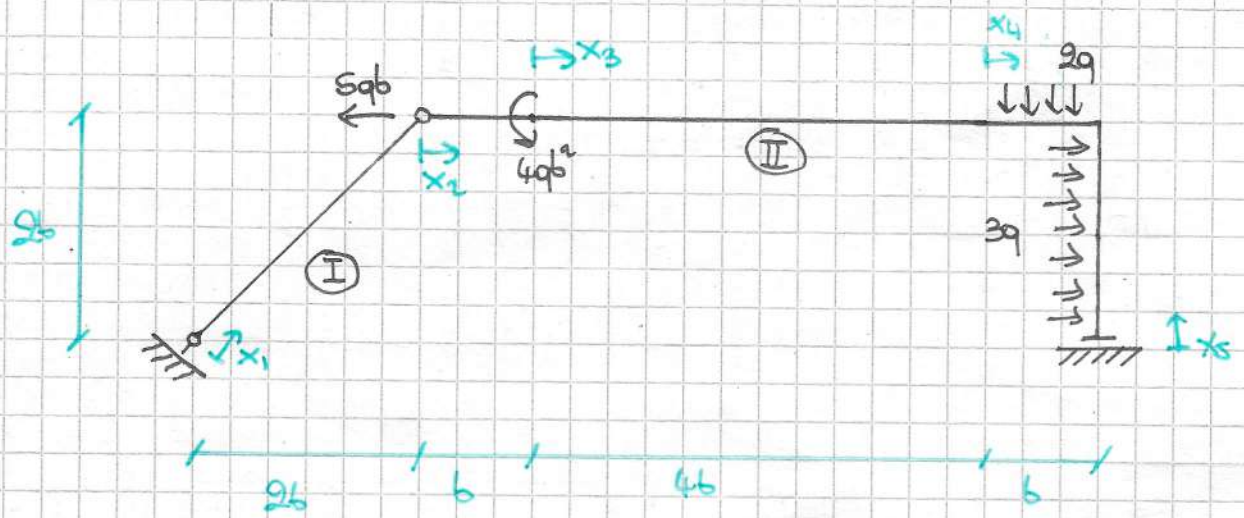
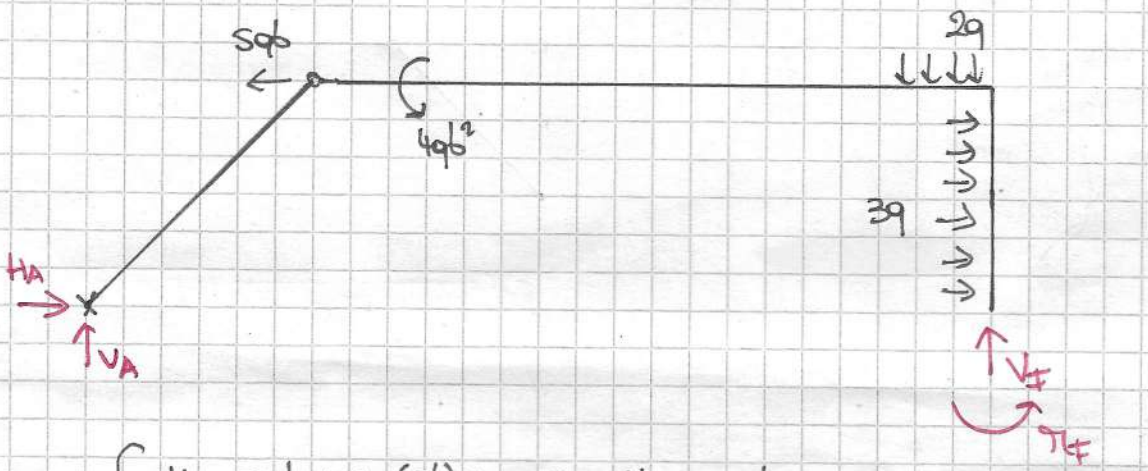
ESERCIZIO 1 - TRACCIA 1 - ESAME 28.01.2025





