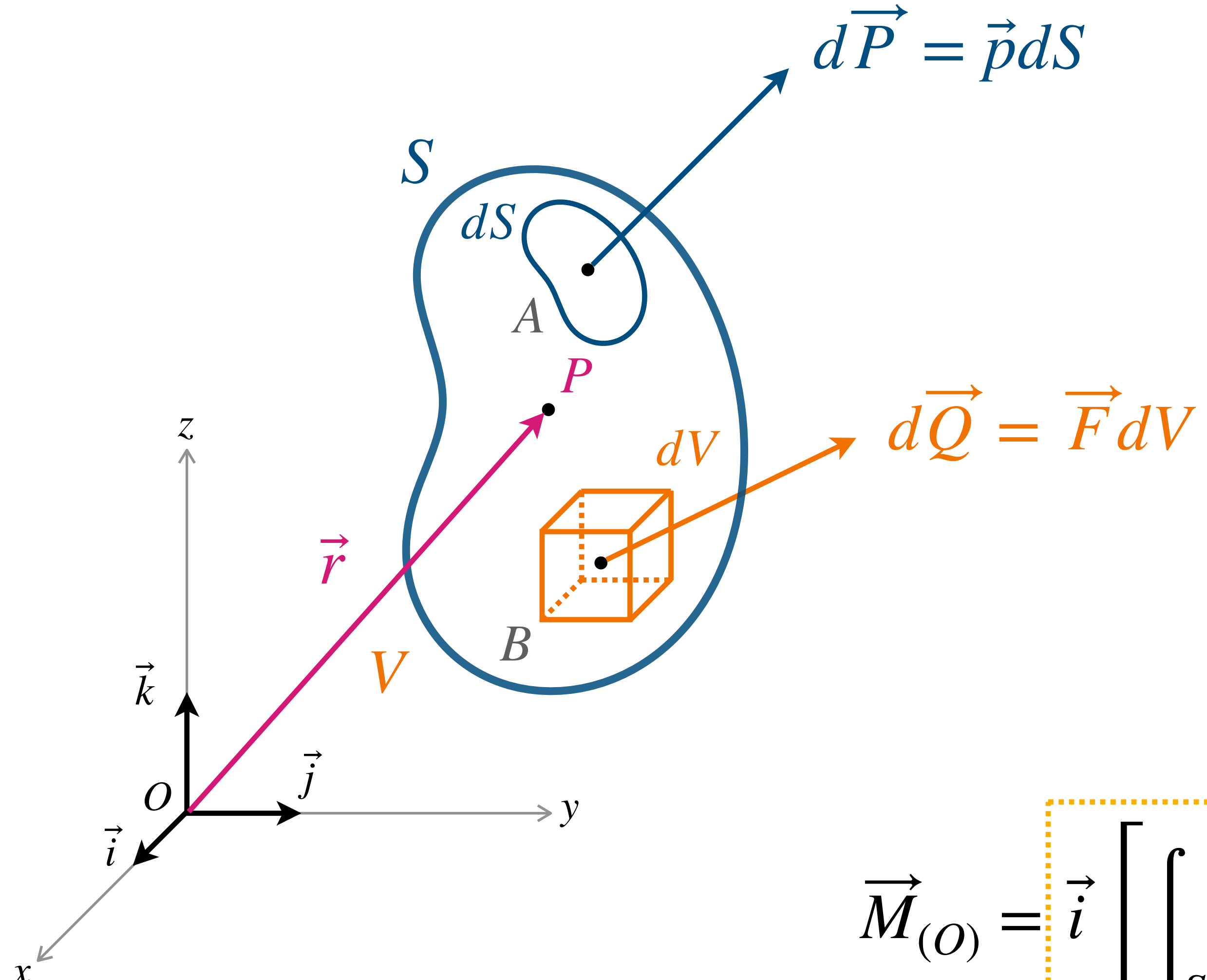

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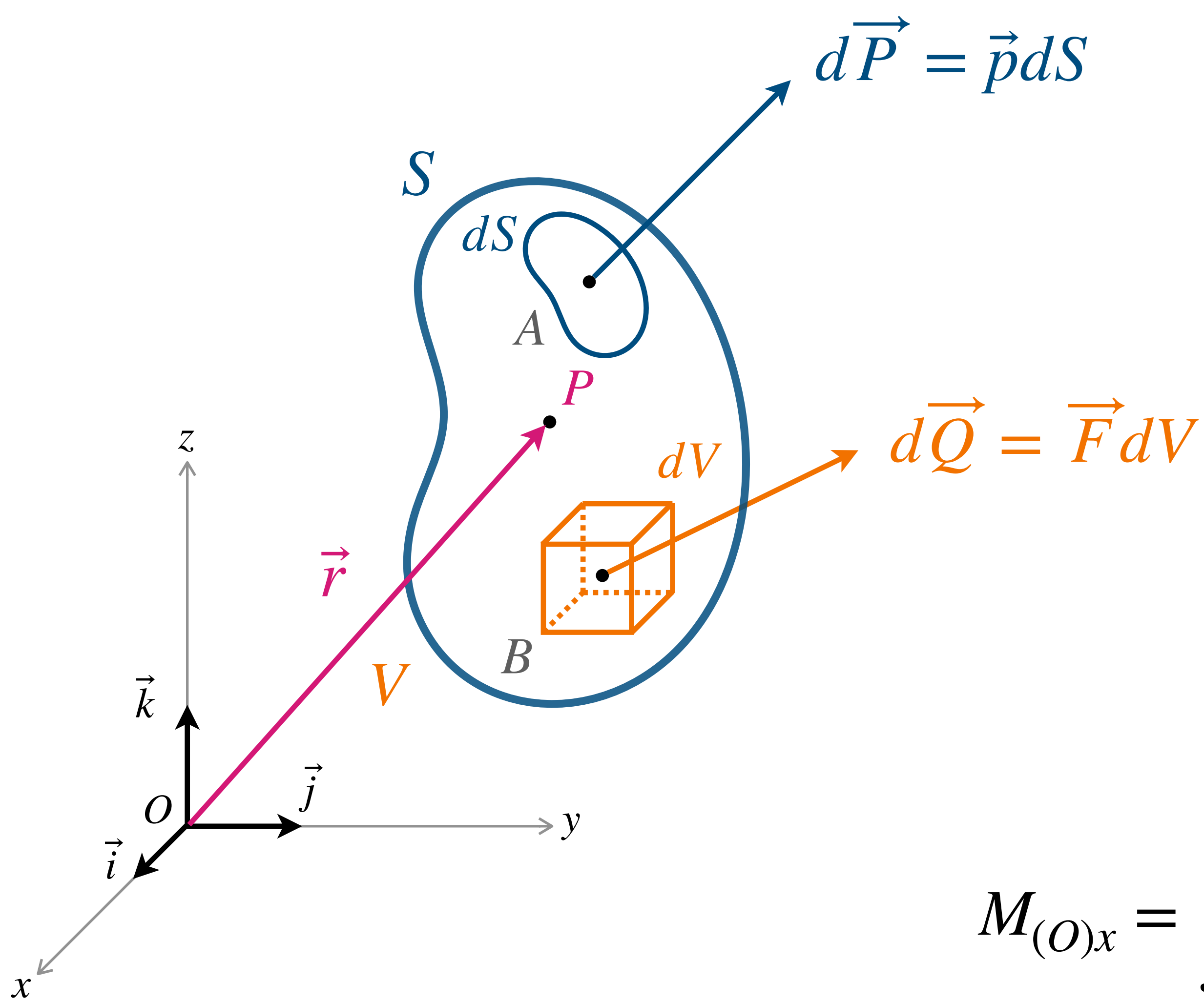

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CONDIZIONI DI EQUILIBRIO

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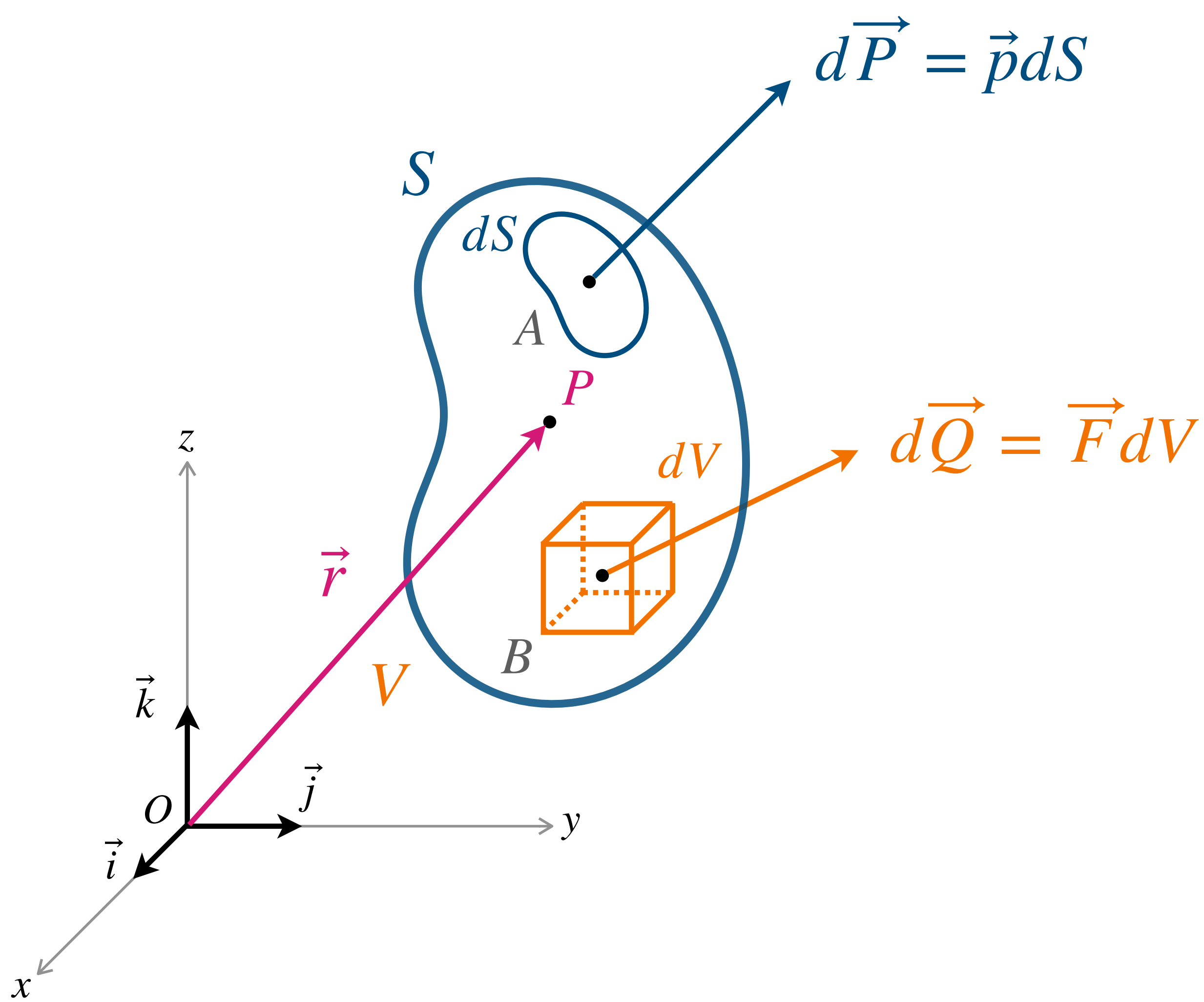
$$\overrightarrow{M}_{(O)} = \int_S \vec{r} \wedge \vec{p}dS + \int_V \vec{r} \wedge \overrightarrow{F}dV = \vec{0}$$

$$M_{(O)x} = \int_S \left(p_z y - p_y z\right) dS + \int_V \left(F_z y - F_y z\right) dV = 0$$

$$M_{(O)y} = \int_S \left(p_x z - p_z x\right) dS + \int_V \left(F_x z - F_z x\right) dV = 0$$

$$M_{(O)z} = \int_S \left(p_y x - p_x y\right) dS + \int_V \left(F_y x - F_x y\right) dV = 0$$

_ Azioni esterne



$$P = (x, y, z)$$

$$\vec{r} = \{x, y, z\}$$

CONDIZIONI DI EQUILIBRIO

$$\vec{R} = \vec{0} \qquad \& \qquad \vec{M}_{(O)} = \vec{0}$$

$$\vec{R} = \vec{0} \quad \begin{cases} R_x = 0 \\ R_y = 0 \\ R_z = 0 \end{cases}$$

$$R_x = \int_S p_x dS + \int_V F_x dV = 0$$

$$R_y = \int_S p_y dS + \int_V F_y dV = 0$$

$$R_z = \int_S p_z dS + \int_V F_z dV = 0$$

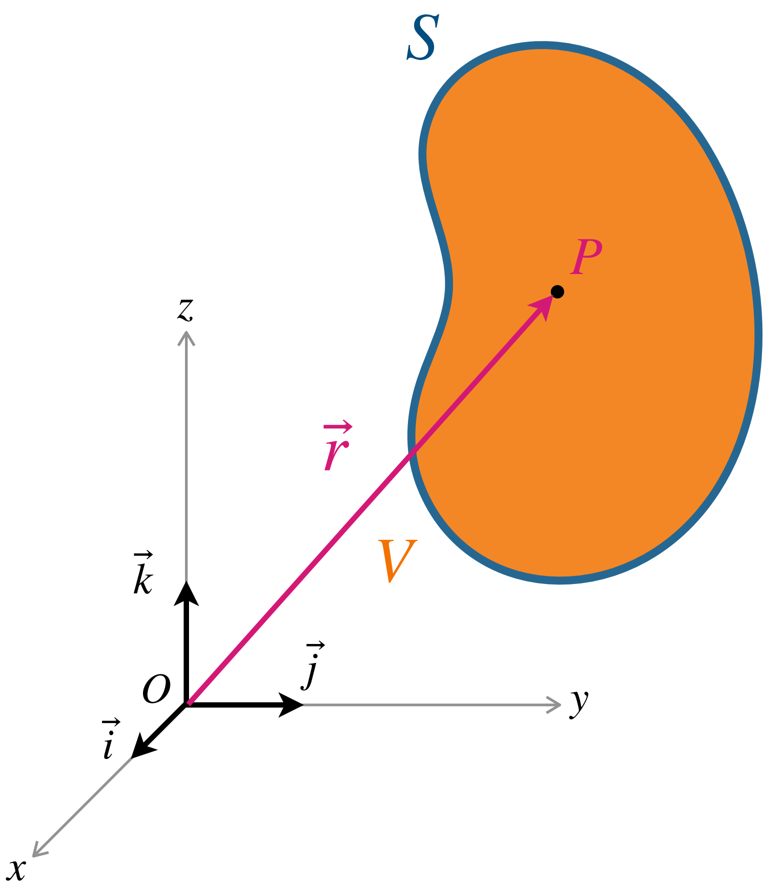
$$\vec{M}_{(O)} = \vec{0} \quad \begin{cases} M_{(O)x} = 0 \\ M_{(O)y} = 0 \\ M_{(O)z} = 0 \end{cases}$$

$$M_{(O)x} = \int_S \left(p_z y - p_y z\right) dS + \int_V \left(F_z y - F_y z\right) dV = 0$$

$$M_{(O)y} = \int_S \left(p_x z - p_z x\right) dS + \int_V \left(F_x z - F_z x\right) dV = 0$$

$$M_{(O)z} = \int_S \left(p_y x - p_x y\right) dS + \int_V \left(F_y x - F_x y\right) dV = 0$$

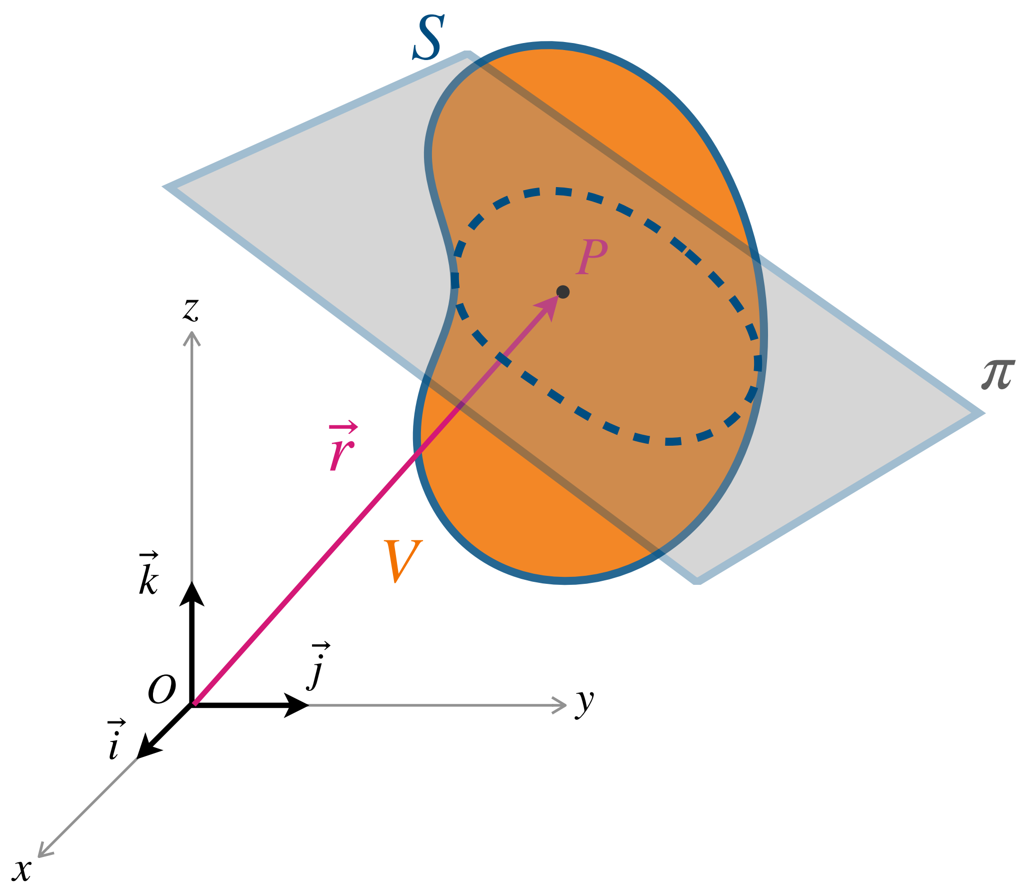
_ Azioni interne



$$P = \left(x, y, z\right)$$

$$\vec{r} = \left\{x, y, z\right\}$$

_ Azioni interne



$$P = \left(x, y, z\right)$$

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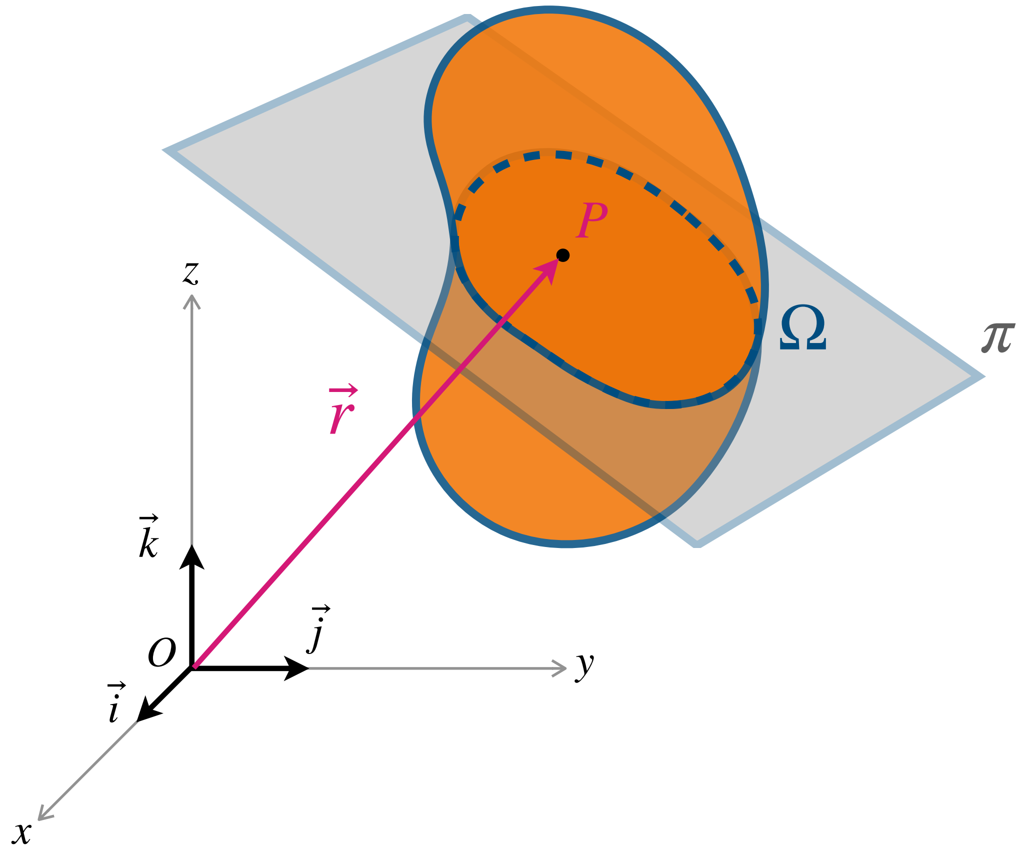
_ Azioni interne



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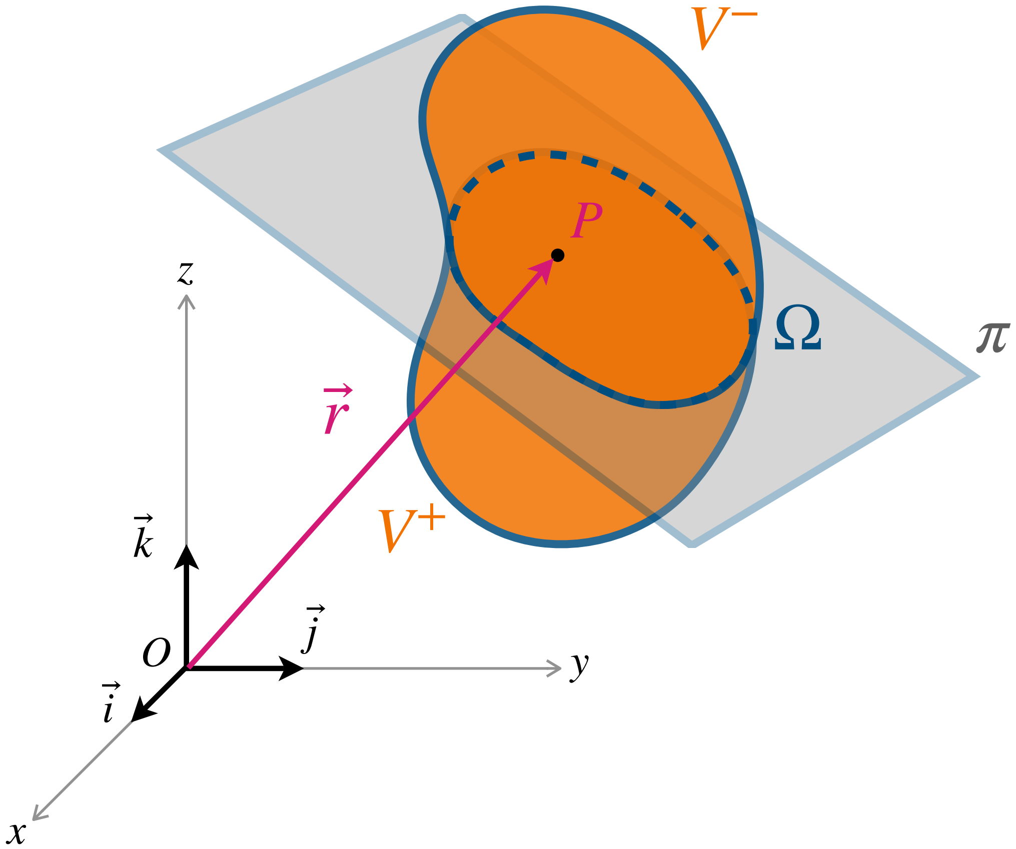

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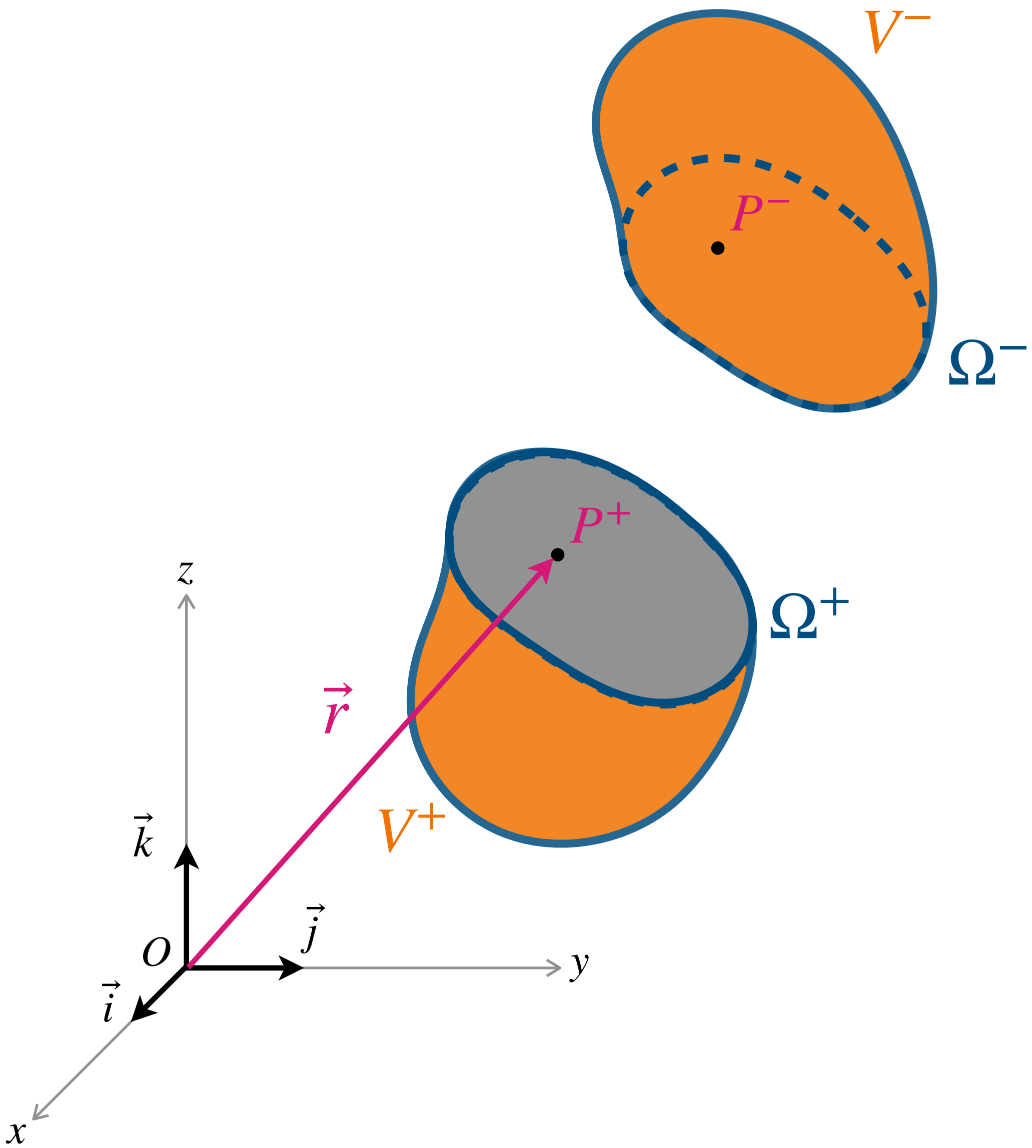

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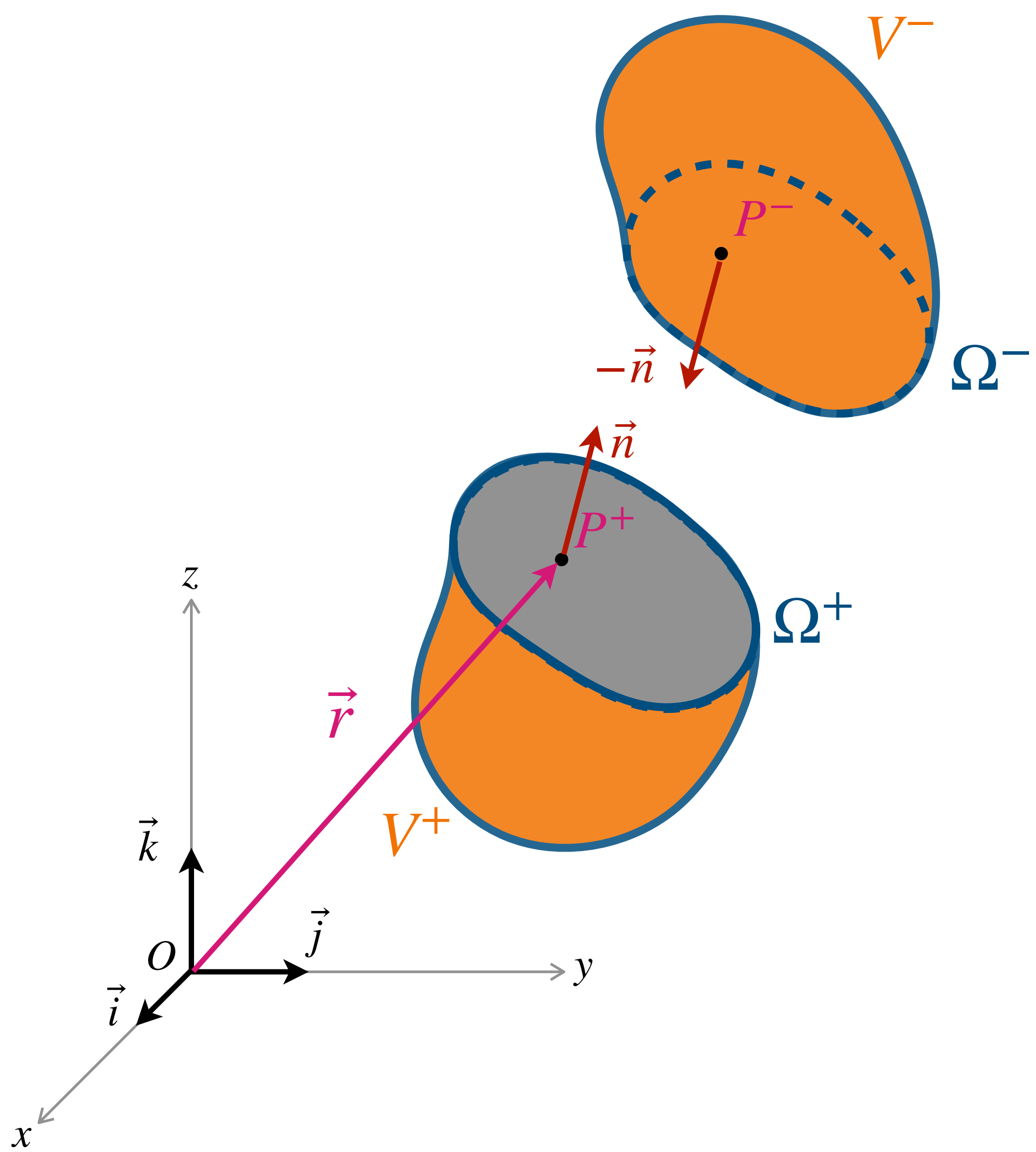
_ Azioni interne



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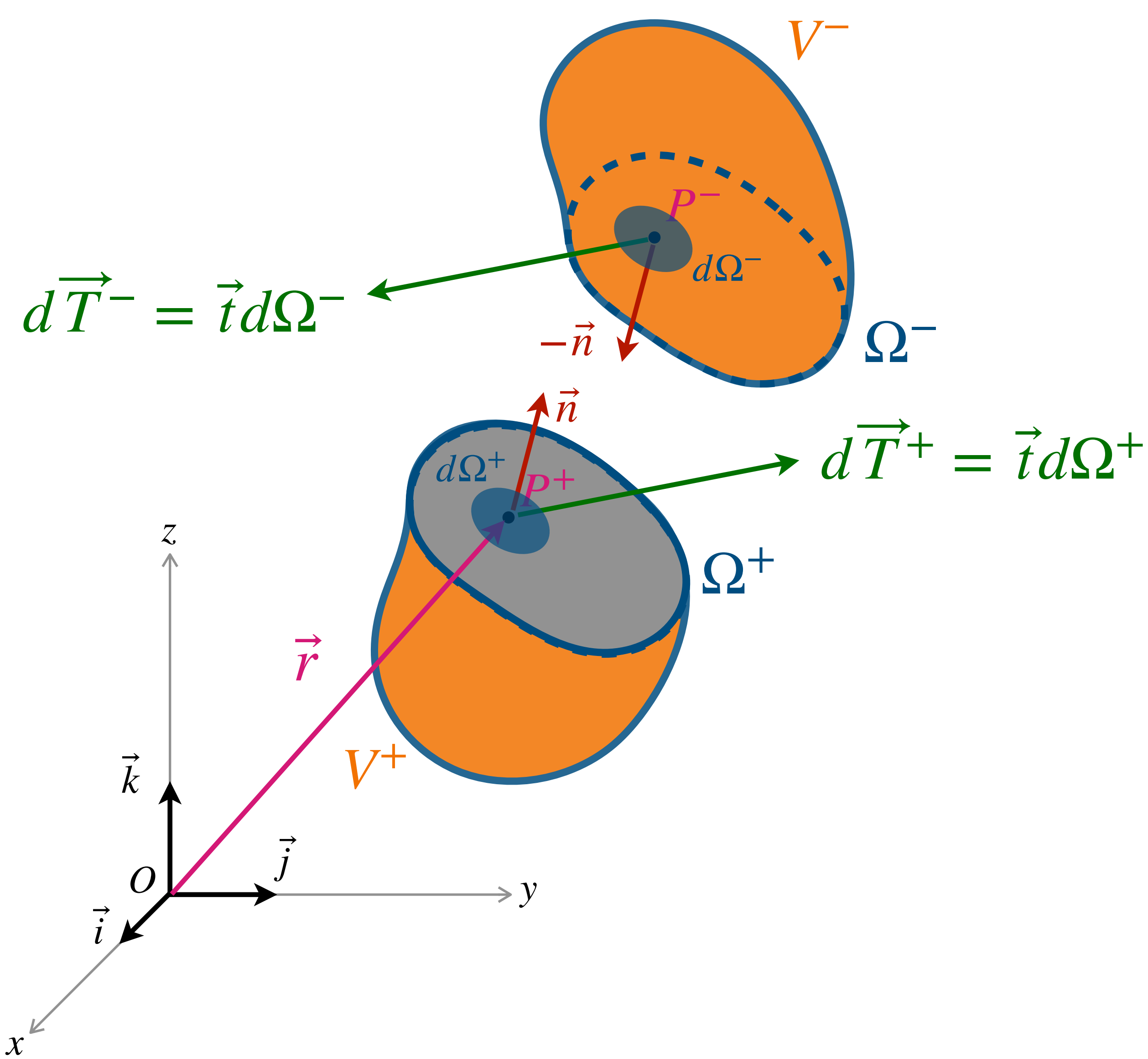
_ Azioni interne



$$P = (x, y, z)$$

$$\vec{r} = \{x, y, z\}$$

FORZE INTERNE $\vec{t}$

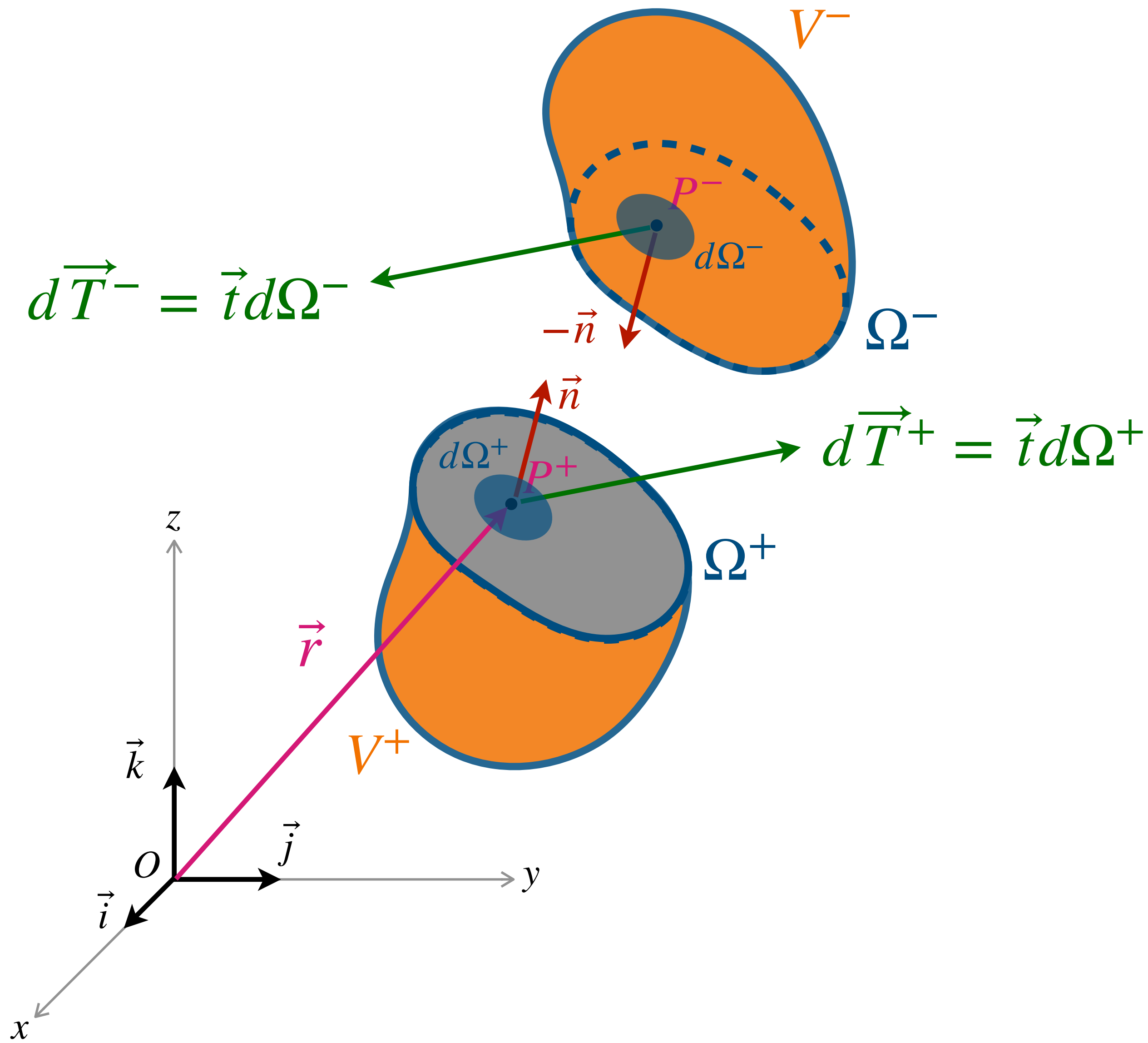
$$\vec{t} = t_x \vec{i} + t_y \vec{j} + t_z \vec{k} \qquad \vec{t} = \{t_x, t_y, t_z\}$$

$$t = \frac{[N]}{[L]^2}$$

PRESSIONE

$$Pa = N/m^2$$

_ Azioni interne



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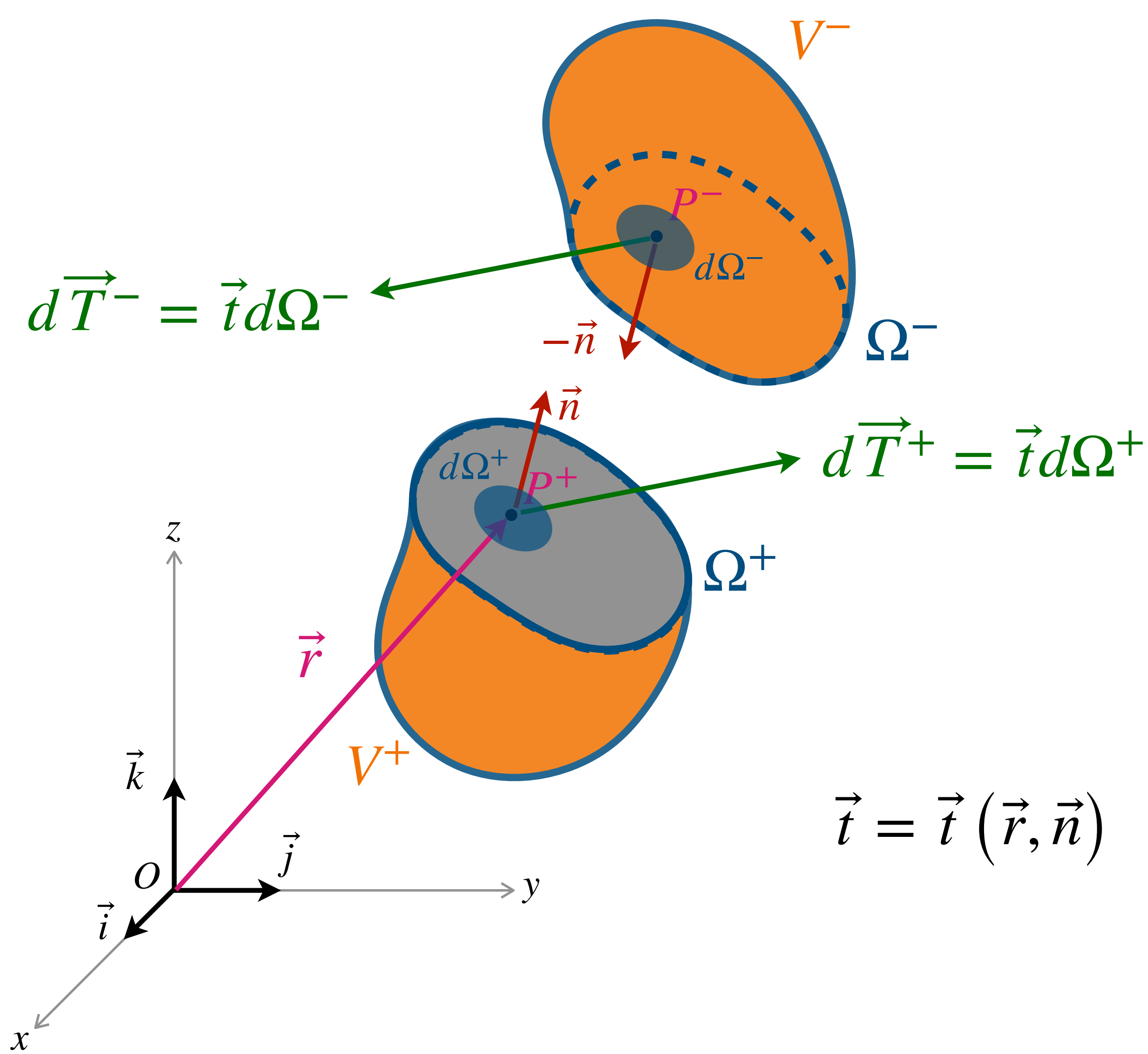
$\vec{t}$ ⇨ VETTORE SFORZO o TENSIONE

_ Azioni interne



$$\vec{t} = \vec{t}\left(\vec{r}, \vec{n}\right)$$

FORZE INTERNE $\vec{t}$

$$\vec{t} = t_x \vec{i} + t_y \vec{j} + t_z \vec{k} \qquad \vec{t} = \left\{ t_x, t_y, t_z \right\}$$

$$t = \frac{[N]}{[L]^2}$$ PRESSIONE $$Pa = N/m^2$$

$\vec{t}$ ⇒ VETTORE SFORZO o TENSIONE

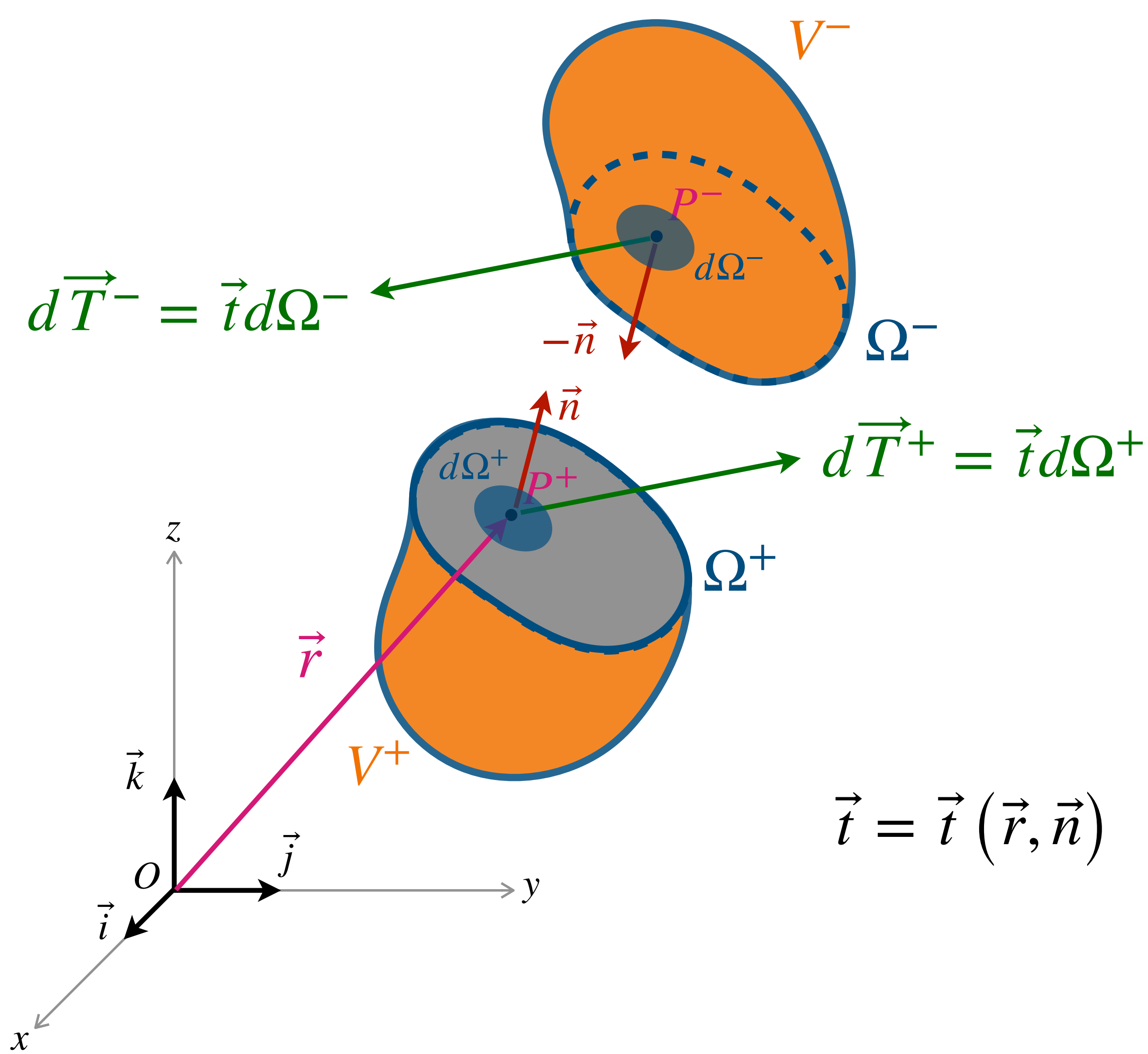
$$\vec{r} = \left\{ x, y, z \right\}$$

$$\vec{n} = \left\{ n_x, n_y, n_z \right\}$$

$$P = \left(x, y, z\right)$$

$$\vec{r} = \left\{ x, y, z \right\}$$

_ Azioni interne



$$\vec{t} = \vec{t}\,(\vec{r}, \vec{n})$$

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$$\vec{t} = t_x \vec{i} + t_y \vec{j} + t_z \vec{k} \qquad \vec{t} = \{t_x, t_y, t_z\}$$

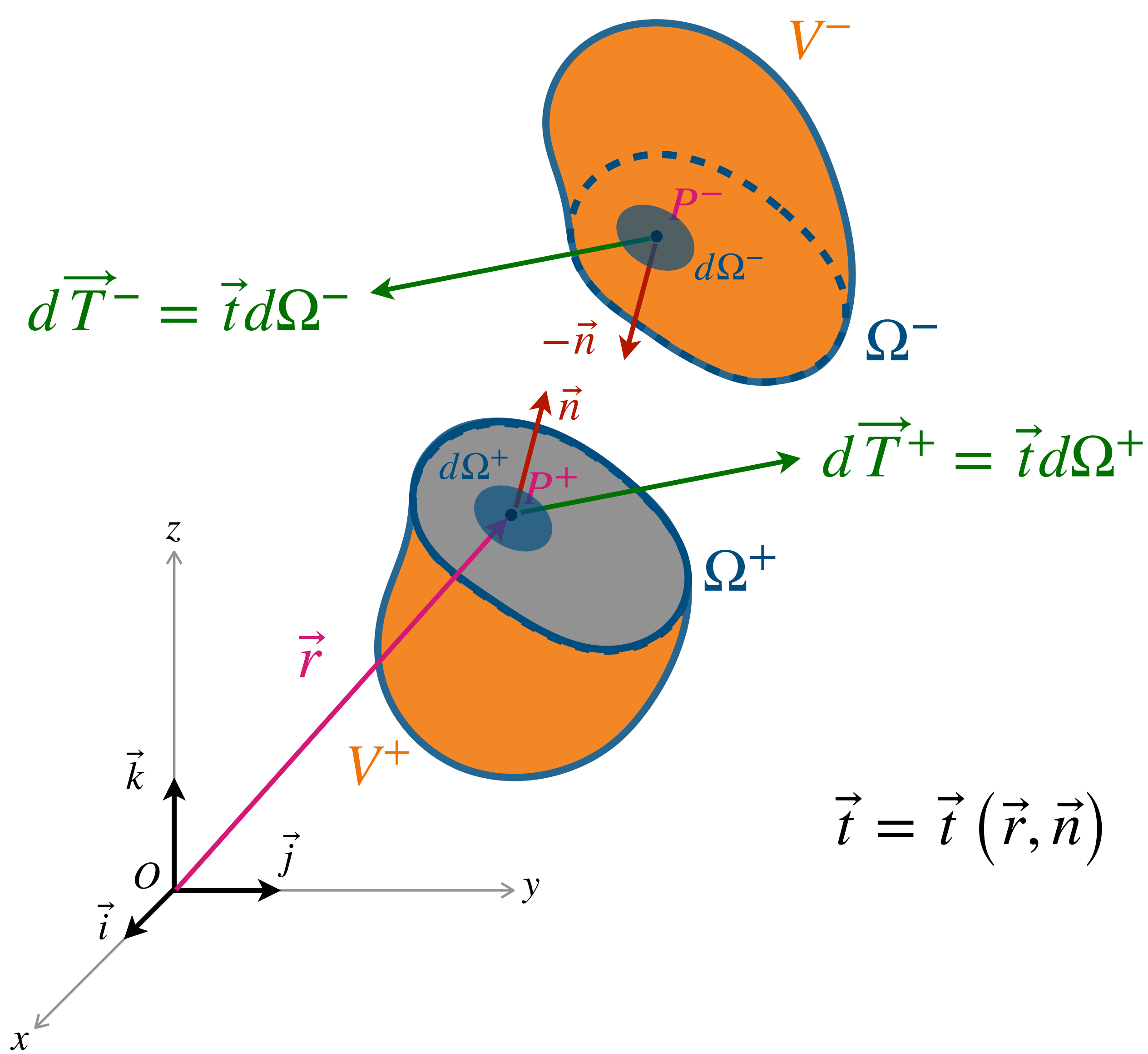
$$t = \frac{[N]}{[L]^2}$$ PRESSIONE $$Pa = N/m^2$$

$\vec{t}$ ⇨ VETTORE SFORZO o TENSIONE

$$\boxed{\vec{r} = \{x, y, z\}}$$

$$\vec{n} = \{n_x, n_y, n_z\}$$

_ Azioni interne



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$$\vec{r} = \left\{ x, y, z \right\}$$