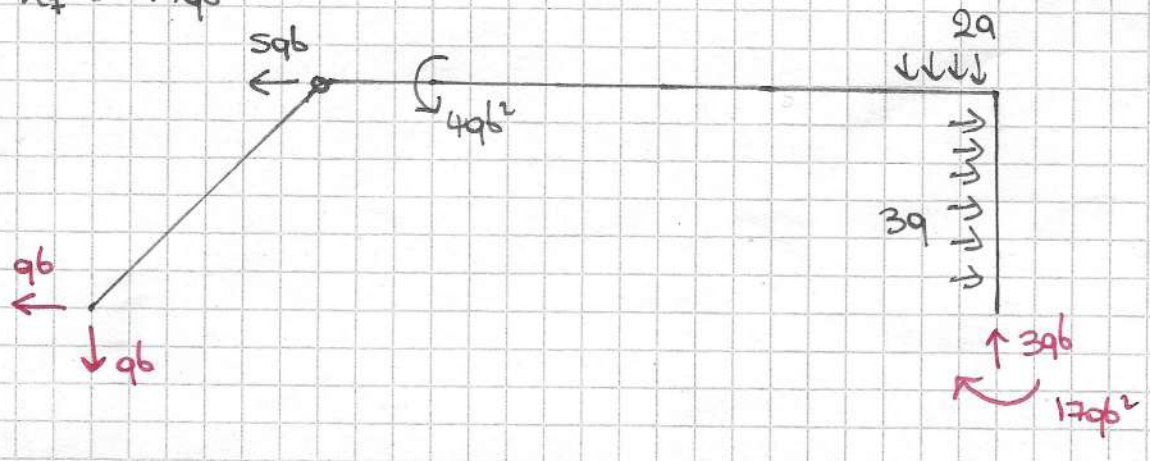
$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \pi_{z(A)} = 0 \end{cases} \quad \begin{cases} H_A - 5qb + 3q(2b) = 0 \Rightarrow H_A = -qb \\ V_A + V_F - 2qb = 0 \Rightarrow V_F = 3qb \\ 5qb(2b) + 4qb^2 - 2q(b)\left(7b + \frac{b}{2}\right) - 3q(2b)(b) + \pi_F + V_F(8b) = 0 \quad [*] \end{cases}$$

eq. aux.

$$\begin{cases} \pi^{(I)}_{z(B)} = 0 \end{cases} \quad \begin{cases} V_A(2b) - H_A(2b) = 0 \Rightarrow 2bV_A = -2qb^2 \Rightarrow V_A = -qb \end{cases}$$

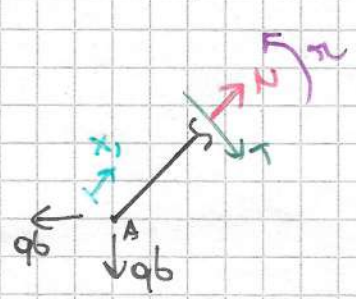
$[*]$ $10qb^2 + 4qb^2 - 15qb^2 - 6qb^2 + \pi_F + 24qb^2 = 0$

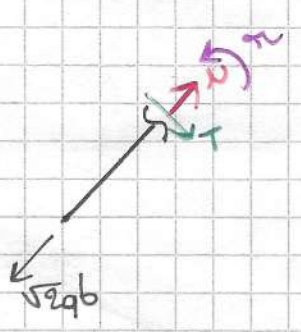
$\pi_F = -17qb^2$



Azioni interne

A → B $\quad 0 \le x_1 \le \sqrt{2}(2b)$



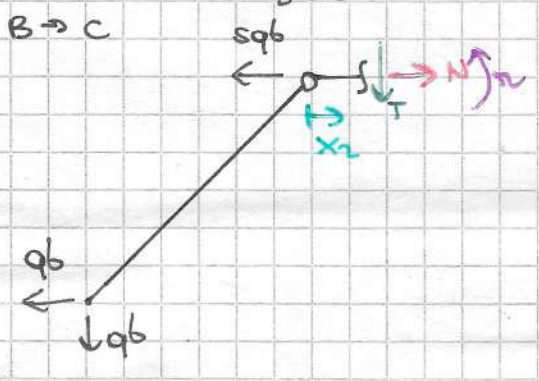


$N_{(x_1)} = \sqrt{2}qb$

$T_{(x_1)} = 0$

$\pi_{(x_1)} = 0$

B → C $\quad 0 \le x_2 \le b$



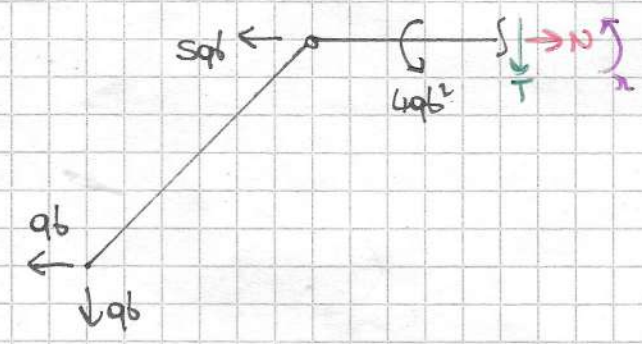
$N_{(x_2)} = 6qb$

$T_{(x_2)} = -qb$

$\pi_{(x_2)} = qb(2b) - qb(2b + x_2)$

$\pi_{(x_2)} = -x_2 qb$

C → D $\quad 0 \le x_3 \le 4b$



$N_{(x_3)} = 6qb$

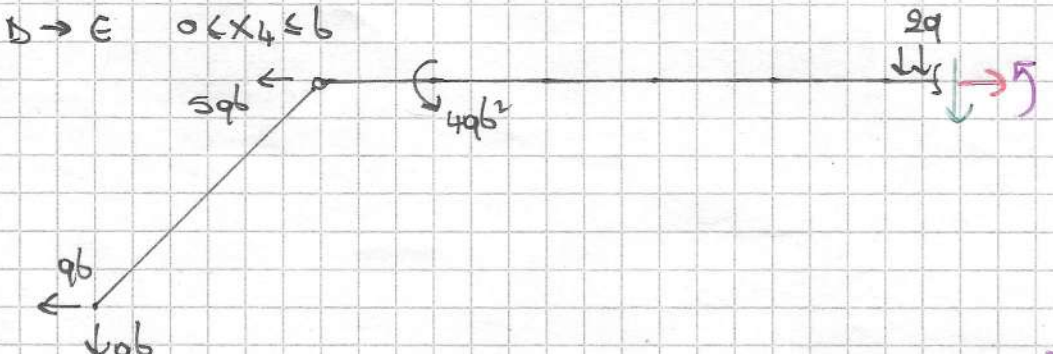
$T_{(x_3)} = -qb$

$\pi_{(x_3)} = qb(2b) - qb(3b + x_3) - 4qb^2$

$= -qb^2 - qbx_3 - 4qb^2$

$\pi_{(x_3)} = -5qb^2 - qbx_3$

D → E $\quad 0 < x_4 \le b$



$N_{(x_4)} = 6qb$

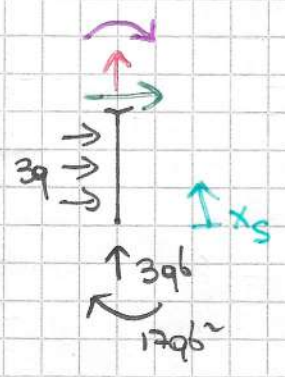