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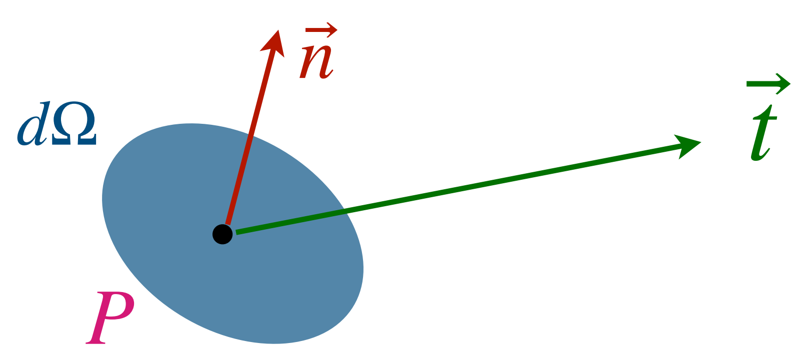
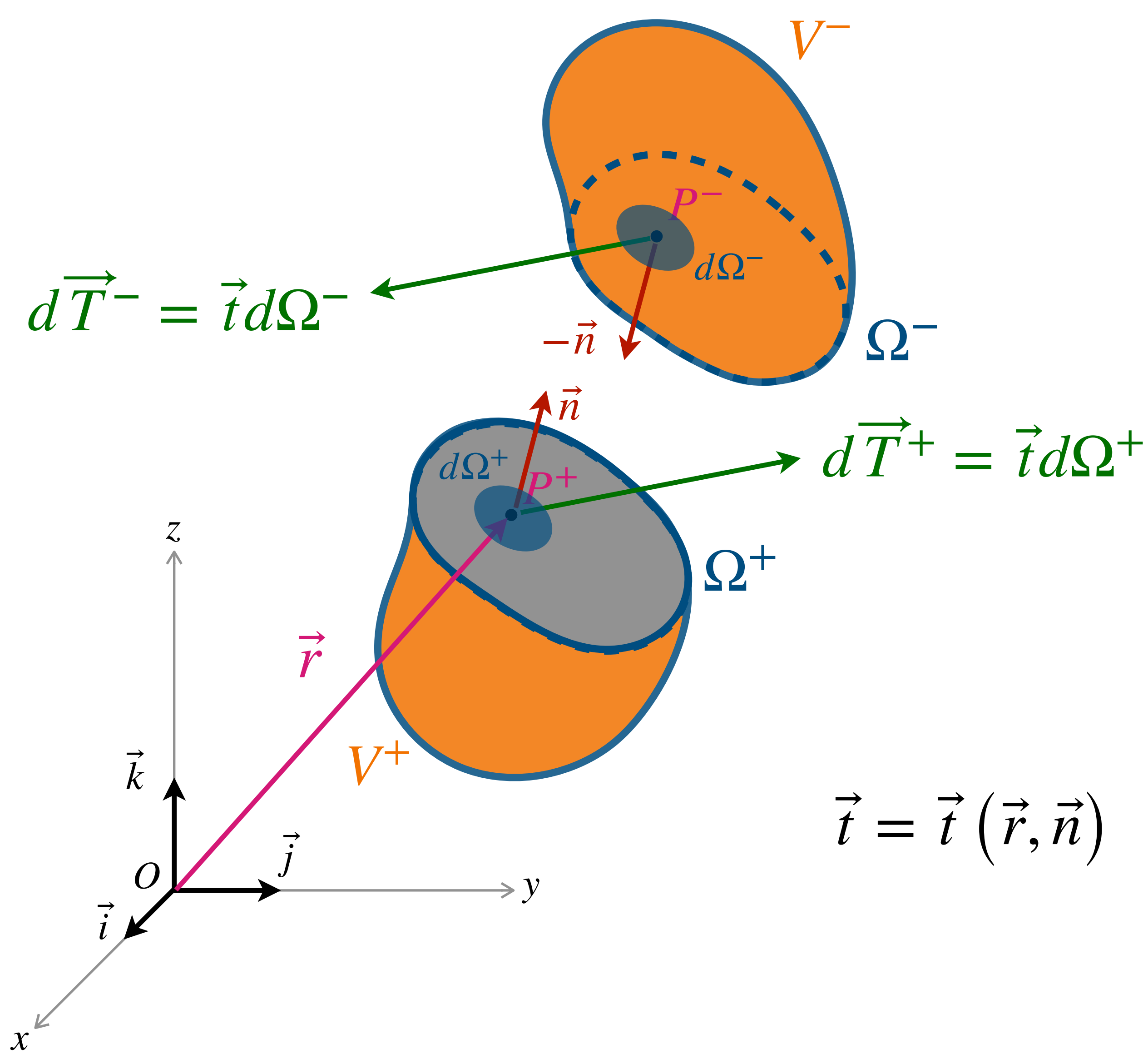
$$\boxed{\begin{array}{l} P = (x, y, z) \\ \vec{r} = \left\{ x, y, z \right\} \end{array}}$$



_ Azioni interne



FORZE INTERNE $\vec{t}$

$$\vec{t} = t_x\vec{i} + t_y\vec{j} + t_z\vec{k} \qquad \vec{t} = \left\{t_x, t_y, t_z\right\}$$

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$\vec{t}$ ⇨ VETTORE SFORZO o TENSIONE

$$\vec{t} = \vec{t}\left(\vec{r}, \vec{n}\right)$$

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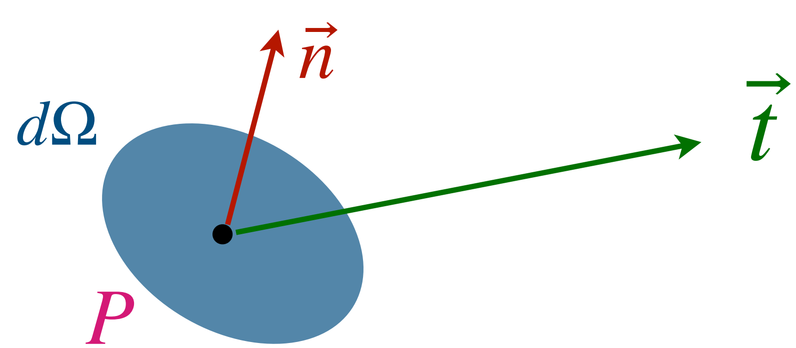


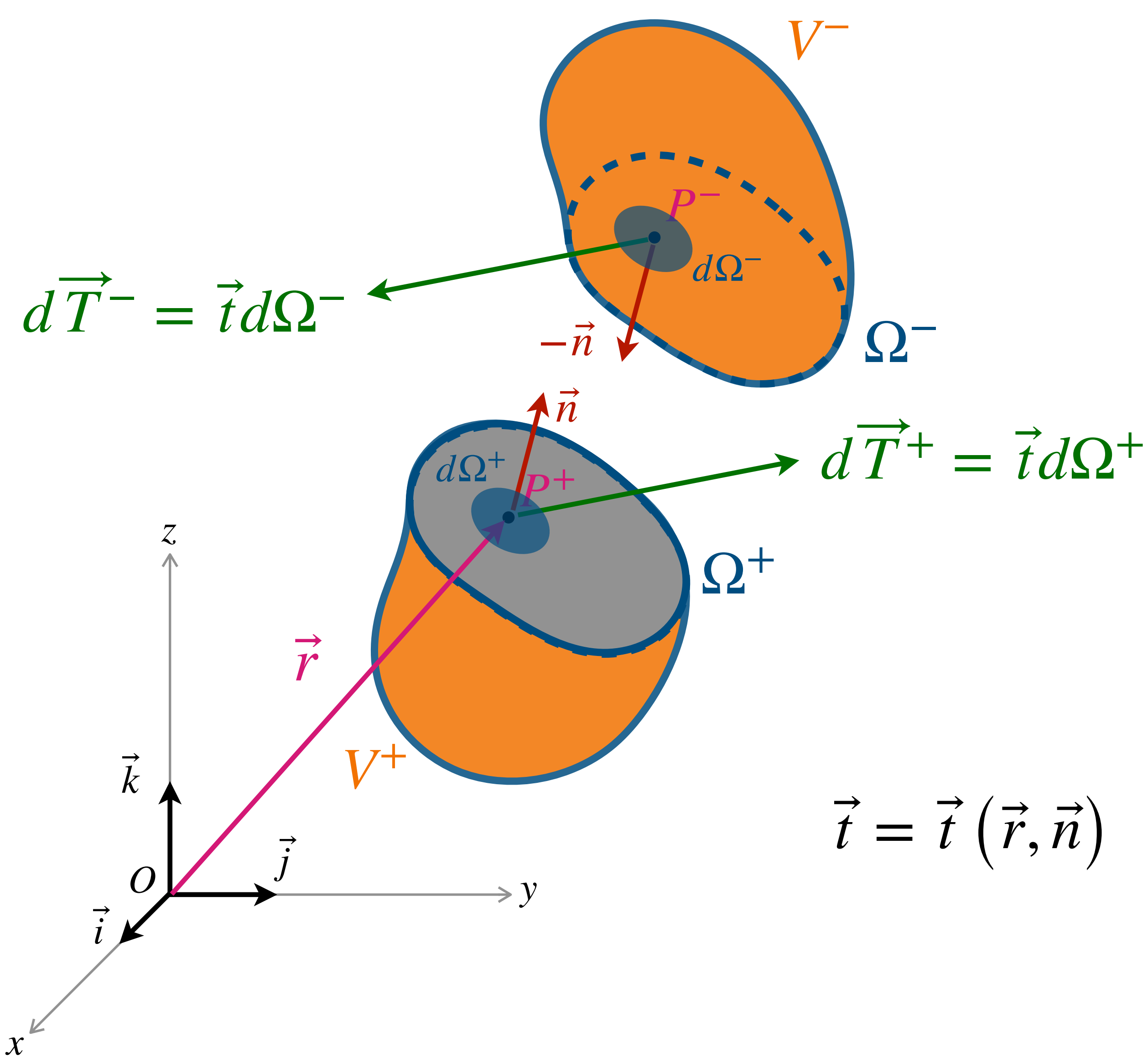
$$\alpha_x = \vec{n} \times \vec{i} \qquad \alpha_x = \cos\left\{\widehat{\vec{n}, \vec{i}}\right\}$$

$$\alpha_y = \vec{n} \times \vec{j} \qquad \alpha_y = \cos\left\{\widehat{\vec{n}, \vec{j}}\right\}$$

$$\alpha_z = \vec{n} \times \vec{k} \qquad \alpha_z = \cos\left\{\widehat{\vec{n}, \vec{k}}\right\}$$

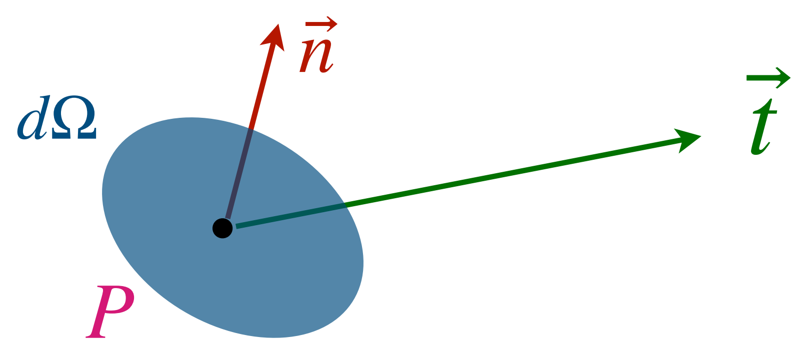
$$\left(\alpha_x^2 + \alpha_y^2 + \alpha_z^2\right) = |\vec{n}|^2 = 1$$

_ Azioni interne



$$\vec{t} = \vec{t}\left(\vec{r}, \vec{n}\right)$$

$$\boxed{\begin{array}{l} P = \left(x, y, z\right) \\ \vec{r} = \left\{x, y, z\right\} \end{array}}$$



FORZE INTERNE $\vec{t}$

$$\vec{t} = t_x\vec{i} + t_y\vec{j} + t_z\vec{k} \qquad \vec{t} = \left\{t_x, t_y, t_z\right\}$$

$$t = \frac{[N]}{[L]^2}$$ PRESSIONE $$Pa = N/m^2$$

$\vec{t}$ ⇨ VETTORE SFORZO o TENSIONE

$$\vec{r} = \left\{x, y, z\right\}$$

$$\vec{n} = \left\{\alpha_x, \alpha_y, \alpha_z\right\} \qquad \vec{n} = \alpha_x\vec{i} + \alpha_y\vec{j} + \alpha_z\vec{k}$$

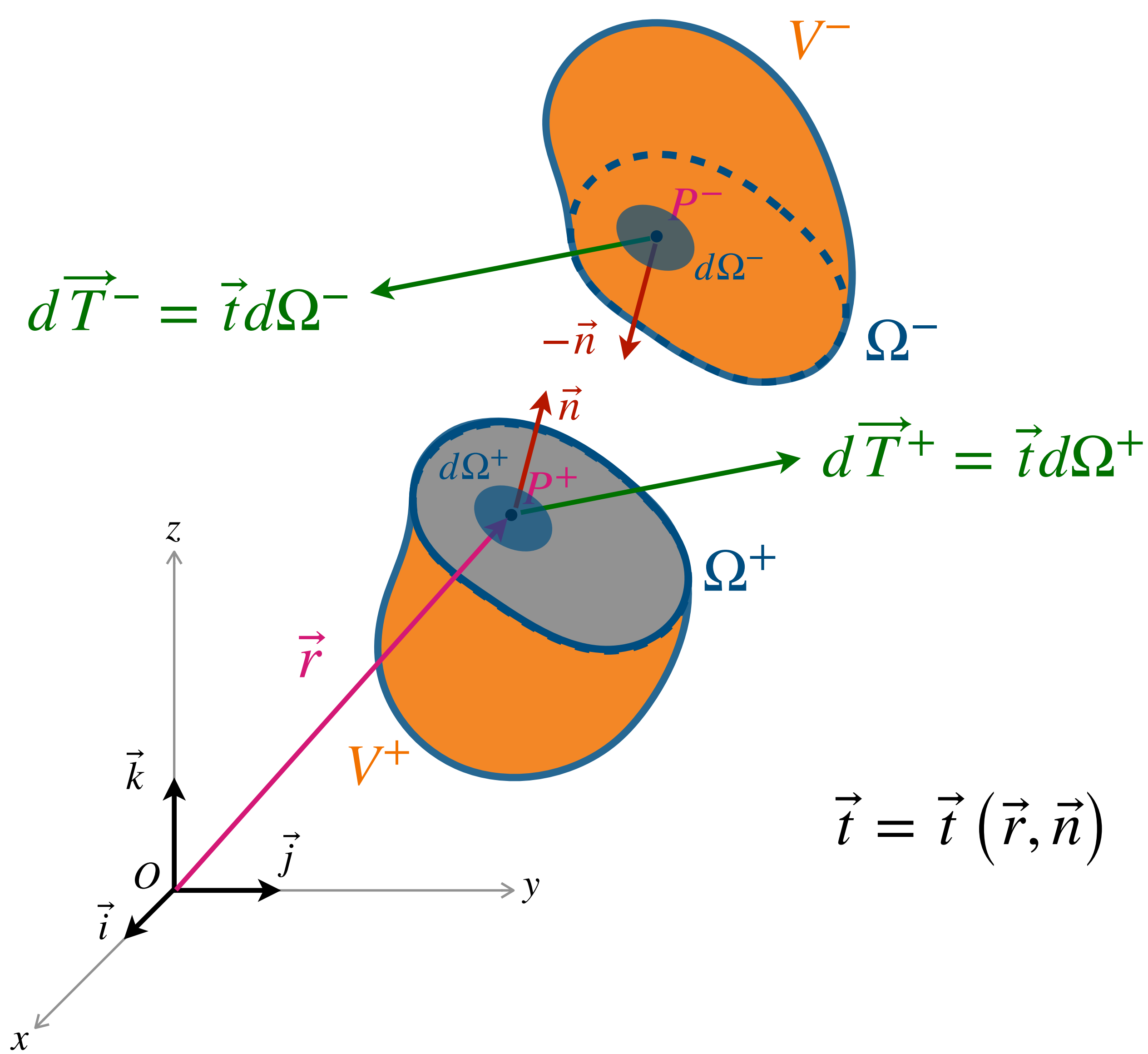
$$\alpha_x = \vec{n} \times \vec{i} \qquad \alpha_x = \cos\left\{\widehat{\vec{n}, \vec{i}}\right\}$$

$$\alpha_y = \vec{n} \times \vec{j} \qquad \alpha_y = \cos\left\{\widehat{\vec{n}, \vec{j}}\right\}$$

$$\alpha_z = \vec{n} \times \vec{k} \qquad \alpha_z = \cos\left\{\widehat{\vec{n}, \vec{k}}\right\}$$

$$\left(\alpha_x^2 + \alpha_y^2 + \alpha_z^2\right) = |\vec{n}|^2 = 1$$

_ Azioni interne



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$$t = \frac{[N]}{[L]^2}$$

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$\vec{t}$ ⇨ VETTORE SFORZO o TENSIONE

$$\vec{r} = \left\{x, y, z\right\}$$

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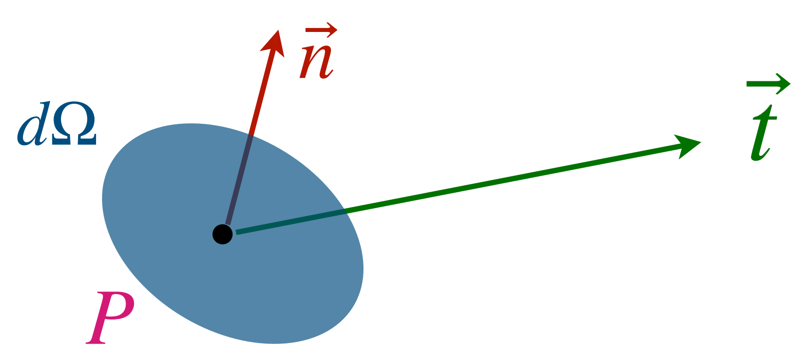
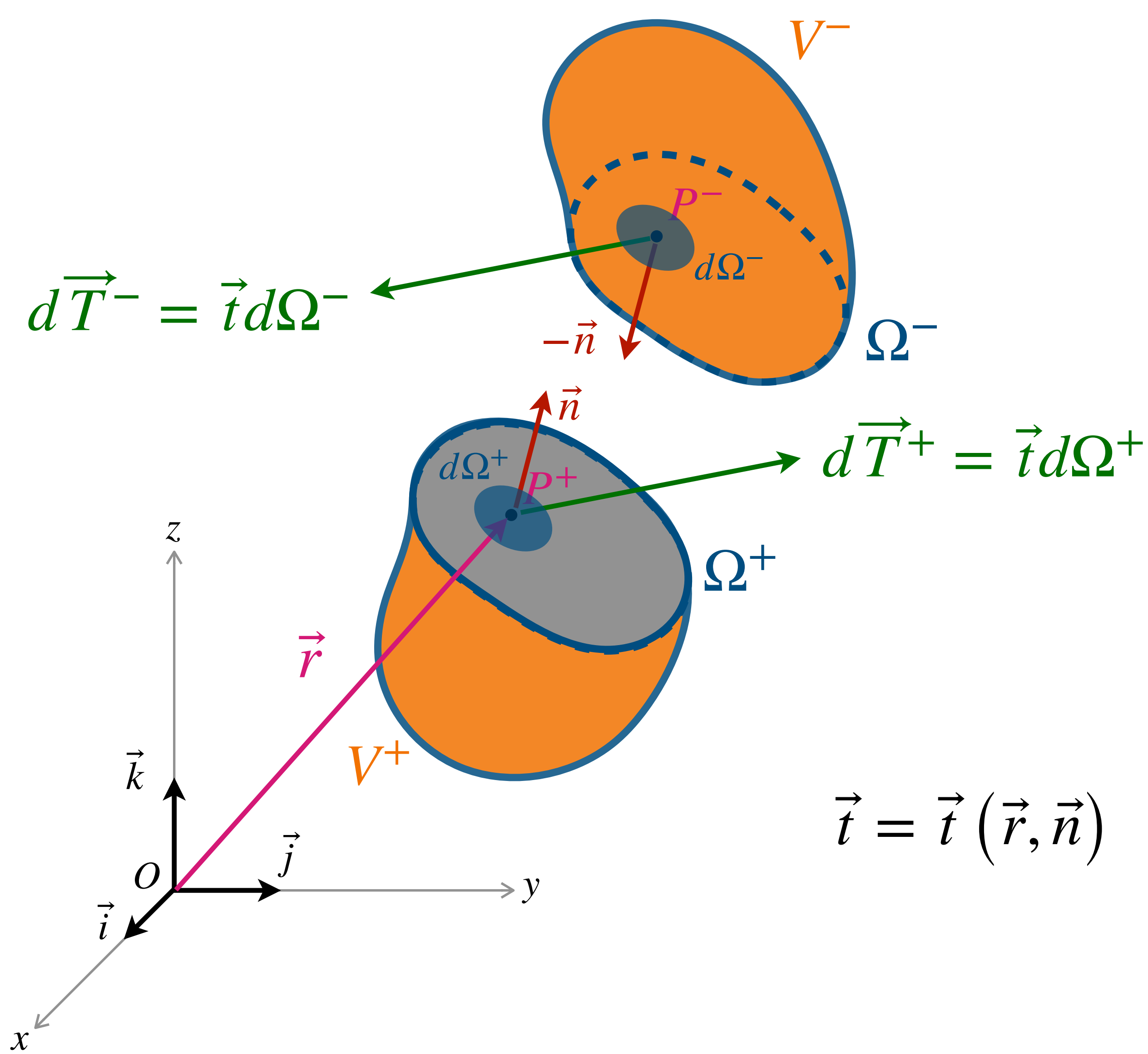


_ Azioni interne



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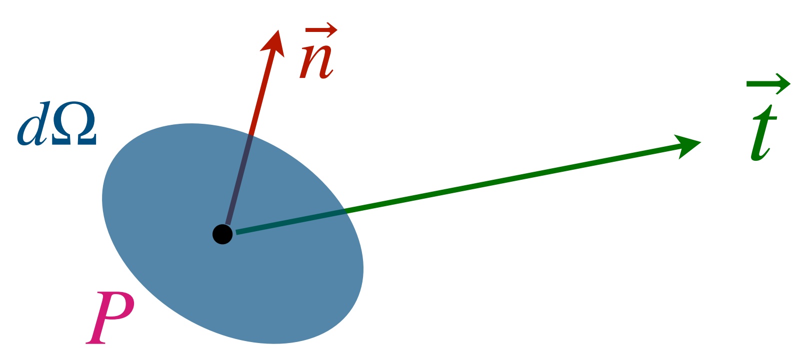
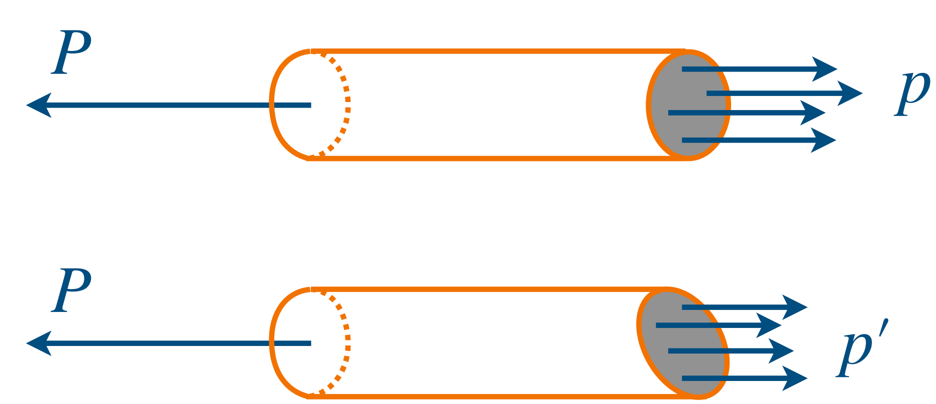
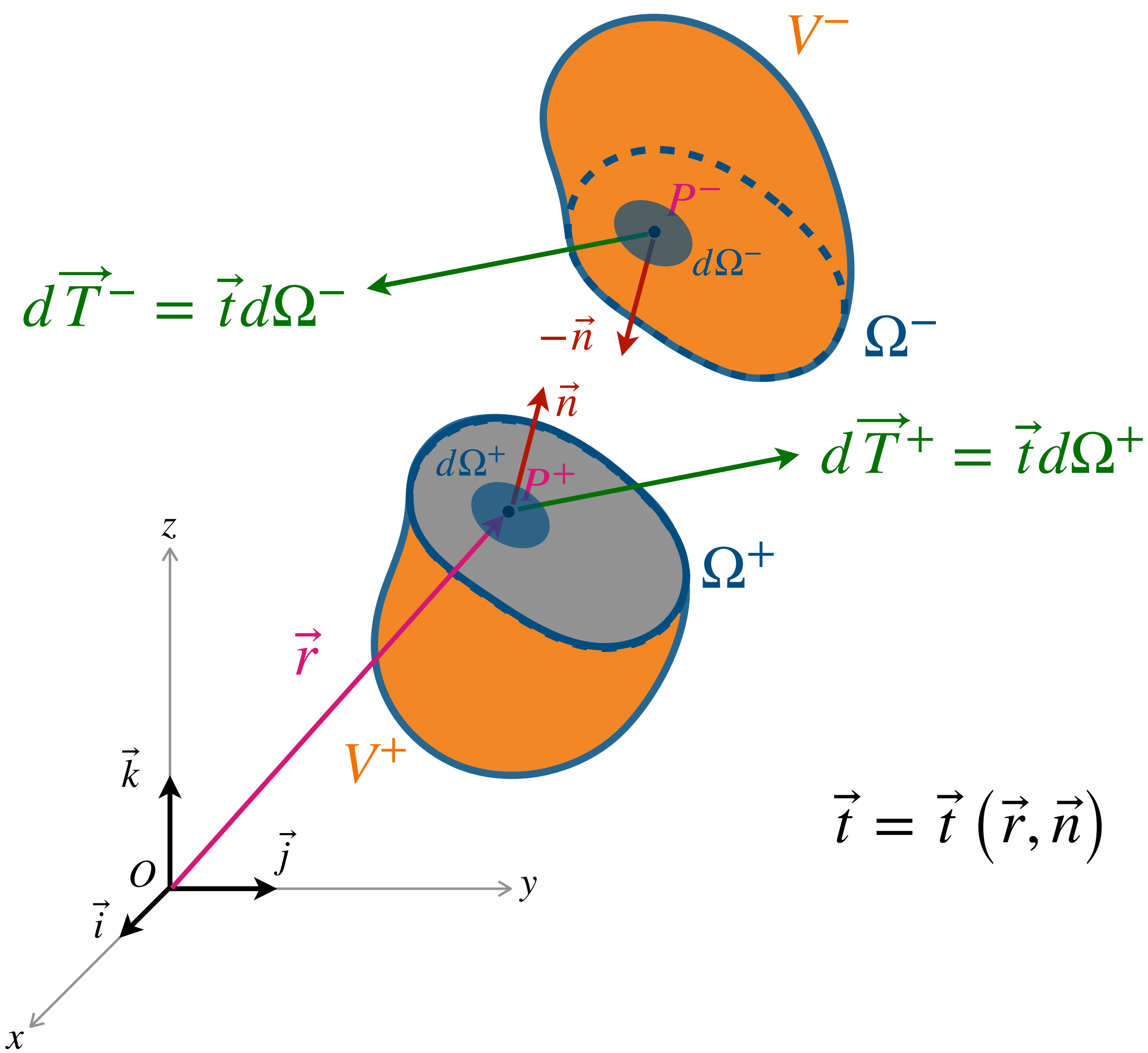




_ Azioni interne



FORZE INTERNE $\vec{t}$

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$$\vec{r} = \left\{ x, y, z \right\}$$

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$$P = \left(x, y, z\right)$$

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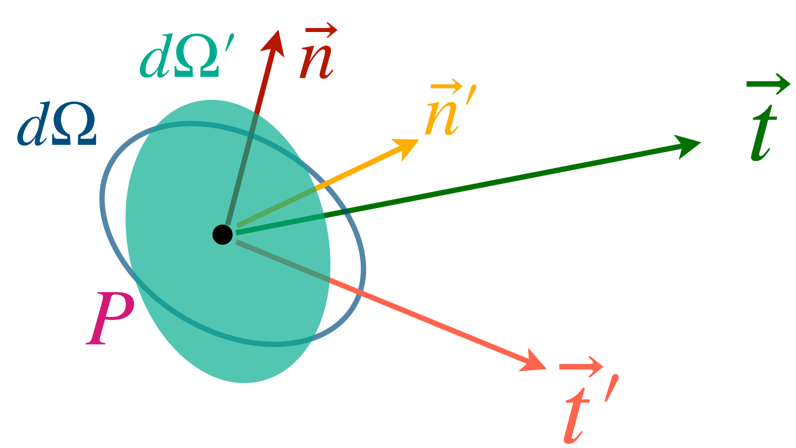
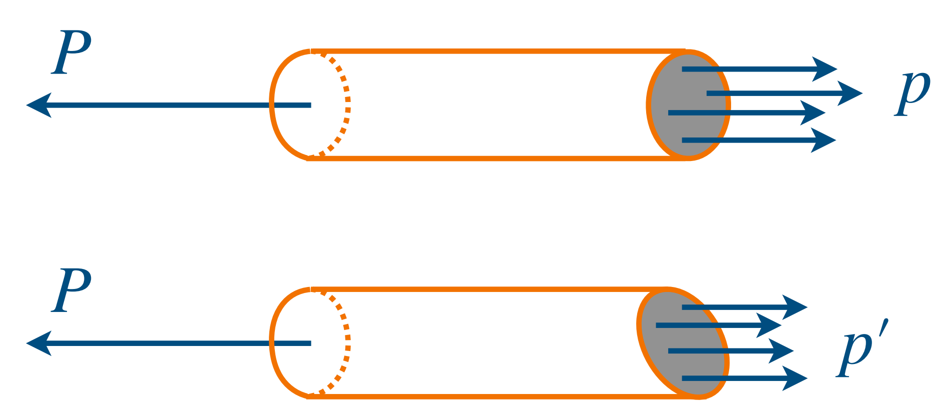
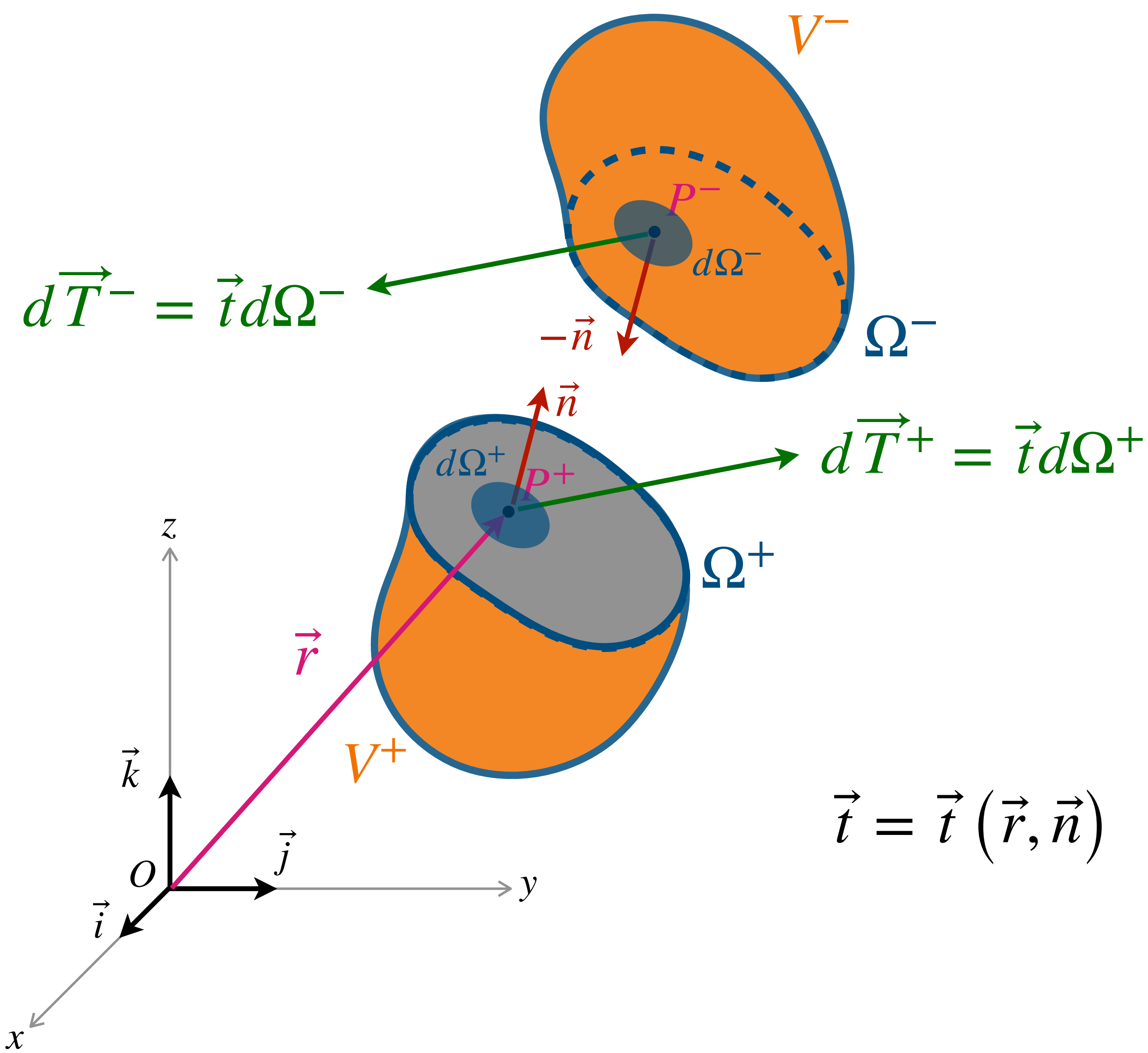




_ Azioni interne



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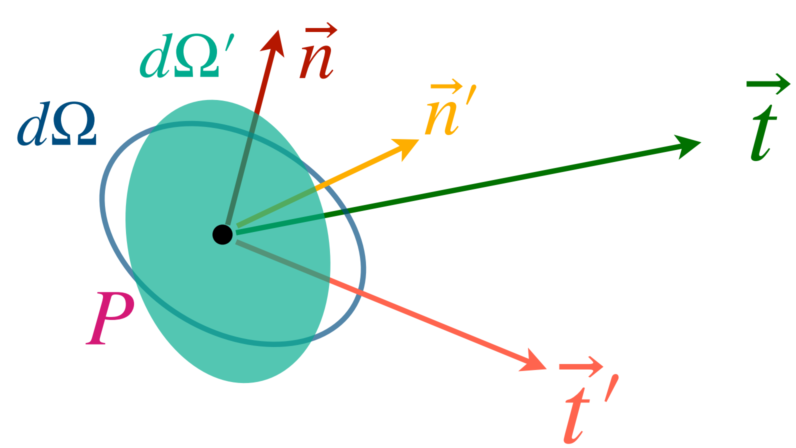
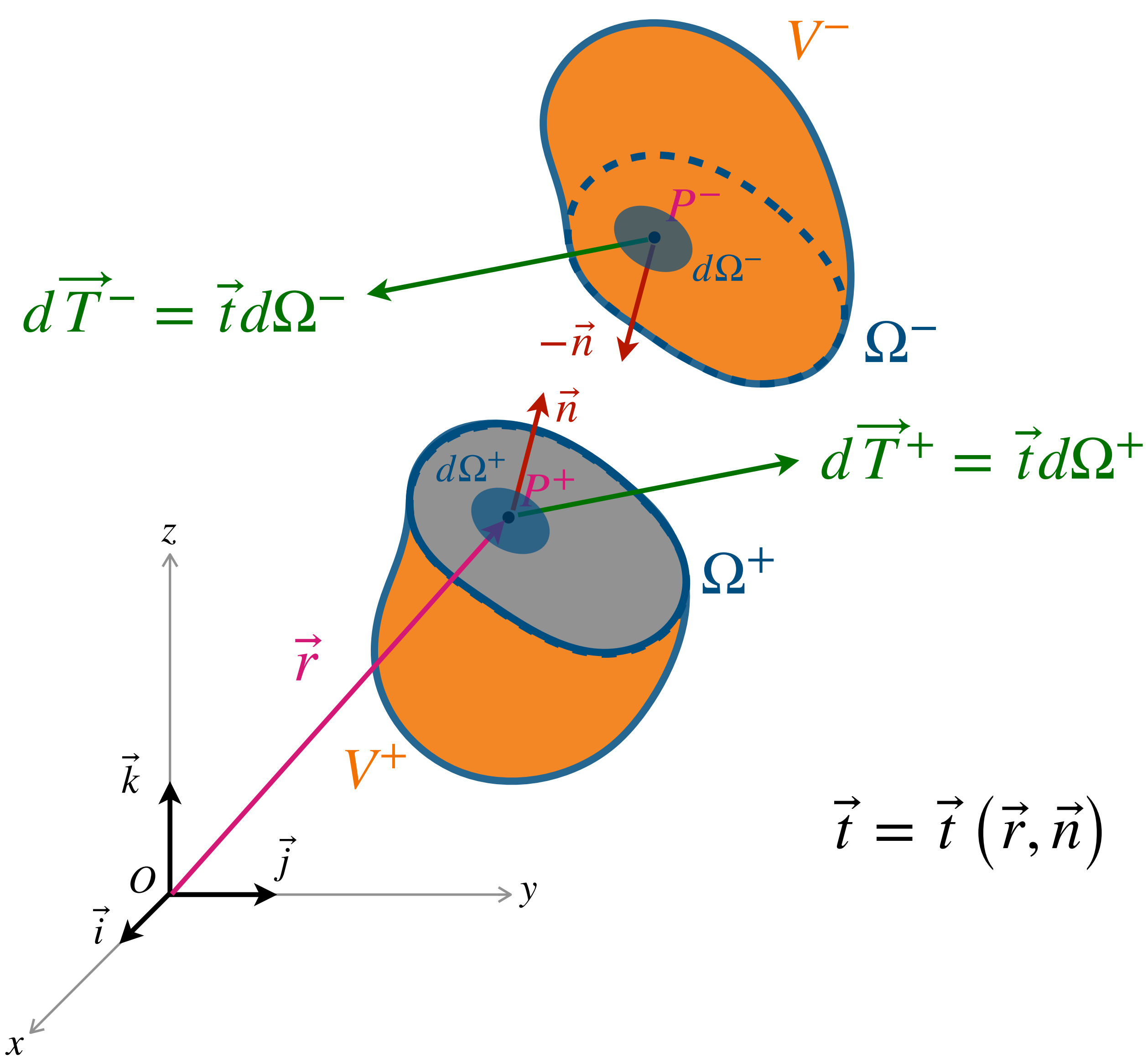


$$\vec{t} = \vec{t}\left(\vec{r}, \vec{n}\right) \neq \vec{t}' = \vec{t}'\left(\vec{r}, \vec{n}'\right)$$

_ Azioni interne



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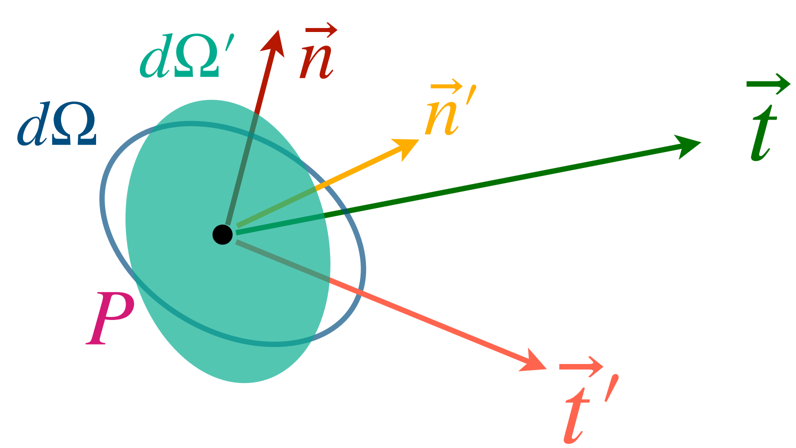
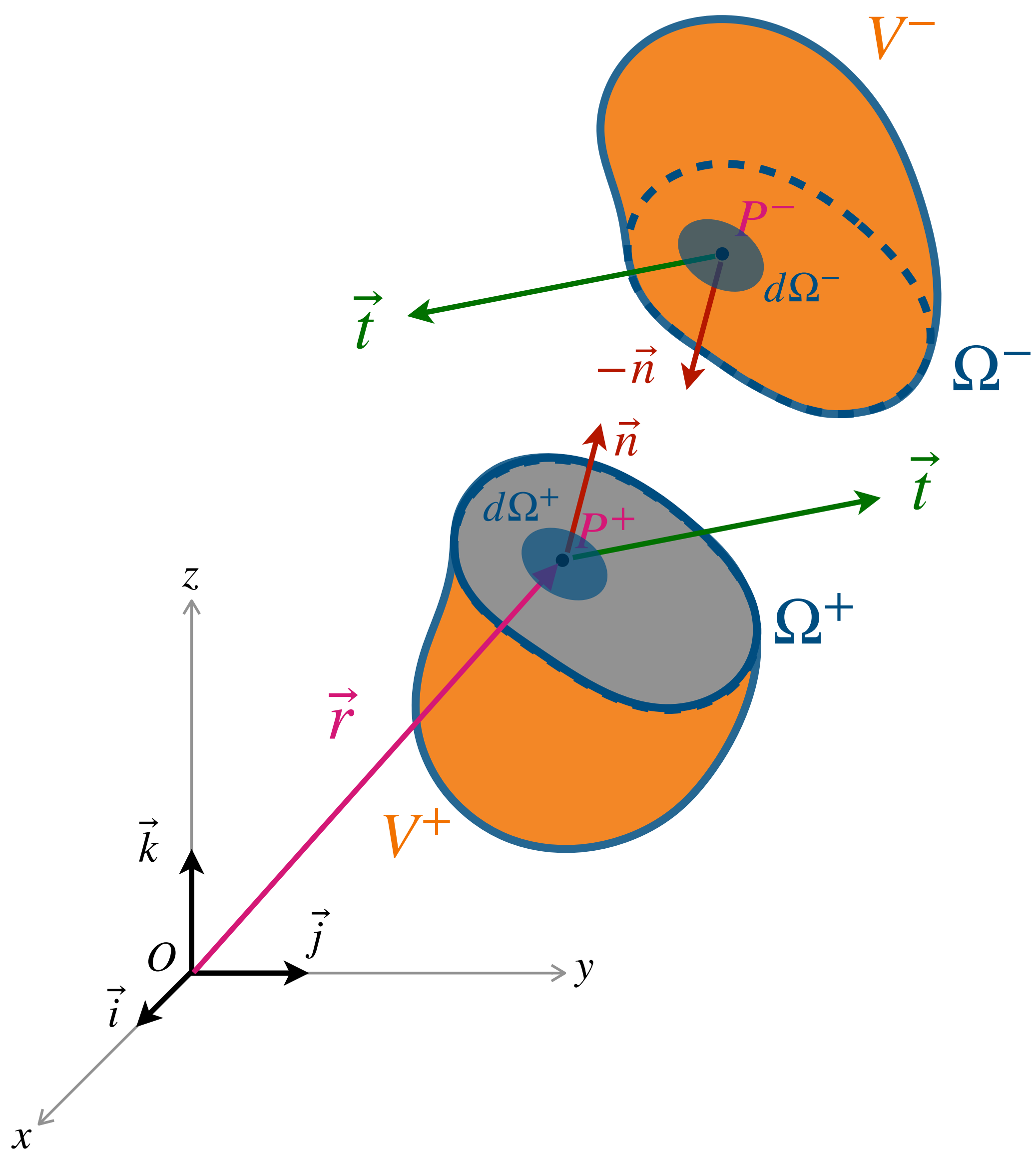


$$\vec{t} = \vec{t}\left(\vec{r}, \vec{n}\right) \neq \vec{t}' = \vec{t}'\left(\vec{r}, \vec{n}'\right)$$

Per ogni giacitura si ha un vettore sforzo!

_ Azioni interne



$$P = (x, y, z)$$

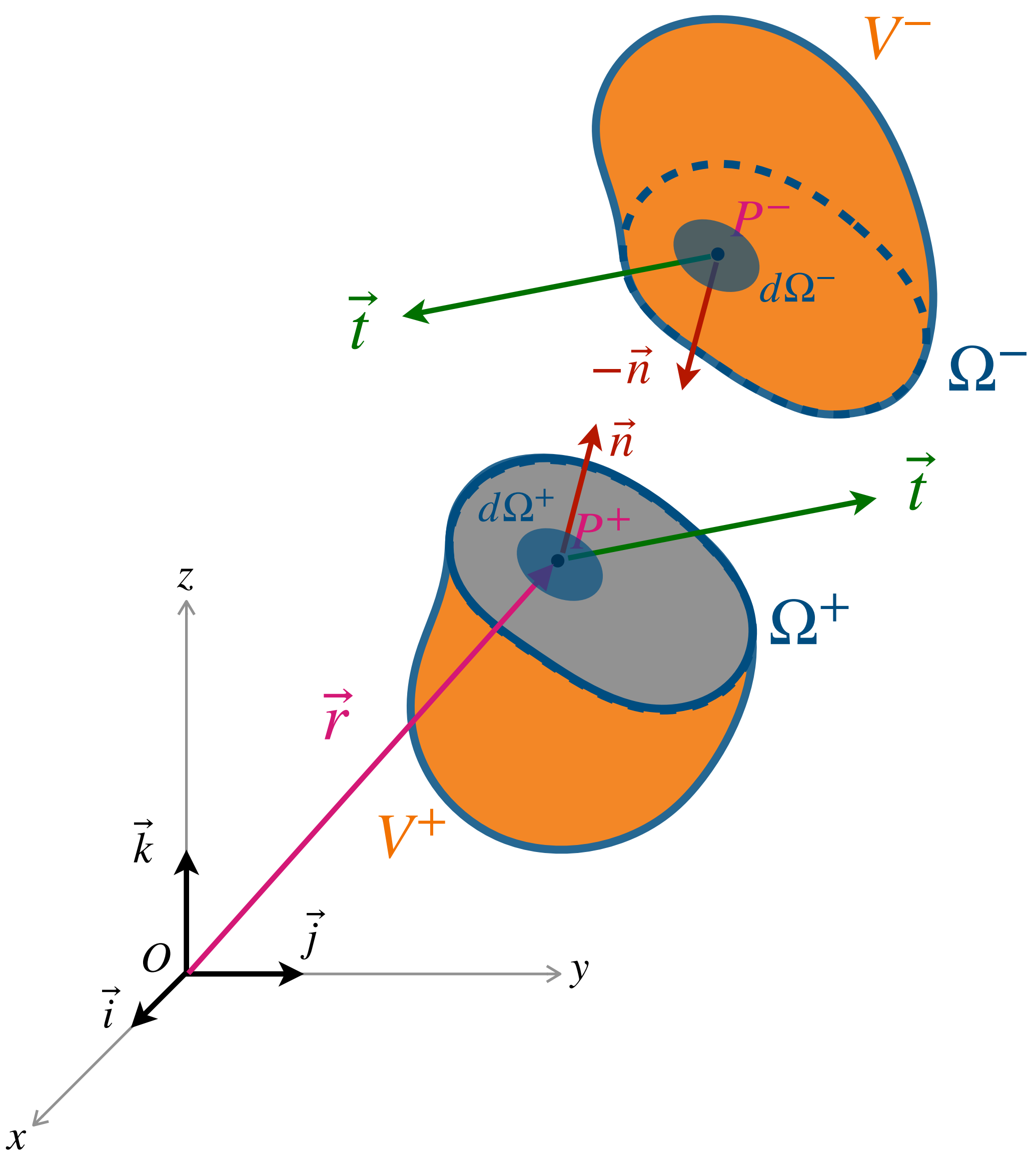
$$\vec{r} = \{x, y, z\}$$

FORZE INTERNE $\vec{t}$

Devono rispettare il principio di reciprocità

$$\vec{t}\,(\vec{r}, \vec{n})\, d\Omega^+ + \vec{t}\,(\vec{r}, -\vec{n})\, d\Omega^- = \vec{0}$$

_ Azioni interne



FORZE INTERNE $\vec{t}$

Devono rispettare il principio di reciprocità

$$\underbrace{\vec{t}\left(\vec{r},\vec{n}\right)d\Omega^+}_{\vec{n}} + \underbrace{\vec{t}\left(\vec{r},-\vec{n}\right)d\Omega^-}_{-\vec{n}} = \vec{0}$$

$$P = \left(x, y, z\right)$$

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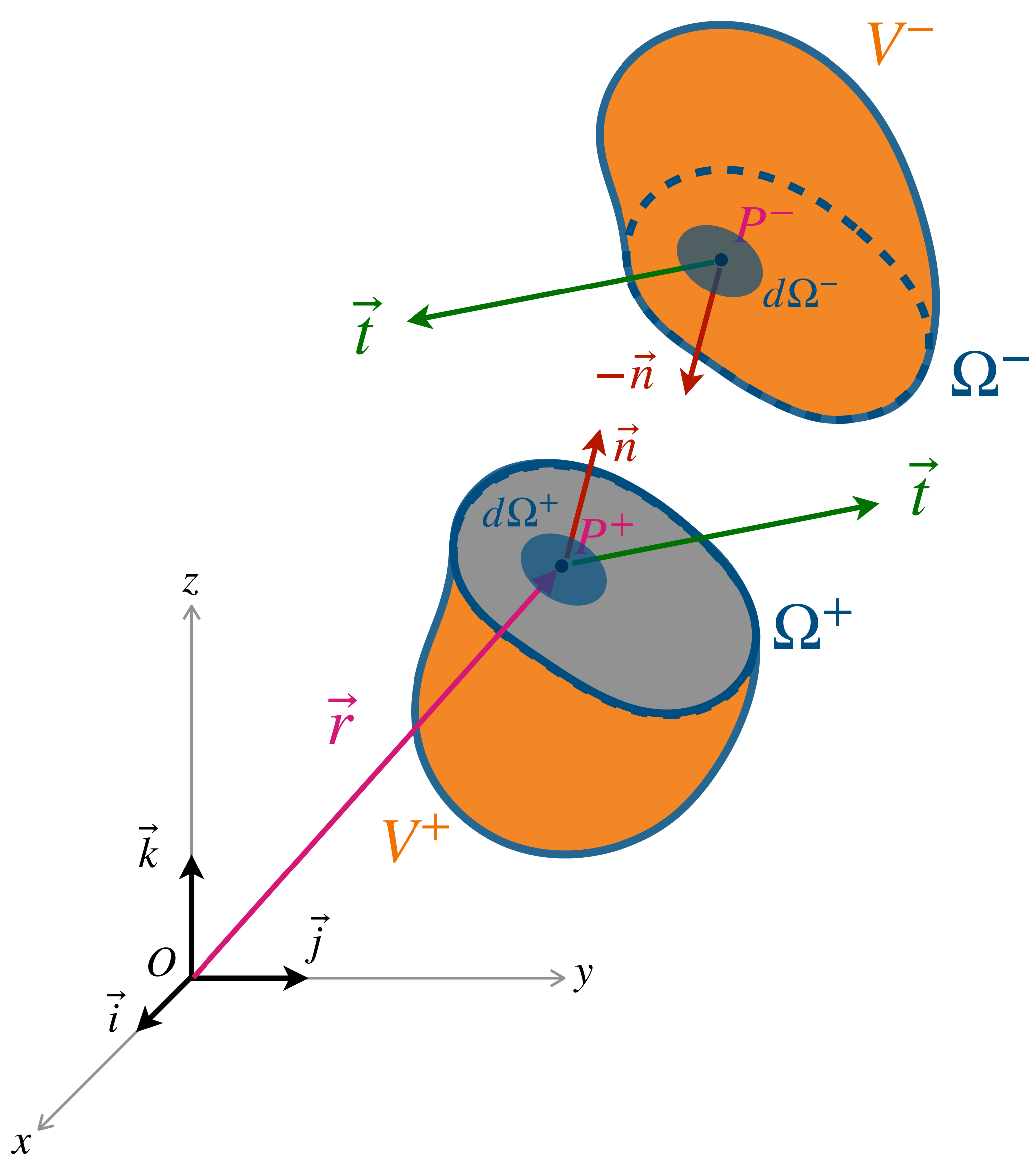
_ Azioni interne



$$P = (x, y, z)$$

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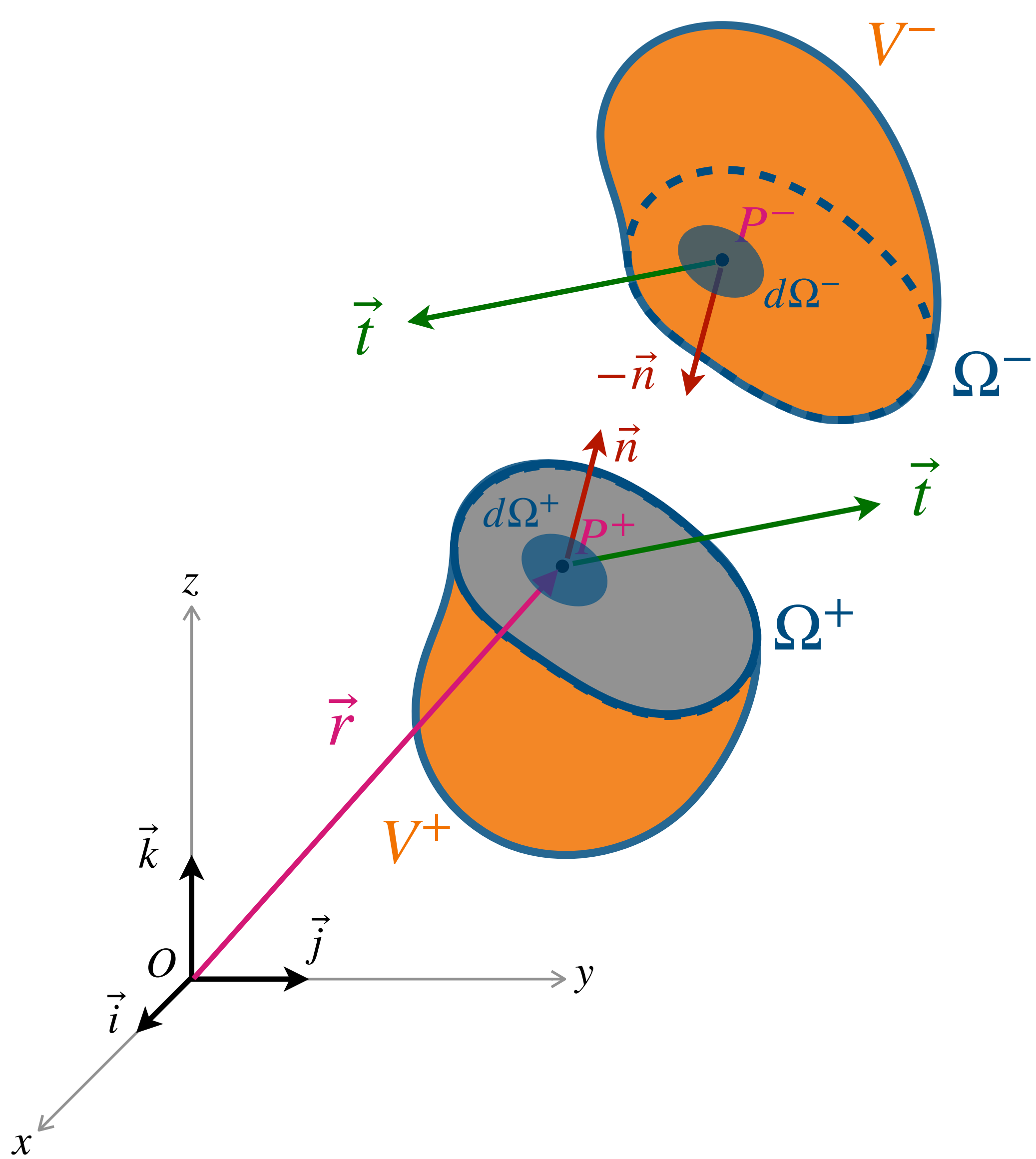
$$\underbrace{\vec{t}\,(\vec{r}, \vec{n})\, d\Omega^+}_{\vec{n}} + \underbrace{\vec{t}\,(\vec{r}, -\vec{n})\, d\Omega^-}_{-\vec{n}} = \vec{0}$$

$$\vec{t}\,(\vec{r}, \vec{n}) + \vec{t}\,(\vec{r}, -\vec{n}) = \vec{0}$$

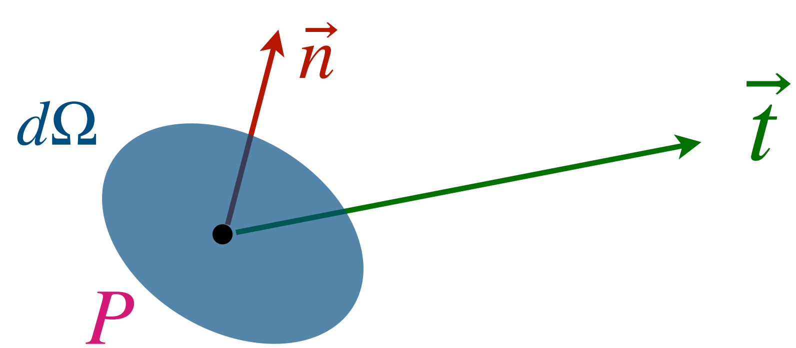
$$\vec{t}\,(\vec{r}, \vec{n}) = -\,\vec{t}\,(\vec{r}, -\vec{n})$$

_ Azioni interne



$$P = (x, y, z)$$

$$\vec{r} = \{x, y, z\}$$



FORZE INTERNE $\vec{t}$

Devono rispettare il principio di reciprocità

$$\underbrace{\vec{t}\,(\vec{r}, \vec{n})\, d\Omega^+}_{\vec{n}} + \underbrace{\vec{t}\,(\vec{r}, -\vec{n})\, d\Omega^-}_{-\vec{n}} = \vec{0}$$

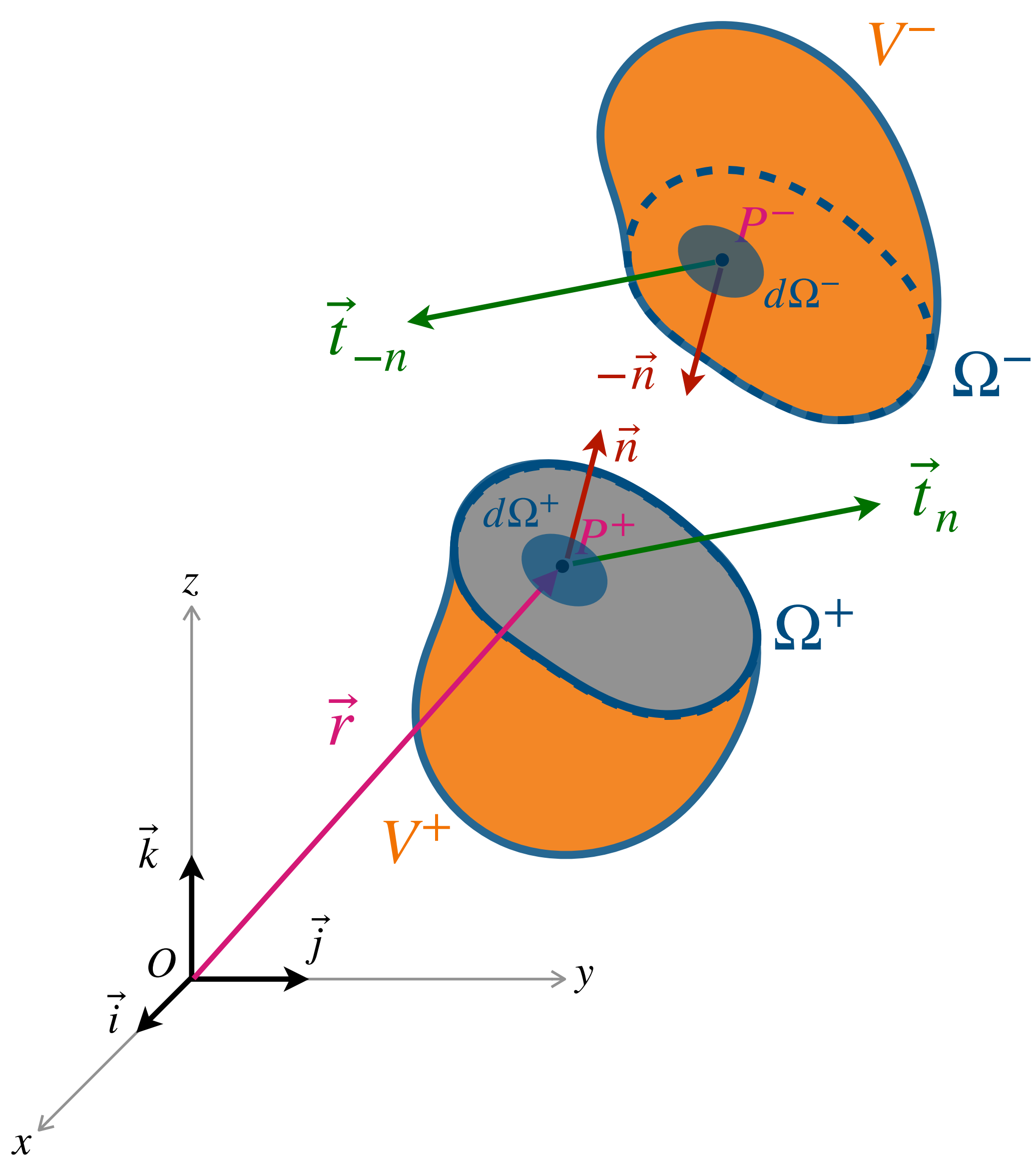
$$\vec{t}\,(\vec{r}, \vec{n}) + \vec{t}\,(\vec{r}, -\vec{n}) = \vec{0}$$

$$\vec{t}\,(\vec{r}, \vec{n}) = -\,\vec{t}\,(\vec{r}, -\vec{n})$$

Fissato P risulta fissato $\vec{r}$:
$\vec{t}$ (nel punto P) dipende solo da $\vec{n}$

$$\vec{t}\,(\circ, \vec{n}) \quad \Rightarrow \quad \vec{t}_n$$

_ Azioni interne



$$P = (x, y, z)$$

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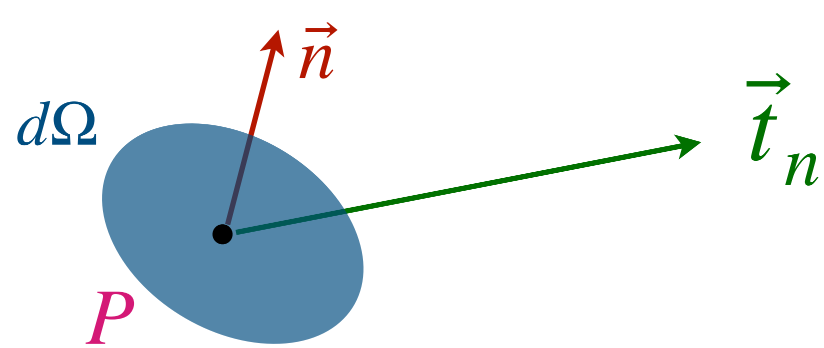

FORZE INTERNE $\vec{t}$

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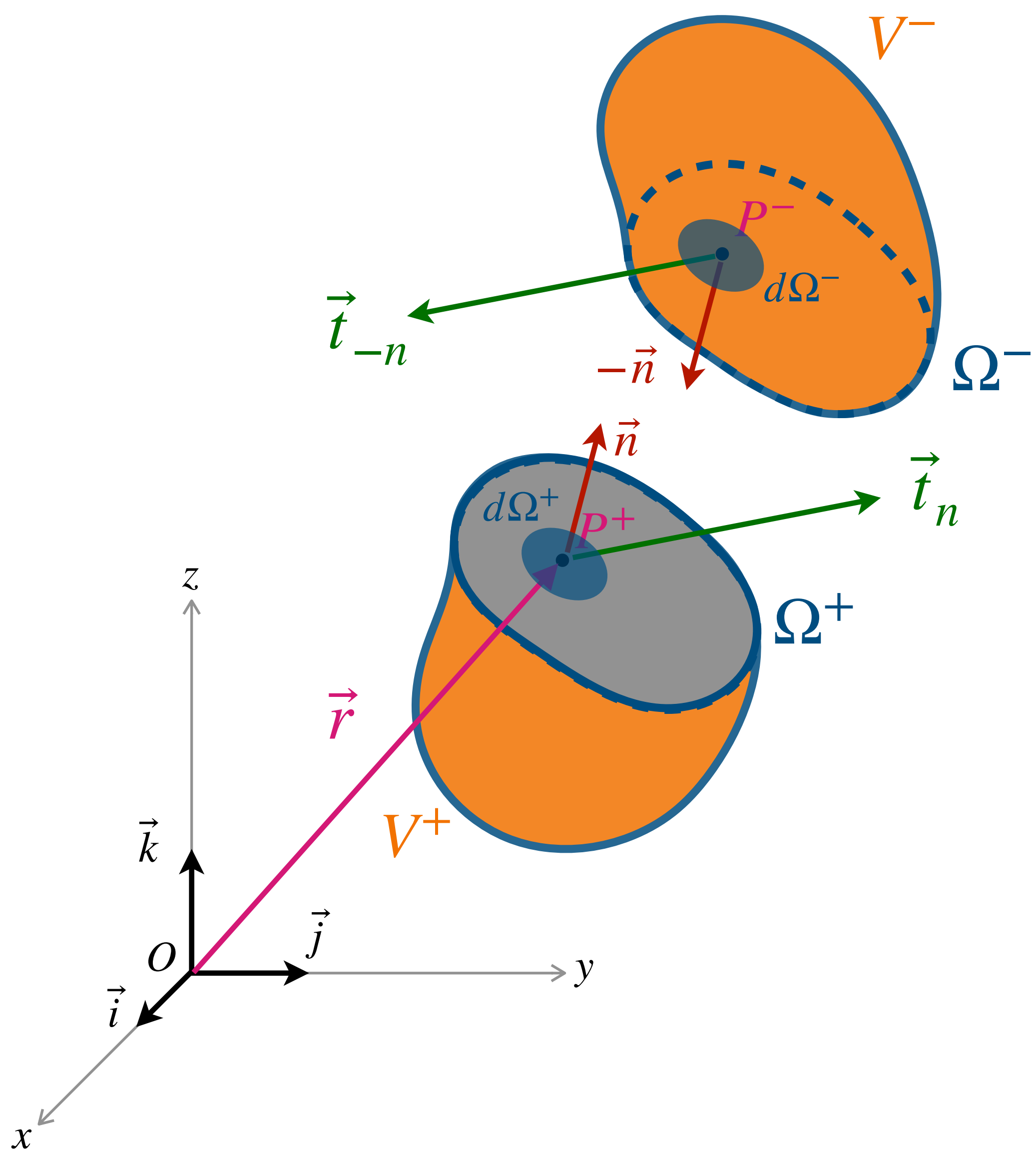
Per il principio di reciprocità $\boxed{\vec{t}_n = -\,\vec{t}_{-n}}$

_ Tensione secondo Cauchy



$$P = (x, y, z)$$

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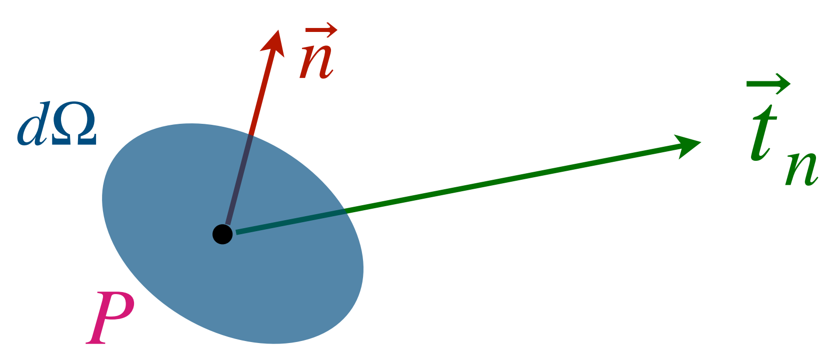
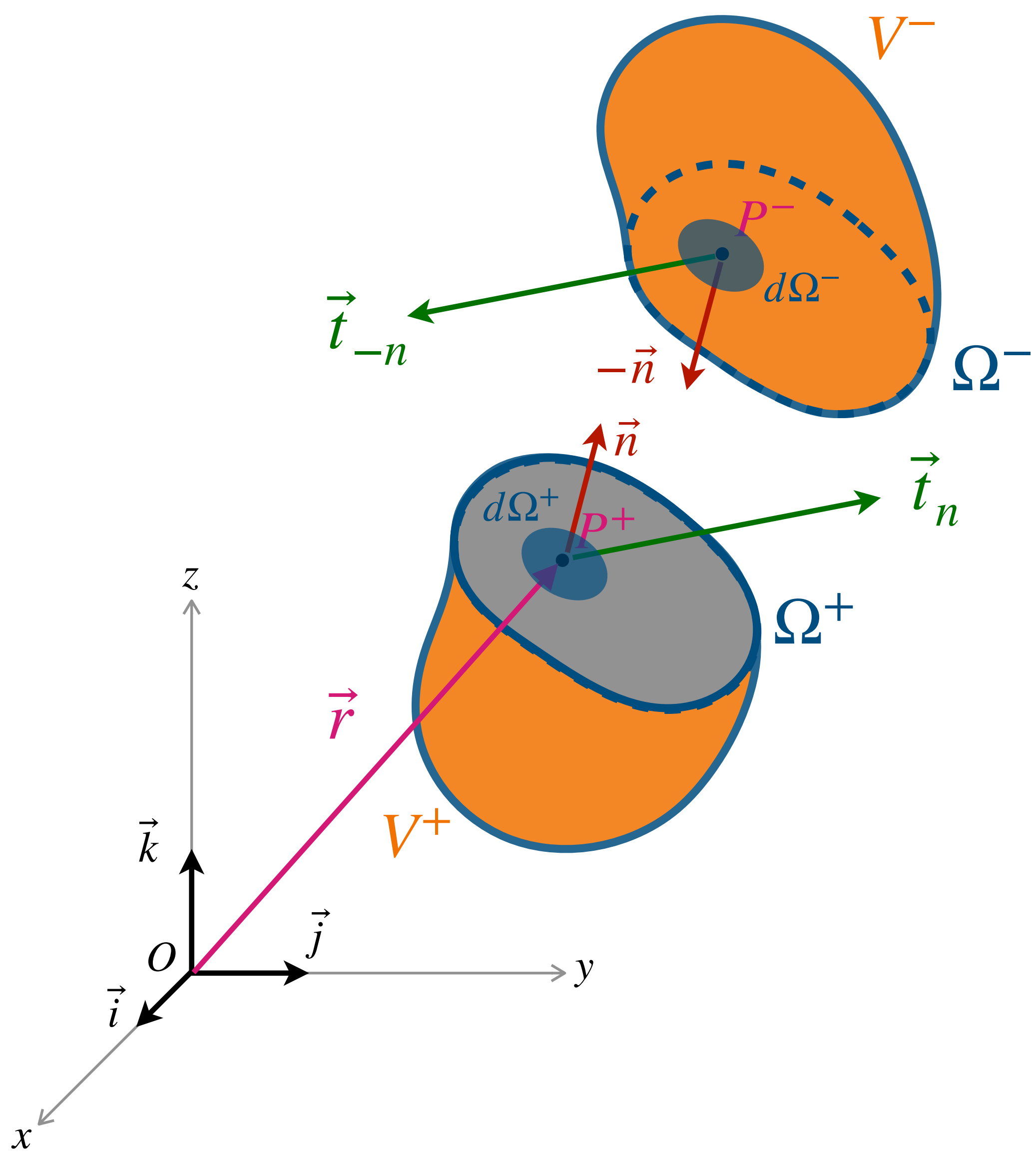


POSTULATO DI CAUCHY

Sia ΔT la risultante delle forze che il tronco V^+ trasmette al tronco V^-, si assume che:

$$\lim_{\Delta\Omega \to 0} \frac{\Delta \overrightarrow{T}}{\Delta\Omega} = \frac{d\overrightarrow{T}}{d\Omega} = \vec{t}_n$$

_ Tensione secondo Cauchy



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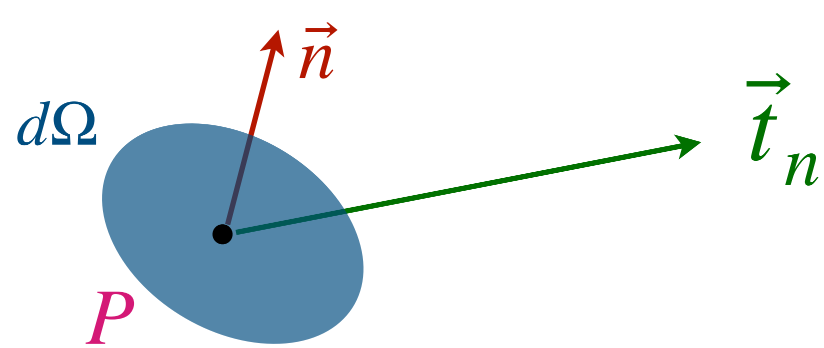

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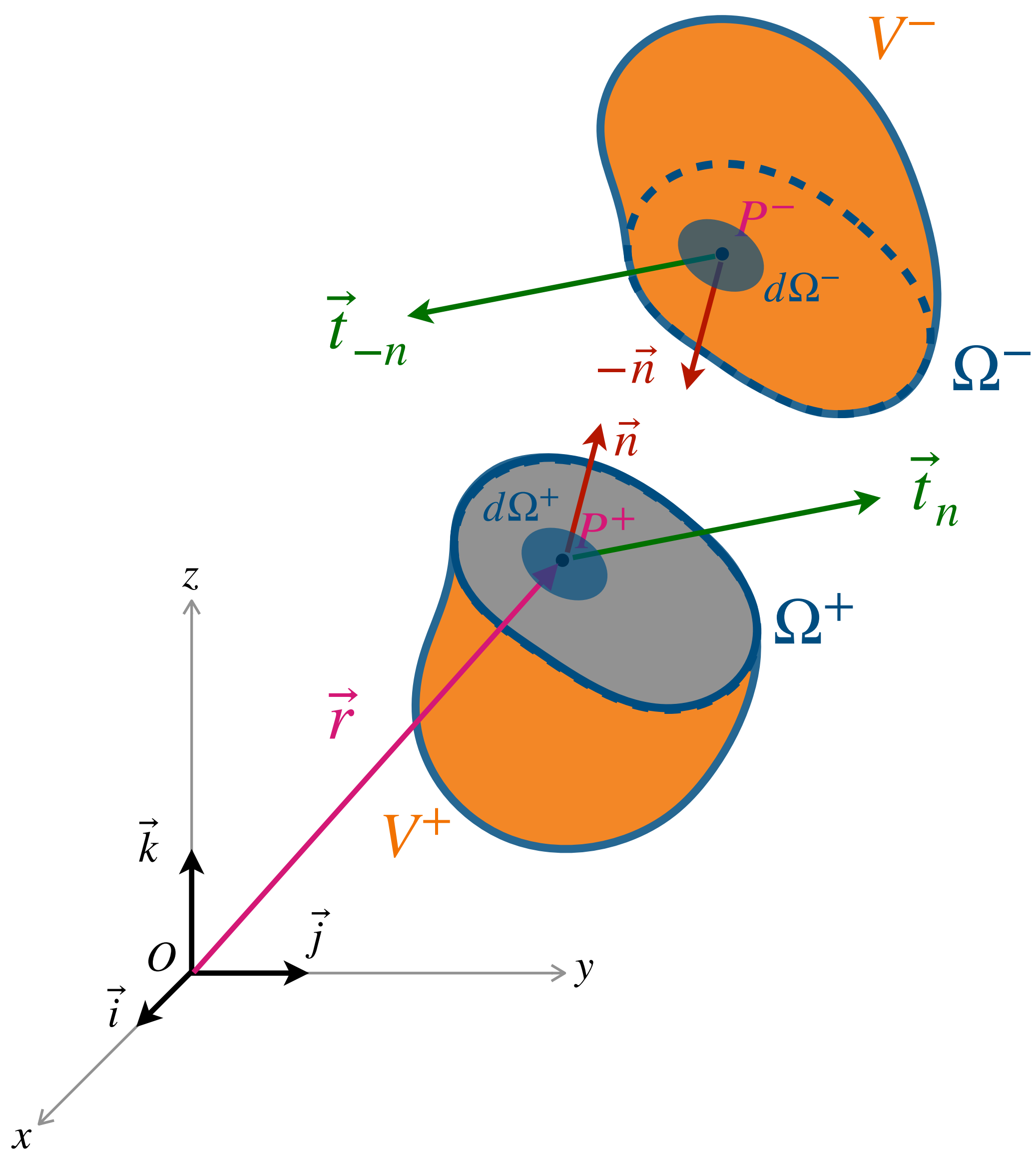
$$\boxed{\vec{t}_n = \lim_{\Delta\Omega \to 0} \frac{\Delta \overrightarrow{T}}{\Delta\Omega}}$$

Vettore **sforzo** secondo Cauchy

_ Tensione secondo Cauchy



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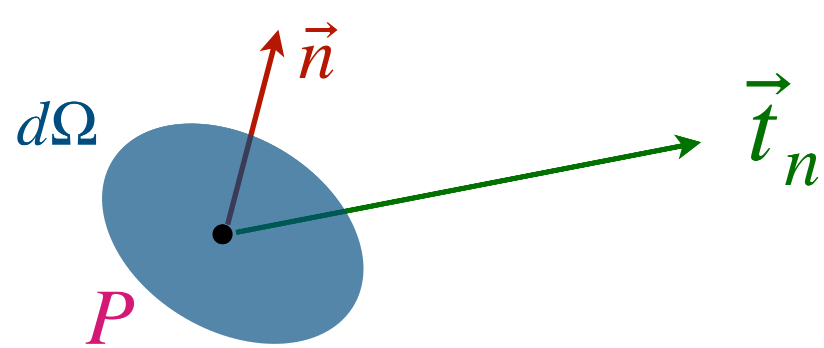

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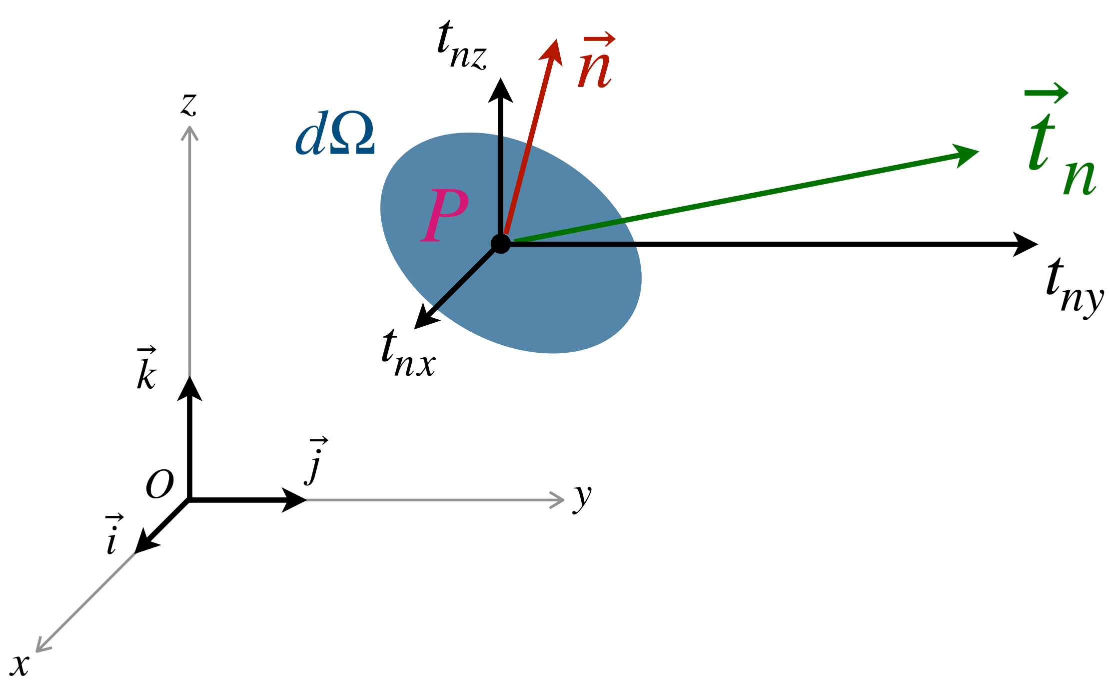
$$\boxed{\vec{t}_n = \lim_{\Delta\Omega \to 0} \frac{\Delta \overrightarrow{T}}{\Delta\Omega}}$$

Vettore **sforzo** secondo Cauchy

$$\boxed{\vec{t}_n = -\vec{t}_{-n}}$$

LEMMA di Cauchy

_ Tensione secondo Cauchy

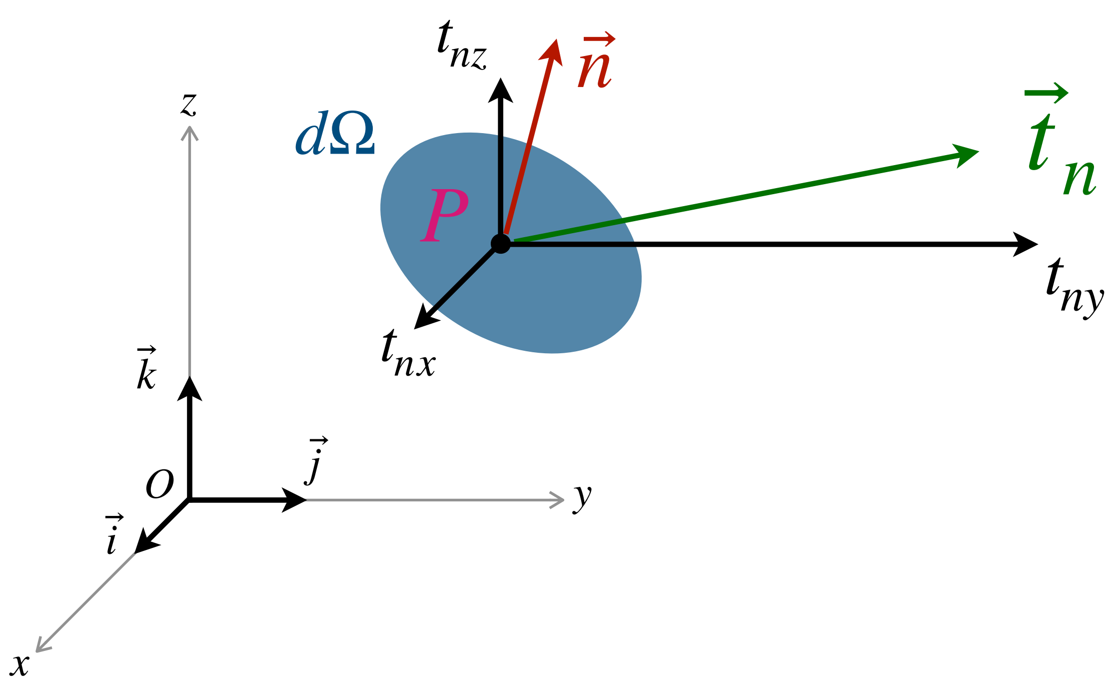


COMPONENTI CARTESIANE DI TENSIONE

$\vec{i},\ \vec{j},\ \vec{k}$

$$\vec{t}_n = t_{nx}\vec{i} + t_{ny}\vec{j} + t_{nz}\vec{k} \qquad \vec{t}_n = \left\{ t_{n_x}, t_{n_y}, t_{n_z} \right\}$$

_ Tensione secondo Cauchy



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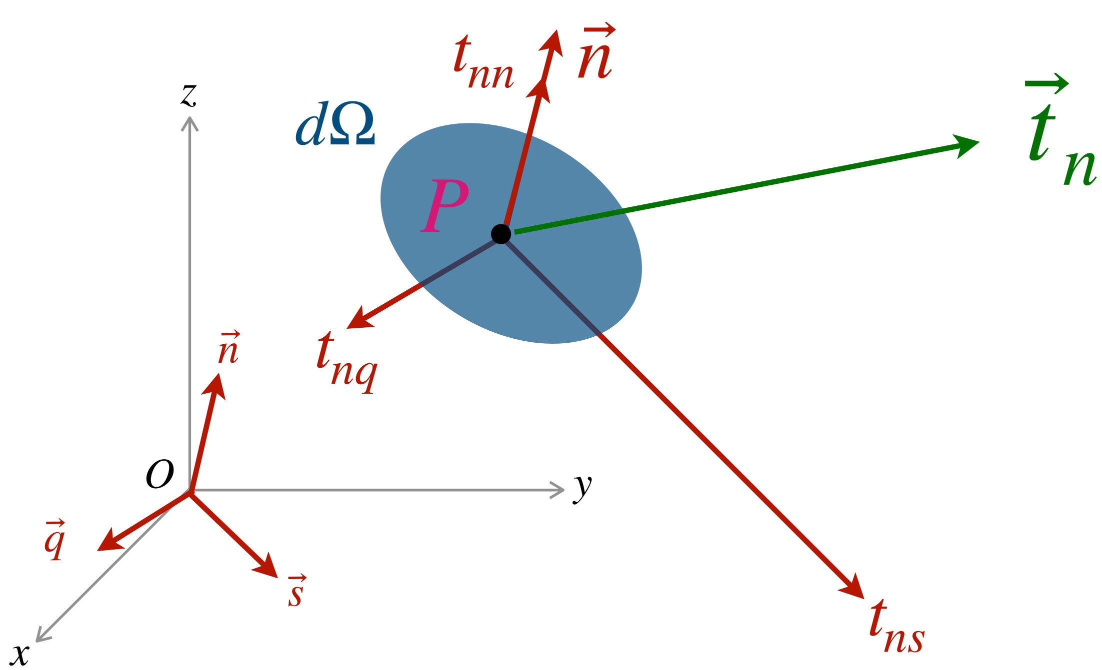
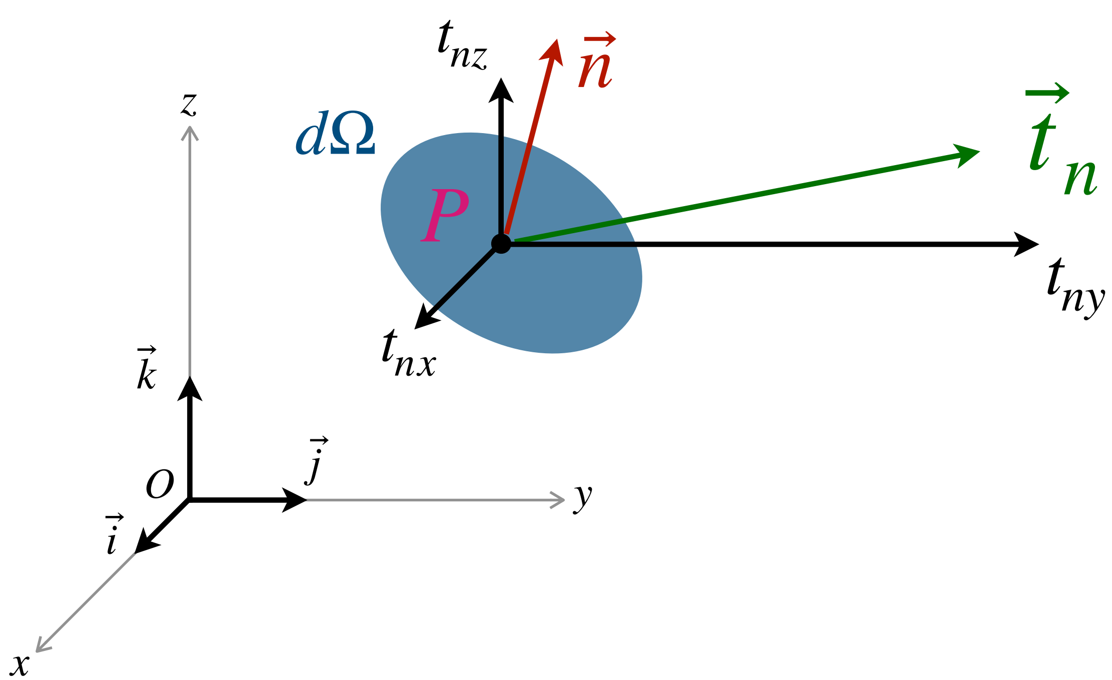


COMPONENTI SPECIALI DI TENSIONE

$\vec{n},\ \vec{q},\ \vec{s}$

$$\vec{t}_n = t_{nn}\vec{n} + t_{nq}\vec{q} + t_{ns}\vec{s} \qquad \vec{t}_n = \left\{ t_{nn}, t_{nq}, t_{ns} \right\}$$

_ Tensione secondo Cauchy



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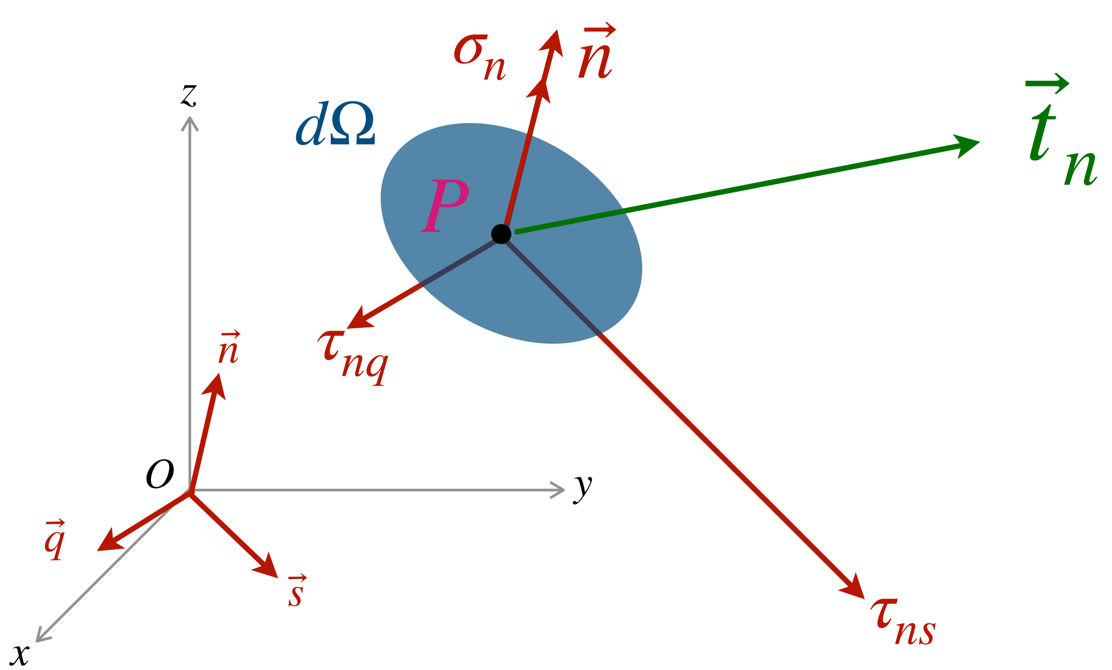

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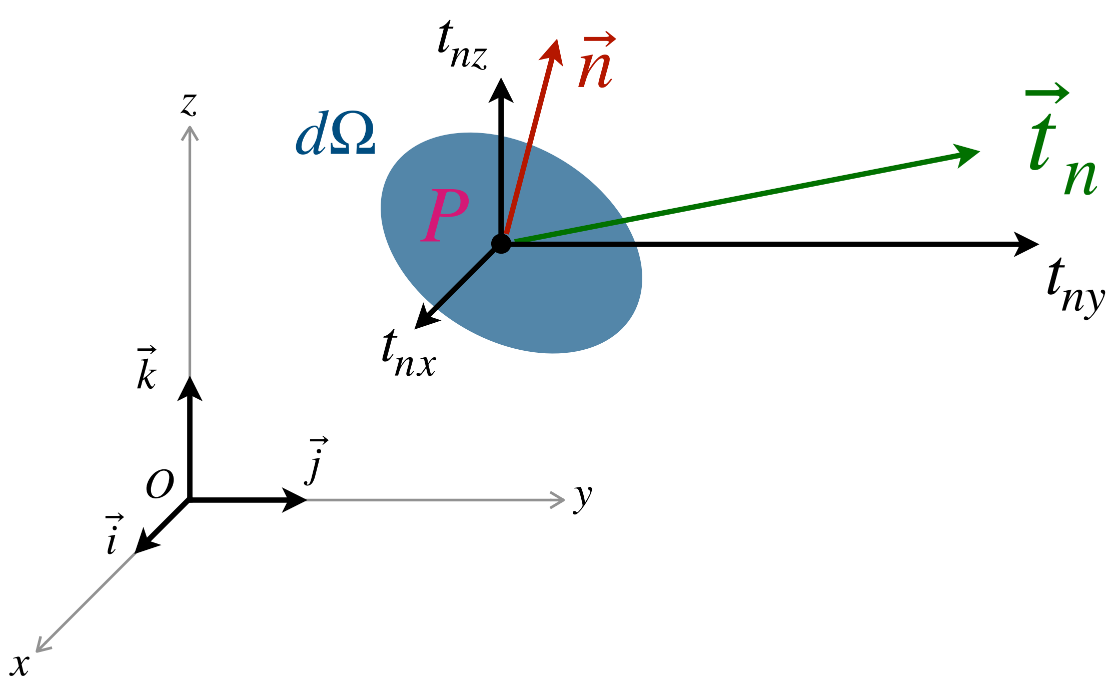
$$\vec{t}_n = t_{nn}\vec{n} + t_{nq}\vec{q} + t_{ns}\vec{s} \qquad \vec{t}_n = \left\{ t_{nn}, t_{nq}, t_{ns} \right\}$$

σ_n - componente normale

τ_{nq} e τ_{ns} - componenti tangenziali

_ Tensione secondo Cauchy



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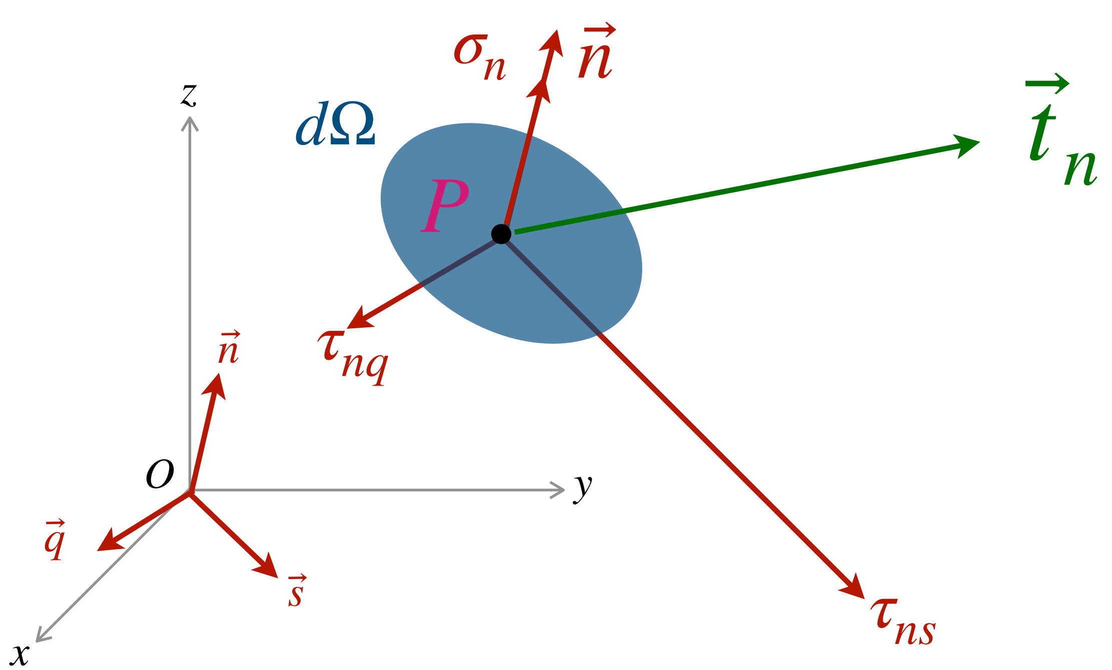


COMPONENTI SPECIALI DI TENSIONE

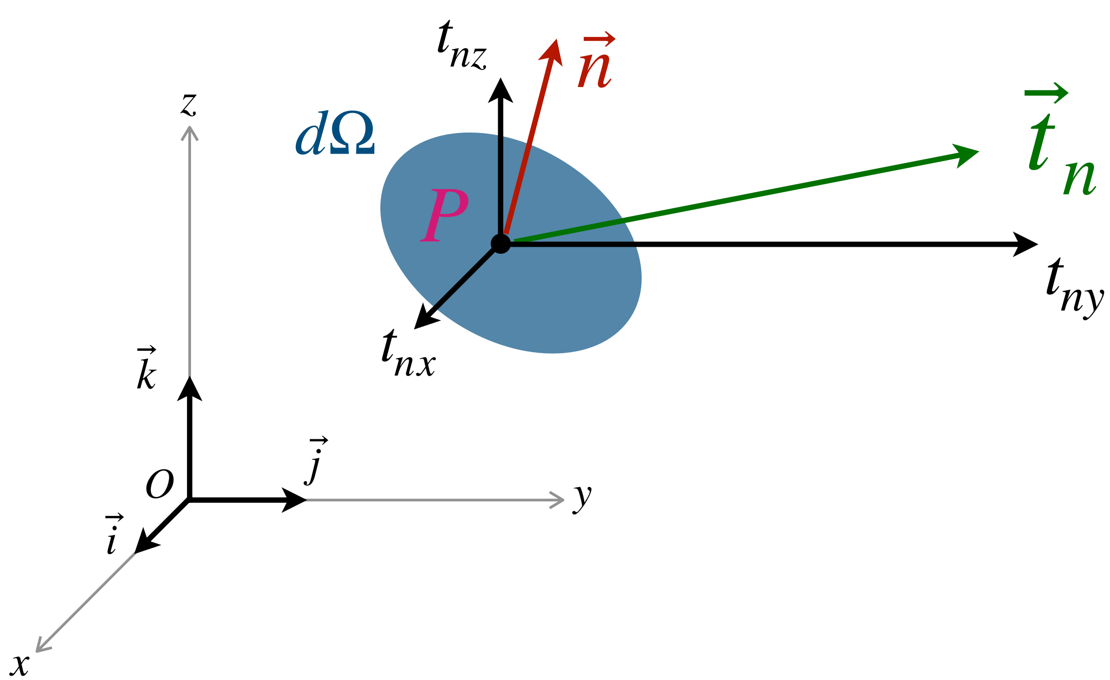
$\vec{n},\ \vec{q},\ \vec{s}$

$$\vec{t}_n = \sigma_n\vec{n} + \tau_{nq}\vec{q} + \tau_{ns}\vec{s} \qquad \boxed{\vec{t}_n = \left\{ \sigma_n, \tau_{nq}, \tau_{ns} \right\}}$$

σ_n - componente normale

τ_{nq} e τ_{ns} - componenti tangenziali

_ Tensione secondo Cauchy



COMPONENTI CARTESIANE DI TENSIONE

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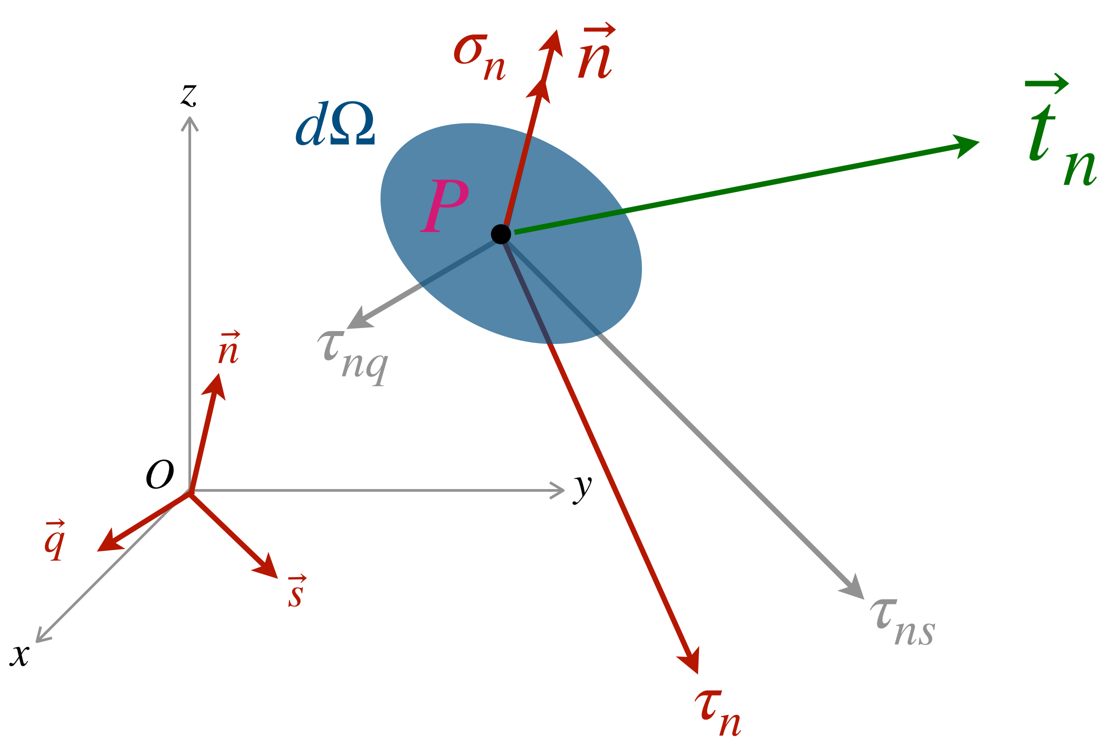


COMPONENTI SPECIALI DI TENSIONE

$$\vec{n},\ \vec{q},\ \vec{s}$$

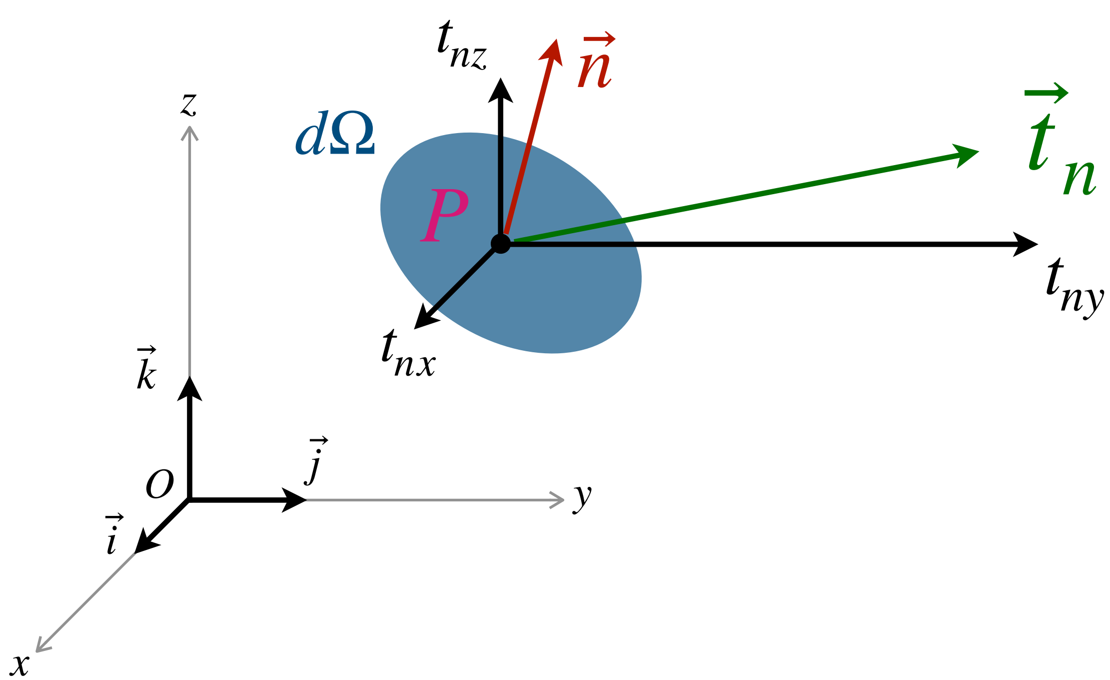
$$\vec{t}_n = \sigma_n\vec{n} + \tau_{nq}\vec{q} + \tau_{ns}\vec{s} \qquad \boxed{\vec{t}_n = \left\{ \sigma_n, \tau_{nq}, \tau_{ns} \right\}}$$

σ_n - componente normale

τ_n - componente tangenziale totale

$$\tau_n = \sqrt{\tau_{nq}^2 + \tau_{ns}^2}$$

_ Tensione secondo Cauchy



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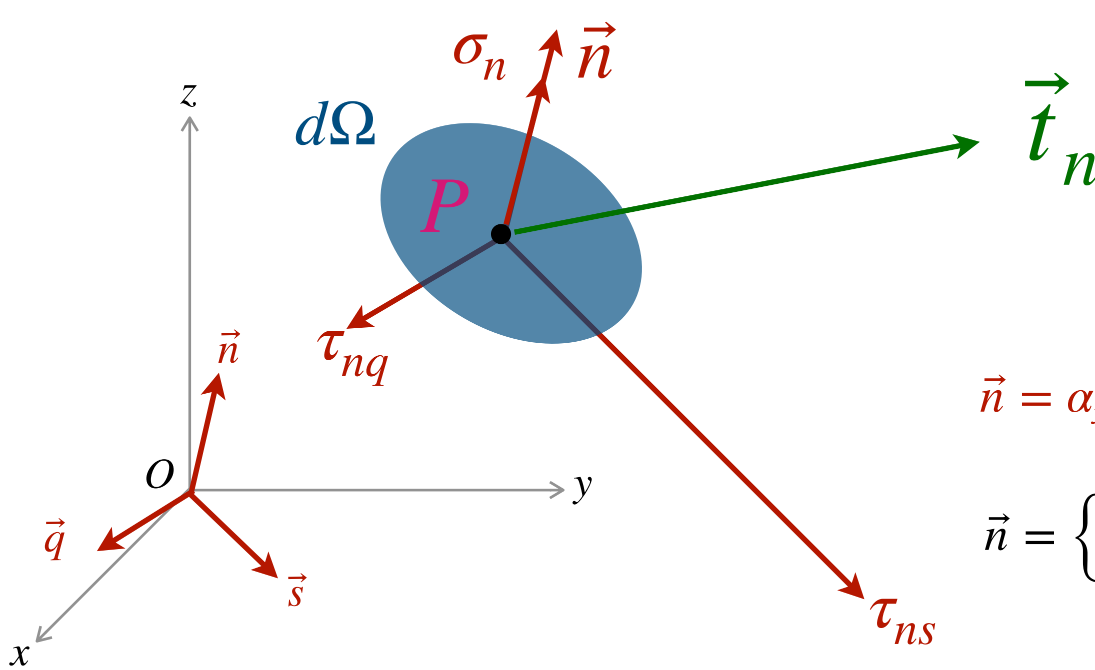

COMPONENTI SPECIALI DI TENSIONE

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$$\vec{t}_n = \sigma_n\vec{n} + \tau_{nq}\vec{q} + \tau_{ns}\vec{s} \qquad \boxed{\vec{t}_n = \left\{ \sigma_n, \tau_{nq}, \tau_{ns} \right\}}$$

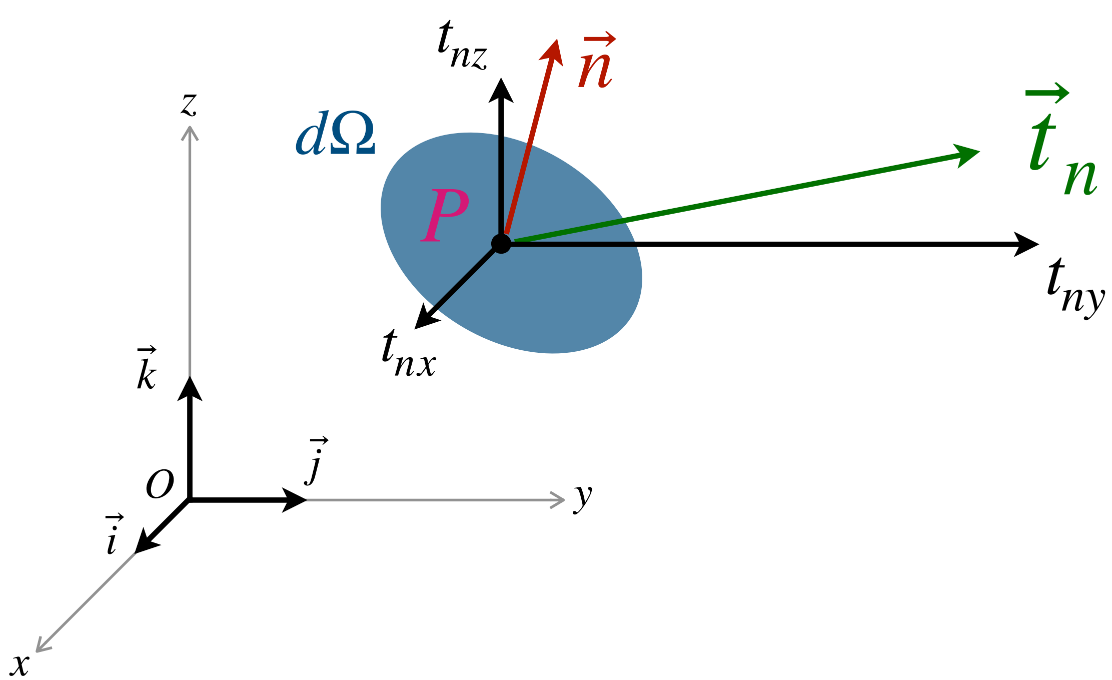
$$\vec{n} = \alpha_x\vec{i} + \alpha_y\vec{j} + \alpha_z\vec{k}$$

$$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\} \qquad \alpha_x = \cos\left\{ \widehat{\vec{n}, \vec{i}} \right\} \qquad \alpha_y = \cos\left\{ \widehat{\vec{n}, \vec{j}} \right\} \qquad \alpha_z = \cos\left\{ \widehat{\vec{n}, \vec{k}} \right\}$$

$$\alpha_x = \vec{n} \times \vec{i} \qquad \alpha_y = \vec{n} \times \vec{j} \qquad \alpha_z = \vec{n} \times \vec{k}$$

$$\left(\alpha_x^2 + \alpha_y^2 + \alpha_z^2 \right) = |\vec{n}|^2 = 1$$

_ Tensione secondo Cauchy



COMPONENTI CARTESIANE DI TENSIONE

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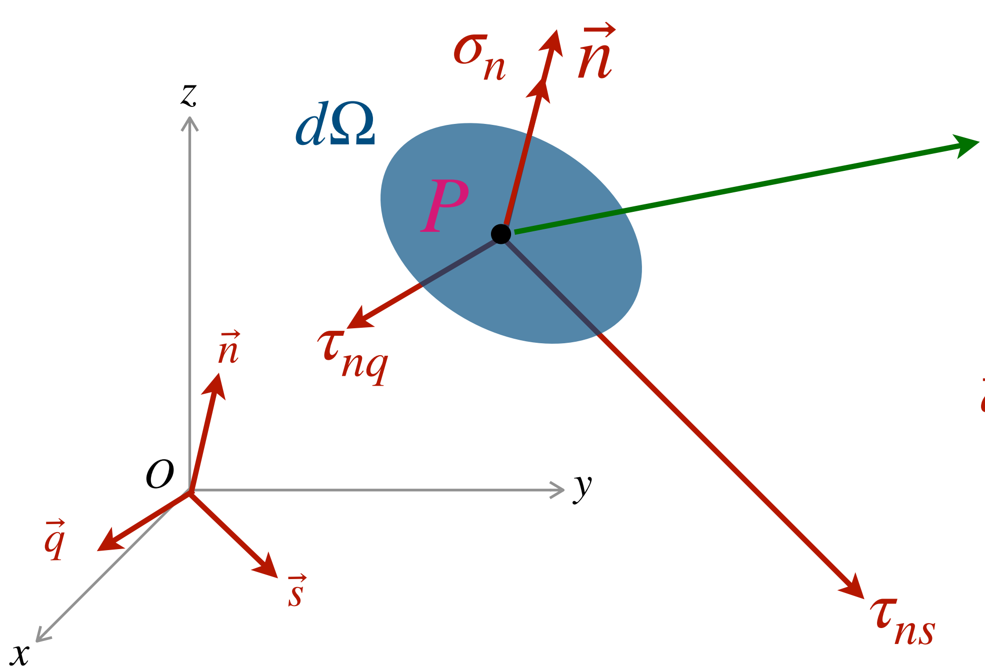
$$\vec{t}_n = t_{nx}\vec{i} + t_{ny}\vec{j} + t_{nz}\vec{k} \qquad \vec{t}_n = \left\{ t_{n_x}, t_{n_y}, t_{n_z} \right\}$$



COMPONENTI SPECIALI DI TENSIONE

$\vec{n},\ \vec{q},\ \vec{s}$

$$\vec{t}_n = \sigma_n\vec{n} + \tau_{nq}\vec{q} + \tau_{ns}\vec{s} \qquad \boxed{\vec{t}_n = \left\{ \sigma_n, \tau_{nq}, \tau_{ns} \right\}}$$

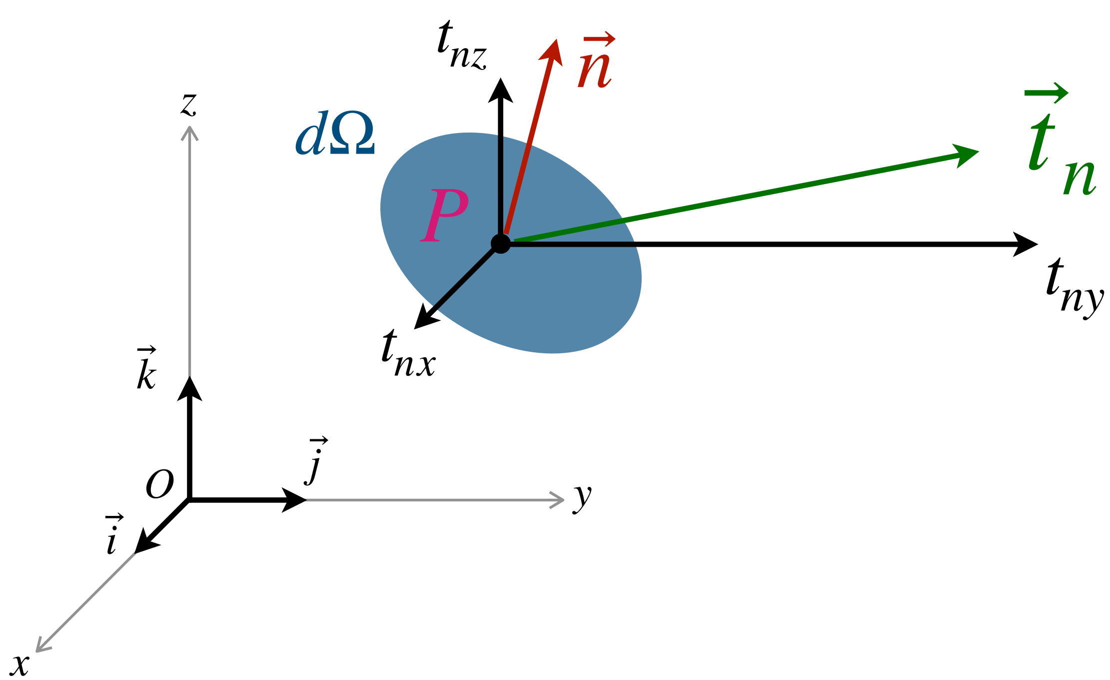
$$\vec{q} = \beta_x\vec{i} + \beta_y\vec{j} + \beta_z\vec{k}$$

$$\vec{q} = \left\{ \beta_x, \beta_y, \beta_z \right\} \qquad \beta_x = \cos\left\{ \vec{q}\,\hat{,}\,\vec{i} \right\} \qquad \beta_y = \cos\left\{ \vec{q}\,\hat{,}\,\vec{j} \right\} \qquad \beta_z = \cos\left\{ \vec{q}\,\hat{,}\,\vec{k} \right\}$$

$$\beta_x = \vec{q} \times \vec{i} \qquad \beta_y = \vec{q} \times \vec{j} \qquad \beta_z = \vec{q} \times \vec{k}$$

$$\left(\beta_x^2 + \beta_y^2 + \beta_z^2 \right) = |\vec{q}\,|^2 = 1$$

_ Tensione secondo Cauchy



COMPONENTI CARTESIANE DI TENSIONE

$\vec{i},\ \vec{j},\ \vec{k}$

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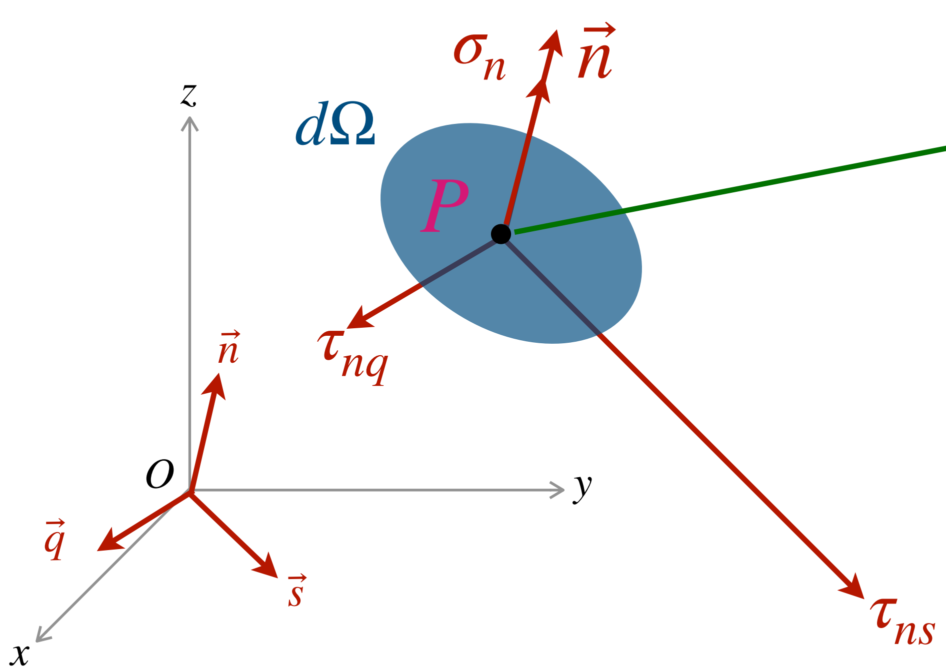


COMPONENTI SPECIALI DI TENSIONE

$\vec{n},\ \vec{q},\ \vec{s}$

$$\vec{t}_n = \sigma_n\vec{n} + \tau_{nq}\vec{q} + \tau_{ns}\vec{s} \qquad \boxed{\vec{t}_n = \left\{ \sigma_n, \tau_{nq}, \tau_{ns} \right\}}$$

$$\vec{s} = \gamma_x\vec{i} + \gamma_y\vec{j} + \gamma_z\vec{k}$$

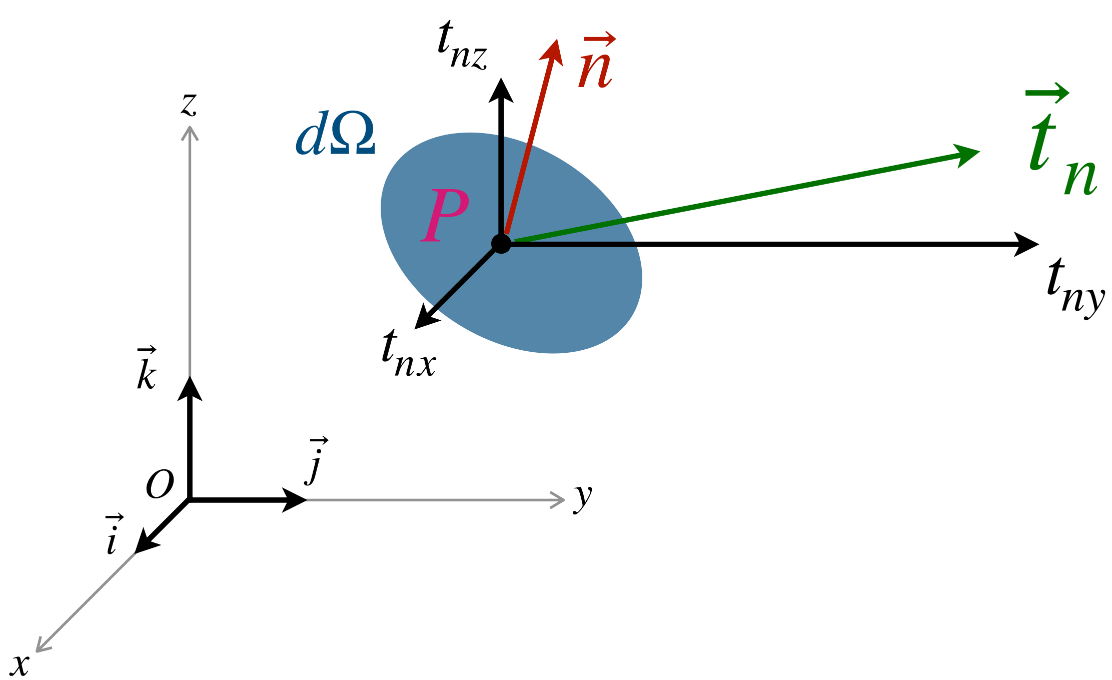
$$\vec{s} = \left\{ \gamma_x, \gamma_y, \gamma_z \right\} \qquad \gamma_x = \cos\left\{ \widehat{\vec{s}, \vec{i}} \right\} \qquad \gamma_y = \cos\left\{ \widehat{\vec{s}, \vec{j}} \right\} \qquad \gamma_z = \cos\left\{ \widehat{\vec{s}, \vec{k}} \right\}$$

$$\gamma_x = \vec{s} \times \vec{i} \qquad \gamma_y = \vec{s} \times \vec{j} \qquad \gamma_z = \vec{s} \times \vec{k}$$

$$\left(\gamma_x^2 + \gamma_y^2 + \gamma_z^2 \right) = |\vec{s}|^2 = 1$$

_ Tensione secondo Cauchy

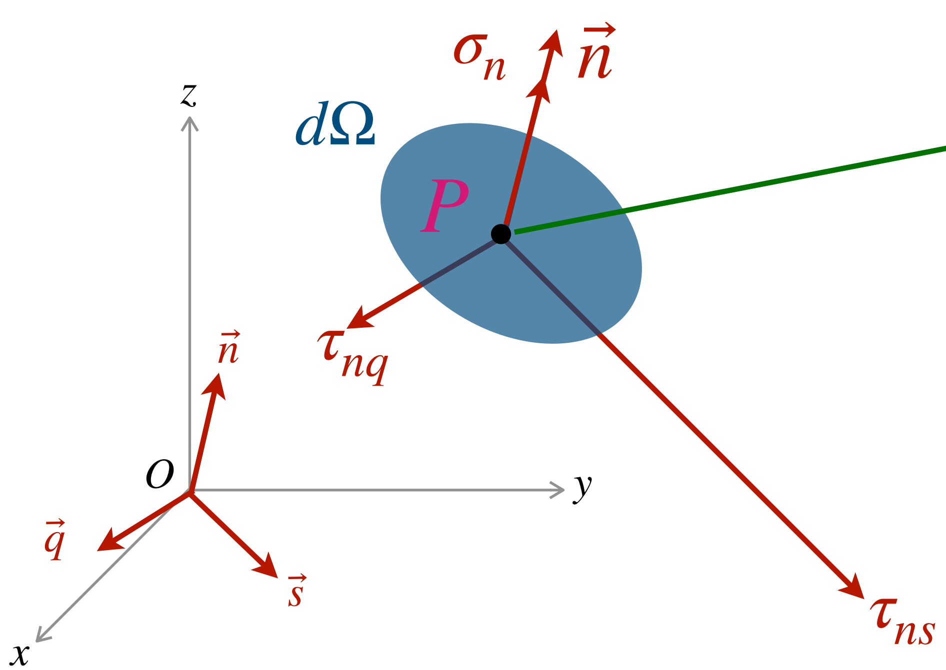
COMPONENTI CARTESIANE DI TENSIONE



$\vec{i},\ \vec{j},\ \vec{k}$

$$\vec{t}_n = t_{nx}\vec{i} + t_{ny}\vec{j} + t_{nz}\vec{k} \qquad \vec{t}_n = \left\{ t_{n_x}, t_{n_y}, t_{n_z} \right\}$$

COMPONENTI SPECIALI DI TENSIONE



$\vec{n},\ \vec{q},\ \vec{s}$

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$$\vec{n} = \alpha_x\vec{i} + \alpha_y\vec{j} + \alpha_z\vec{k}$$

$$\vec{q} = \beta_x\vec{i} + \beta_y\vec{j} + \beta_z\vec{k}$$

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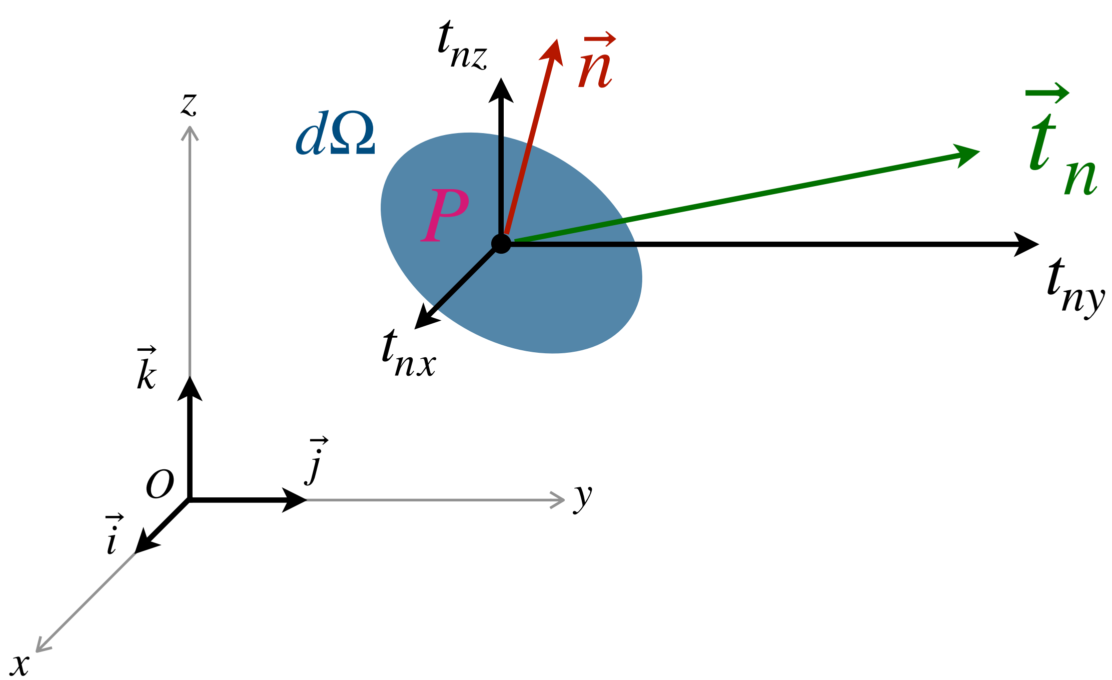
_ Tensione secondo Cauchy



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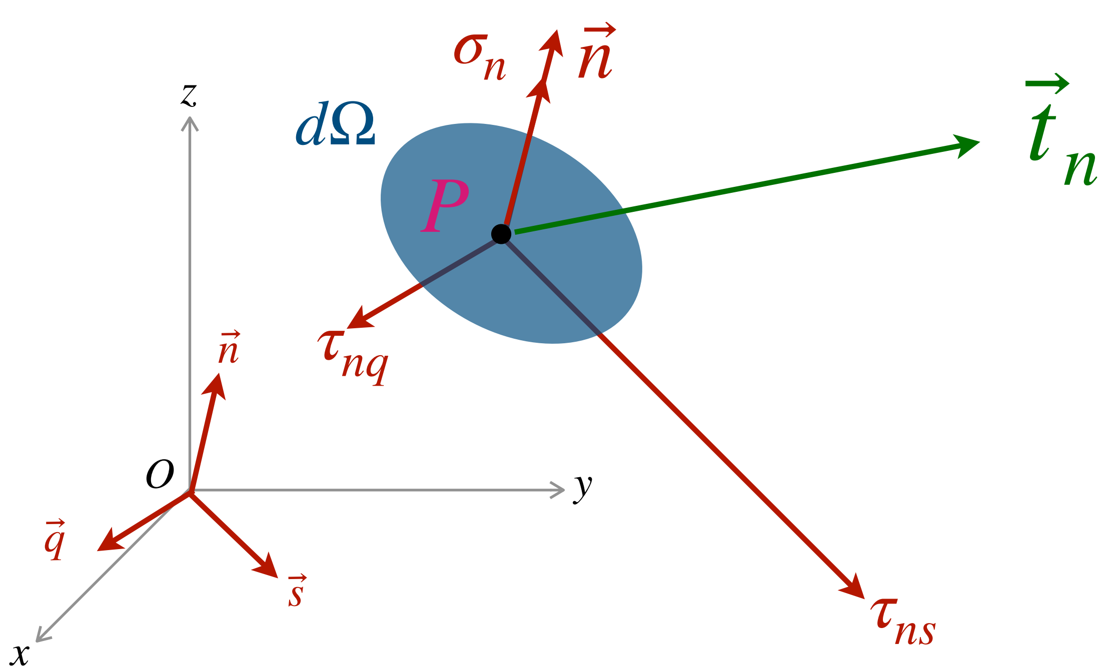

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$$\vec{n} = \alpha_x\vec{i} + \alpha_y\vec{j} + \alpha_z\vec{k}$$

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$$\vec{s} = \gamma_x\vec{i} + \gamma_y\vec{j} + \gamma_z\vec{k}$$

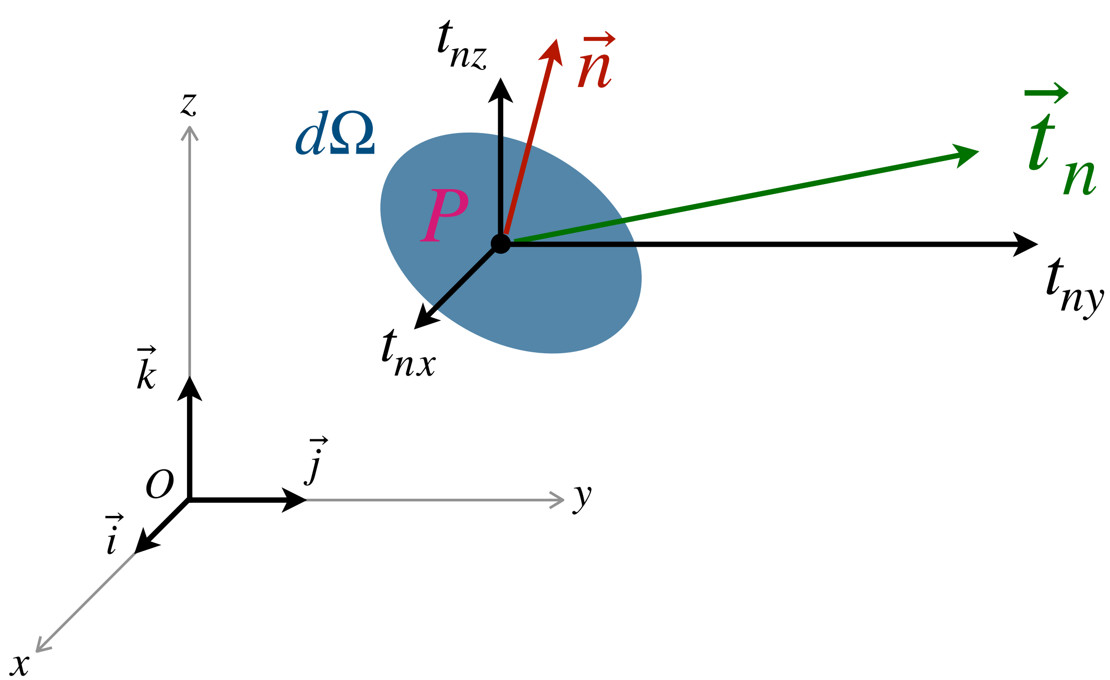
$$\Rightarrow$$

$$\sigma_n = t_{nx}\alpha_x + t_{ny}\alpha_y + t_{nz}\alpha_z$$

$$\tau_{nq} = t_{nx}\beta_x + t_{ny}\beta_y + t_{nz}\beta_z$$

$$\tau_{ns} = t_{nx}\gamma_x + t_{ny}\gamma_y + t_{nz}\gamma_z$$

_ Tensione secondo Cauchy



COMPONENTI CARTESIANE DI TENSIONE

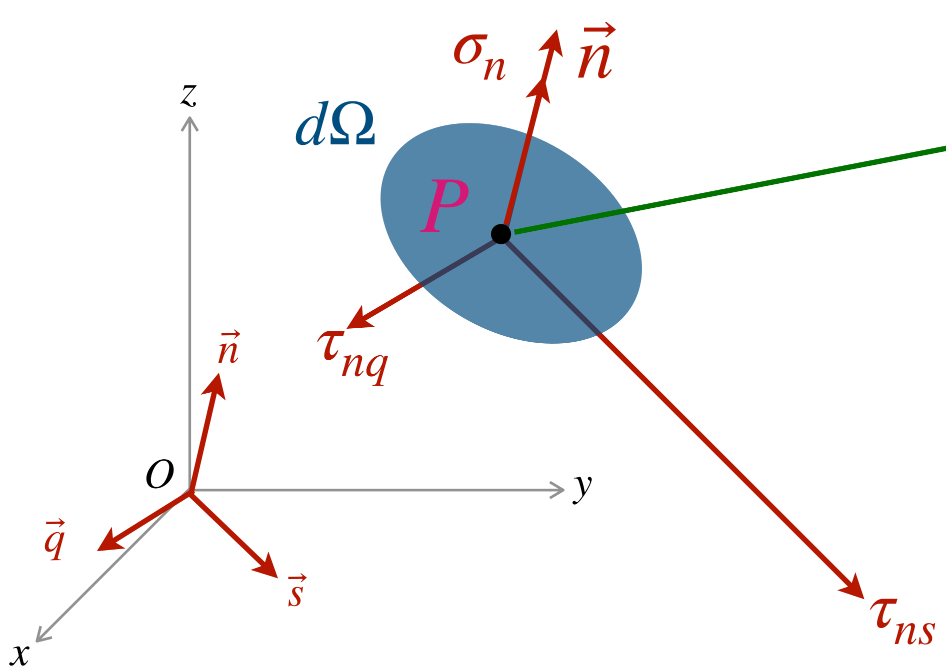
$\vec{i},\ \vec{j},\ \vec{k}$

$$\vec{t}_n = t_{nx}\vec{i} + t_{ny}\vec{j} + t_{nz}\vec{k} \qquad \vec{t}_n = \left\{t_{n_x}, t_{n_y}, t_{n_z}\right\}$$

$$\vec{i} = \vec{n}\alpha_x + \vec{q}\beta_x + \vec{s}\gamma_x$$

$$\vec{j} = \vec{n}\alpha_y + \vec{q}\beta_y + \vec{s}\gamma_y$$

$$\vec{k} = \vec{n}\alpha_z + \vec{q}\beta_z + \vec{s}\gamma_z$$



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$$\vec{n} = \alpha_x\vec{i} + \alpha_y\vec{j} + \alpha_z\vec{k}$$

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$$\vec{s} = \gamma_x\vec{i} + \gamma_y\vec{j} + \gamma_z\vec{k}$$

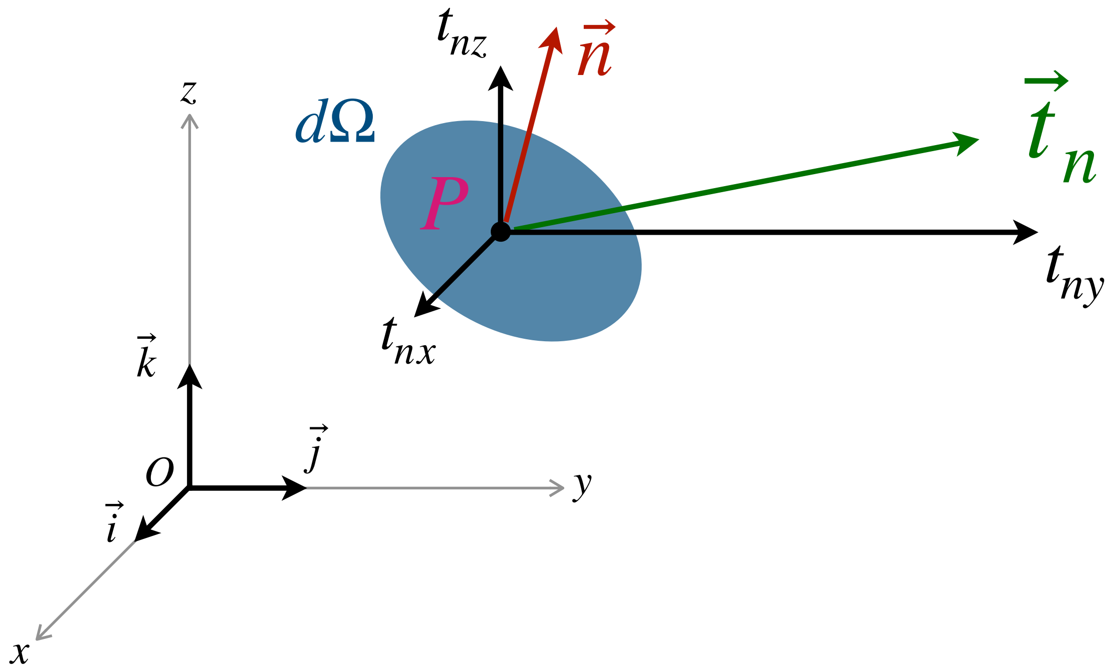
$$\Rightarrow$$

$$\sigma_n = t_{nx}\alpha_x + t_{ny}\alpha_y + t_{nz}\alpha_z$$

$$\tau_{nq} = t_{nx}\beta_x + t_{ny}\beta_y + t_{nz}\beta_z$$

$$\tau_{ns} = t_{nx}\gamma_x + t_{ny}\gamma_y + t_{nz}\gamma_z$$

_ Tensione secondo Cauchy



COMPONENTI CARTESIANE DI TENSIONE

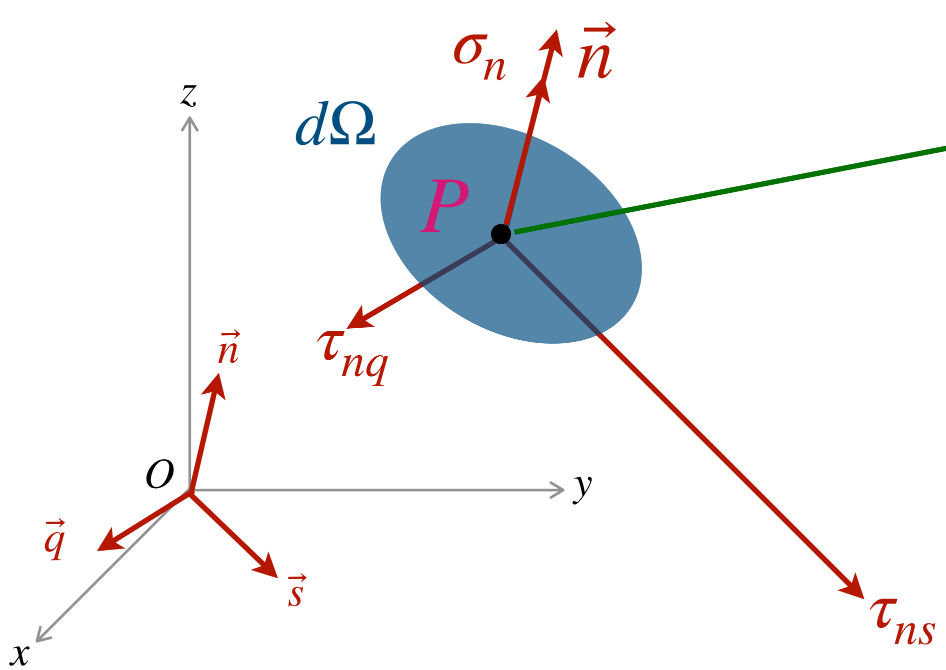
$$\vec{i},\ \vec{j},\ \vec{k}$$

$$\vec{t}_n = t_{nx}\vec{i} + t_{ny}\vec{j} + t_{nz}\vec{k} \qquad \vec{t}_n = \left\{t_{n_x}, t_{n_y}, t_{n_z}\right\}$$

$$\vec{i} = \vec{n}\alpha_x + \vec{q}\beta_x + \vec{s}\gamma_x \qquad t_{nx} = \sigma_n\alpha_x + \tau_{nq}\beta_x + \tau_{ns}\gamma_x$$

$$\vec{j} = \vec{n}\alpha_y + \vec{q}\beta_y + \vec{s}\gamma_y \qquad t_{ny} = \sigma_n\alpha_y + \tau_{nq}\beta_y + \tau_{ns}\gamma_y$$

$$\vec{k} = \vec{n}\alpha_z + \vec{q}\beta_z + \vec{s}\gamma_z \qquad t_{nz} = \sigma_n\alpha_z + \tau_{nq}\beta_z + \tau_{ns}\gamma_z$$



COMPONENTI SPECIALI DI TENSIONE

$$\vec{n},\ \vec{q},\ \vec{s}$$

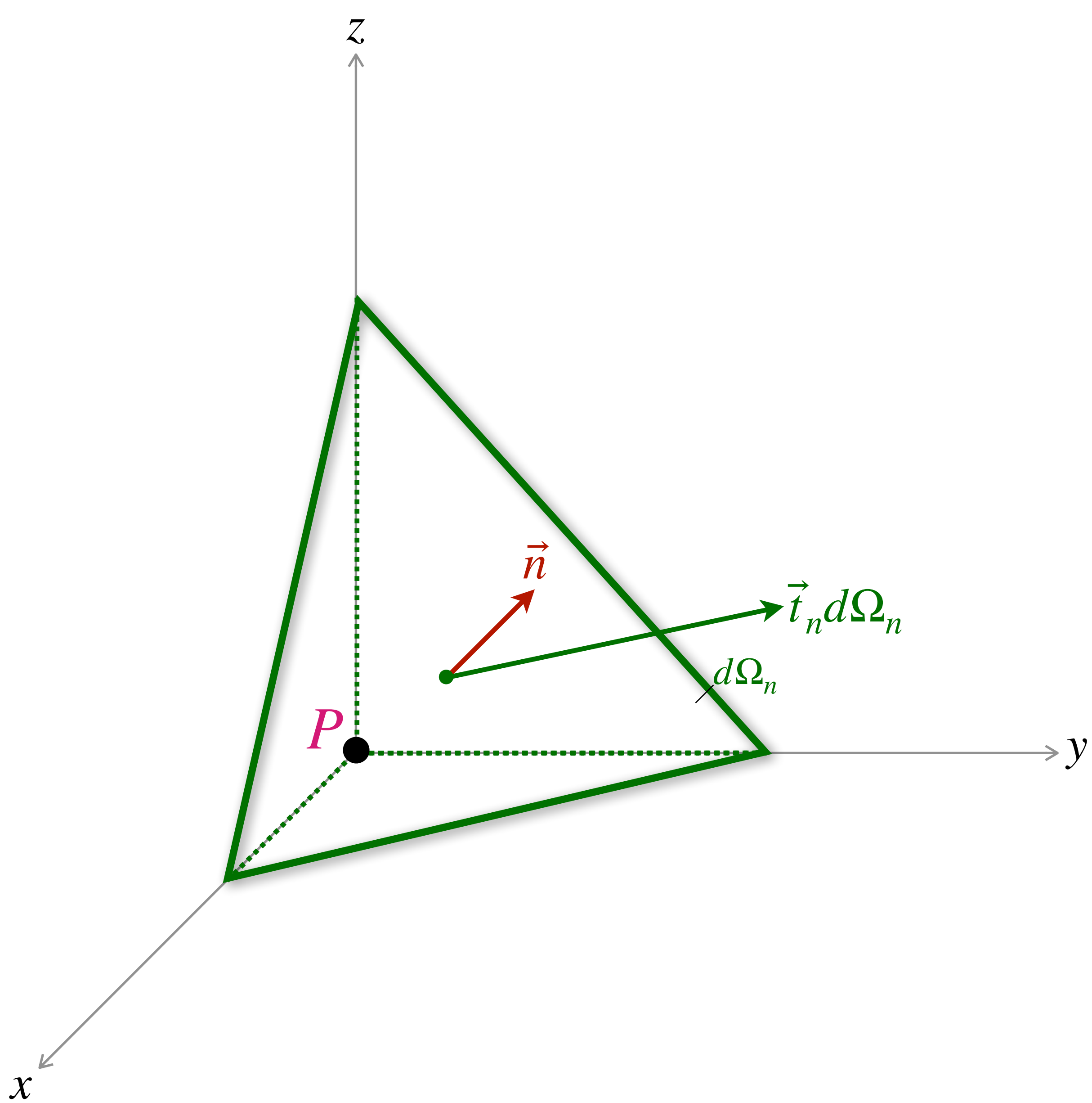
$$\vec{t}_n = \sigma_n\vec{n} + \tau_{nq}\vec{q} + \tau_{ns}\vec{s} \qquad \boxed{\vec{t}_n = \left\{\sigma_n, \tau_{nq}, \tau_{ns}\right\}}$$

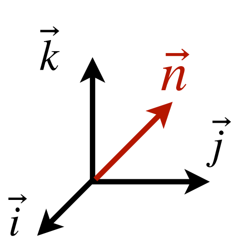
$$\vec{n} = \alpha_x\vec{i} + \alpha_y\vec{j} + \alpha_z\vec{k} \qquad \sigma_n = t_{nx}\alpha_x + t_{ny}\alpha_y + t_{nz}\alpha_z$$

$$\vec{q} = \beta_x\vec{i} + \beta_y\vec{j} + \beta_z\vec{k} \qquad \tau_{nq} = t_{nx}\beta_x + t_{ny}\beta_y + t_{nz}\beta_z$$

$$\vec{s} = \gamma_x\vec{i} + \gamma_y\vec{j} + \gamma_z\vec{k} \qquad \tau_{ns} = t_{nx}\gamma_x + t_{ny}\gamma_y + t_{nz}\gamma_z$$

_ Tensione secondo Cauchy



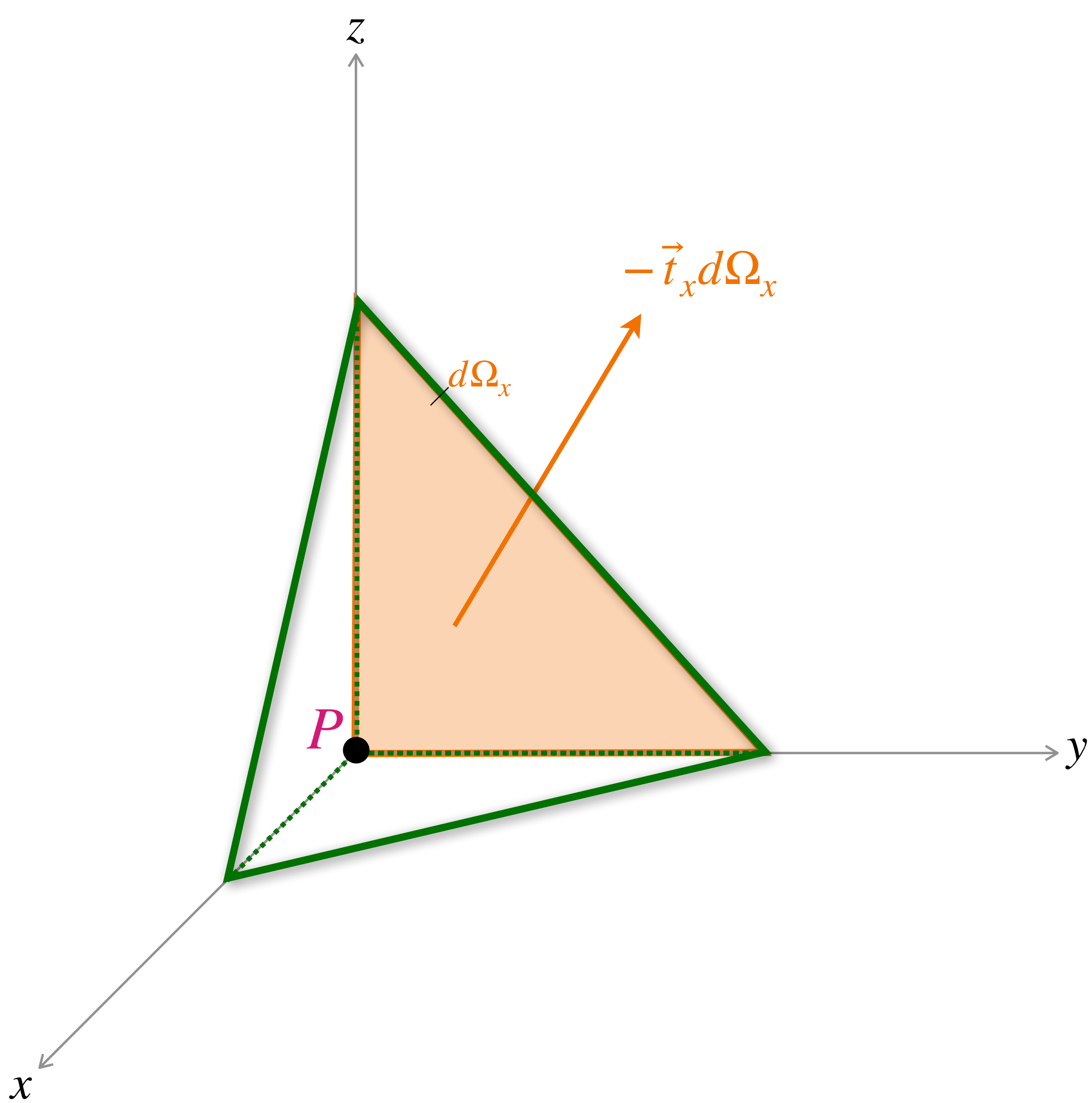


$$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$$

TEOREMA DI CAUCHY

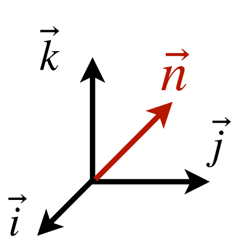
Assegnato un punto P all'interno del corpo, il vettore $\vec{t}_n$ varia al variare della giacitura, ovvero di $\vec{n}$.

_ Tensione secondo Cauchy



TEOREMA DI CAUCHY

É sufficiente conoscere la tensione $\vec{t}_n$ su 3 giaciture ortogonali per poter definire la tensione su qualunque altra giacitura.



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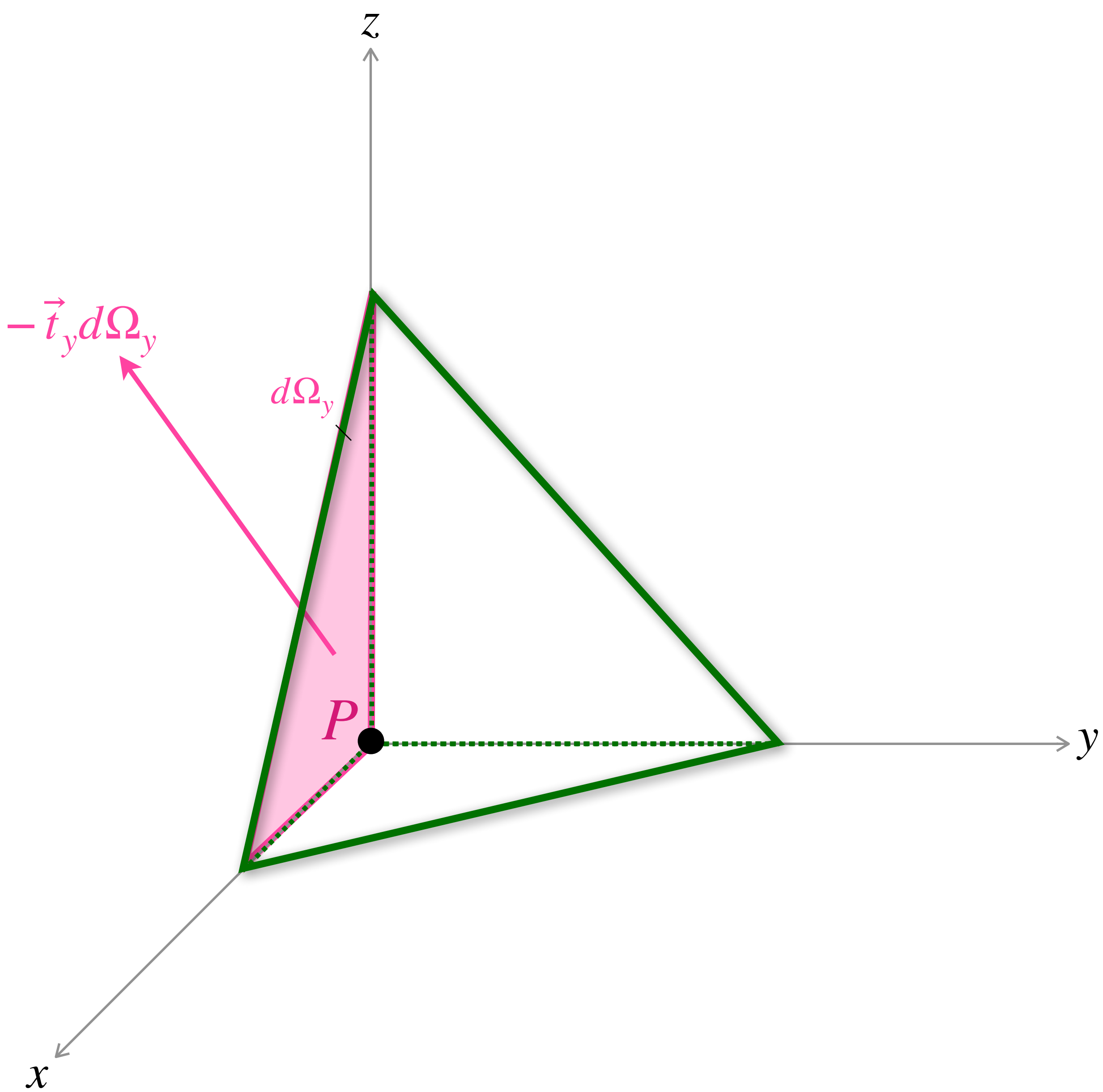

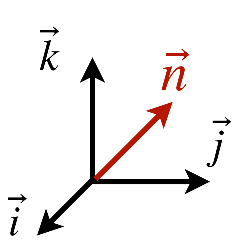

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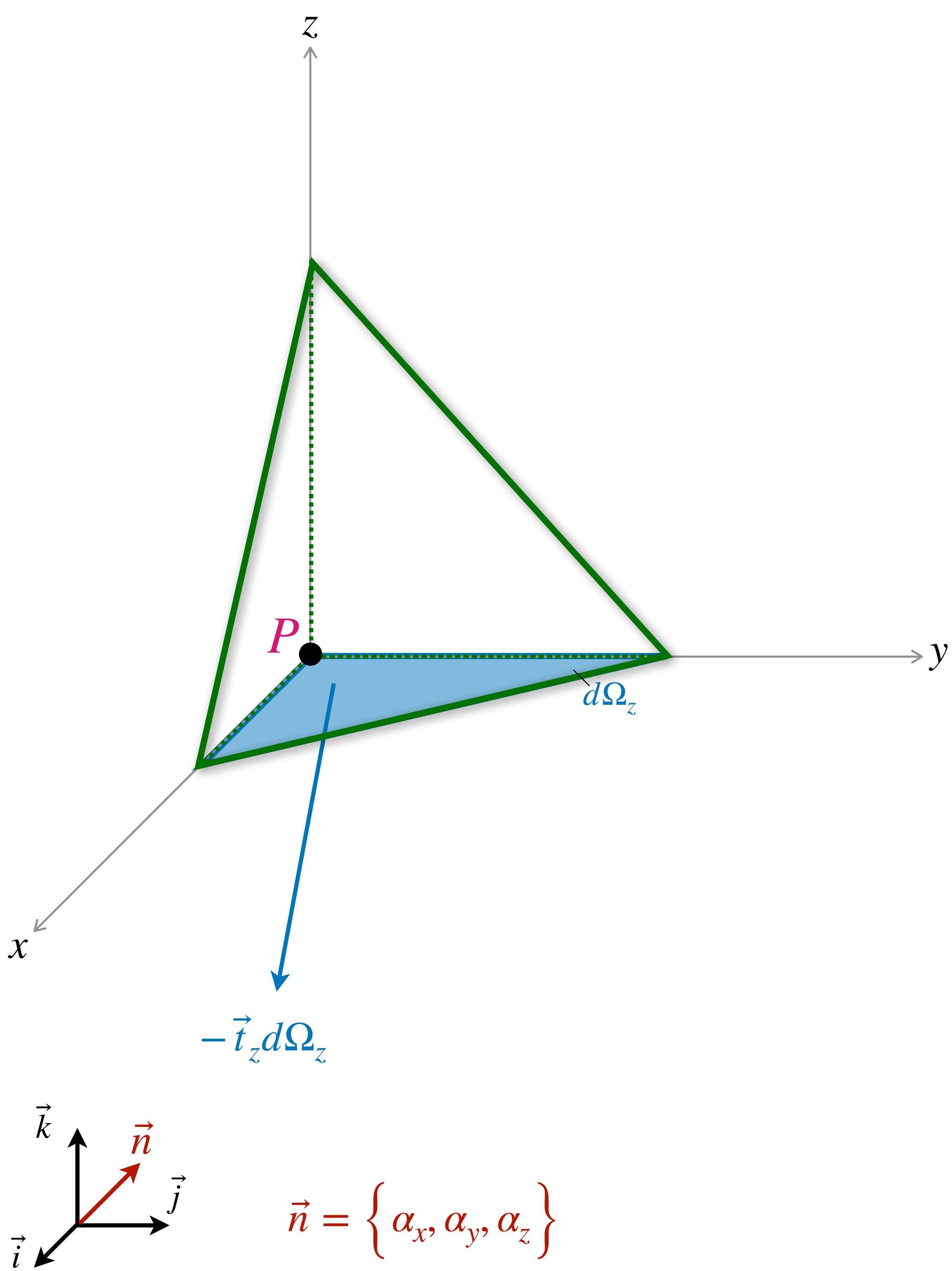

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TEOREMA DI CAUCHY

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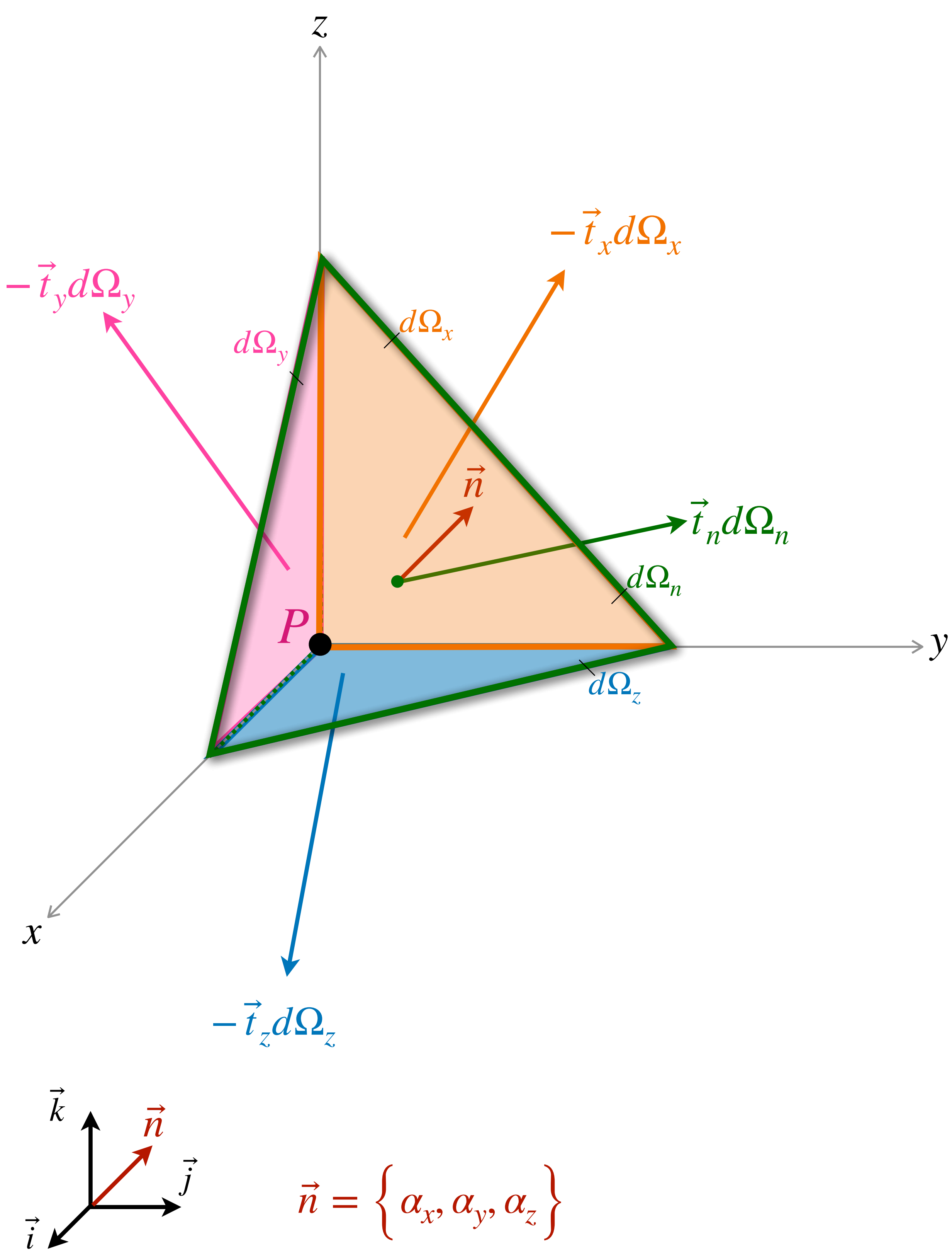

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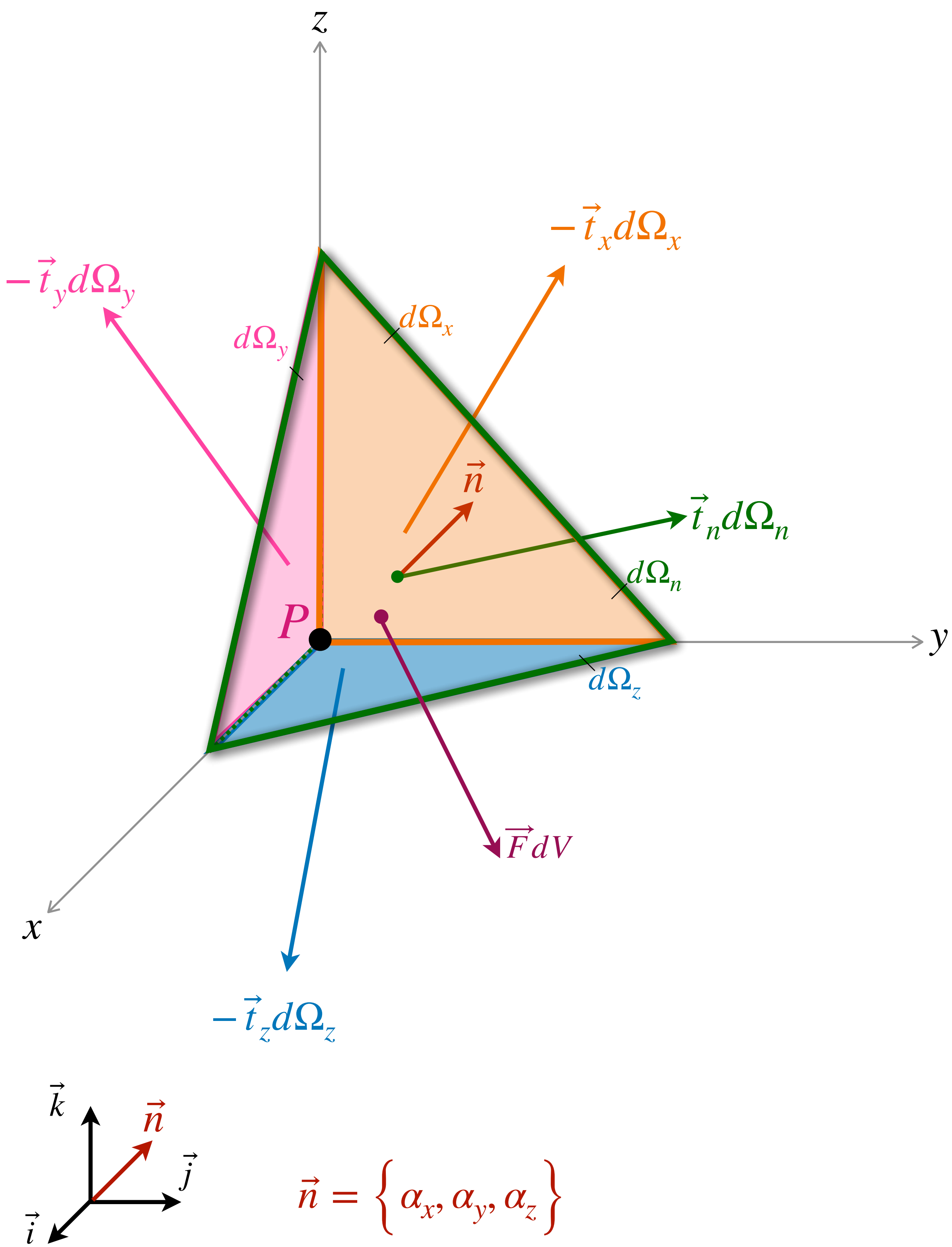

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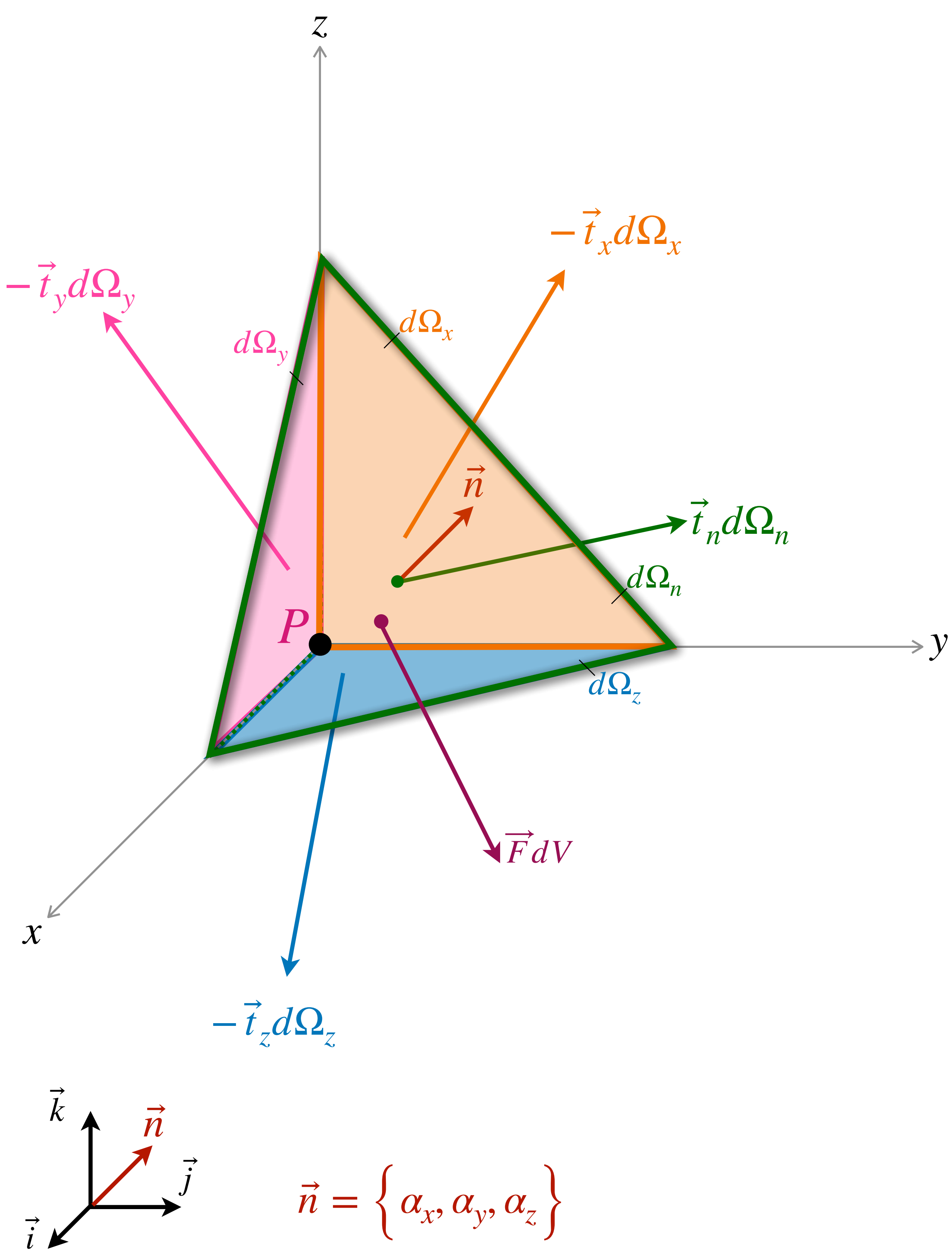


$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$

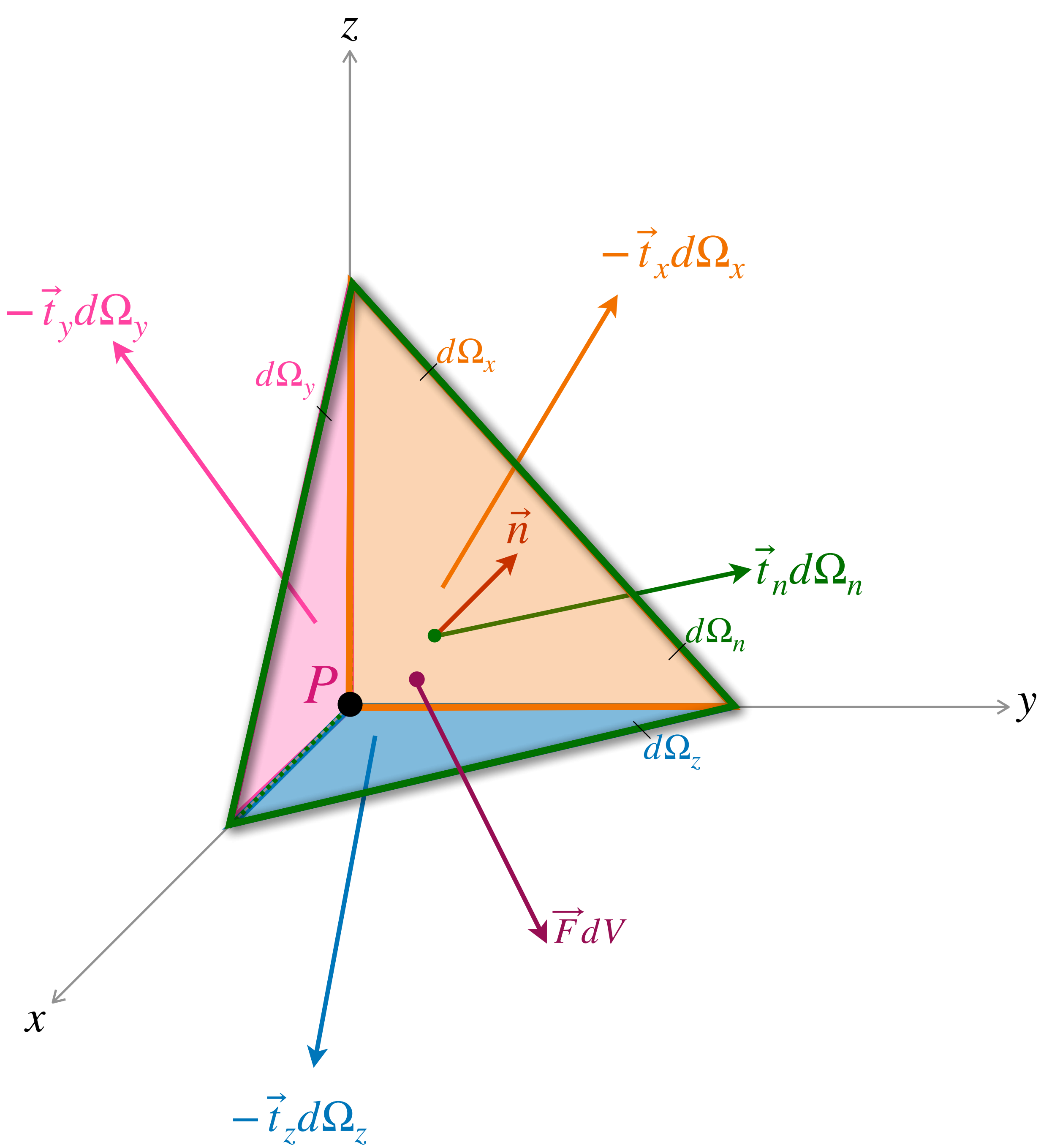
TEOREMA DI CAUCHY

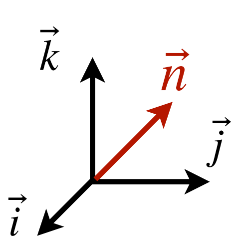
É sufficiente conoscere la tensione $\vec{t}_n$ su 3 giaciture ortogonali per poter definire la tensione su qualunque altra giacitura.

CONDIZIONI DI EQUILIBRIO

$$\vec{R} = \vec{0} \qquad \& \qquad \vec{M}_{(O)} = \vec{0}$$

_ Tensione secondo Cauchy





$$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$$

TEOREMA DI CAUCHY

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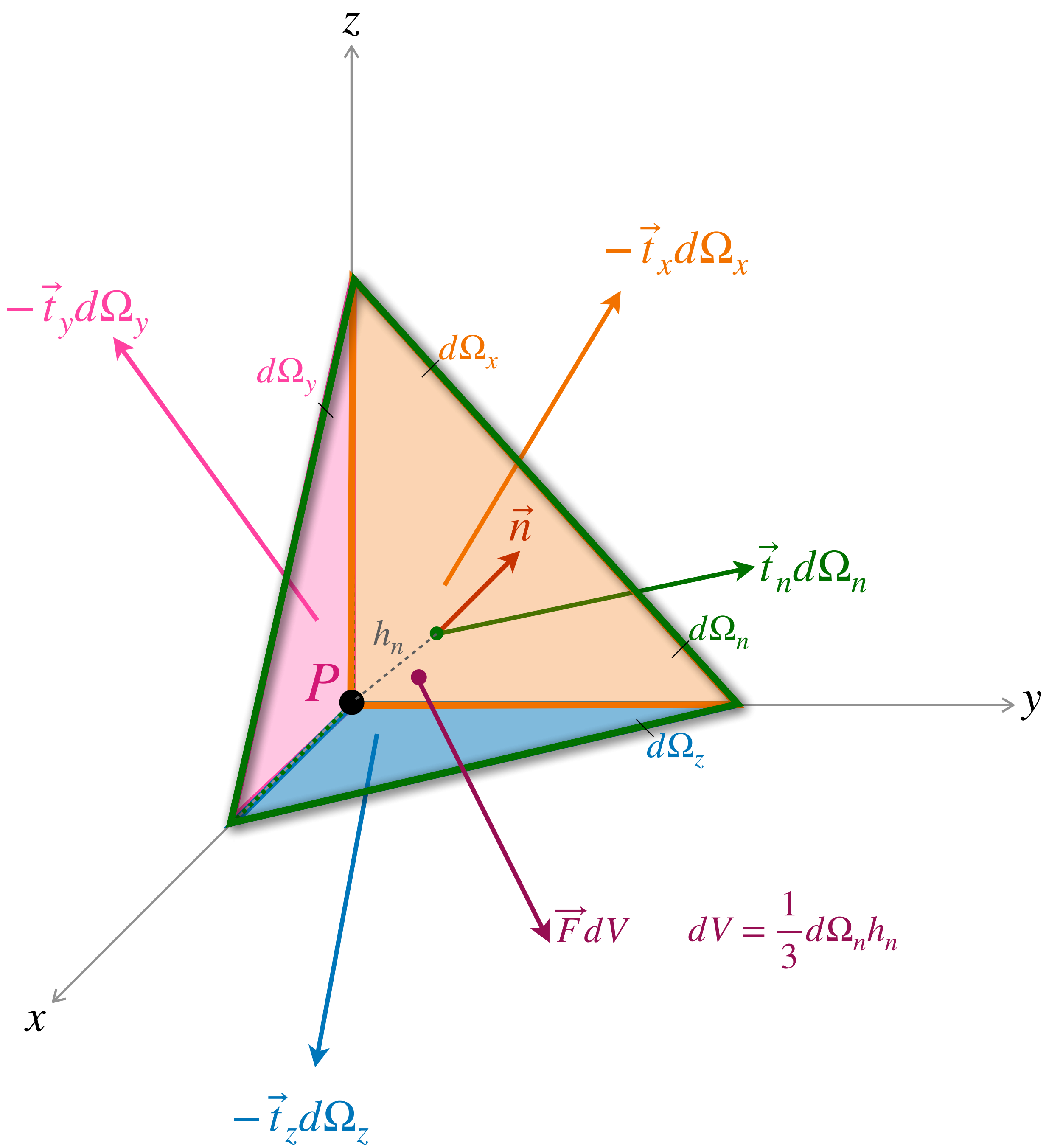
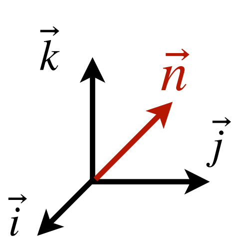
$$\vec{R} = \vec{0} \qquad \& \qquad \vec{M}_{(O)} = \vec{0}$$

$$-\vec{t}_x d\Omega_x - \vec{t}_y d\Omega_y - \vec{t}_z d\Omega_z + \vec{t}_n d\Omega_n + \vec{F} dV = \vec{0}$$

_ Tensione secondo Cauchy





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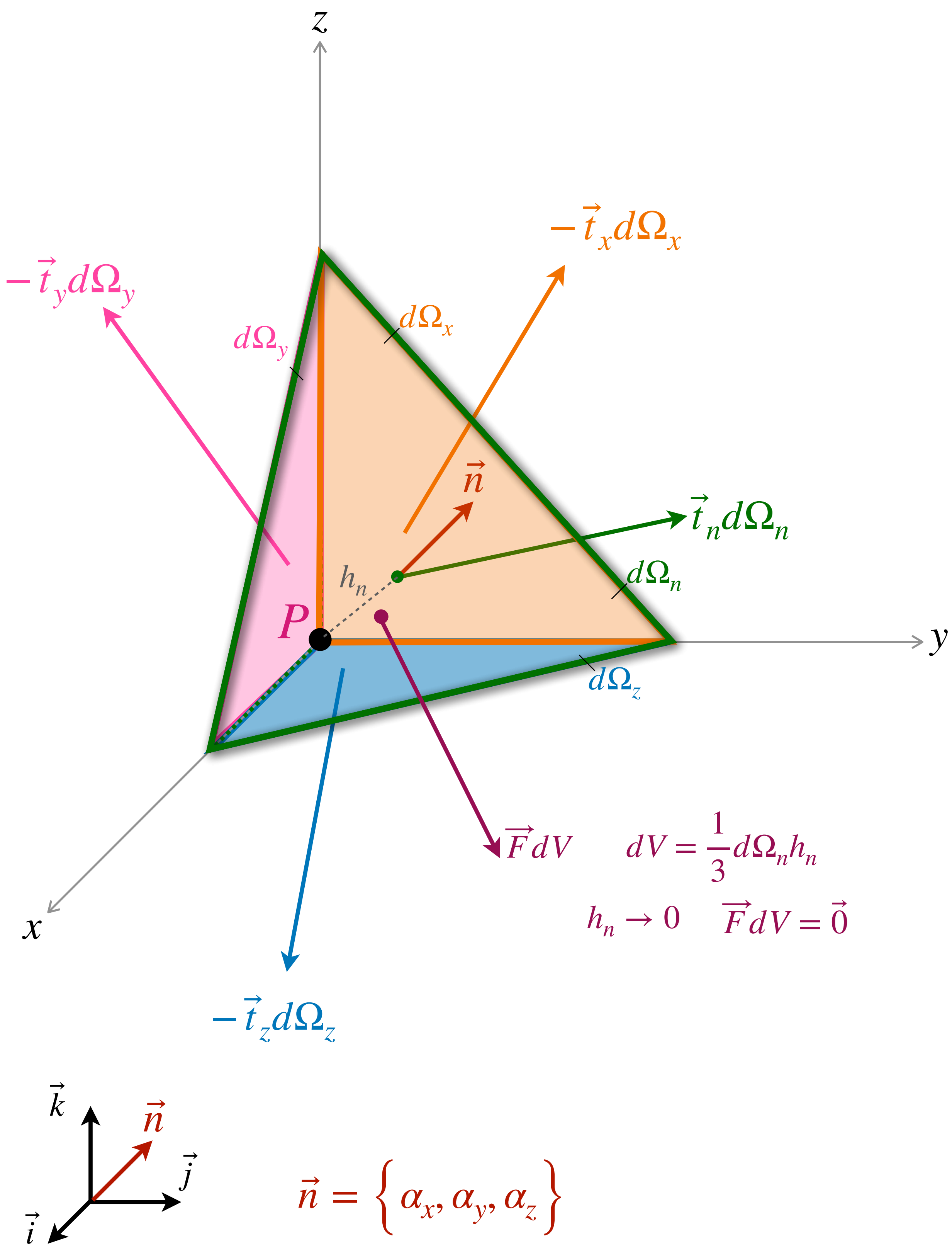
$$h_n \to 0 \qquad dV \to 0 \qquad \vec{F}dV = \vec{F}\frac{1}{3}d\Omega_n h_n = \vec{0}$$

_ Tensione secondo Cauchy



$\vec{n} = \{\alpha_x, \alpha_y, \alpha_z\}$

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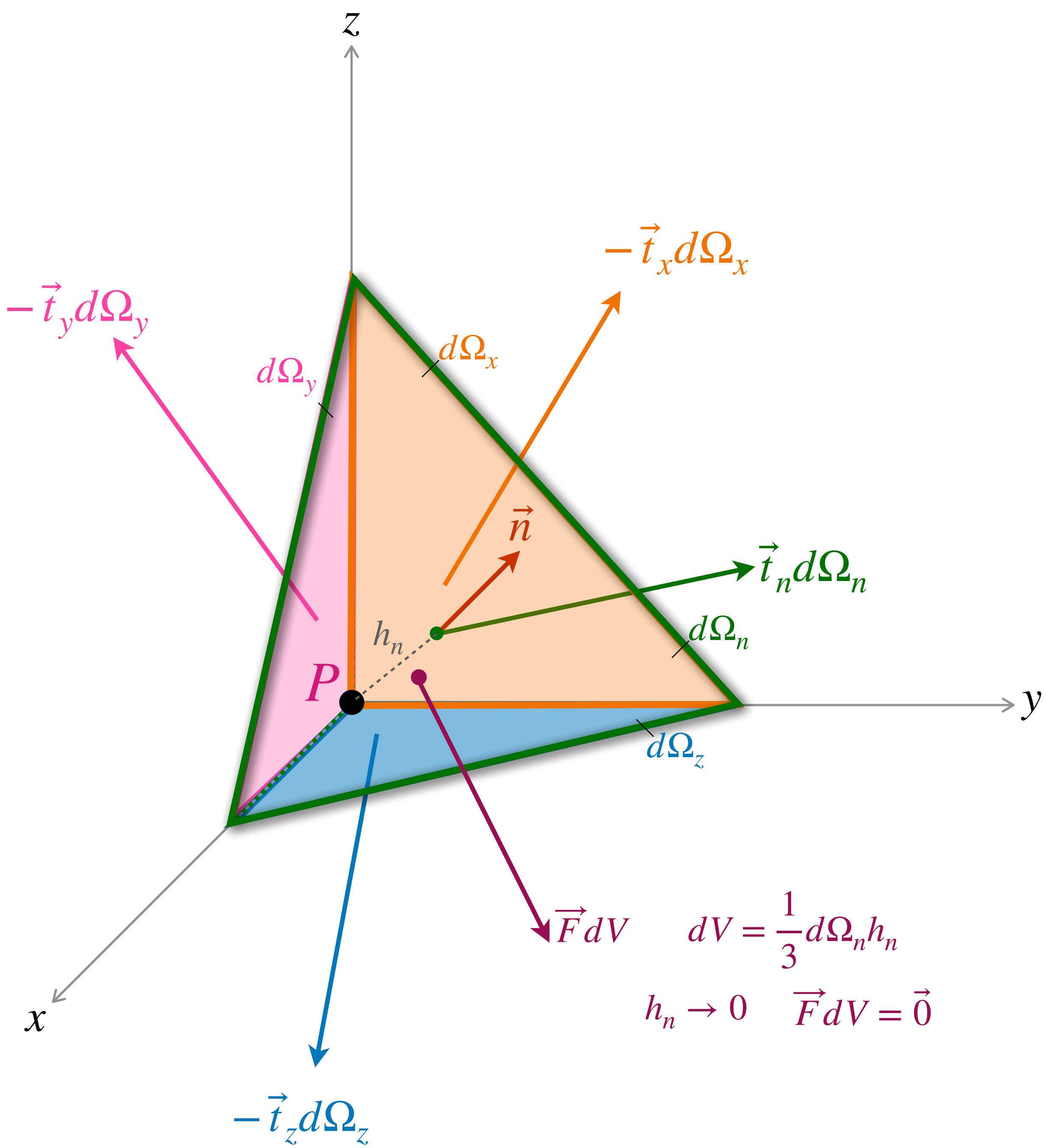

$$dV = \frac{1}{3} d\Omega_n h_n$$

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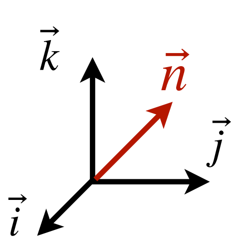

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$$-\vec{t}_x d\Omega_x - \vec{t}_y d\Omega_y - \vec{t}_z d\Omega_z + \vec{t}_n d\Omega_n = \vec{0}$$

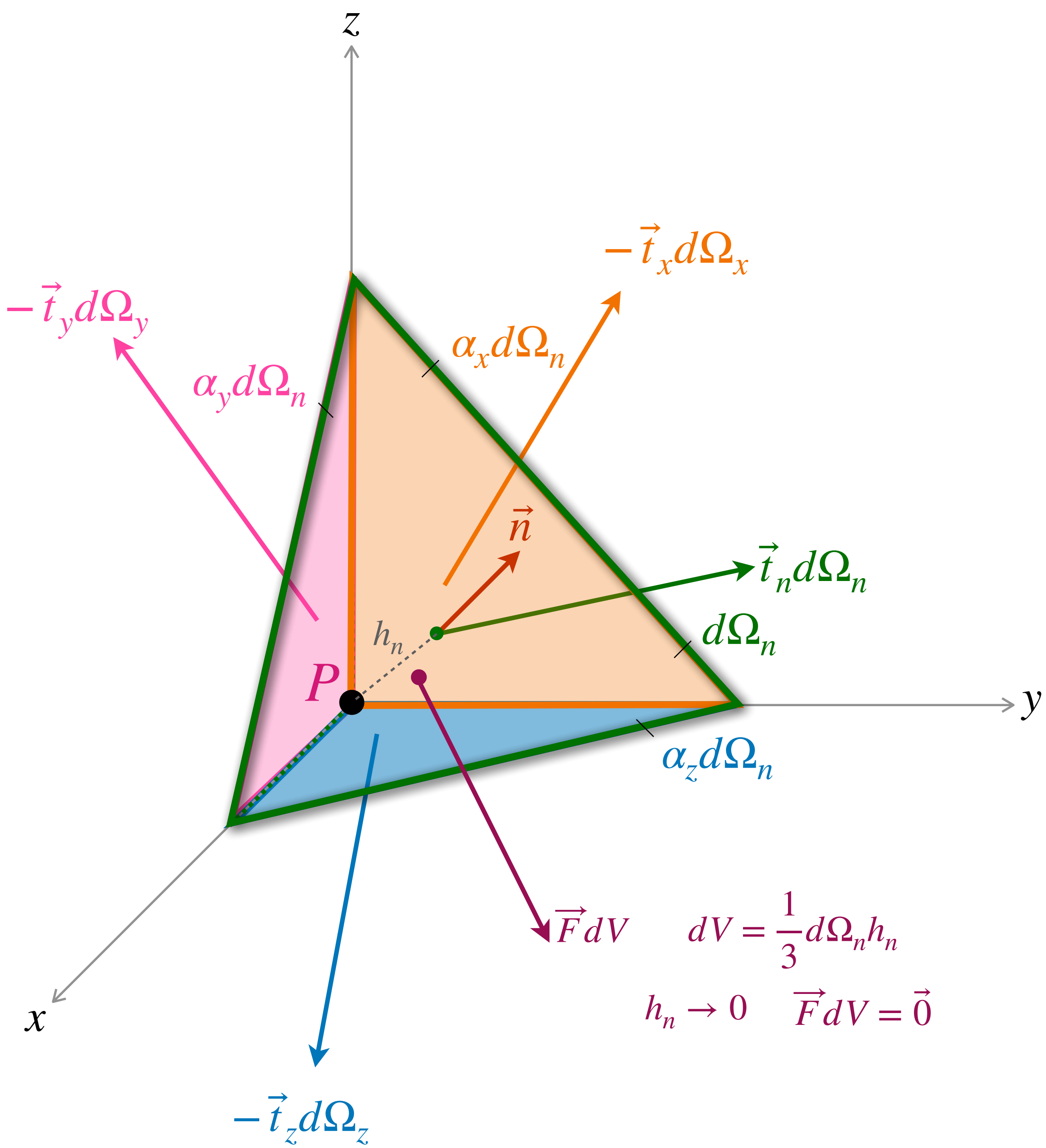
$$\alpha_x = \cos\left\{\hat{\vec{n}, \vec{i}}\right\}$$

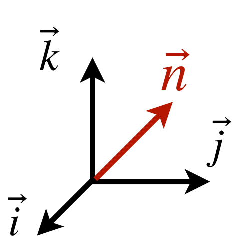
$$\vec{n} = \alpha_x \vec{i} + \alpha_y \vec{j} + \alpha_z \vec{k} \qquad \alpha_y = \cos\left\{\hat{\vec{n}, \vec{j}}\right\}$$

$$\alpha_z = \cos\left\{\hat{\vec{n}, \vec{k}}\right\}$$

$$d\Omega_x = \alpha_x d\Omega_n \qquad d\Omega_y = \alpha_y d\Omega_n \qquad d\Omega_z = \alpha_z d\Omega_n$$

_ Tensione secondo Cauchy





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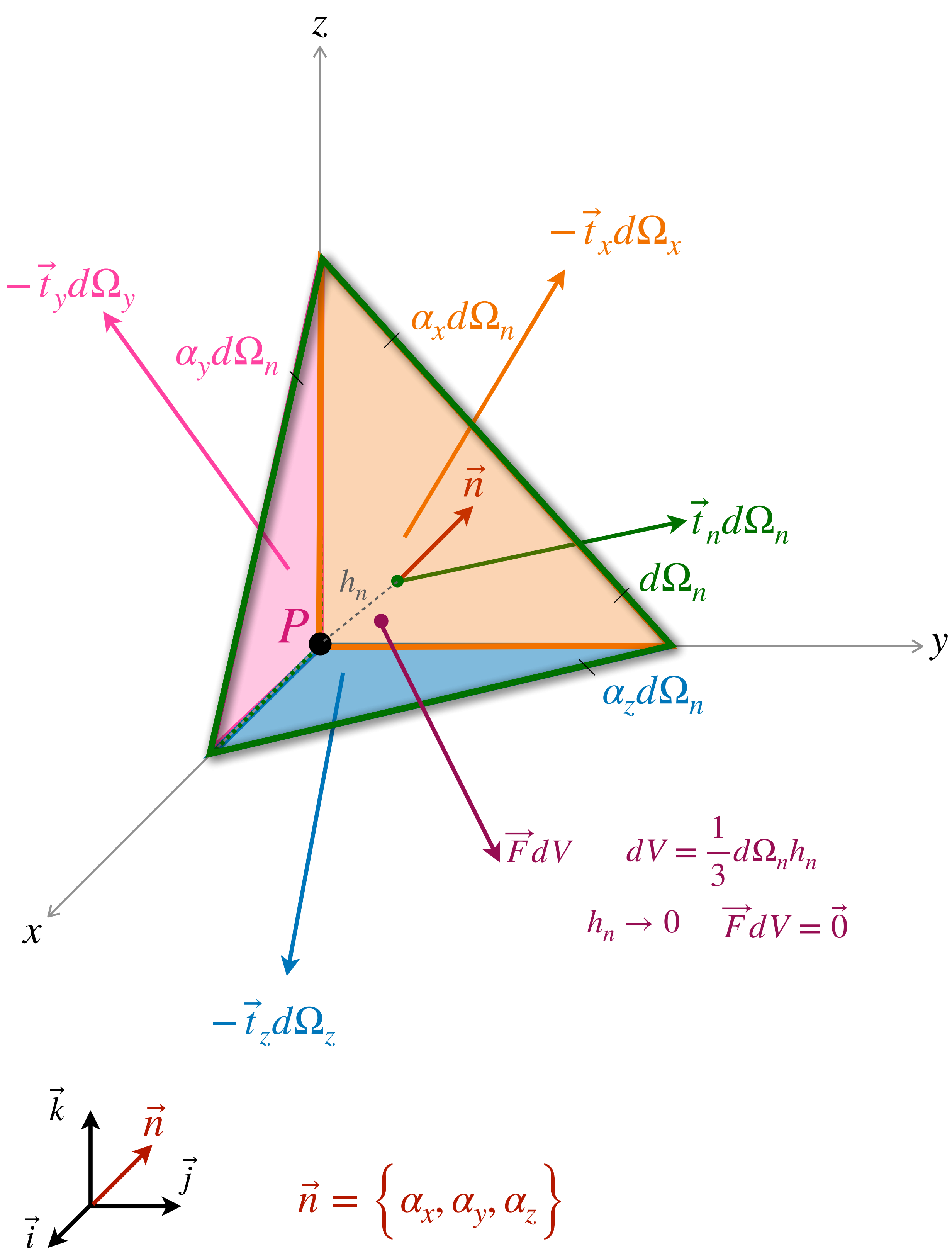
$$-\vec{t}_x \alpha_x d\Omega_n - \vec{t}_y \alpha_y d\Omega_n - \vec{t}_z \alpha_z d\Omega_n + \vec{t}_n d\Omega_n = \vec{0}$$

_ Tensione secondo Cauchy



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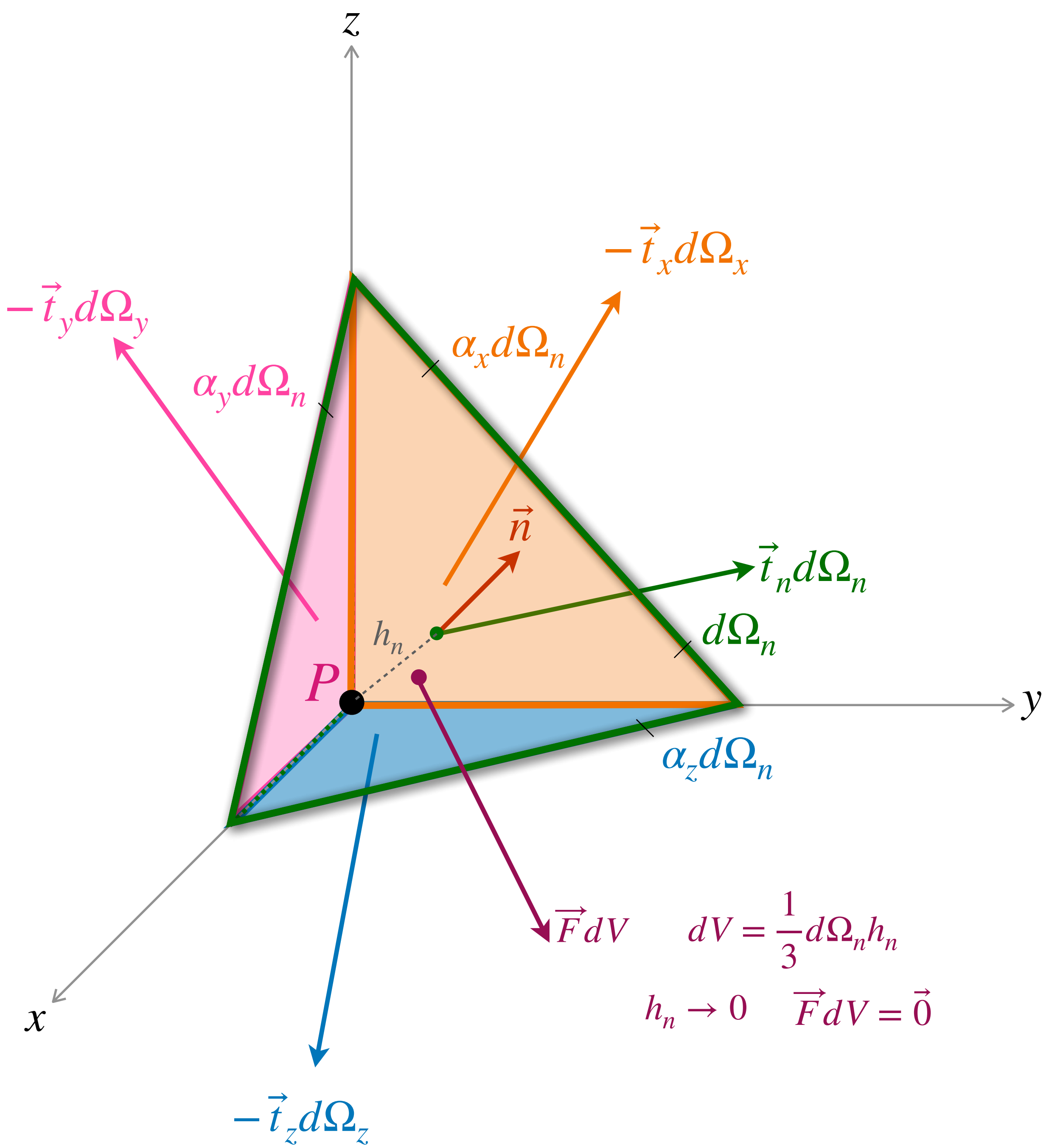
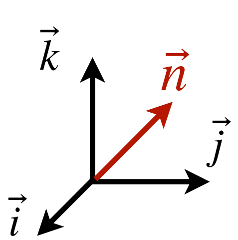
$$-\vec{t}_x \alpha_x d\Omega_n - \vec{t}_y \alpha_y d\Omega_n - \vec{t}_z \alpha_z d\Omega_n + \vec{t}_n d\Omega_n = \vec{0}$$

$$\left(-\vec{t}_x \alpha_x - \vec{t}_y \alpha_y - \vec{t}_z \alpha_z + \vec{t}_n \right) d\Omega_n = \vec{0}$$

_ Tensione secondo Cauchy





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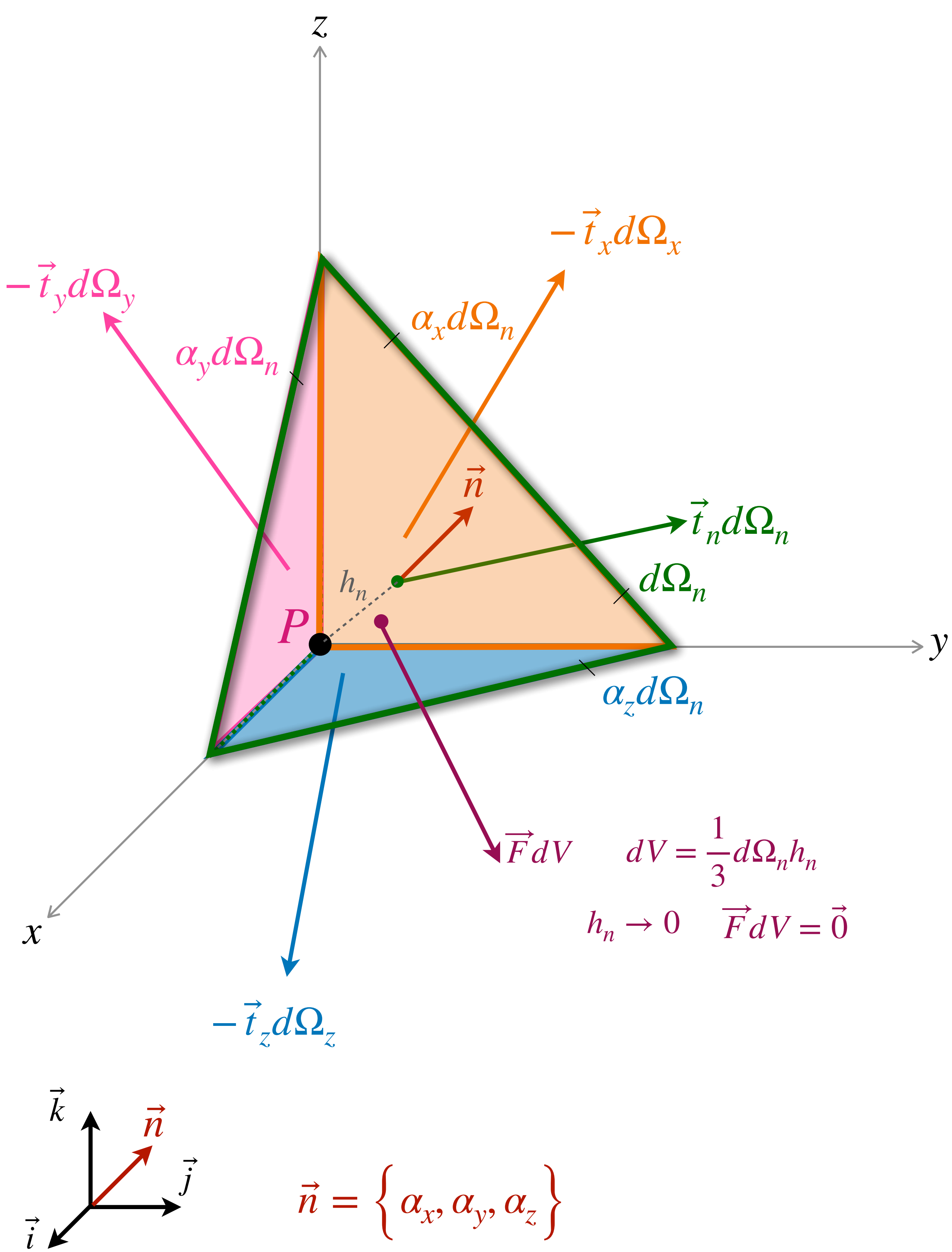
$$-\vec{t}_x \alpha_x d\Omega_n - \vec{t}_y \alpha_y d\Omega_n - \vec{t}_z \alpha_z d\Omega_n + \vec{t}_n d\Omega_n = \vec{0}$$

$$\left(-\vec{t}_x \alpha_x - \vec{t}_y \alpha_y - \vec{t}_z \alpha_z + \vec{t}_n \right) d\Omega_n = \vec{0}$$

$$\boxed{\vec{t}_n = \vec{t}_x \alpha_x + \vec{t}_y \alpha_y + \vec{t}_z \alpha_z}$$

_ Tensione secondo Cauchy



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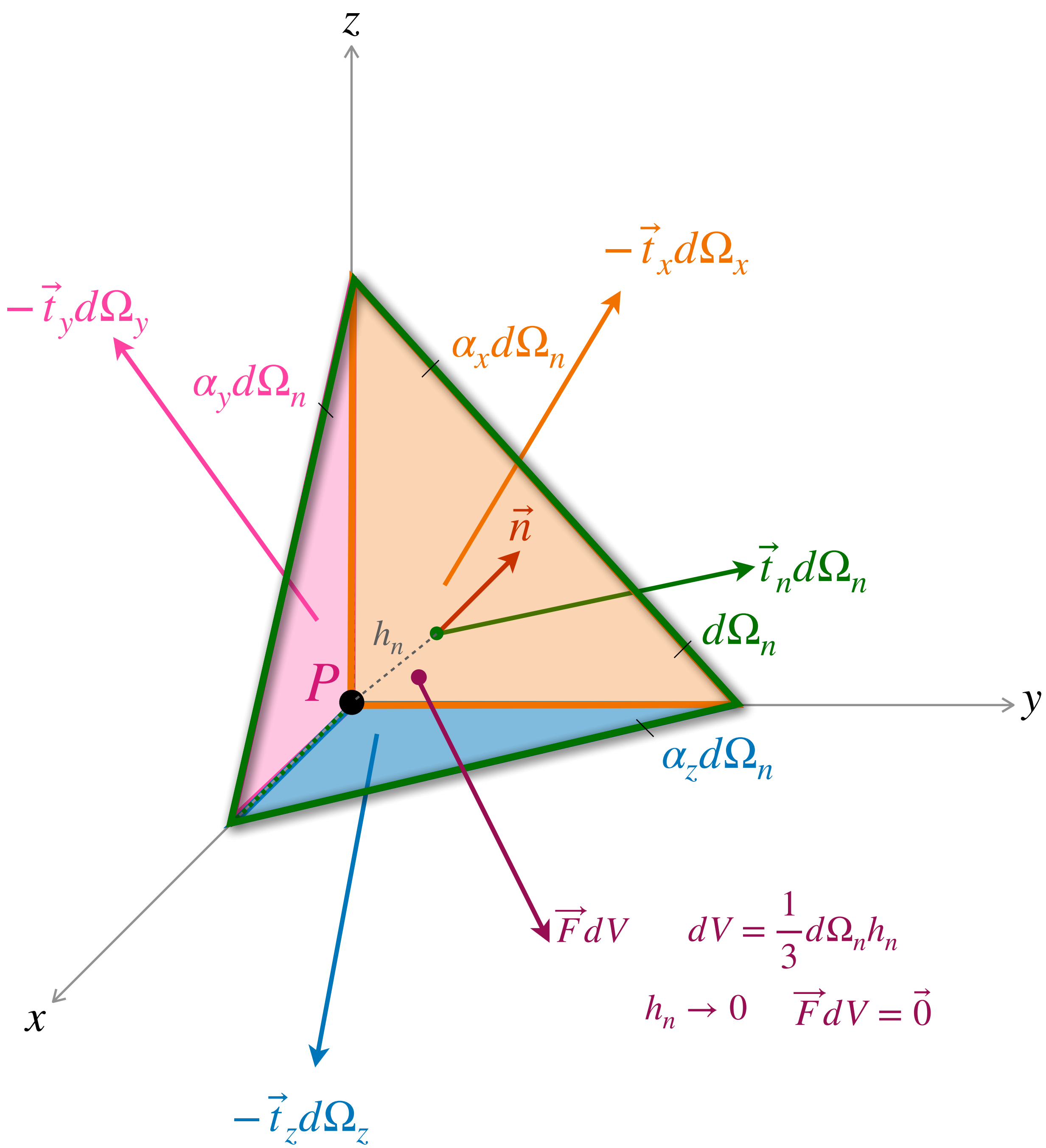

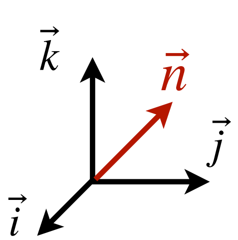

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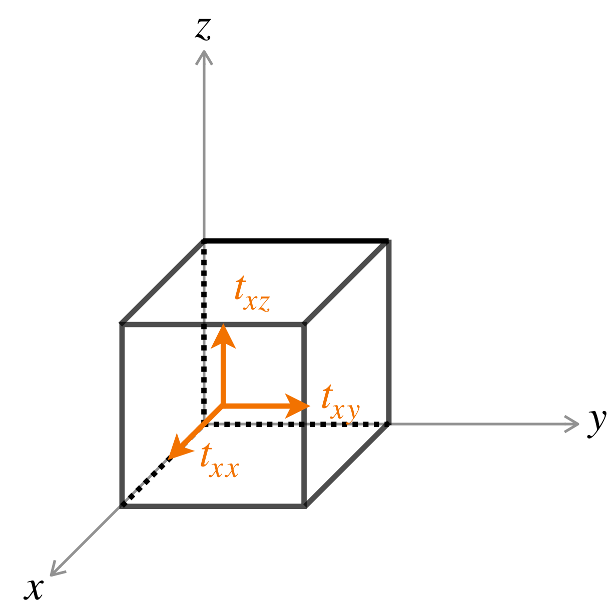

$$\vec{t}_x = t_{xx}\vec{i} + t_{xy}\vec{j} + t_{xz}\vec{k}$$

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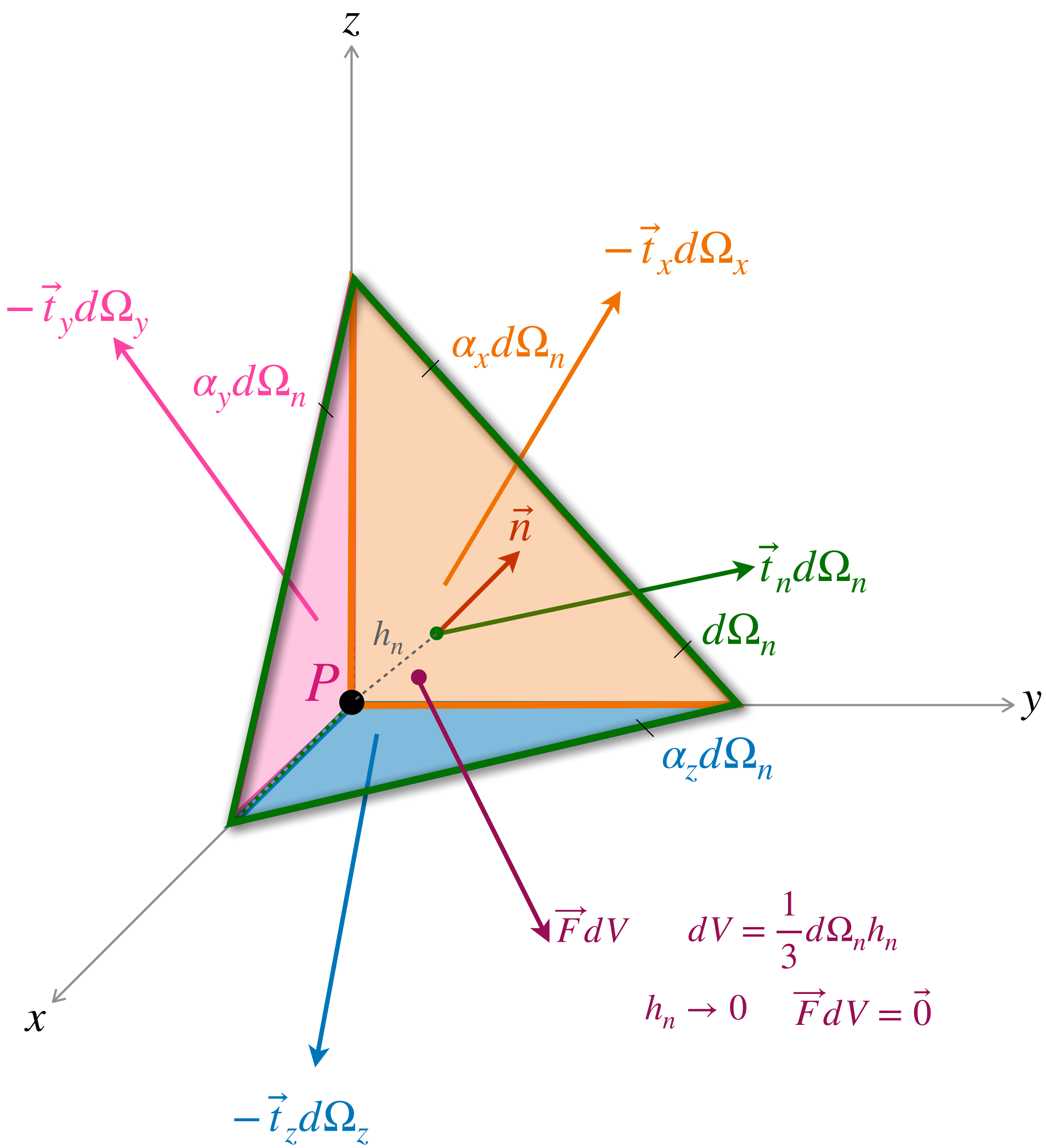

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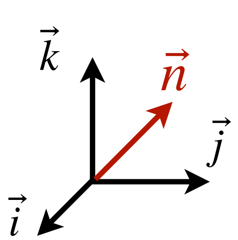

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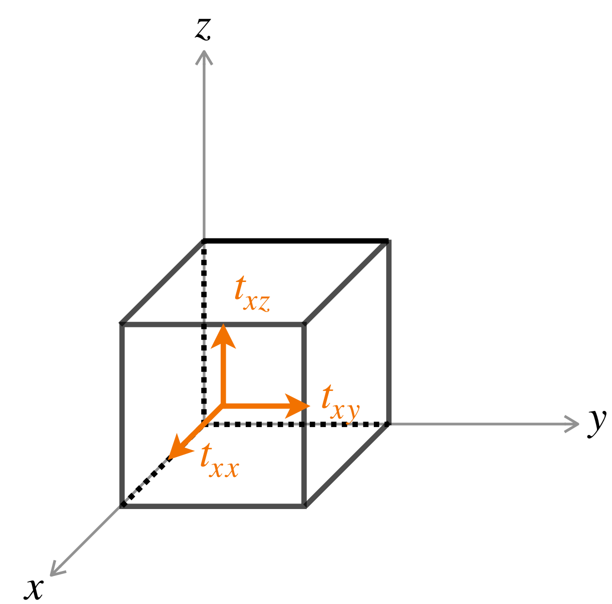


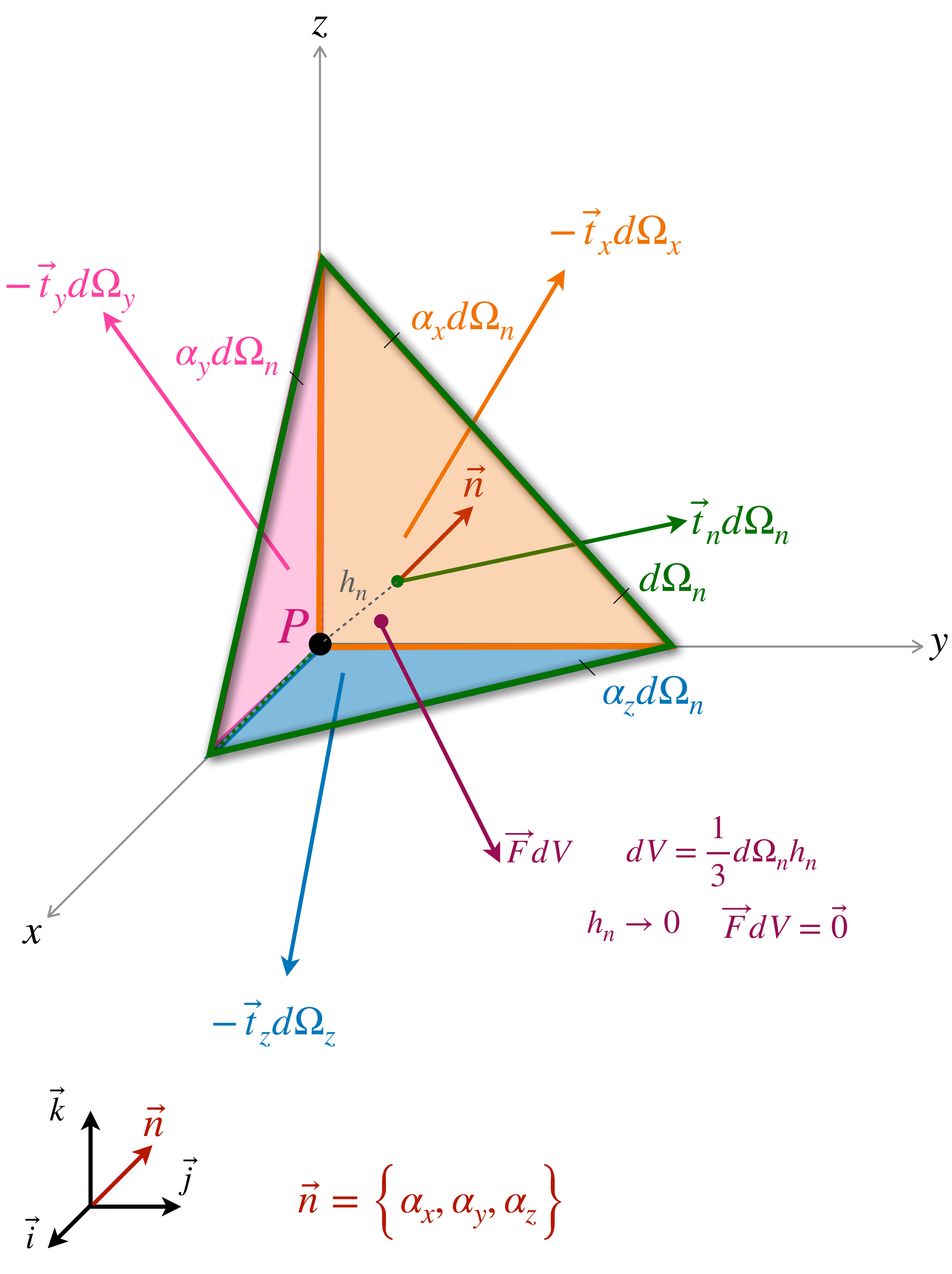
$$\vec{t}_x = t_{xx}\vec{i} + t_{xy}\vec{j} + t_{xz}\vec{k}$$

$$\vec{t}_x = \left\{ t_{xx}, t_{xy}, t_{xz} \right\}$$

$$\vec{t}_y = t_{yx}\vec{i} + t_{yy}\vec{j} + t_{yz}\vec{k} \qquad \vec{t}_y = \left\{ t_{yx}, t_{yy}, t_{yz} \right\}$$

$$\vec{t}_z = t_{zx}\vec{i} + t_{zy}\vec{j} + t_{zz}\vec{k} \qquad \vec{t}_z = \left\{ t_{zx}, t_{zy}, t_{zz} \right\}$$

_ Tensione secondo Cauchy



$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$

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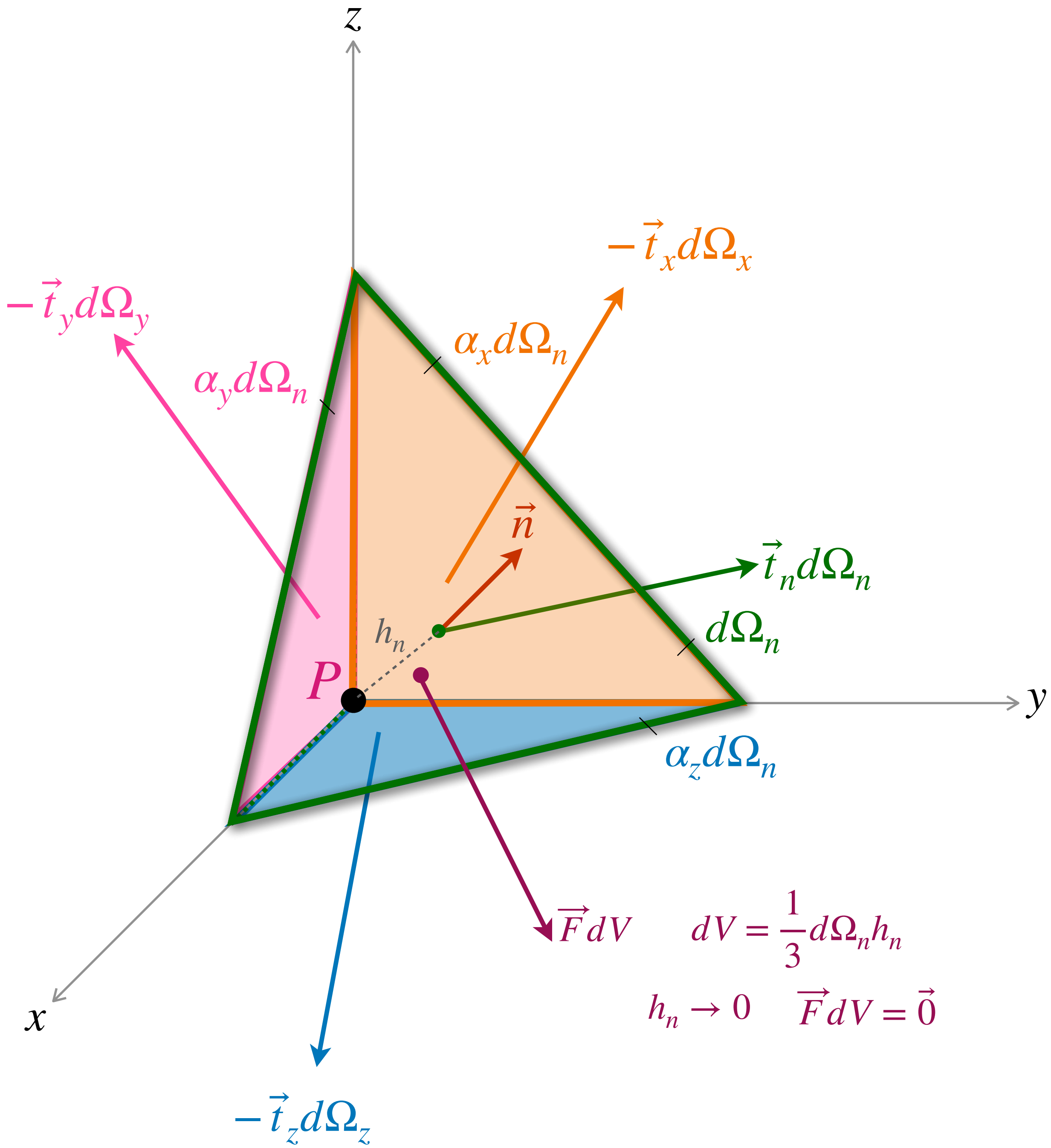
$\vec{t}_x = \left\{ t_{xx}, t_{xy}, t_{xz} \right\} \qquad \vec{t}_y = \left\{ t_{yx}, t_{yy}, t_{yz} \right\} \qquad \vec{t}_z = \left\{ t_{zx}, t_{zy}, t_{zz} \right\}$

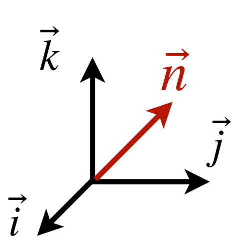
$$t_{nx} = t_{xx}\alpha_x + t_{yx}\alpha_y + t_{zx}\alpha_z$$

$$t_{ny} = t_{xy}\alpha_x + t_{yy}\alpha_y + t_{zy}\alpha_z$$

$$t_{nz} = t_{xz}\alpha_x + t_{yz}\alpha_y + t_{zz}\alpha_z$$

_ Tensione secondo Cauchy





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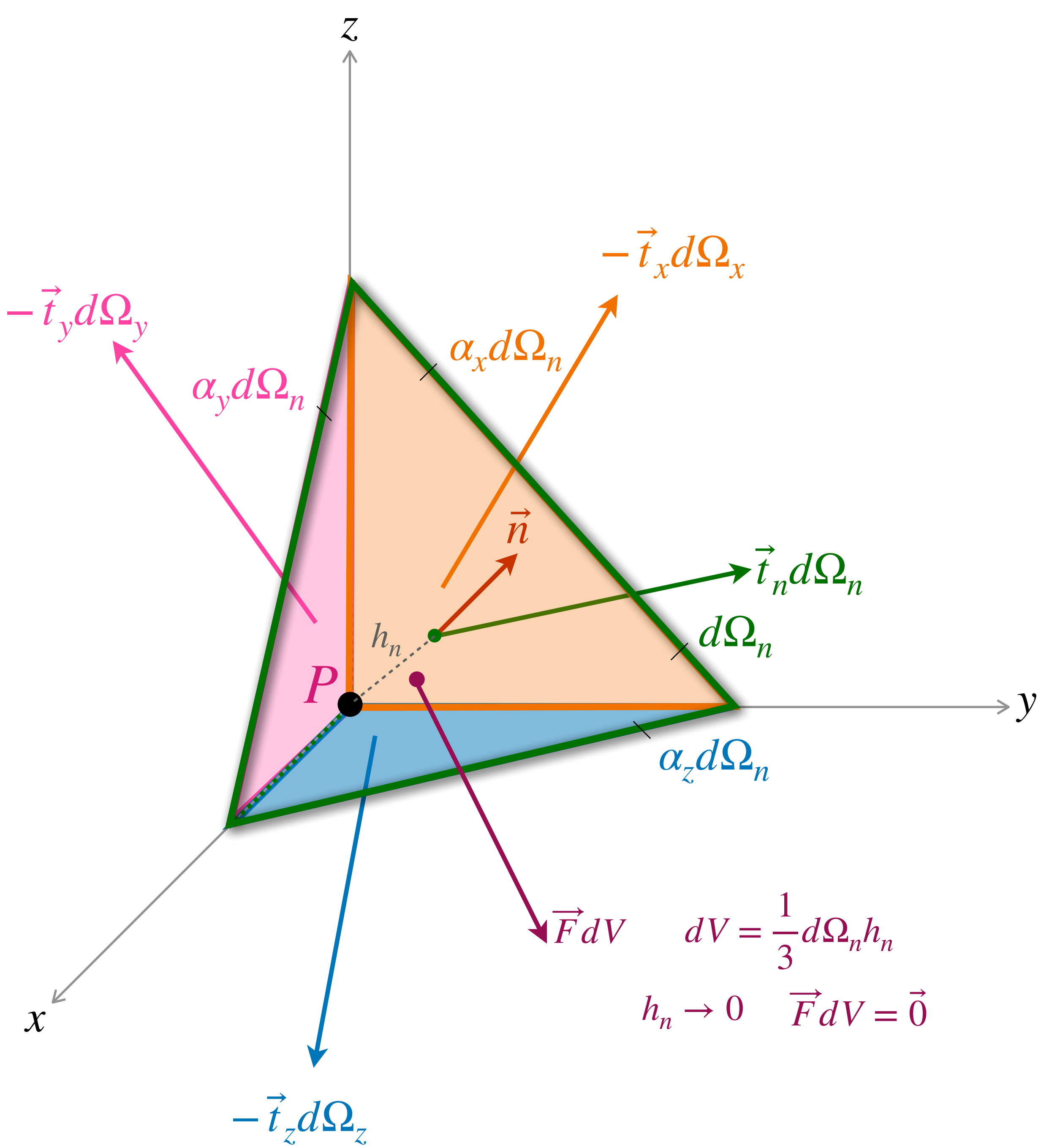
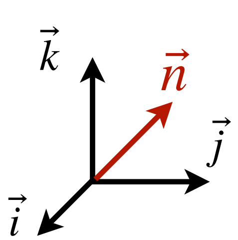
$$\boxed{\vec{t}_n = \vec{t}_x \alpha_x + \vec{t}_y \alpha_y + \vec{t}_z \alpha_z}$$

$$\vec{t}_x = \left\{ t_{xx}, t_{xy}, t_{xz} \right\} \qquad \vec{t}_y = \left\{ t_{yx}, t_{yy}, t_{yz} \right\} \qquad \vec{t}_z = \left\{ t_{zx}, t_{zy}, t_{zz} \right\}$$

$$\begin{array}{lcccccc}
 & & \vec{t}_x & & \vec{t}_y & & \vec{t}_z \\
t_{nx} & = & t_{xx}\alpha_x & + & t_{yx}\alpha_y & + & t_{zx}\alpha_z \\
t_{ny} & = & t_{xy}\alpha_x & + & t_{yy}\alpha_y & + & t_{zy}\alpha_z \\
t_{nz} & = & t_{xz}\alpha_x & + & t_{yz}\alpha_y & + & t_{zz}\alpha_z
\end{array}$$

_ Tensione secondo Cauchy



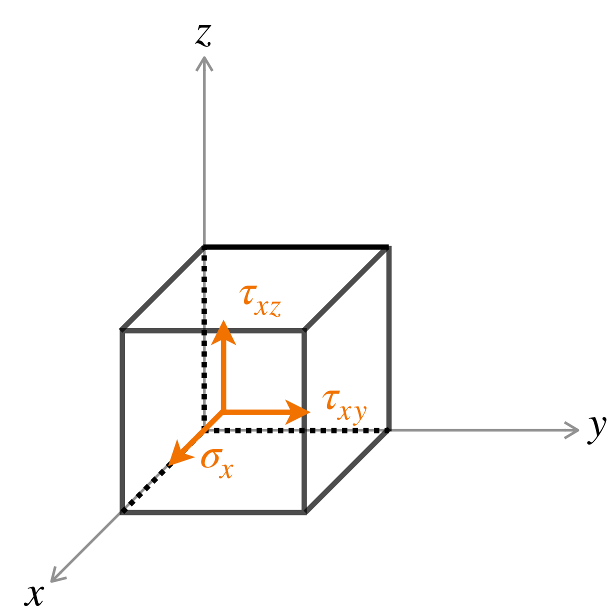


$$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$$

TEOREMA DI CAUCHY

É sufficiente conoscere la tensione $\vec{t}_n$ su 3 giaciture ortogonali per poter definire la tensione su qualunque altra giacitura.

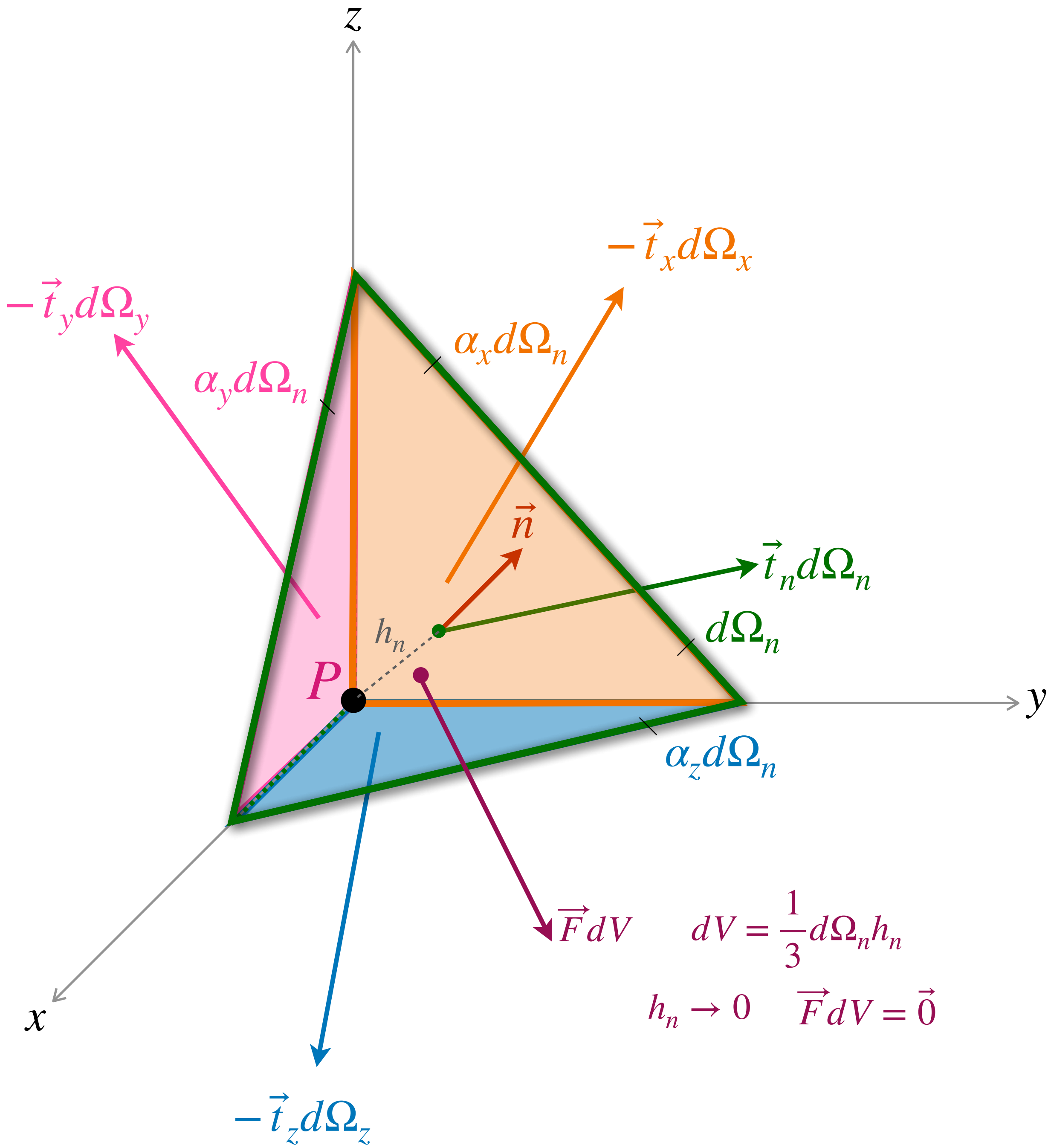
$$\boxed{\vec{t}_n = \vec{t}_x \alpha_x + \vec{t}_y \alpha_y + \vec{t}_z \alpha_z}$$



$$\vec{t}_x = \sigma_x \vec{i} + \tau_{xy} \vec{j} + \tau_{xz} \vec{k}$$

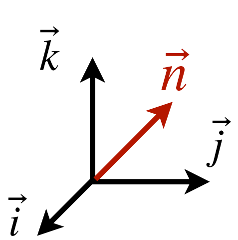
$$\vec{t}_x = \left\{ \sigma_x, \tau_{xy}, \tau_{xz} \right\}$$

_ Tensione secondo Cauchy



$$dV = \frac{1}{3} d\Omega_n h_n$$

$$h_n \to 0 \qquad \vec{F} dV = \vec{0}$$



$$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$$

TEOREMA DI CAUCHY

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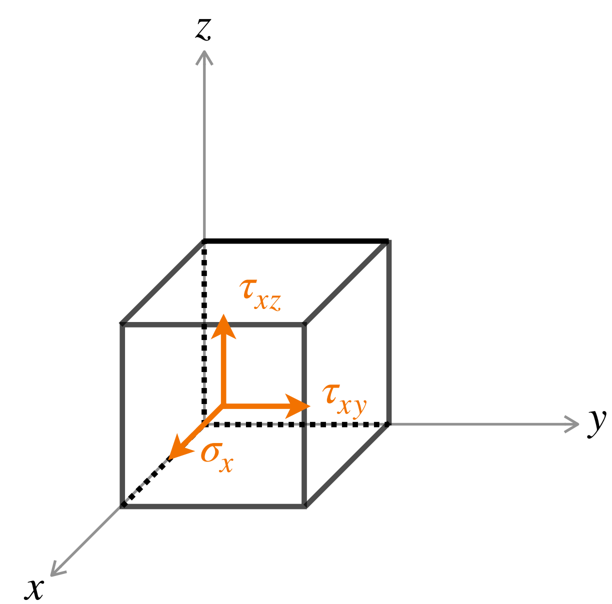


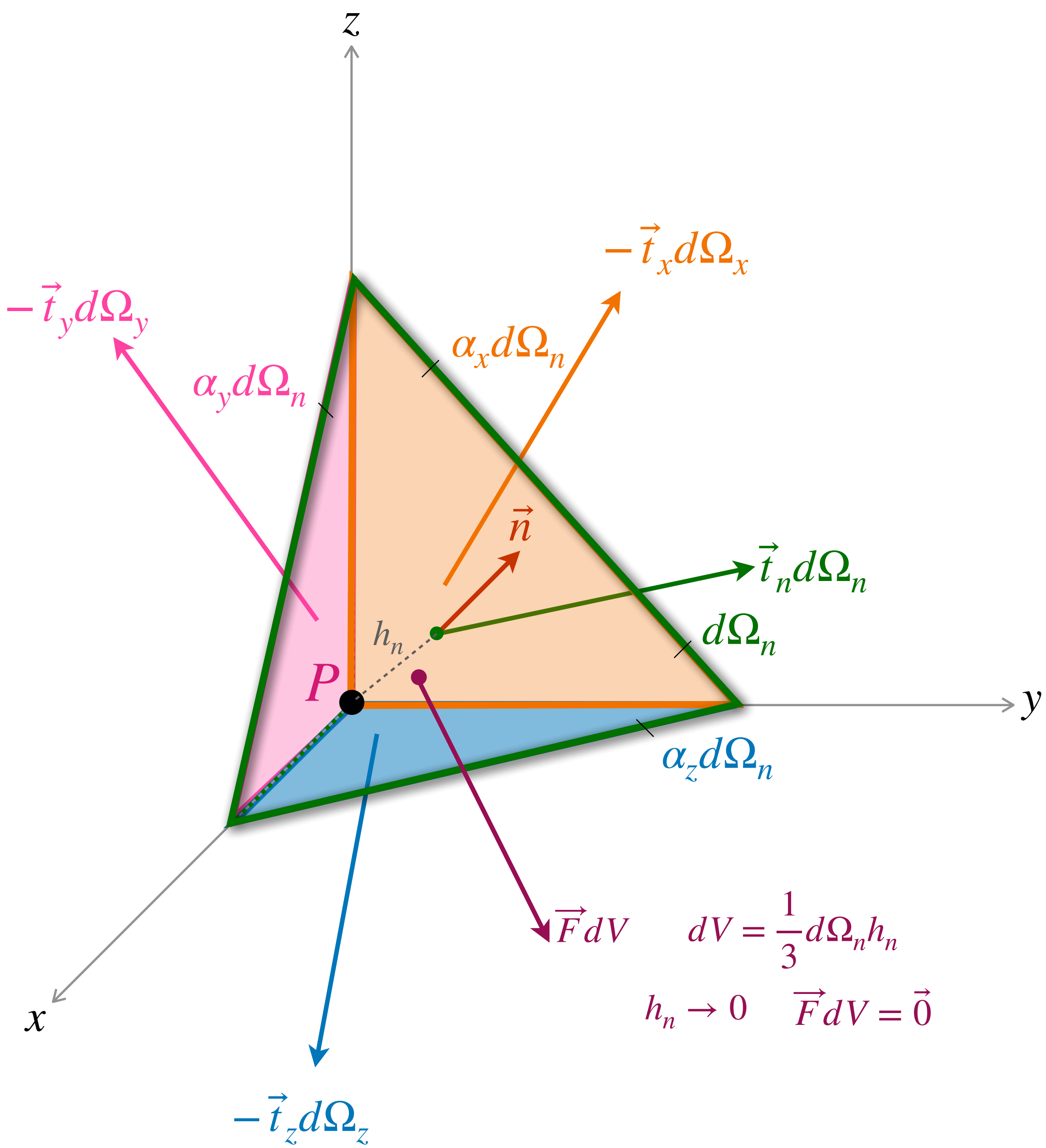
$$\vec{t}_x = \sigma_x \vec{i} + \tau_{xy} \vec{j} + \tau_{xz} \vec{k}$$

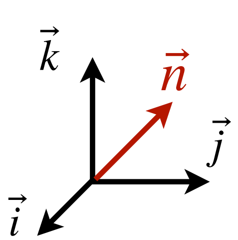
$$\vec{t}_x = \left\{ \sigma_x, \tau_{xy}, \tau_{xz} \right\}$$

$$\vec{t}_y = \tau_{yx} \vec{i} + \sigma_y \vec{j} + \tau_{yz} \vec{k} \qquad \vec{t}_y = \left\{ \tau_{yx}, \sigma_y, \tau_{yz} \right\}$$

$$\vec{t}_z = \tau_{zx} \vec{i} + \tau_{zy} \vec{j} + \sigma_z \vec{k} \qquad \vec{t}_z = \left\{ \tau_{zx}, \tau_{zy}, \sigma_z \right\}$$

_ Tensione secondo Cauchy





$$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$$

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$$t_{nx} = \sigma_x \alpha_x + \tau_{yx} \alpha_y + \tau_{zx} \alpha_z$$

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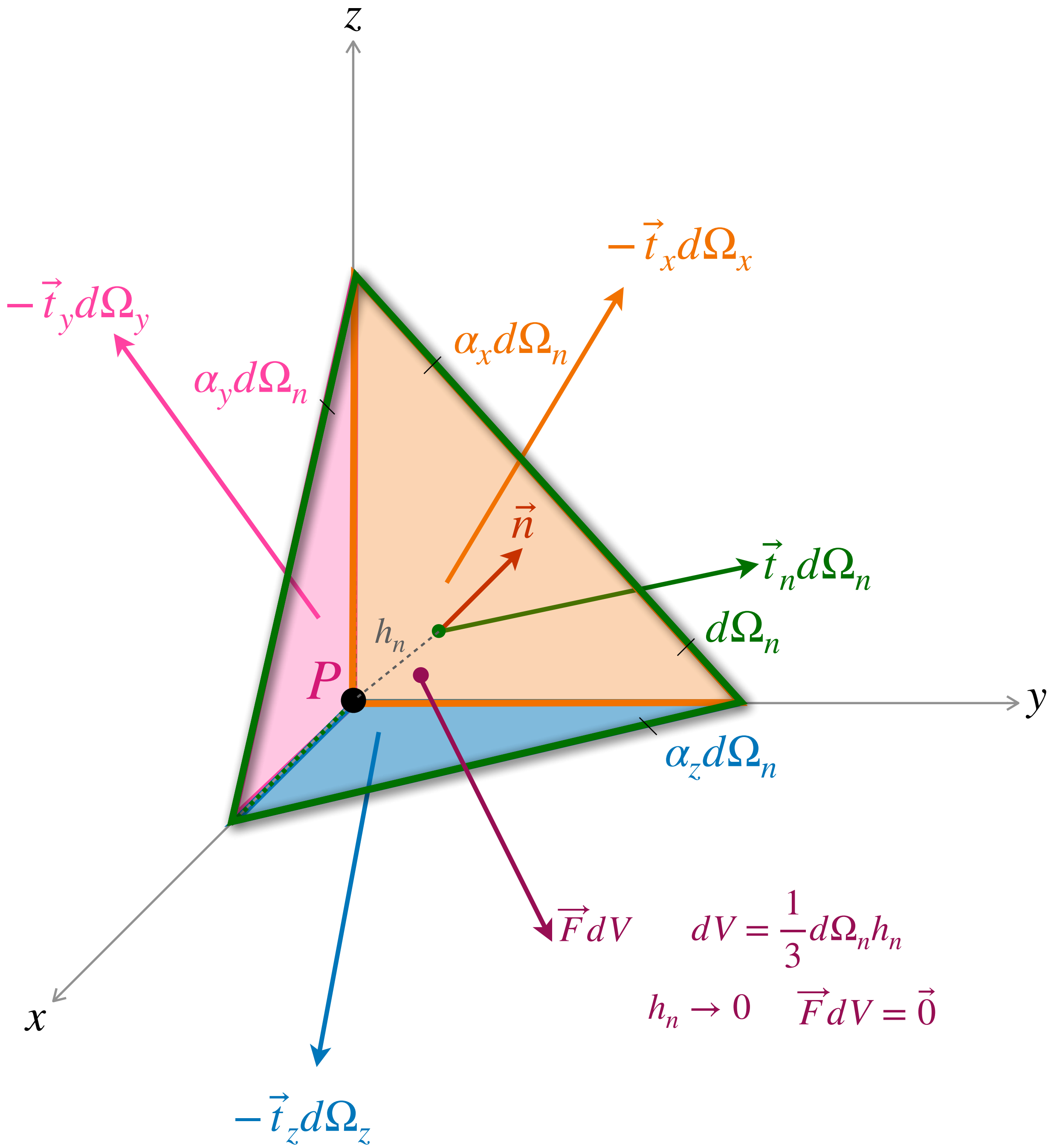
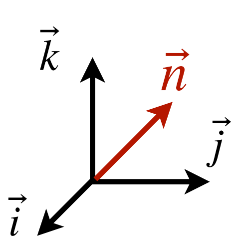
$$t_{nz} = \tau_{xz} \alpha_x + \tau_{yz} \alpha_y + \sigma_z \alpha_z$$

_ Tensione secondo Cauchy





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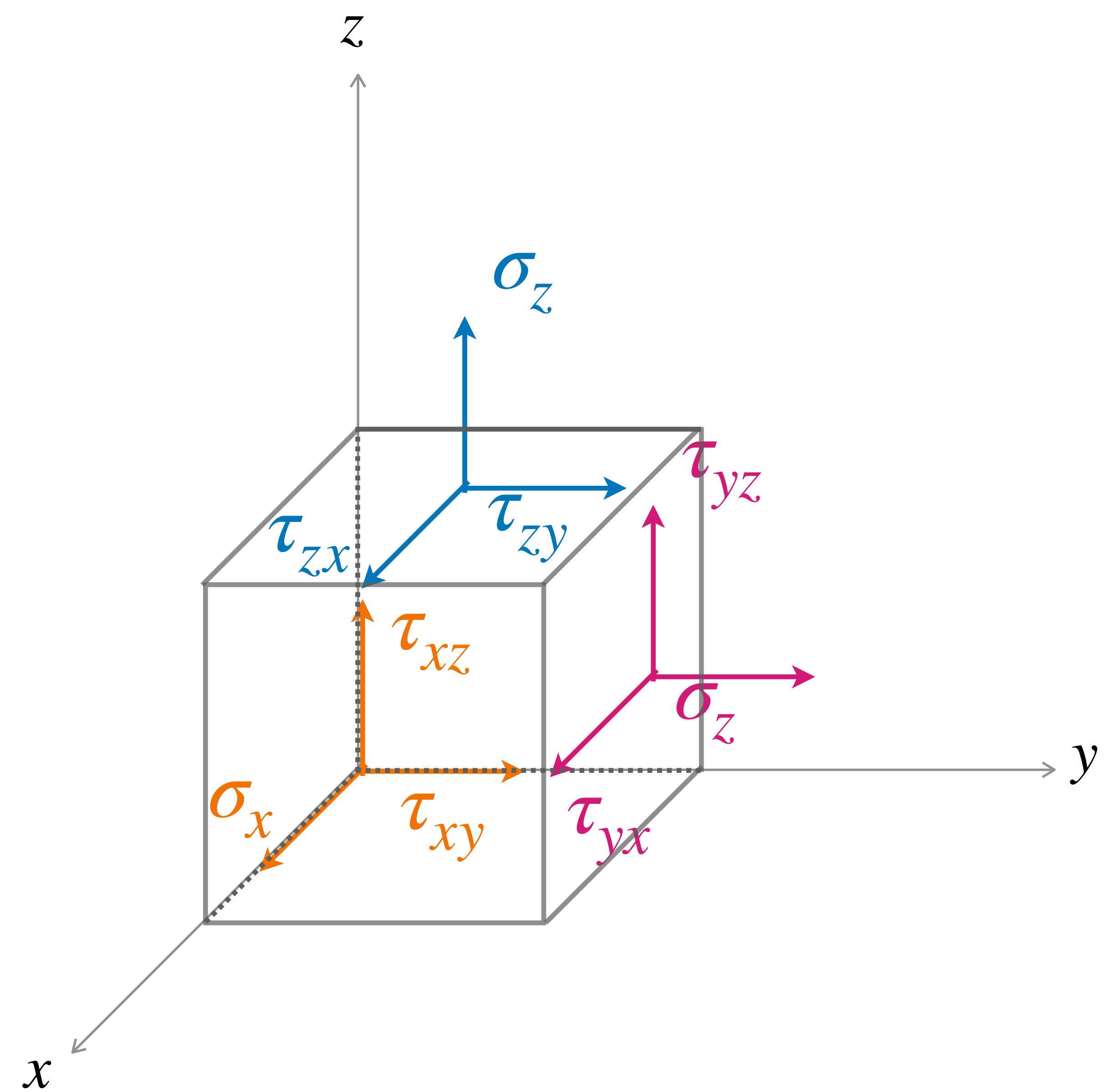
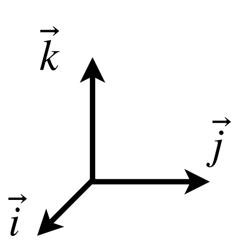
$$\begin{array}{lcccccc}
 & & \vec{t}_x & & \vec{t}_y & & \vec{t}_z \\
t_{nx} & = & \sigma_x \alpha_x & + & \tau_{yx} \alpha_y & + & \tau_{zx} \alpha_z \\
t_{ny} & = & \tau_{xy} \alpha_x & + & \sigma_y \alpha_y & + & \tau_{zy} \alpha_z \\
t_{nz} & = & \tau_{xz} \alpha_x & + & \tau_{yz} \alpha_y & + & \sigma_z \alpha_z
\end{array}$$

_ Tensione secondo Cauchy





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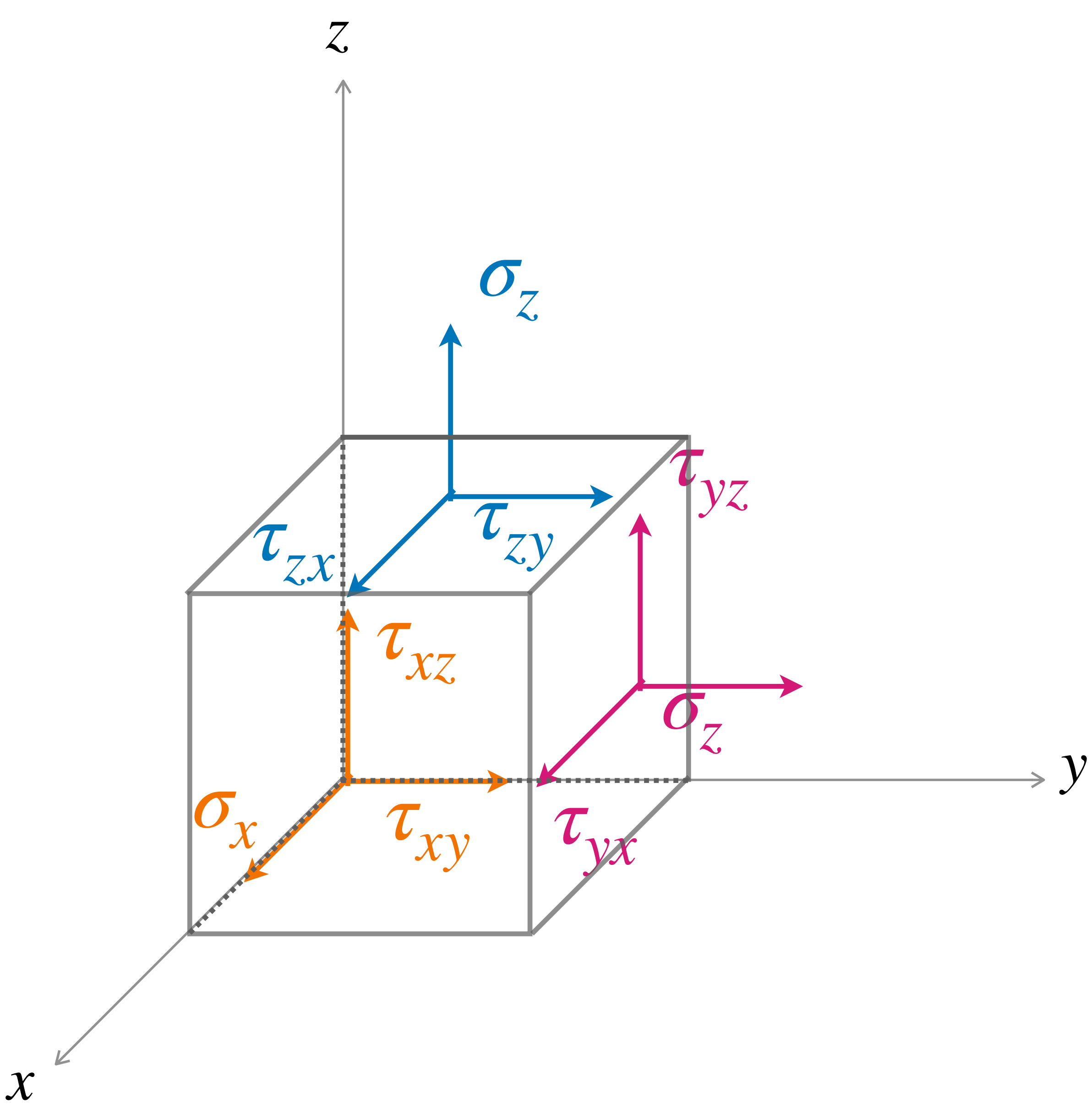
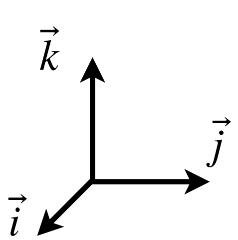
$$\begin{Bmatrix} t_{nx} \\ t_{ny} \\ t_{nz} \end{Bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yx} & \sigma_z \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix}$$

_ Tensione secondo Cauchy





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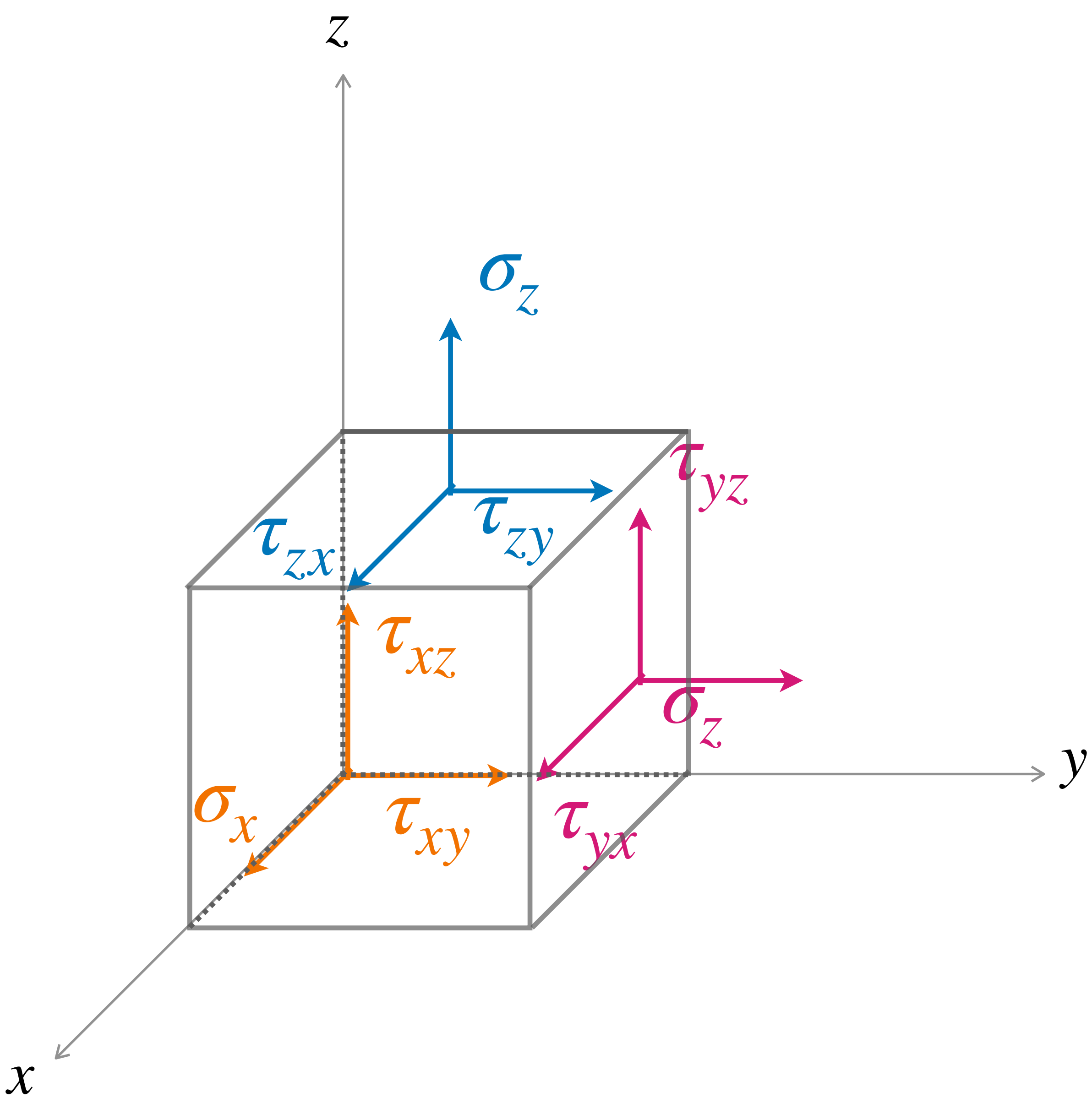
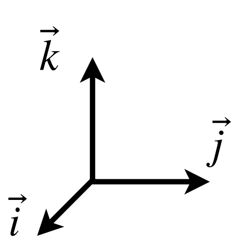
$$\vec{t}_x = \left\{\sigma_x, \tau_{xy}, \tau_{xz}\right\} \qquad \vec{t}_y = \left\{\tau_{yx}, \sigma_y, \tau_{yz}\right\} \qquad \vec{t}_z = \left\{\tau_{zx}, \tau_{zy}, \sigma_z\right\}$$

$$\begin{array}{ccc} & & \begin{array}{ccc} \vec{t}_x & \vec{t}_y & \vec{t}_z \end{array} \\ \begin{Bmatrix} t_{nx} \\ t_{ny} \\ t_{nz} \end{Bmatrix} & = & \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yx} & \sigma_z \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix} \end{array}$$

_ Tensione secondo Cauchy





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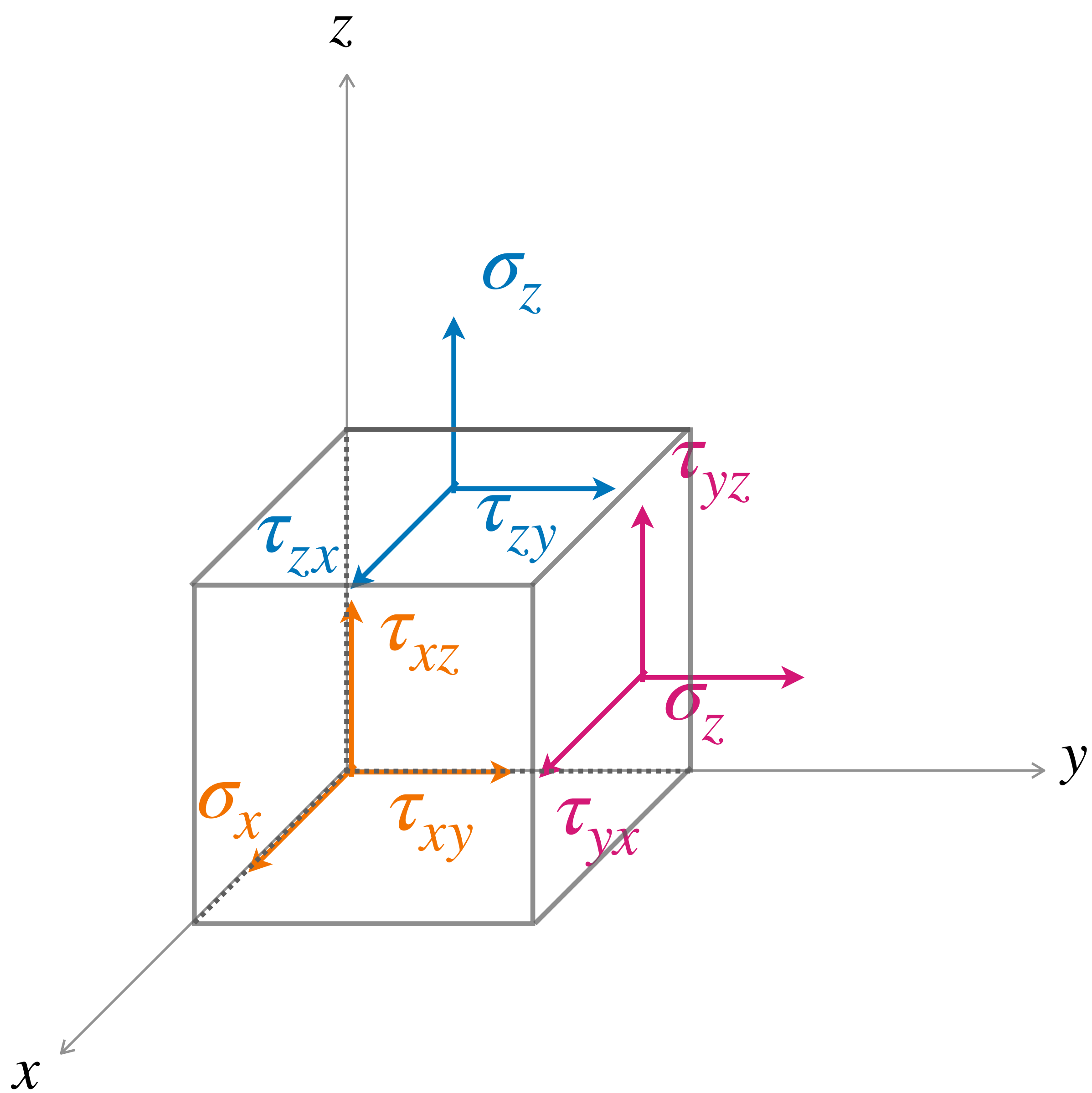
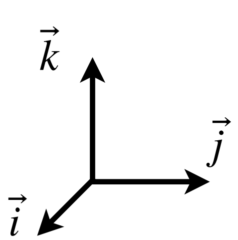
$$\begin{array}{ccccc} \vec{t}_n & & \begin{array}{ccc} \vec{t}_x & \vec{t}_y & \vec{t}_z \end{array} & \vec{n} \\ \begin{Bmatrix} t_{nx} \\ t_{ny} \\ t_{nz} \end{Bmatrix} & = & \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yx} & \sigma_z \end{bmatrix} & \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix} \end{array}$$

_ Tensione secondo Cauchy





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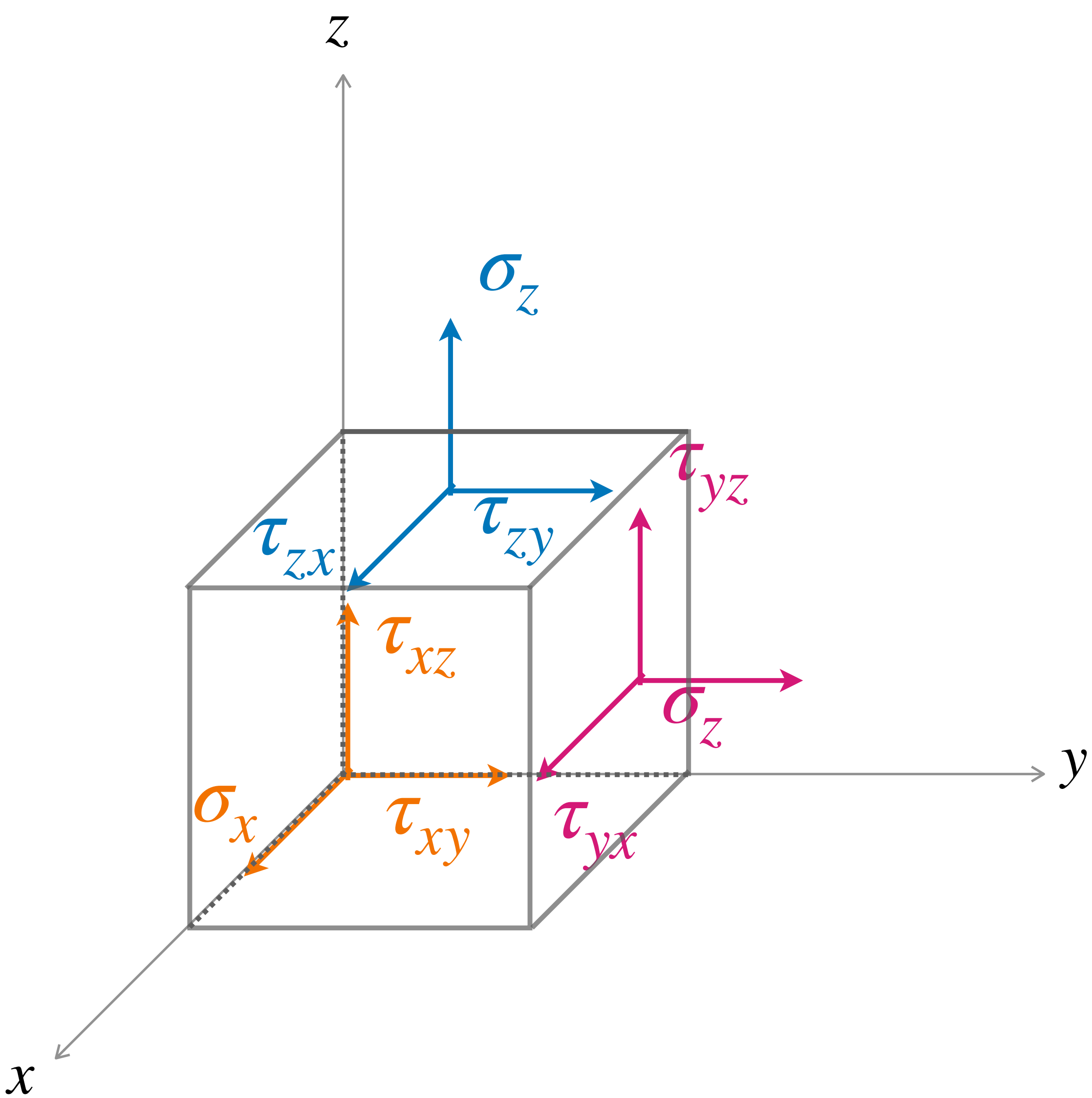
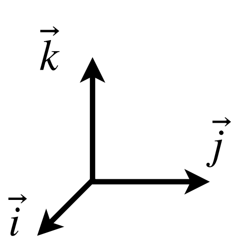
RELAZIONE DI CAUCHY

$$\boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$$

_ Tensione secondo Cauchy





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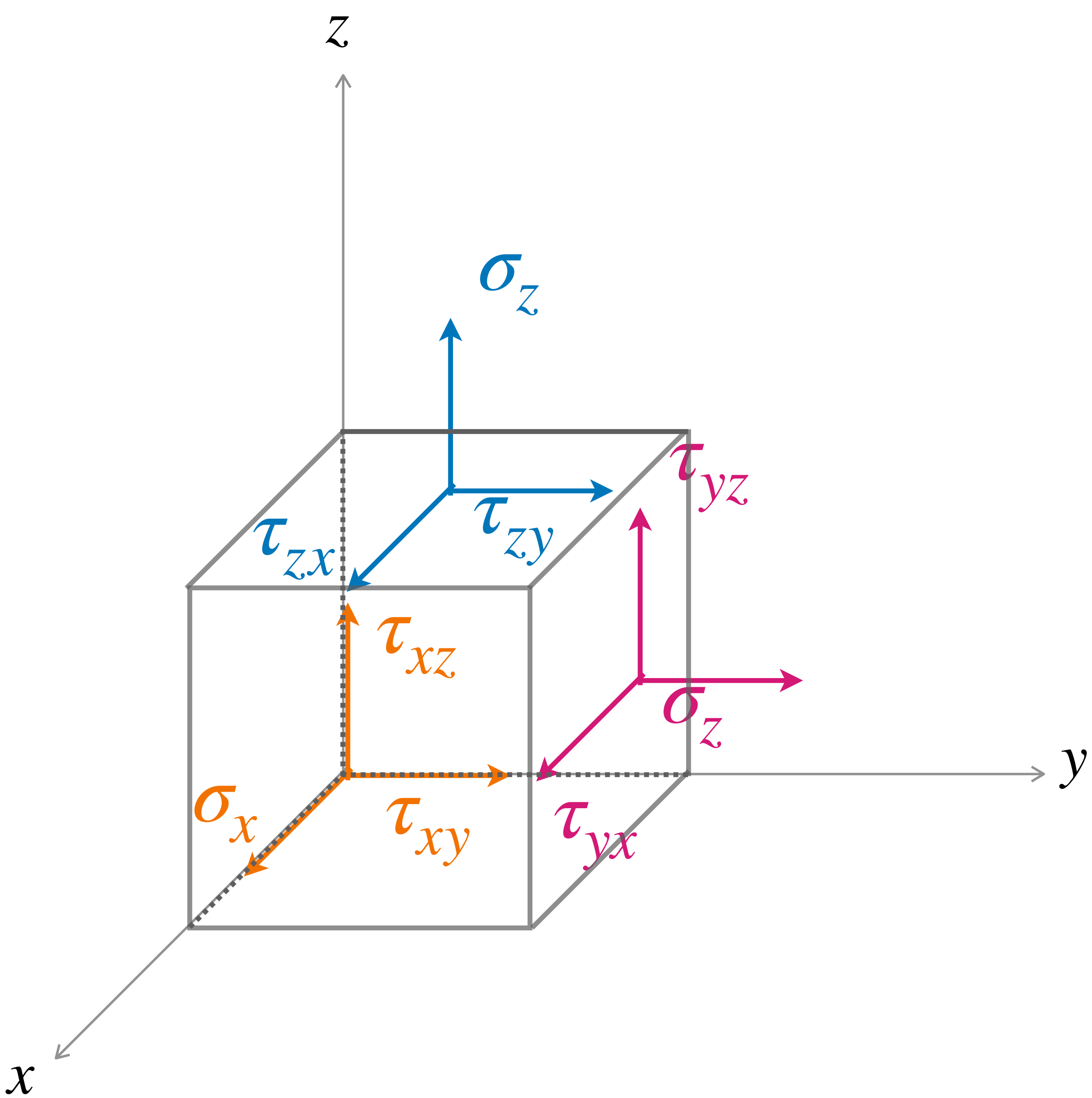
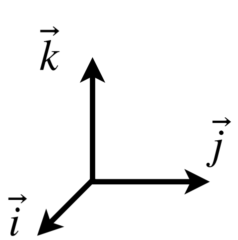
$$\begin{array}{ccc} \vec{t}_n & & \begin{array}{ccc} \vec{t}_x & \vec{t}_y & \vec{t}_z \end{array} & \vec{n} \\ \begin{Bmatrix} t_{nx} \\ t_{ny} \\ t_{nz} \end{Bmatrix} & = & \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yx} & \sigma_z \end{bmatrix} & \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix} \end{array}$$

RELAZIONE DI CAUCHY $\quad \boxed{\vec{t}_n = \underline{\underline{\sigma}} \vec{n}}$

$\underline{\underline{\sigma}}$ **TENSORE DEGLI SFORZI**

_ Tensione secondo Cauchy





TEOREMA DI CAUCHY

É sufficiente conoscere la tensione $\vec{t}_n$ su 3 giaciture ortogonali per poter definire la tensione su qualunque altra giacitura.

RELAZIONE DI CAUCHY

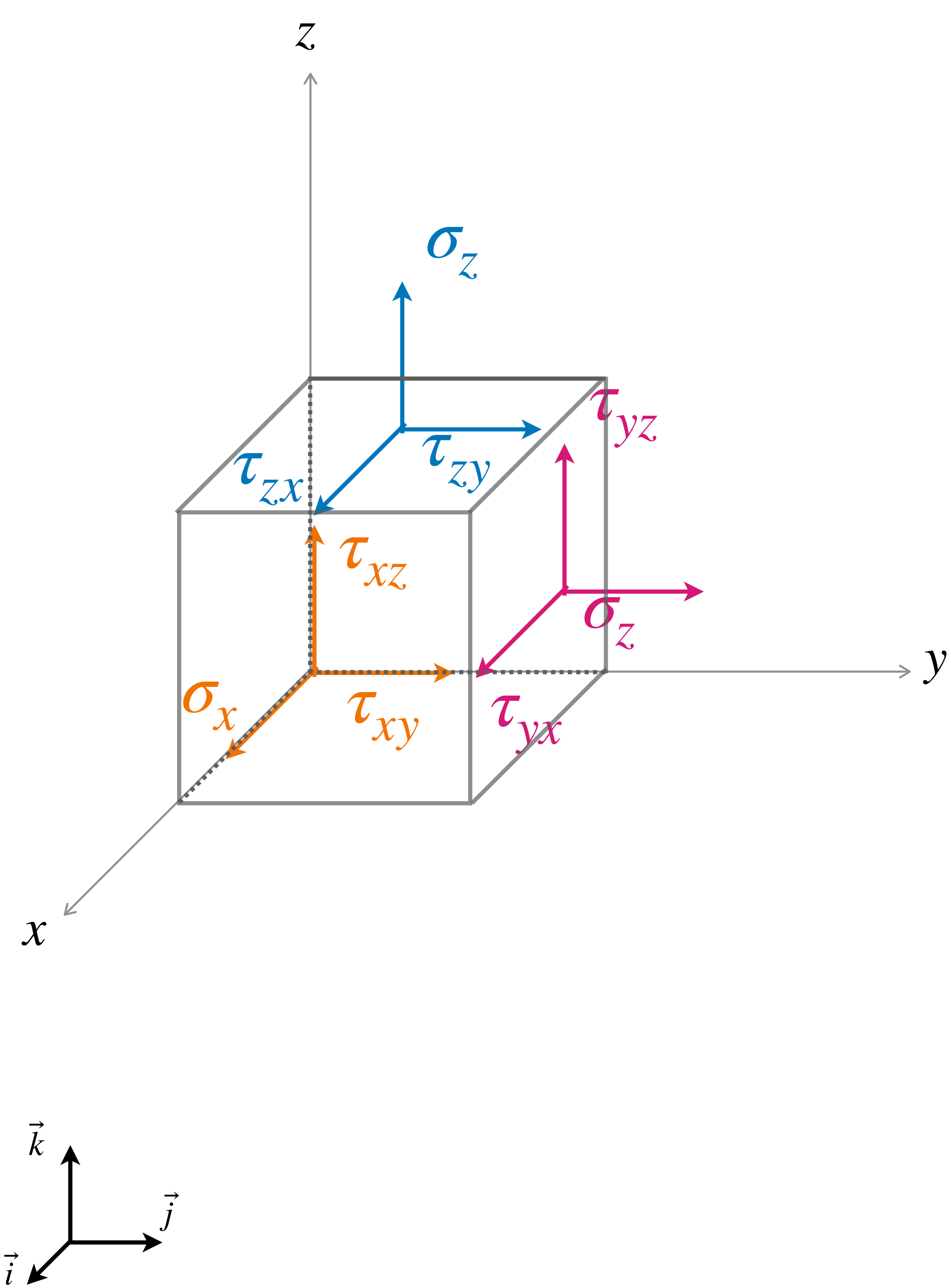
$$\boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$$

9 componenti: σ - 3 componenti normali

$$\begin{Bmatrix} t_{nx} \\ t_{ny} \\ t_{nz} \end{Bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yx} & \sigma_z \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix}$$

$\underline{\underline{\sigma}}$ **TENSORE DEGLI SFORZI**

_ Tensione secondo Cauchy



TEOREMA DI CAUCHY

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RELAZIONE DI CAUCHY

$$\boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$$

9 componenti:

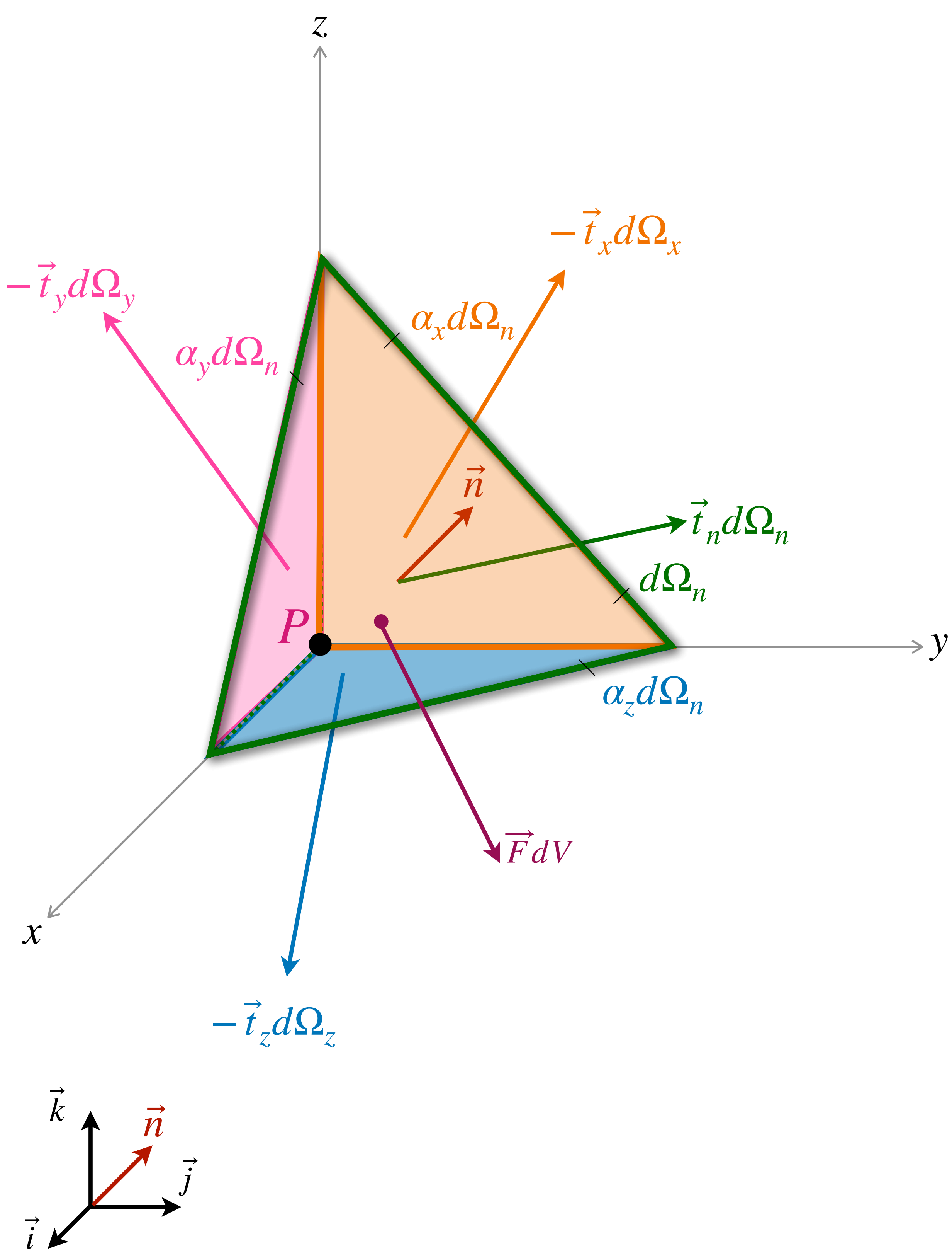
σ - 3 componenti normali

τ - 6 componenti tangenziali

$$\begin{Bmatrix} t_{nx} \\ t_{ny} \\ t_{nz} \end{Bmatrix} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yx} & \sigma_z \end{bmatrix} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix}$$

$\underline{\underline{\sigma}}$ **TENSORE DEGLI SFORZI**

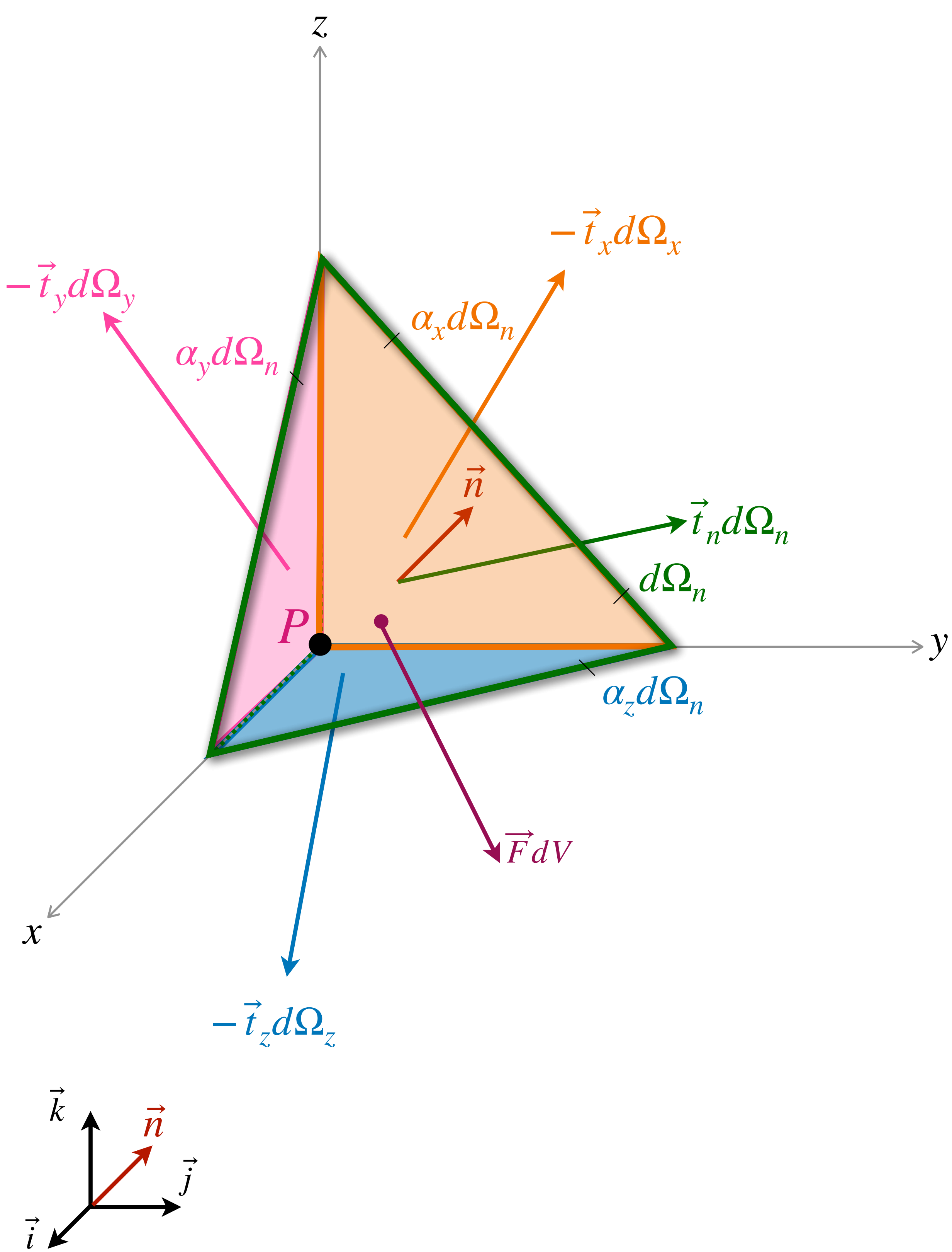
_ Tensione secondo Cauchy



TEOREMA DI CAUCHY

$\vec{R} = \vec{0}$ $\boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$ $\underline{\underline{\sigma}}$ Tensore degli sforzi

_ Tensione secondo Cauchy

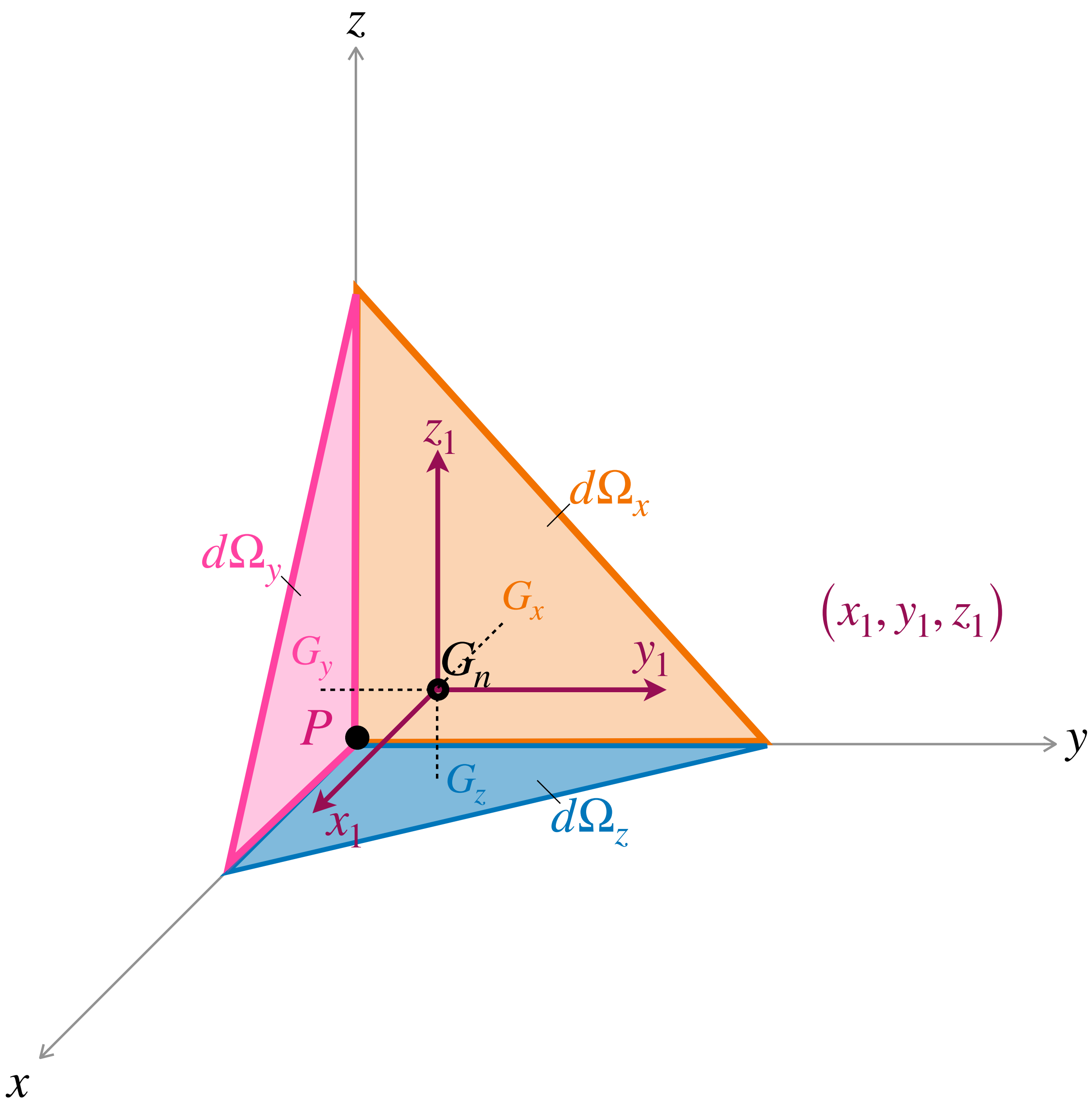


TEOREMA DI CAUCHY

$\vec{R} = \vec{0}$ $\boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$ $\underline{\underline{\sigma}}$ Tensore degli sforzi

$\vec{M} = \vec{0}$

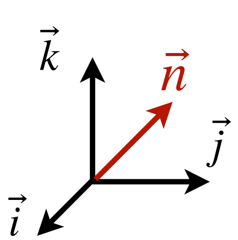
_ Tensione secondo Cauchy



TEOREMA DI CAUCHY

$\vec{R} = \vec{0}$ $\quad$ $\boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$ $\quad$ $\underline{\underline{\sigma}}$ Tensore degli sforzi

$$\vec{M}_{(G_n)} = \vec{0}$$



$$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$$

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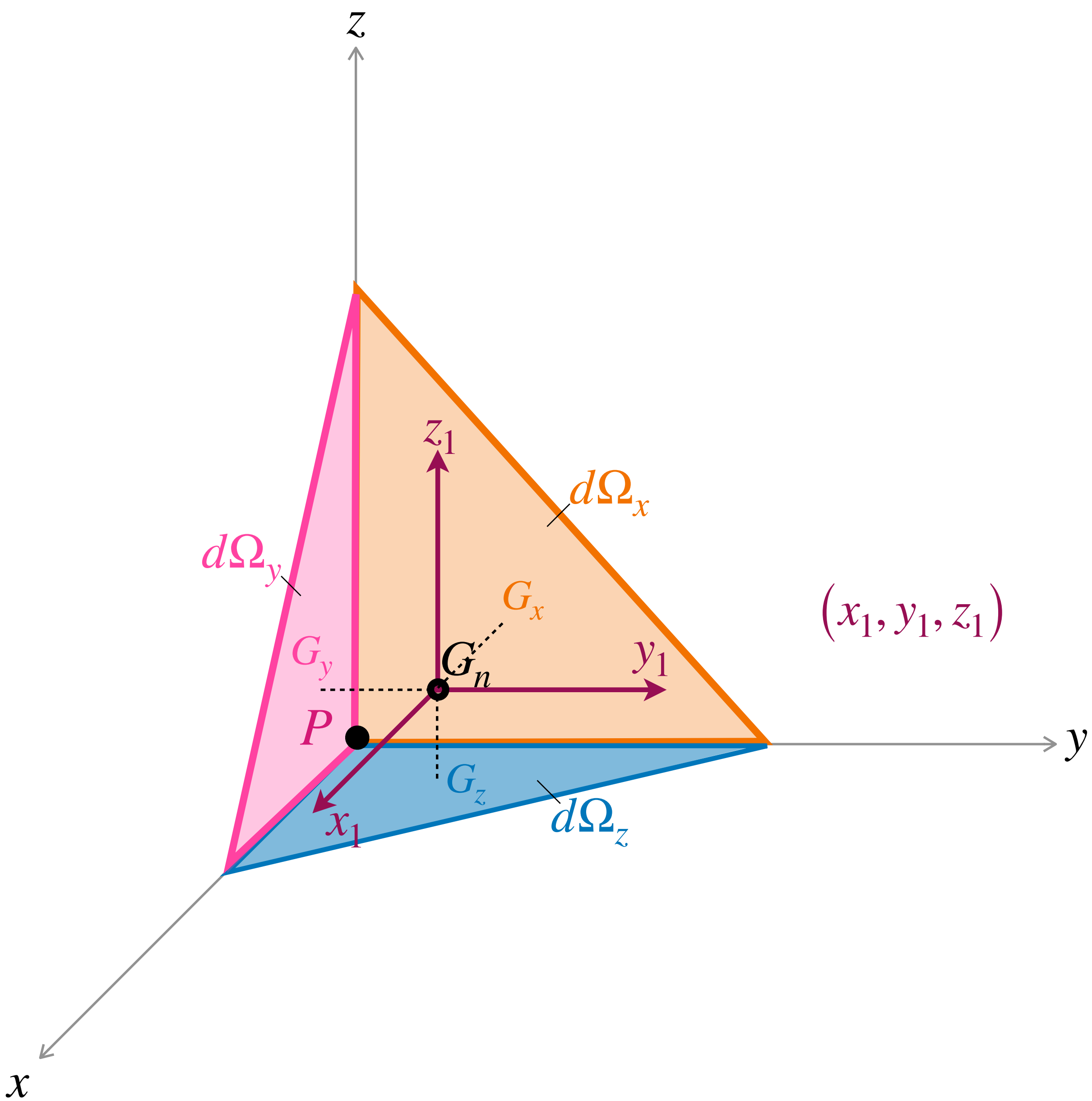
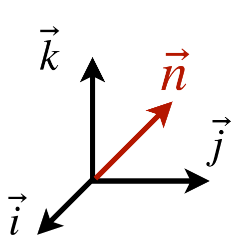


TEOREMA DI CAUCHY

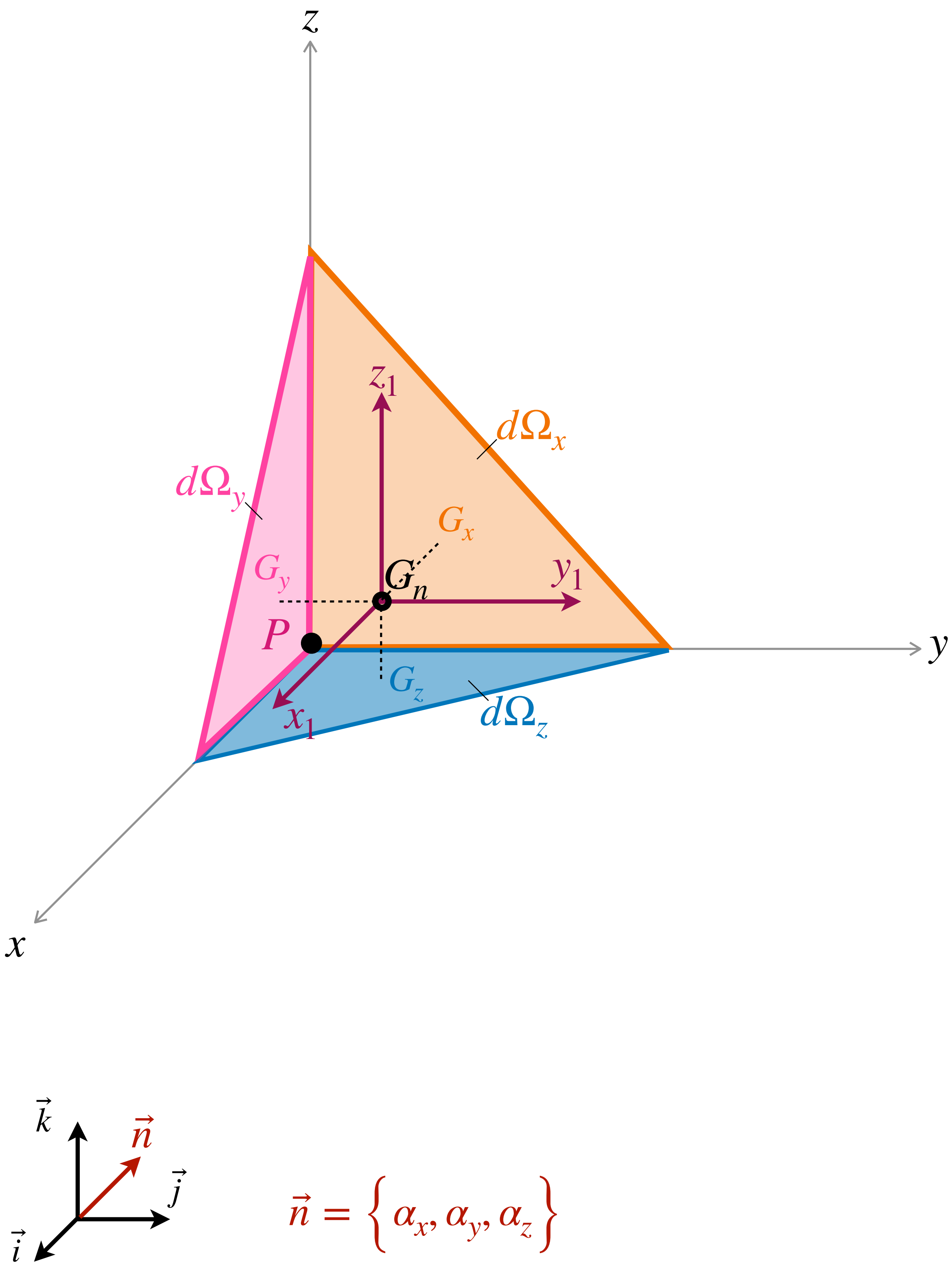
$$\vec{R} = \vec{0} \qquad \boxed{\vec{t}_n = \underline{\underline{\sigma}}\,\vec{n}} \qquad \underline{\underline{\sigma}} \text{ Tensore degli sforzi}$$

$$\vec{M}_{(G_n)} = \vec{0} \qquad \begin{cases} M_{(G_n)x} = 0 \\ M_{(G_n)y} = 0 \\ M_{(G_n)z} = 0 \end{cases}$$



$$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$$

_ Tensione secondo Cauchy



$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$

TEOREMA DI CAUCHY

$\vec{R} = \vec{0}$ $\boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$ $\underline{\underline{\sigma}}$ Tensore degli sforzi

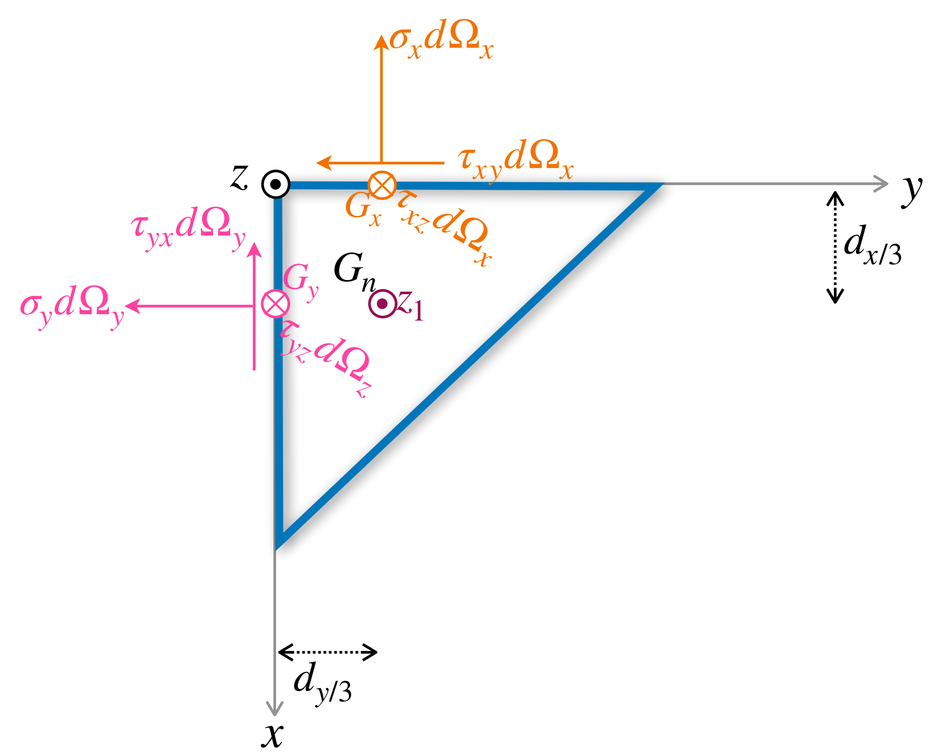
$M_{(G_n)z} = 0$

$$M_{(G_n)z} = \tau_{xy} d\Omega_x \frac{dx}{3} - \tau_{yx} d\Omega_y \frac{dy}{3} = 0$$

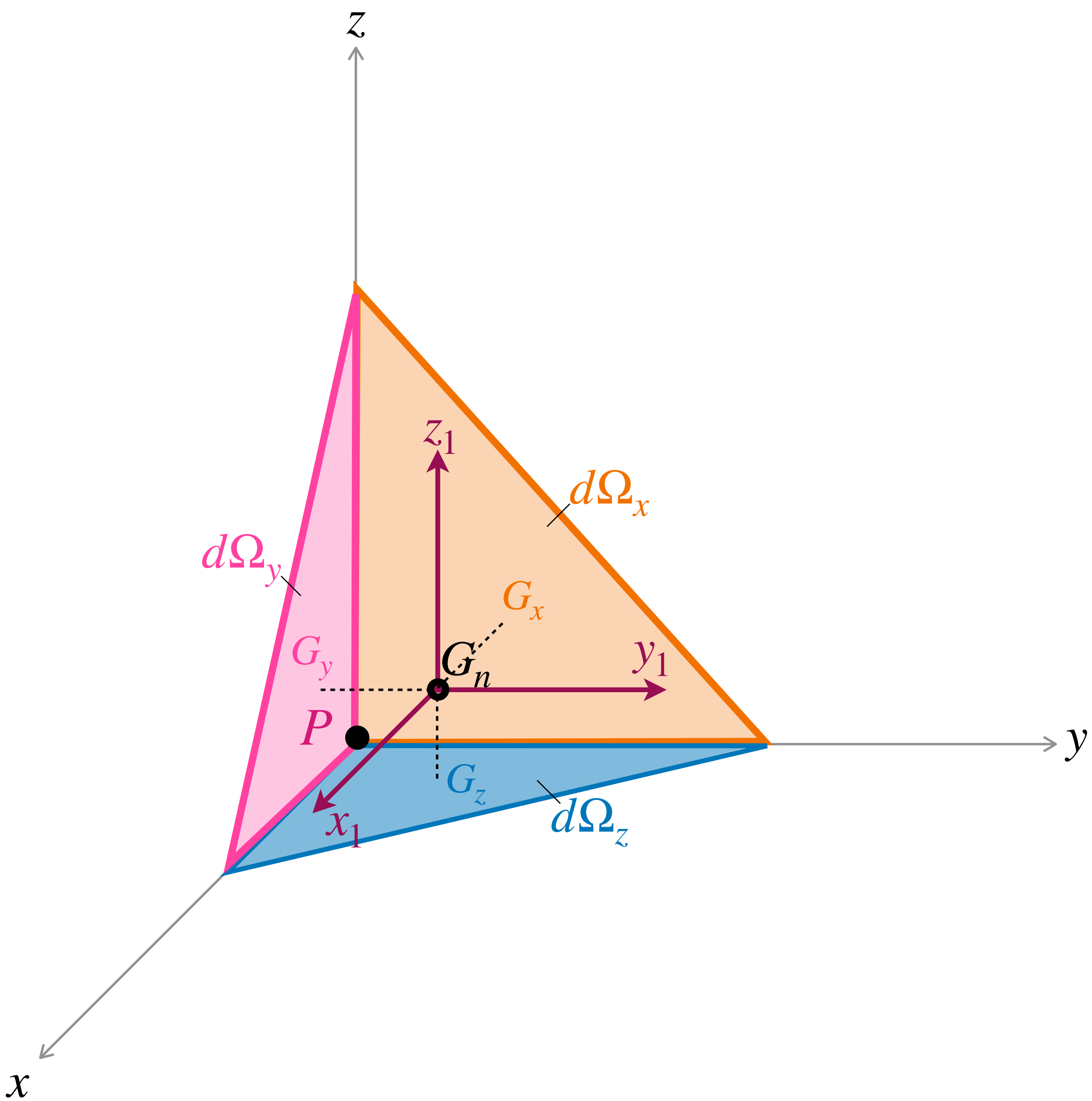
N.B. $dV = d\Omega x \frac{dx}{3} = d\Omega y \frac{dy}{3} = d\Omega z \frac{dz}{3}$

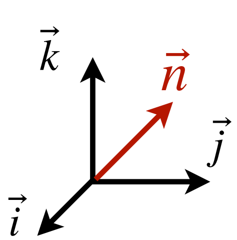
$$M_{(G_n)z} = \tau_{xy} dV - \tau_{yx} dV = 0$$

$$\tau_{xy} - \tau_{yx} = 0$$



_ Tensione secondo Cauchy





$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$

TEOREMA DI CAUCHY

$\vec{R} = \vec{0}$ $\quad \boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$ $\quad \underline{\underline{\sigma}}$ Tensore degli sforzi

$M_{(G_n)z} = 0$

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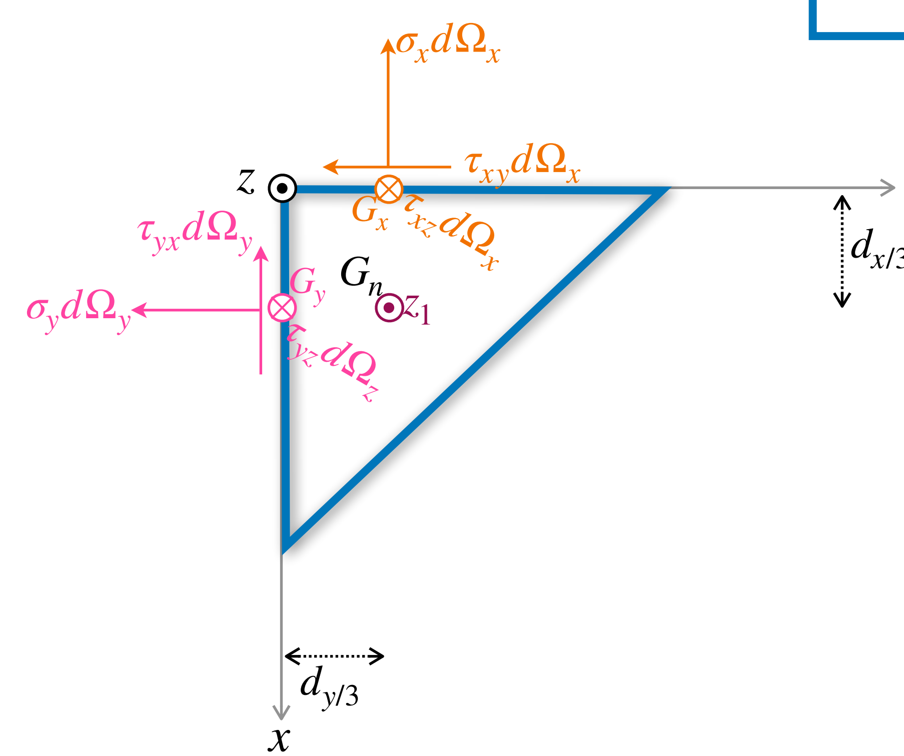
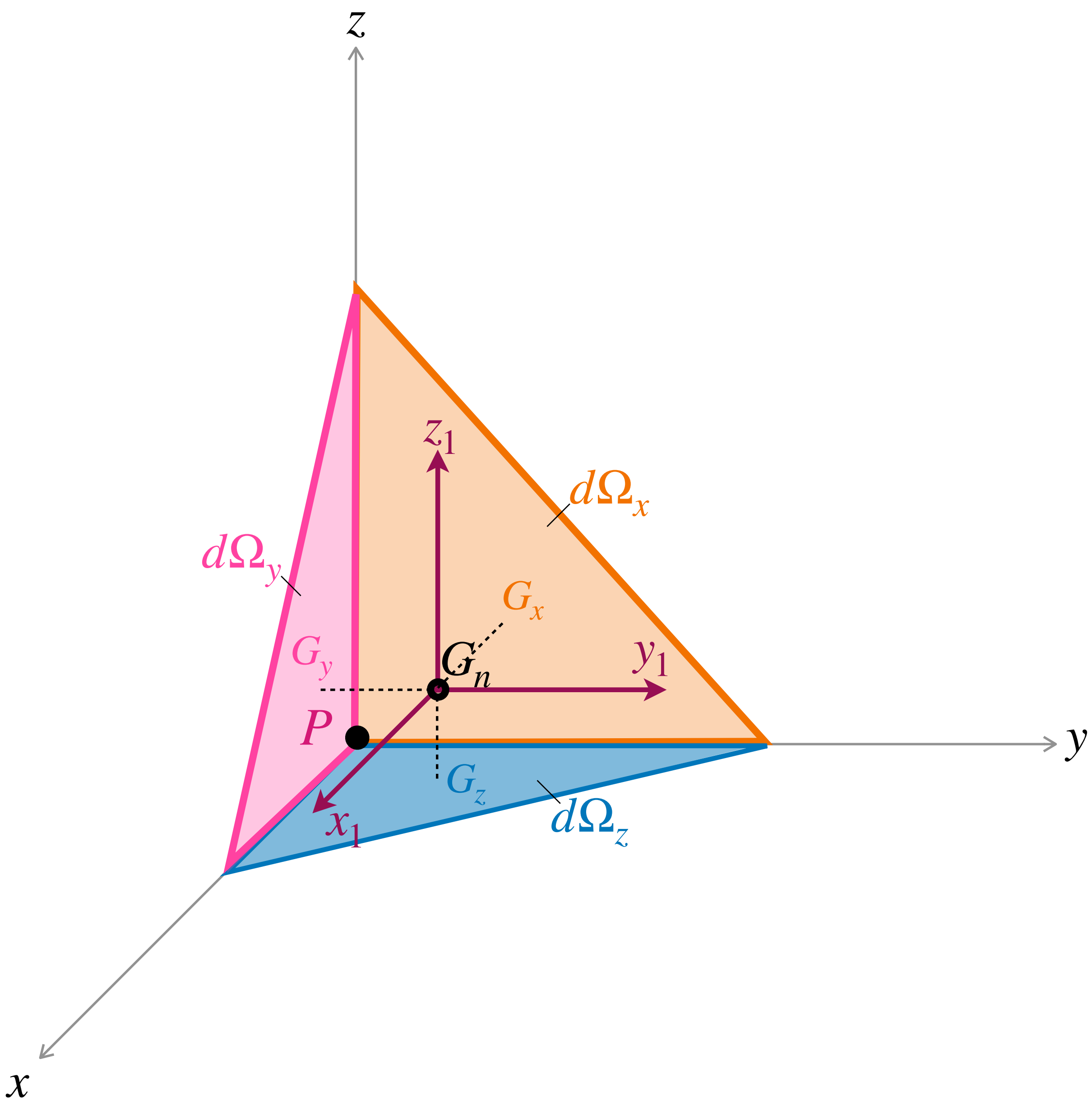
$$M_{(G_n)z} = \tau_{xy} dV - \tau_{yx} dV = 0$$

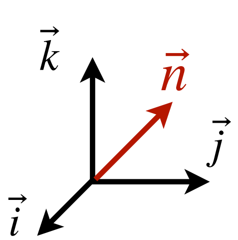
$$\tau_{xy} - \tau_{yx} = 0$$

$$\boxed{\tau_{xy} = \tau_{yx}}$$



_ Tensione secondo Cauchy





$$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$$

TEOREMA DI CAUCHY

$\vec{R} = \vec{0}$ $\boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$ $\underline{\underline{\sigma}}$ Tensore degli sforzi

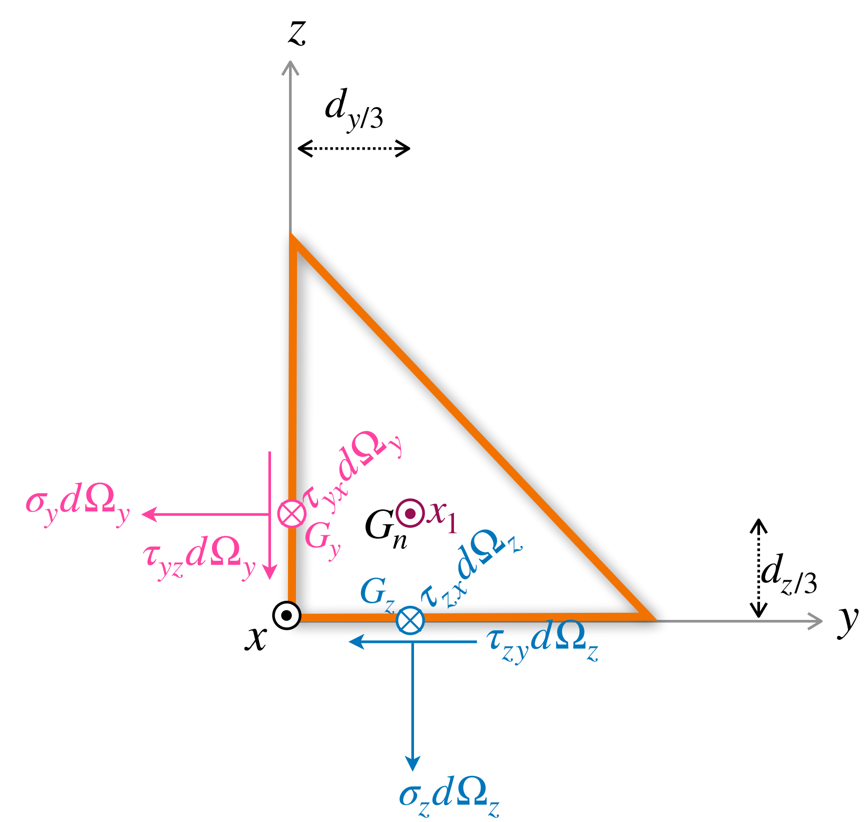
$$M_{(G_n)x} = 0$$

$$M_{(G_n)x} = \tau_{yz} d\Omega_y \frac{dy}{3} - \tau_{zy} d\Omega_z \frac{dz}{3} = 0$$

N.B. $dV = d\Omega x \dfrac{dx}{3} = d\Omega y \dfrac{dy}{3} = d\Omega z \dfrac{dz}{3}$

$$M_{(G_n)x} = \tau_{yz} dV - \tau_{zy} dV = 0$$

$$\tau_{yz} - \tau_{zy} = 0$$



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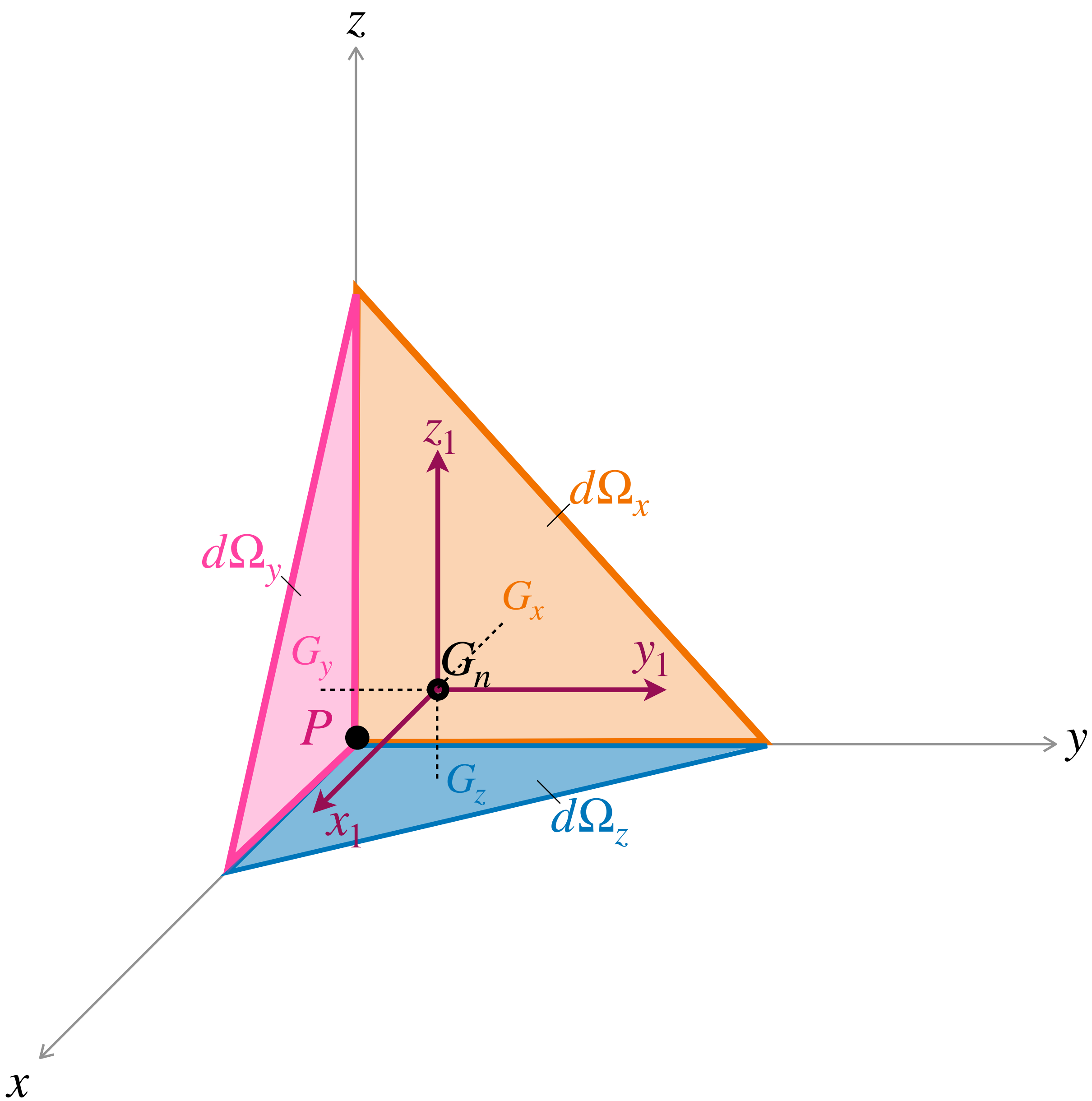

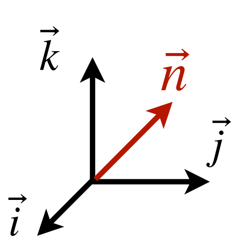

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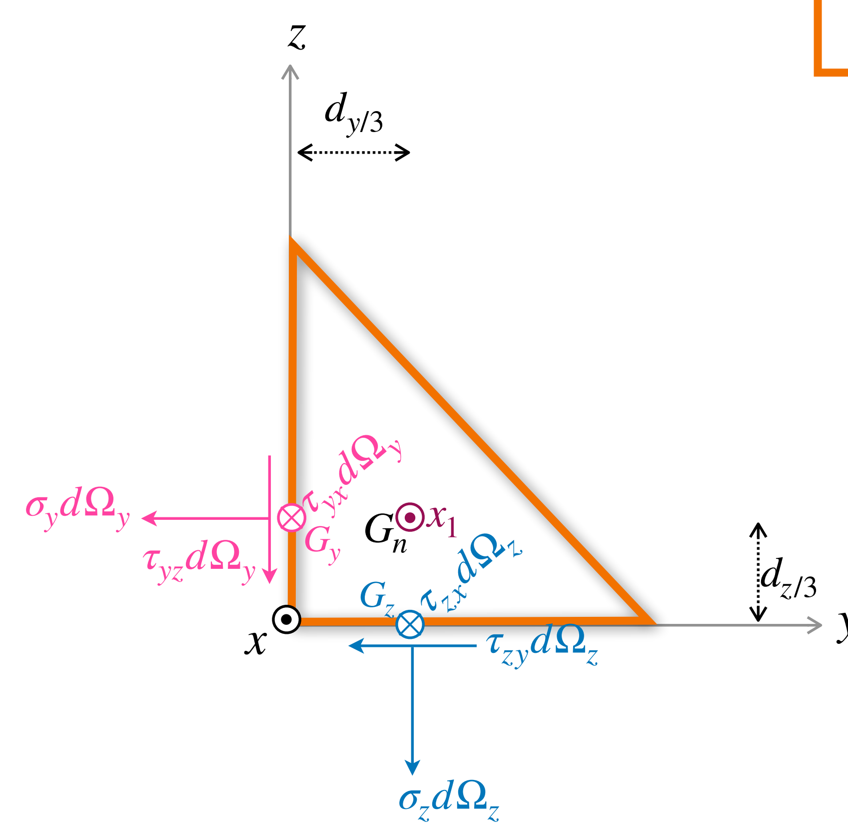

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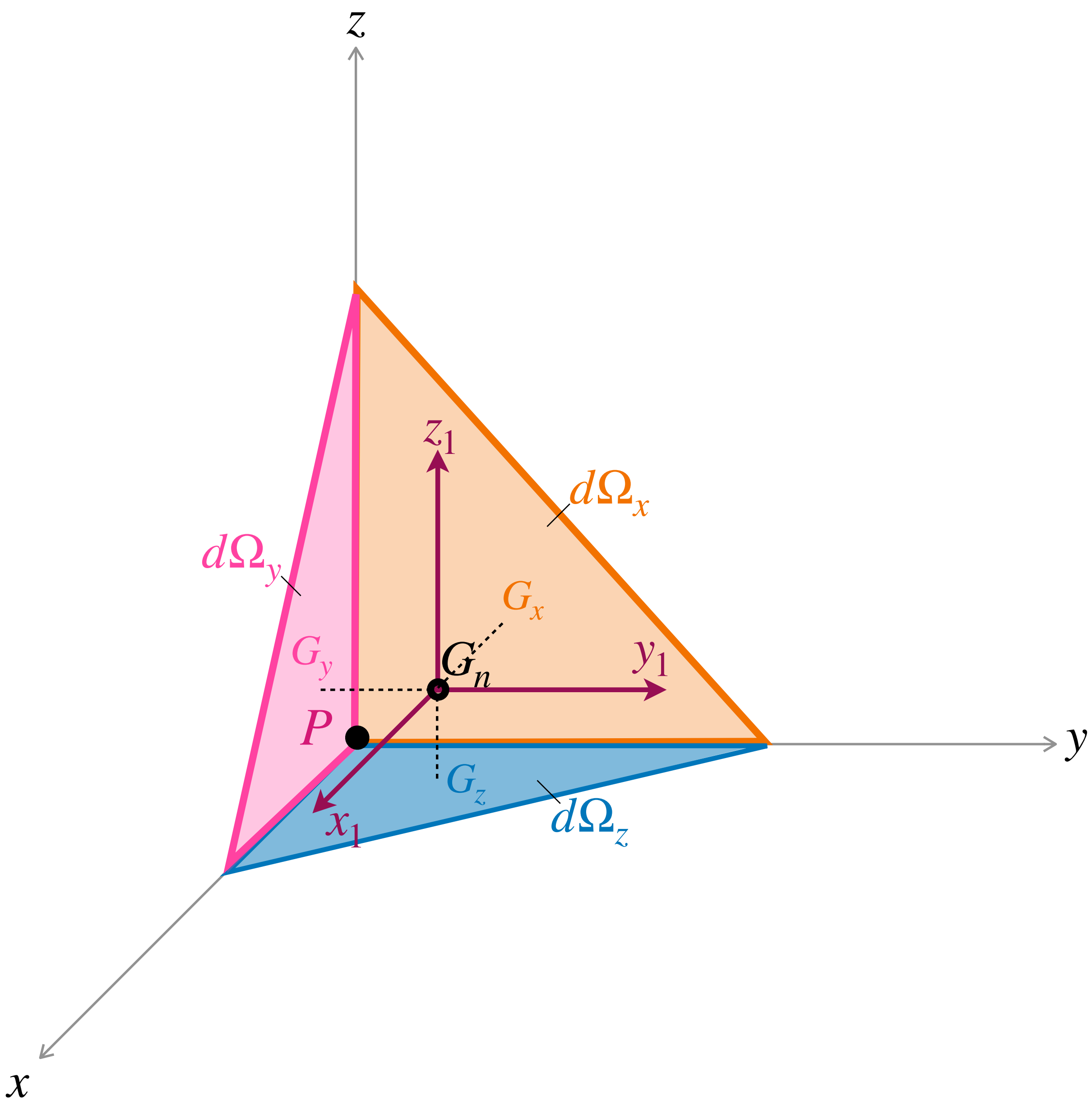
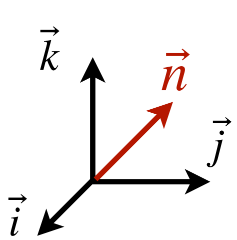




$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$

TEOREMA DI CAUCHY

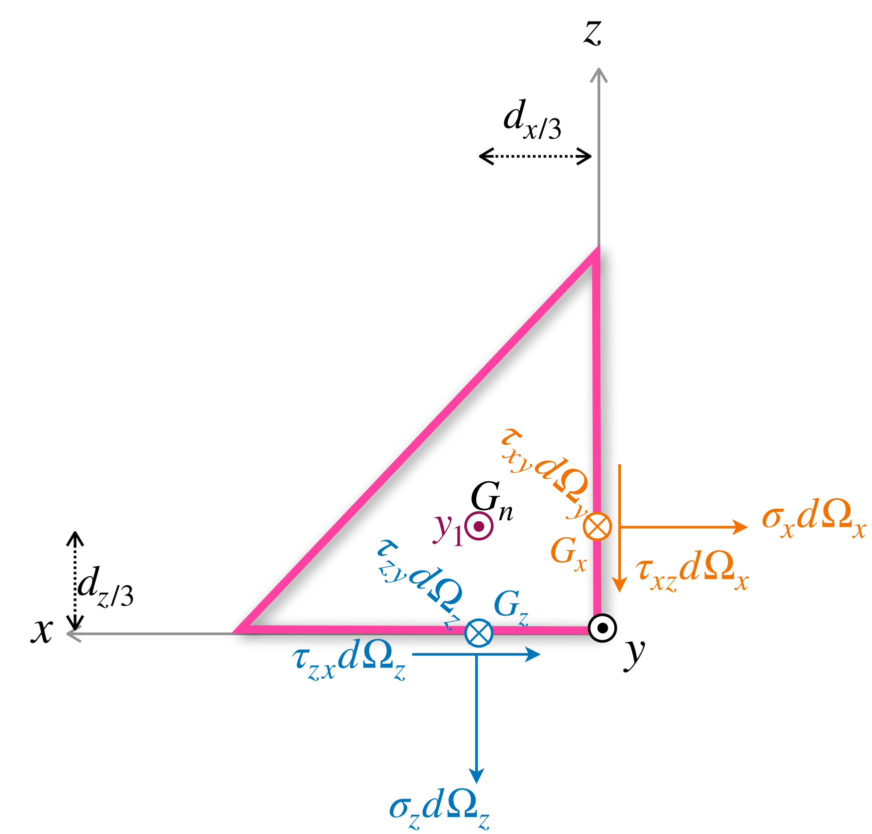
$\vec{R} = \vec{0}$ $\quad \boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$ $\quad \underline{\underline{\sigma}}$ Tensore degli sforzi

$M_{(G_n)y} = 0$ $\quad M_{(G_n)y} = \tau_{zx} d\Omega_z \dfrac{dz}{3} - \tau_{xz} d\Omega_x \dfrac{dx}{3} = 0$

N.B. $\quad dV = d\Omega x \dfrac{dx}{3} = d\Omega y \dfrac{dy}{3} = d\Omega z \dfrac{dz}{3}$

$$M_{(G_n)x} = \tau_{zx} dV - \tau_{xz} dV = 0$$

$$\tau_{zx} - \tau_{xz} = 0$$



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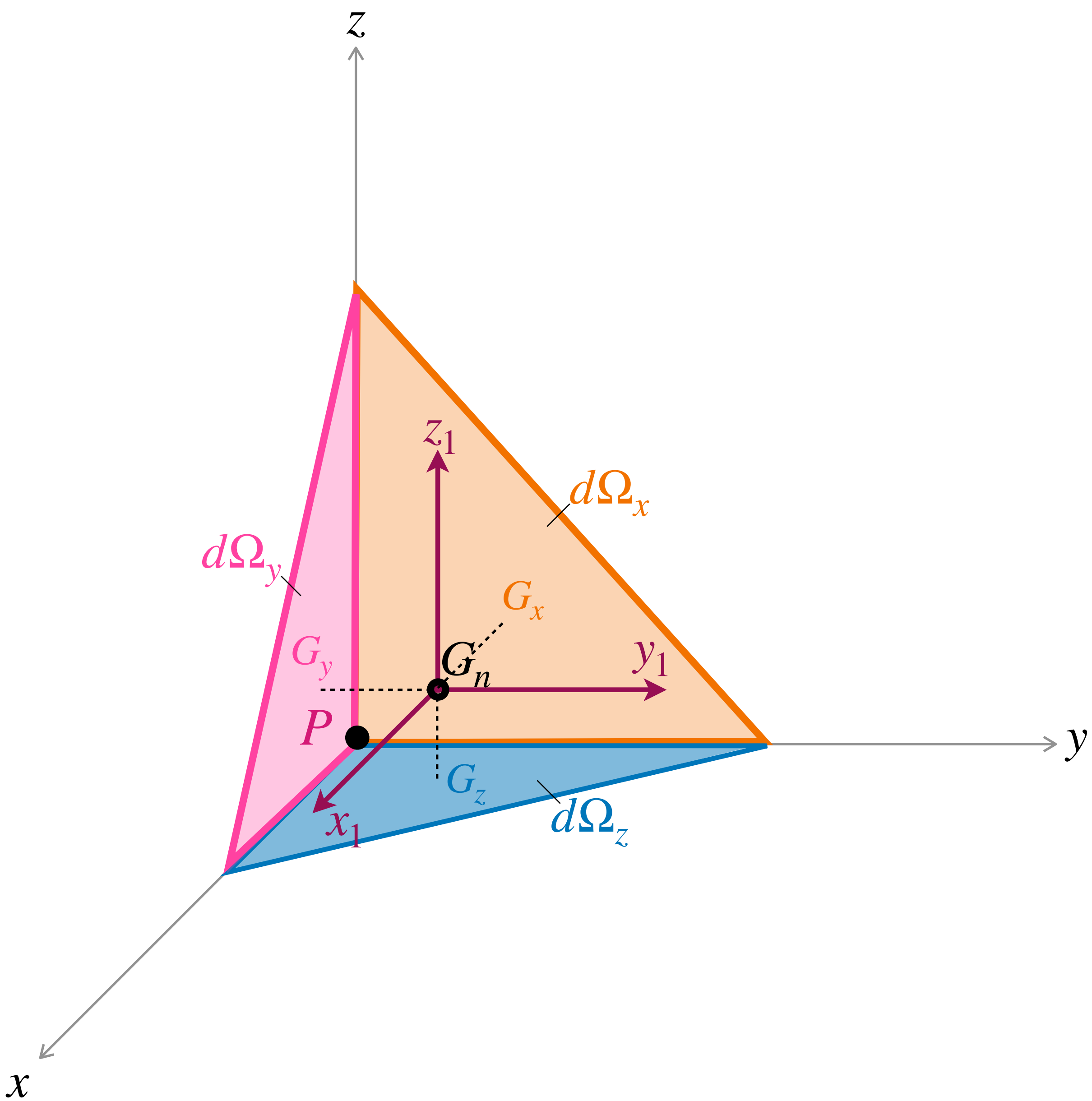
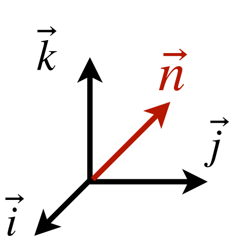




$$\vec{n} = \left\{ \alpha_x, \alpha_y, \alpha_z \right\}$$

TEOREMA DI CAUCHY

$$\vec{R} = \vec{0} \qquad \boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}} \qquad \underline{\underline{\sigma}} \text{ Tensore degli sforzi}$$

$$M_{(G_n)y} = 0$$

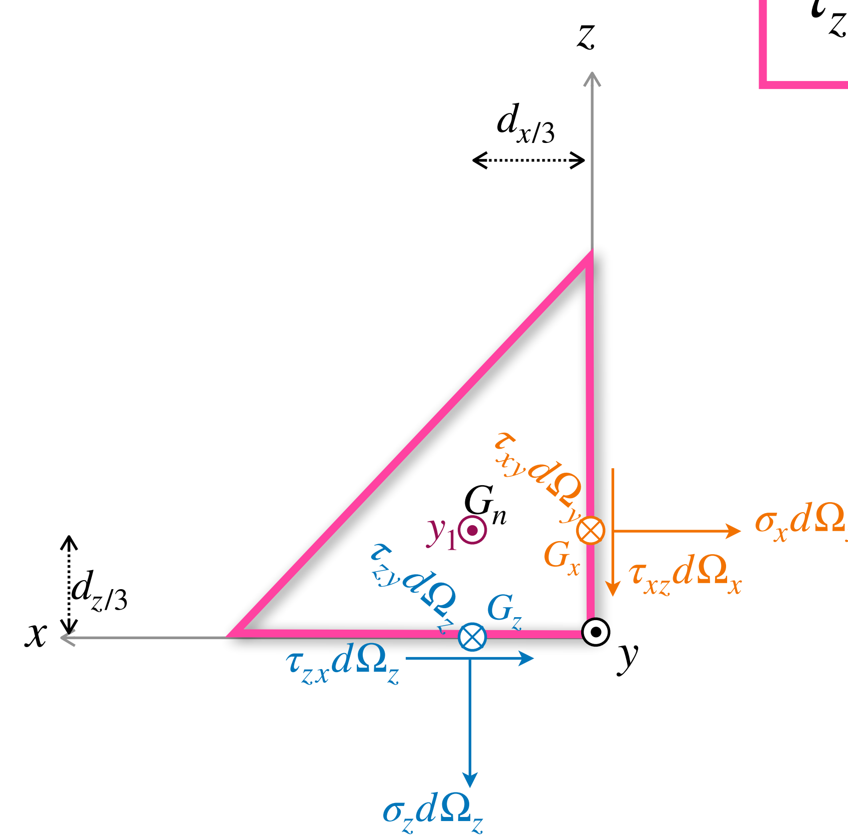
$$M_{(G_n)y} = \tau_{zx} d\Omega_z \frac{dz}{3} - \tau_{xz} d\Omega_x \frac{dx}{3} = 0$$

N.B. $\quad dV = d\Omega x \frac{dx}{3} = d\Omega y \frac{dy}{3} = d\Omega z \frac{dz}{3}$

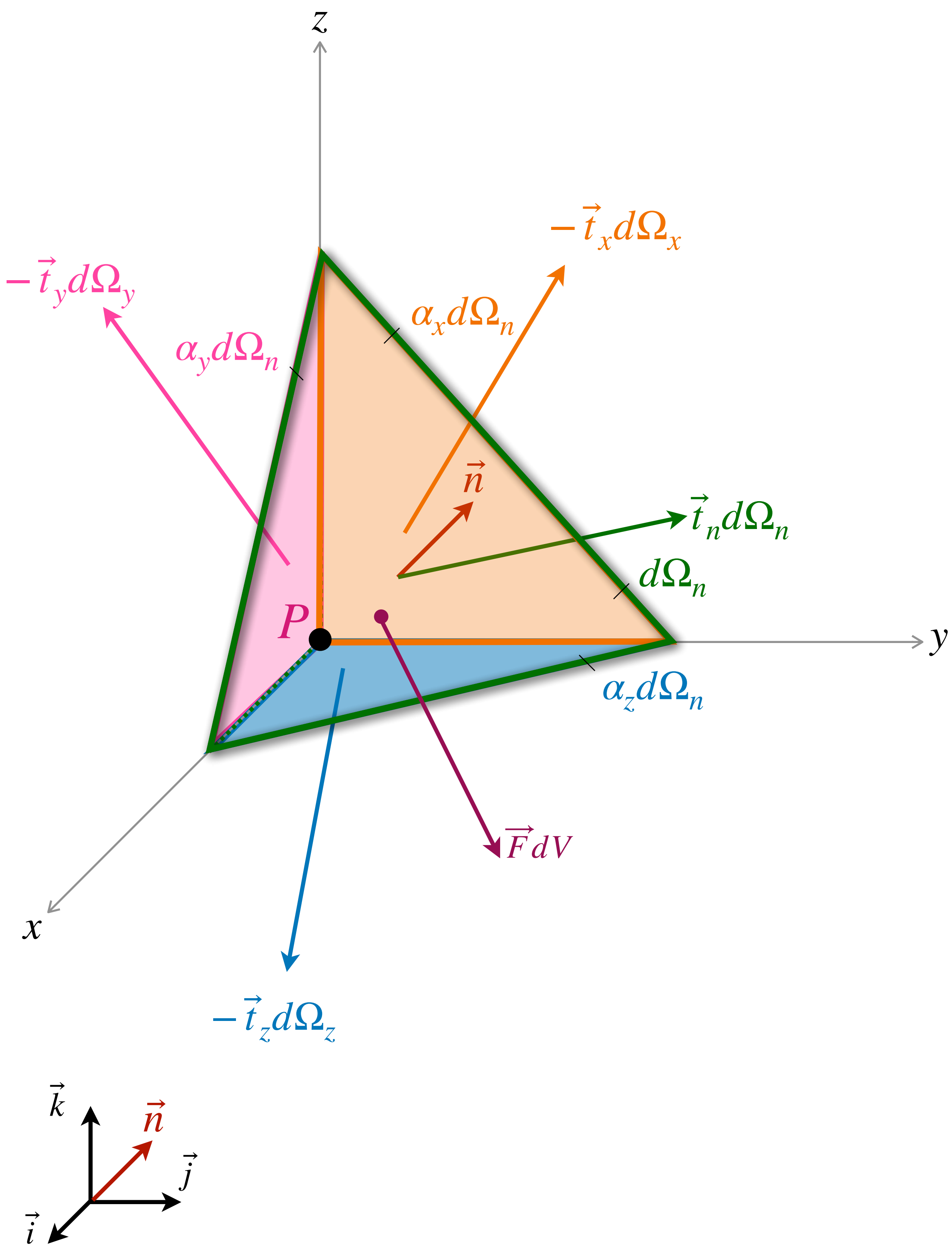
$$M_{(G_n)x} = \tau_{zx} dV - \tau_{xz} dV = 0$$

$$\tau_{zx} - \tau_{xz} = 0$$

$$\boxed{\tau_{zx} = \tau_{xz}}$$



_ Tensione secondo Cauchy

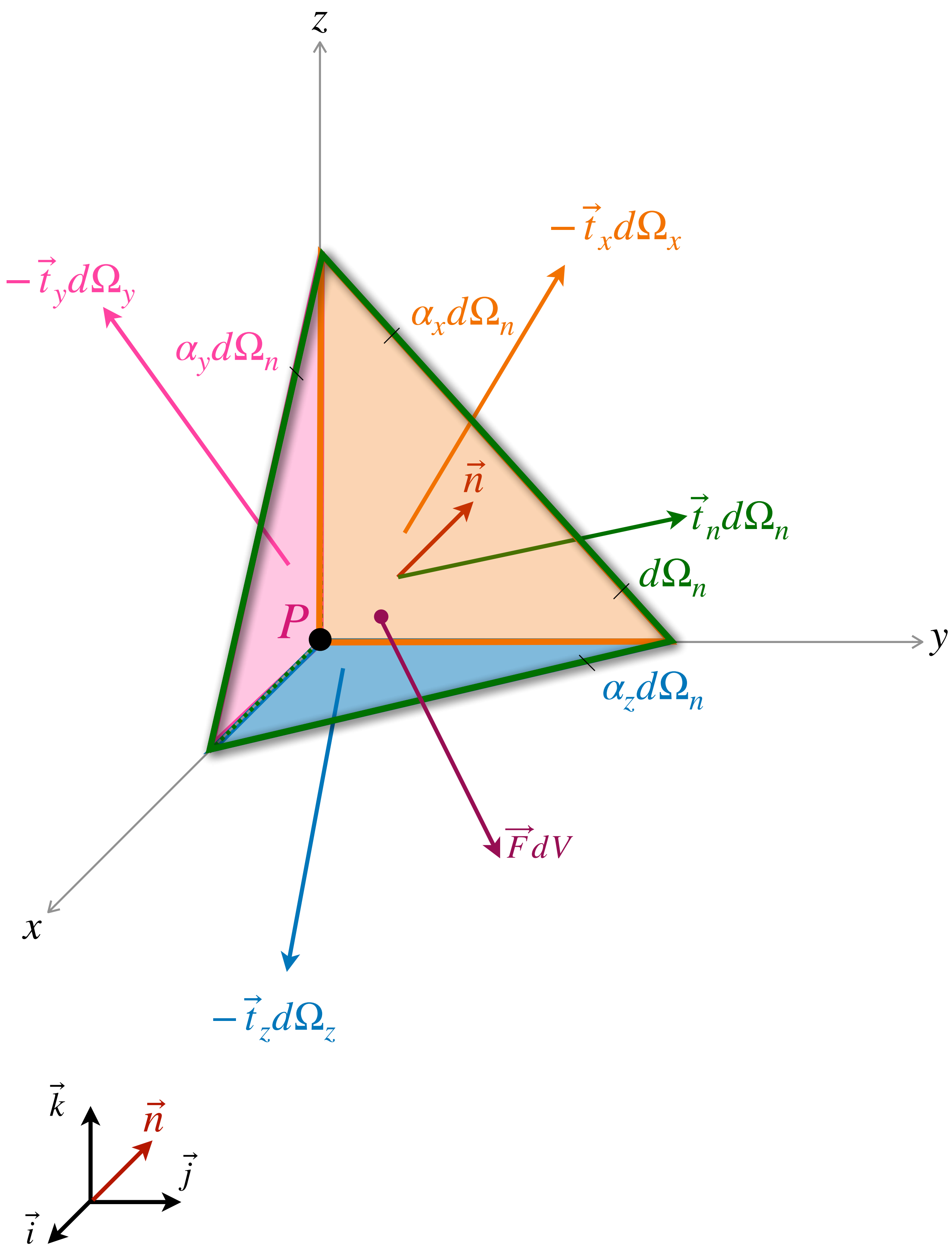


TEOREMA DI CAUCHY

$\vec{R} = \vec{0}$ $\boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$ $\underline{\underline{\sigma}}$ Tensore degli sforzi

$$\vec{M}_{(G_n)} = \vec{0} \qquad \begin{cases} M_{(G_n)x} = 0 \\ M_{(G_n)y} = 0 \\ M_{(G_n)z} = 0 \end{cases}$$

_ Tensione secondo Cauchy



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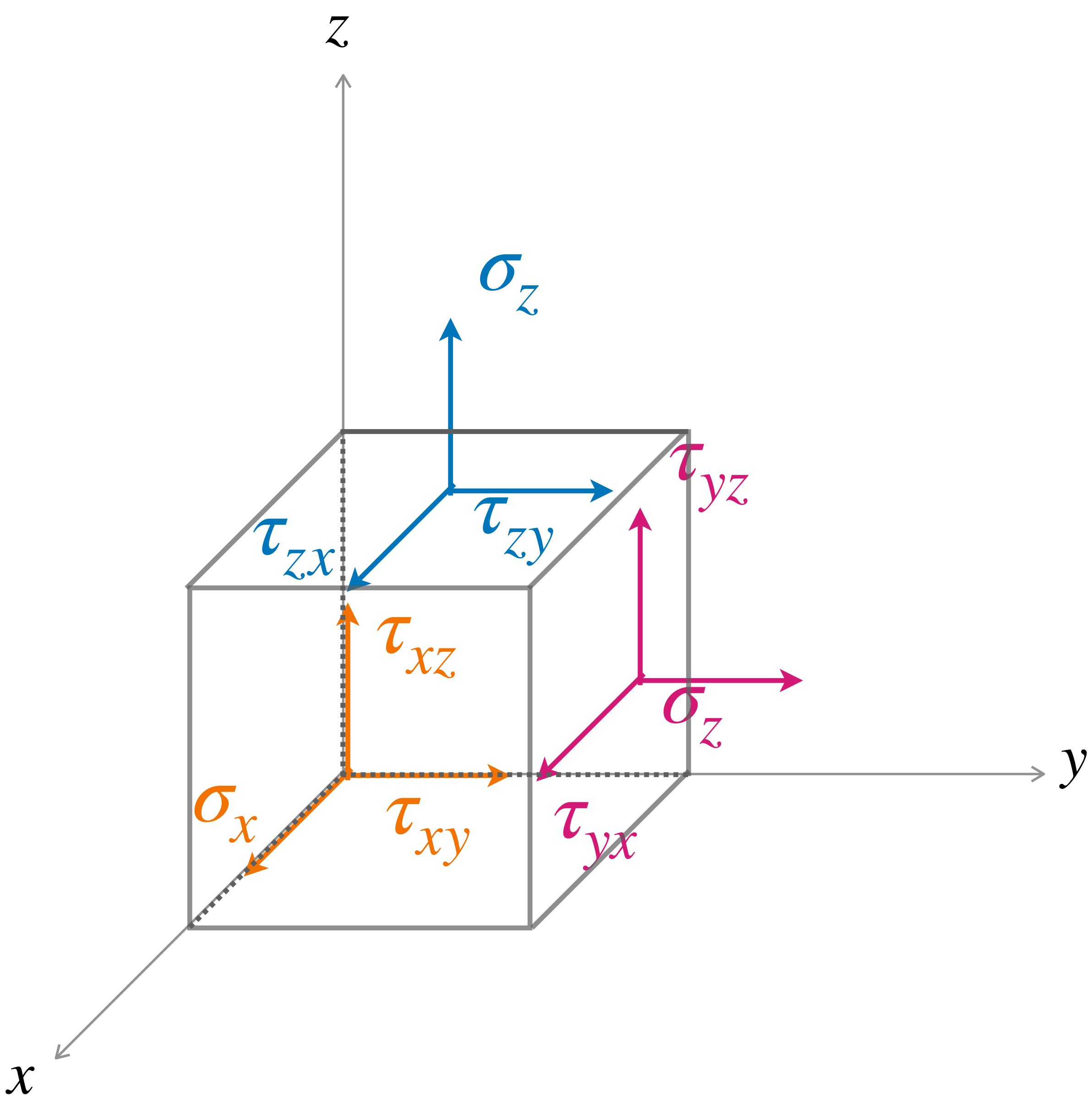
$\boxed{\tau_{yz} = \tau_{zy}}$

$\boxed{\tau_{zx} = \tau_{xz}}$

$\boxed{\tau_{xy} = \tau_{yx}}$

RECIPROCITA' DELLE TENSIONI TANGENZIALI

_ Tensione secondo Cauchy



TEOREMA DI CAUCHY

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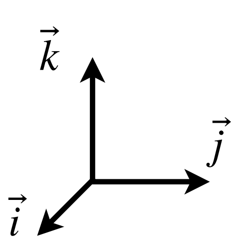
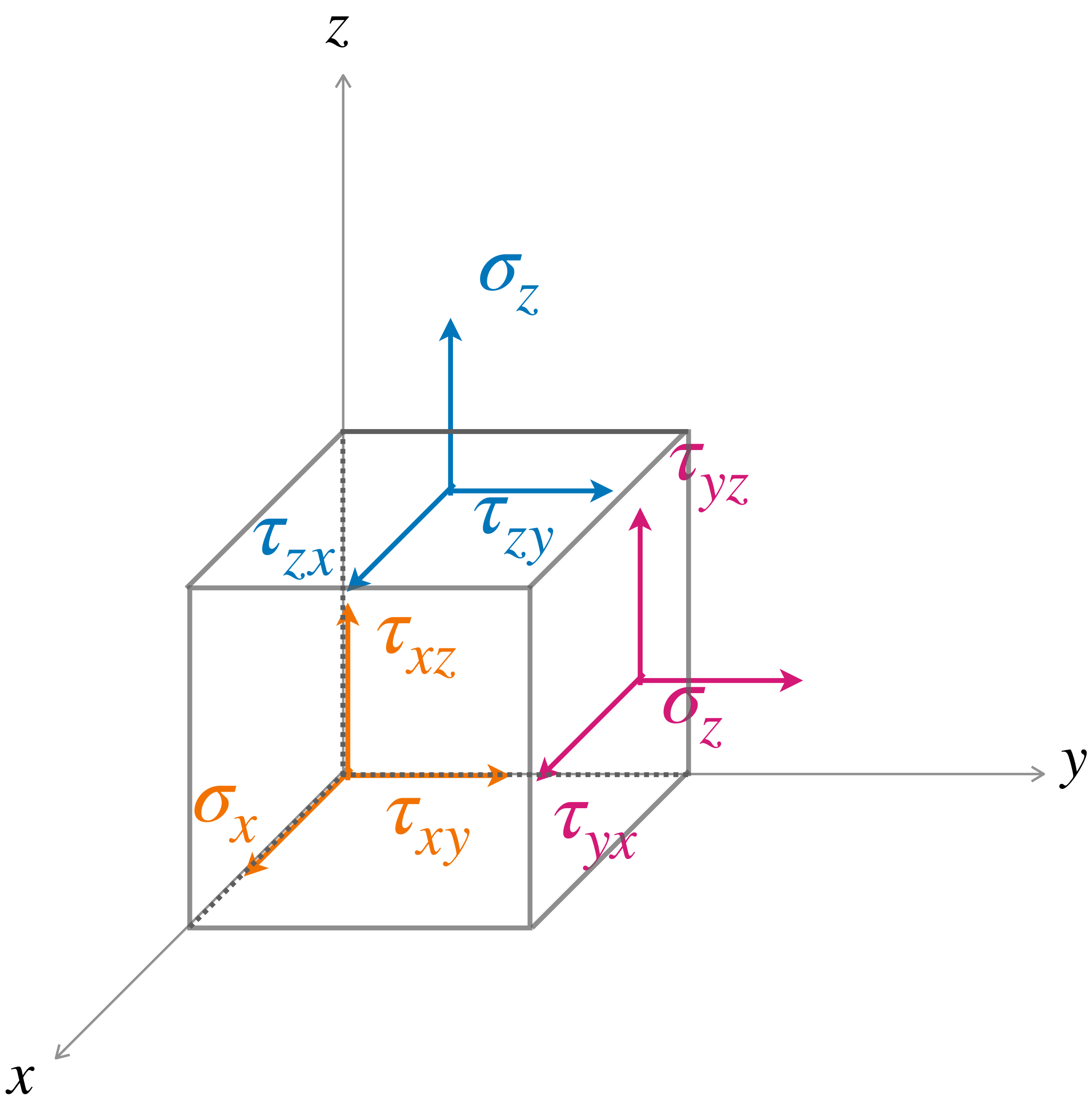
RECIPROCITA'
DELLE TENSIONI
TANGENZIALI

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yx} & \sigma_z \end{bmatrix}$$

9 componenti: σ - 3 componenti normali
τ - 6 componenti tangenziali



_ Tensione secondo Cauchy



TEOREMA DI CAUCHY

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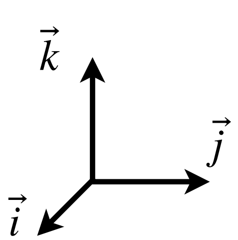
RECIPROCITA' DELLE TENSIONI TANGENZIALI

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yx} & \sigma_z \end{bmatrix}$$

$\underline{\underline{\sigma}}$ è simmetrico

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^T$$



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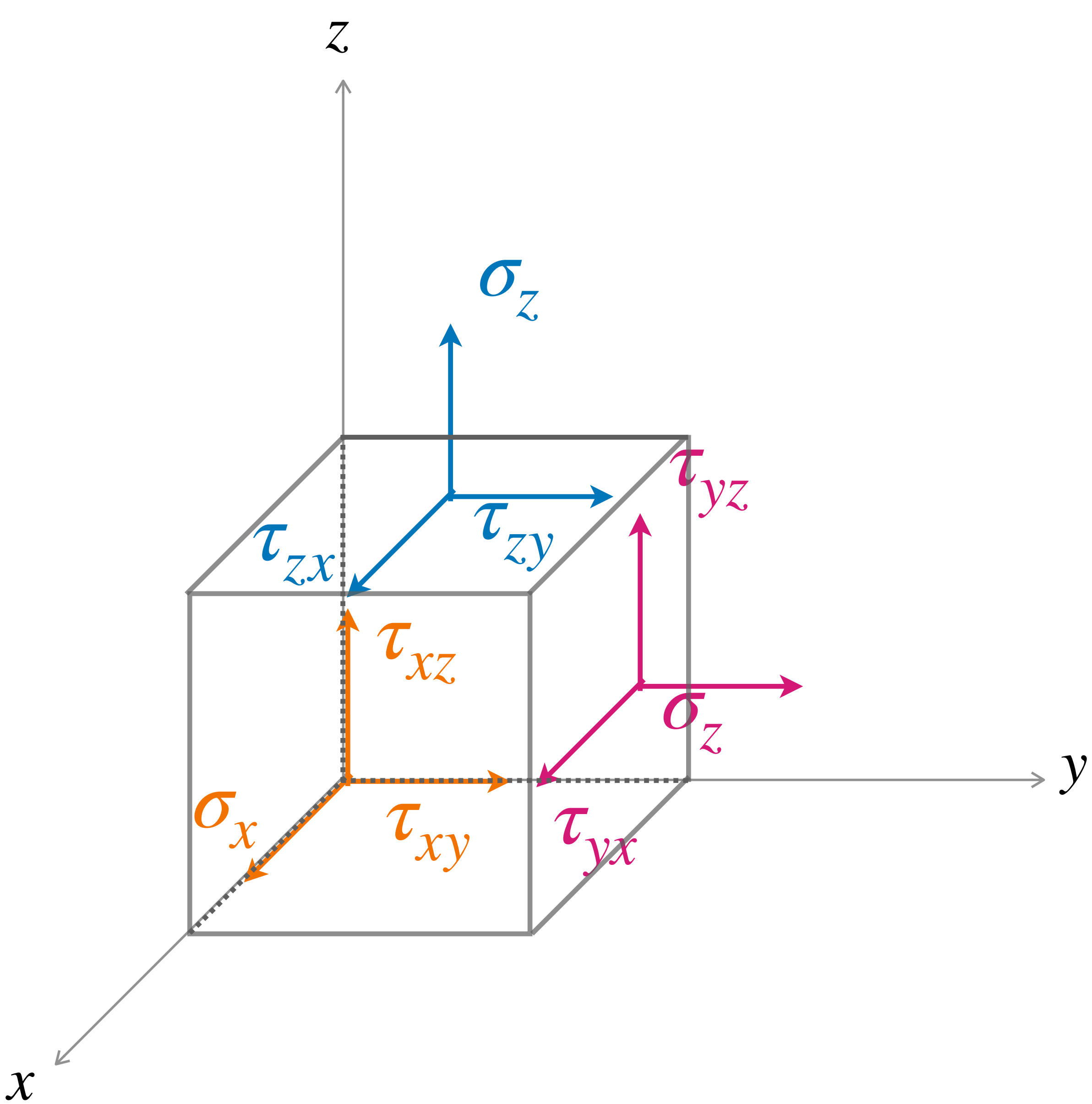

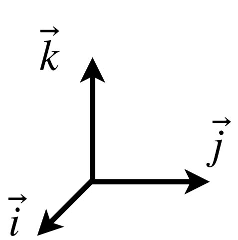

TEOREMA DI CAUCHY

$\vec{R} = \vec{0}$ $\quad \boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$ $\quad \underline{\underline{\sigma}}$ Tensore degli sforzi

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RECIPROCITA' DELLE TENSIONI TANGENZIALI

$$\underline{\underline{\sigma}} = \begin{bmatrix} \sigma_x & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \sigma_y & \tau_{zy} \\ \tau_{xz} & \tau_{yx} & \sigma_z \end{bmatrix}$$

$\underline{\underline{\sigma}}$ è simmetrico

$$\underline{\underline{\sigma}} = \underline{\underline{\sigma}}^T$$

6 componenti indipendenti:

σ - 3 componenti normali

τ - 3 componenti tangenziali

_ Altri modelli di tensione?

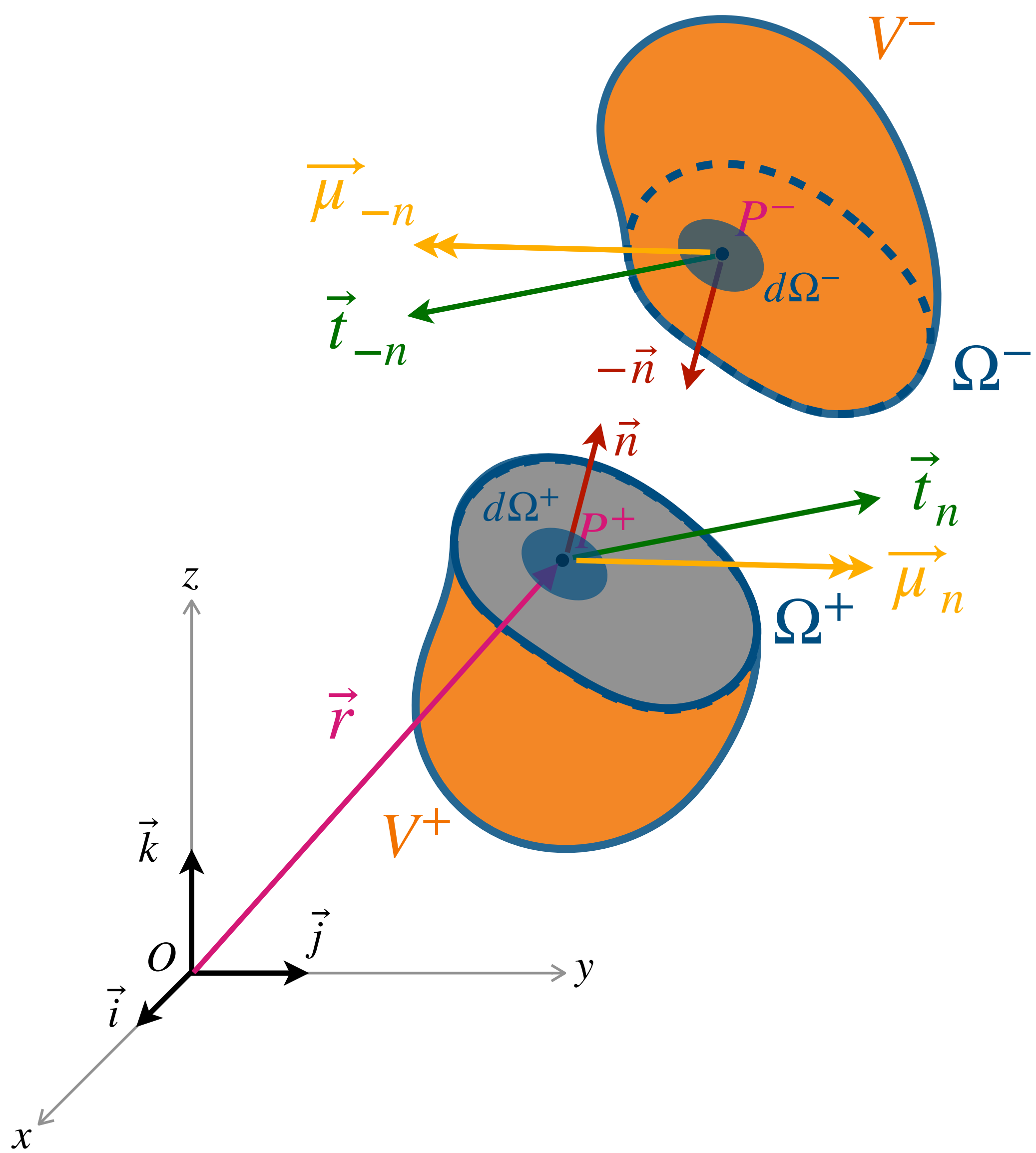
_ Altri modelli di tensione?

François Cosserat
(Douai 1852 - 1914)

Eugène Cosserat
(Amiens 1866, Tolosa 1931)

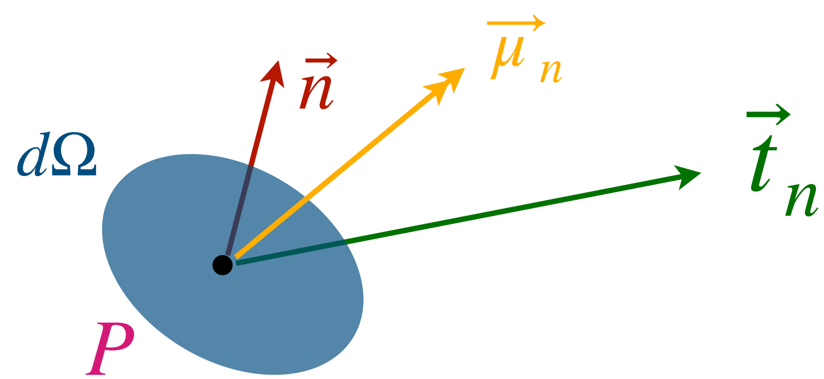
Cosserat, Eugène, et Cosserat, François (1909) *Théorie des corps déformables*, Hermann et fils, Paris.

_ Altri modelli di tensione

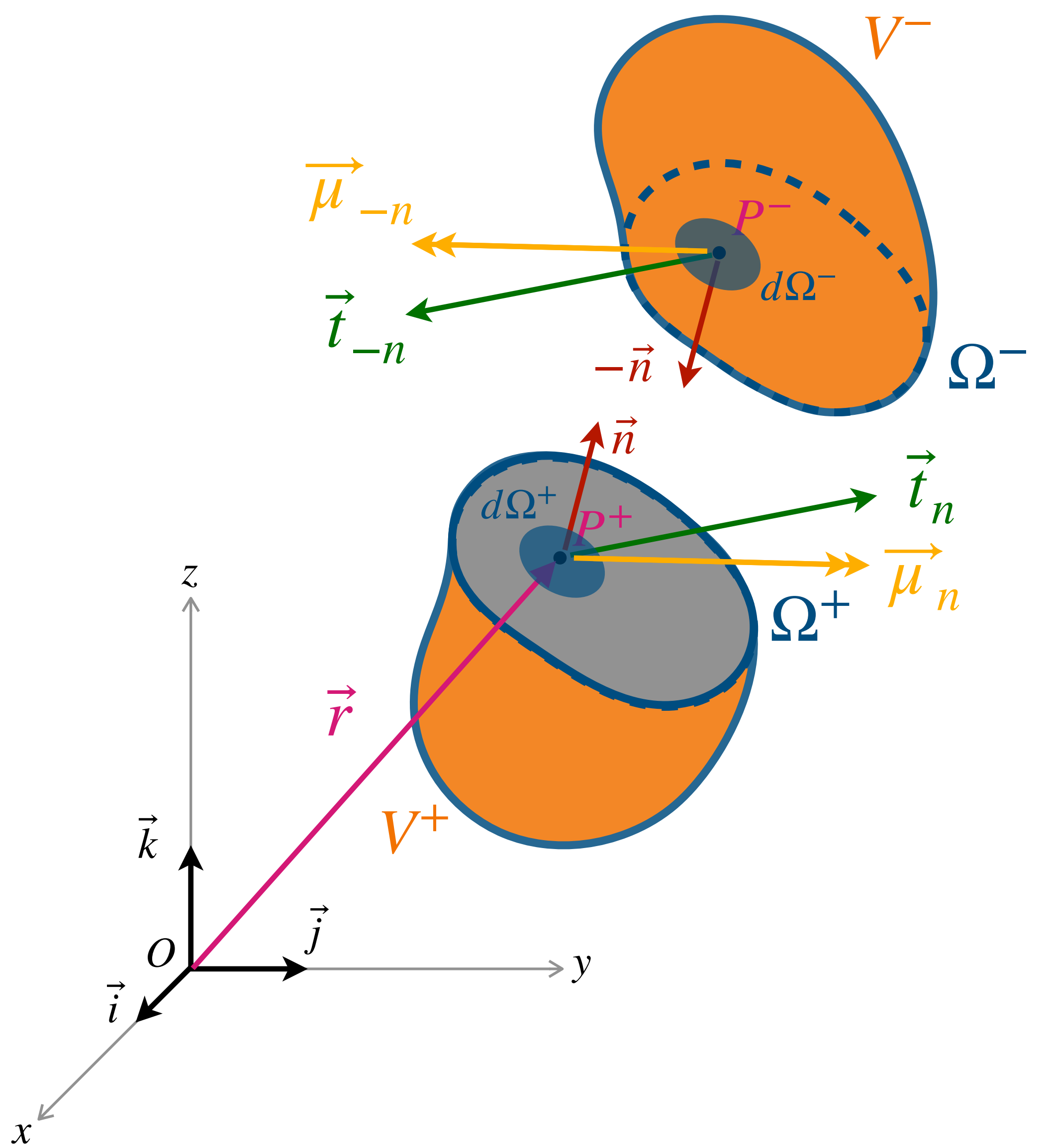


$$P = (x, y, z)$$

$$\vec{r} = \{x, y, z\}$$



_ Altri modelli di tensione



TENSIONE ALLA COSSERAT

Siano ΔT e ΔM la risultante delle forze e il momento risultante che il tronco V^+ trasmette al tronco V^-, si assume che:

$$\lim_{\Delta\Omega \to 0} \frac{\Delta \overrightarrow{T}}{\Delta\Omega} = \frac{d\overrightarrow{T}}{d\Omega} = \vec{t}_n$$

$$\lim_{\Delta\Omega \to 0} \frac{\Delta \overrightarrow{M}}{\Delta\Omega} = \frac{d\overrightarrow{M}}{d\Omega} = \vec{\mu}_n$$

$$P = (x, y, z)$$

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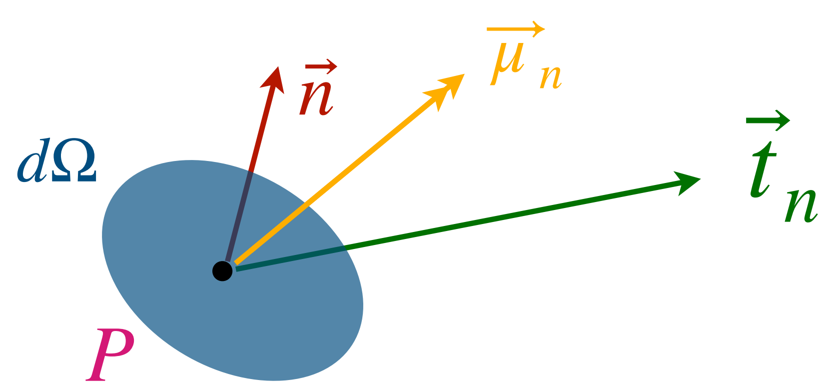

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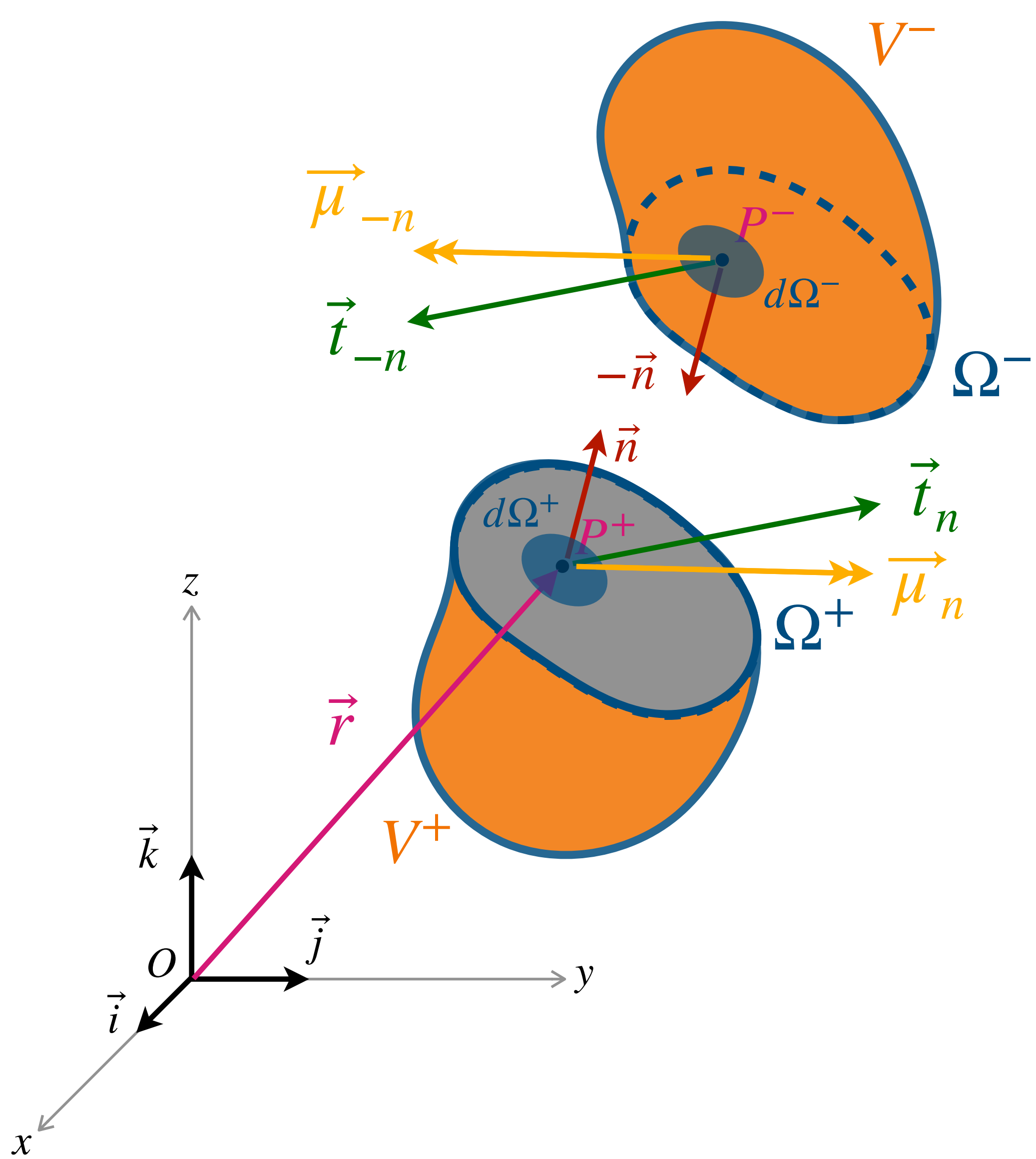

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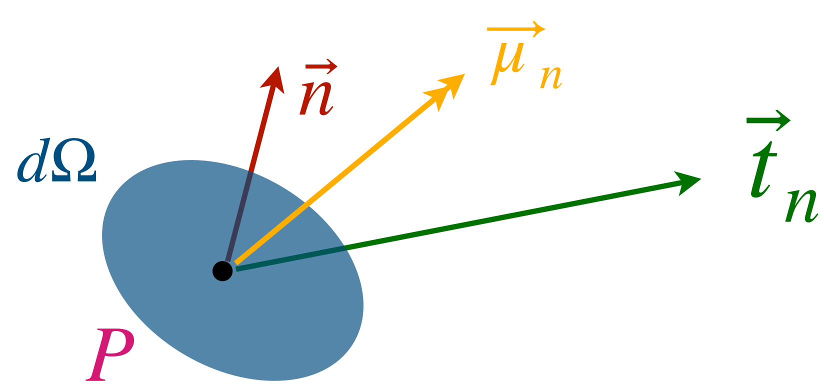

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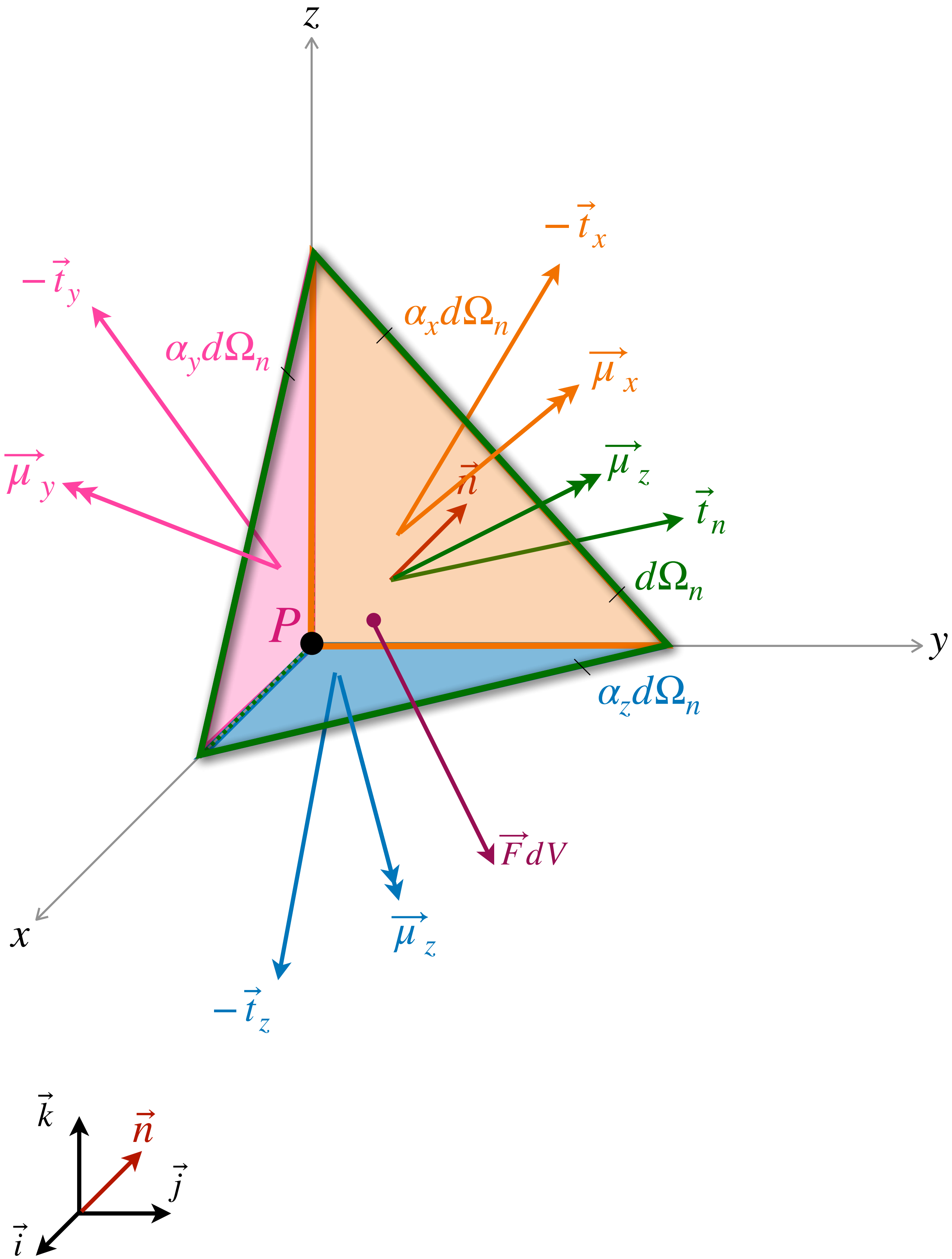
$$\boxed{\vec{t}_n = \lim_{\Delta\Omega \to 0} \frac{\Delta \vec{T}}{\Delta\Omega}}$$

$$\boxed{\vec{\mu}_n = \lim_{\Delta\Omega \to 0} \frac{\Delta \vec{M}}{\Delta\Omega}}$$

$$\boxed{\vec{t}_n = -\,\vec{t}_{-n}} \qquad \boxed{\vec{\mu}_n = -\,\vec{\mu}_{-n}}$$

_ Altri modelli di tensione



TENSIONE ALLA COSSERAT

CONDIZIONI DI EQUILIBRIO

$$\vec{R} = \vec{0} \qquad \& \qquad \vec{M}_{(O)} = \vec{0}$$

$$\vec{R} = \vec{0} \qquad \boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}} \qquad \underline{\underline{\sigma}}$$ Tensore degli sforzi

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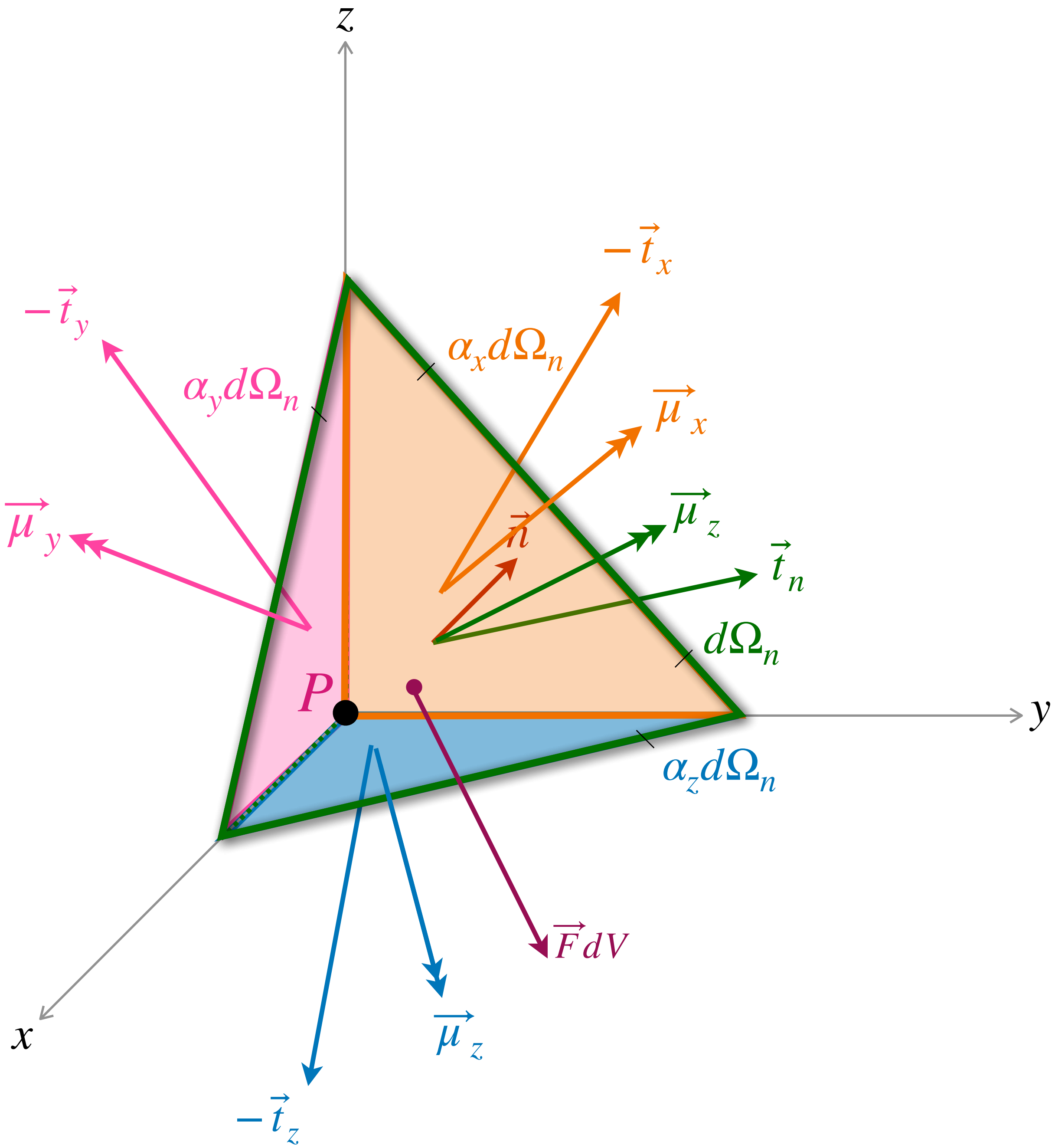
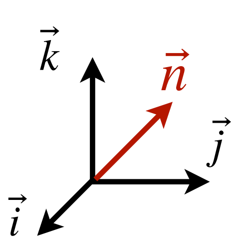
~~RECIPROCITA' DELLE TENSIONI TANGENZIALI~~

_ Altri modelli di tensione





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$$\vec{R} = \vec{0} \qquad \boxed{\vec{t}_n = \underline{\underline{\sigma}}\vec{n}}$$

$\underline{\underline{\sigma}}$ Tensore degli sforzi

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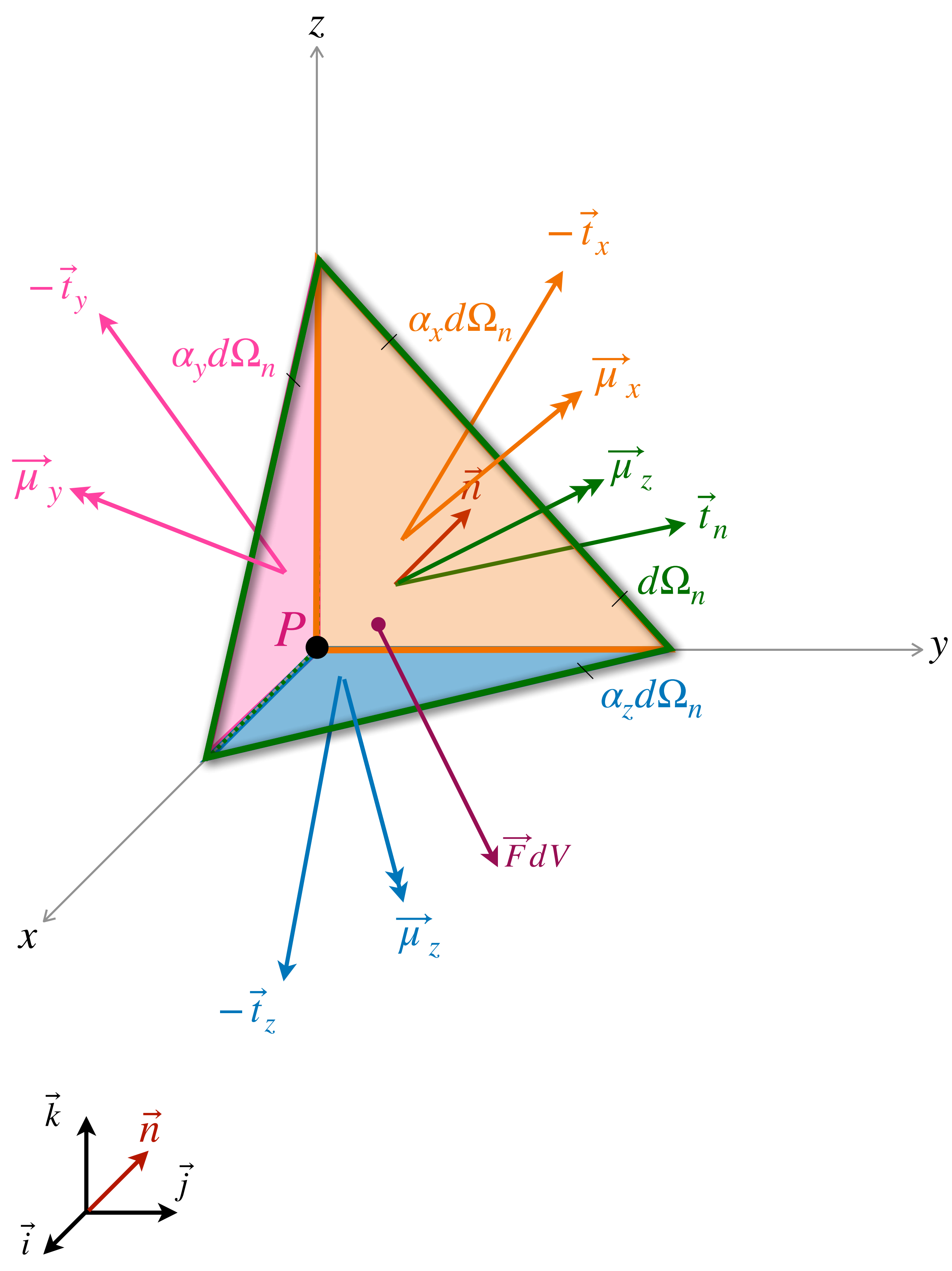
$\underline{\underline{\sigma}}$ non è simmetrico!

9 componenti indipendenti

_ Altri modelli di tensione



TENSIONE ALLA COSSERAT

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$\underline{\underline{\sigma}}$ non è simmetrico!

9 componenti indipendenti

$$\underline{\underline{\mu}} = \begin{bmatrix} \mu_{xx} & \mu_{yx} & \mu_{zx} \\ \mu_{xy} & \mu_{yy} & \mu_{zy} \\ \mu_{xz} & \mu_{yx} & \mu_{zz} \end{bmatrix}$$

$\underline{\underline{\mu}}$

+ 9 componenti indipendenti