$T_{(x_4)} = -qb - 2q(x_4)$

$\pi_{(x_4)} = qb(2b) - qb(7b + x_4) +$

$-4qb^2 - 2q(x_4)\left(\frac{x_4}{2}\right)$

$\pi_{(x_4)} = -9qb^2 - qbx_4 - qx_4^2$

$F \to \in \quad 0 \le x_S \le 2b$

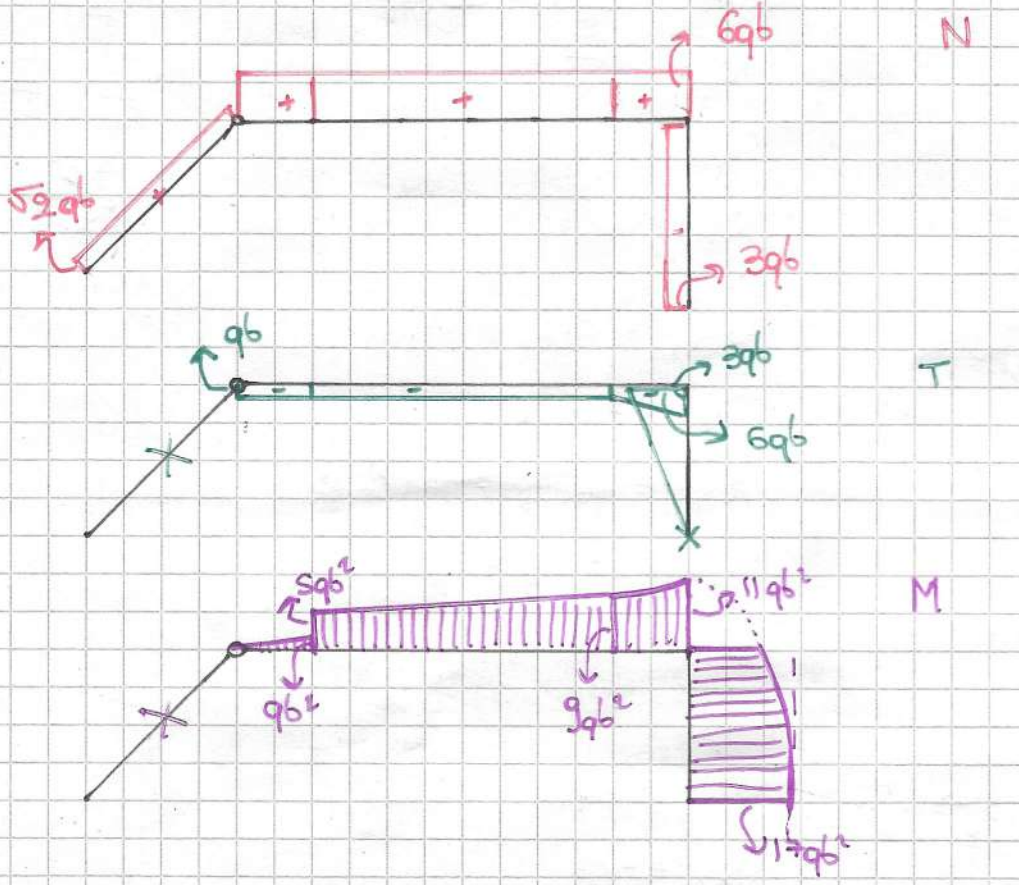


$$N_{(x_S)} = -3qb$$

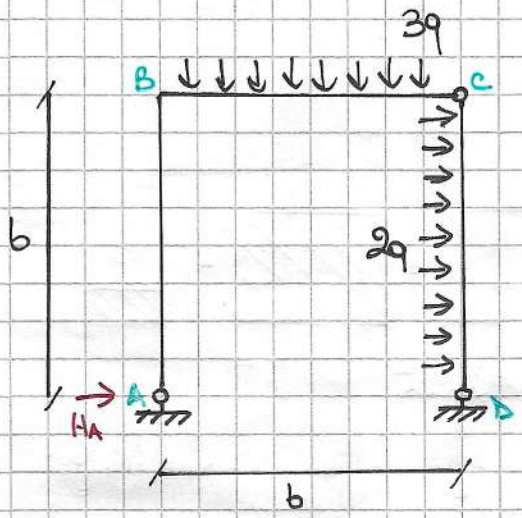
$$T_{(x_S)} = -3qx_S$$

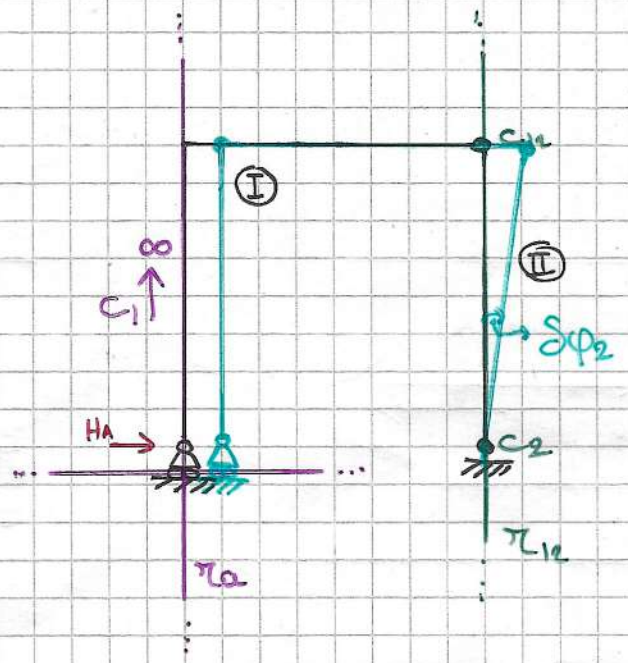
$$M_{(x_S)} = 3qx_S\left(\frac{x_S}{2}\right) - 17qb^2$$

$$= \frac{3}{2}qx_S^2 - 17qb^2$$



Esercizio 2.





STRUTTURA 1 VOLTA IPOSTATICA.

$C_1 \in r_a$

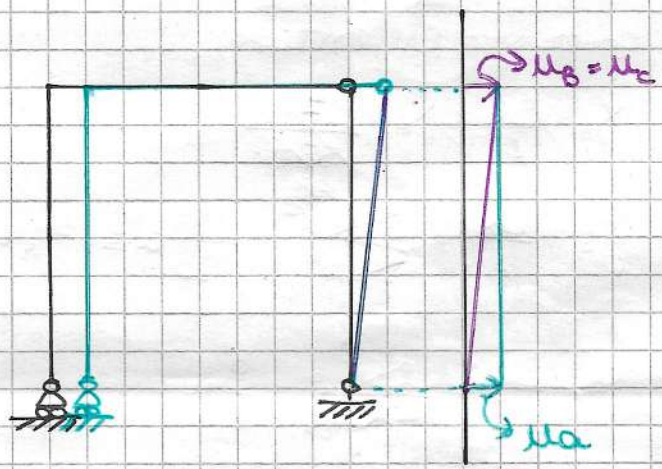
$C_2 = D = (b;0)$

$C_{12} = C = (b;b)$

$C_1 \leftrightarrow C_{12} \leftrightarrow C_2$

$C_{12} \in r_{12}$

$C_1 = (\infty;\infty)$

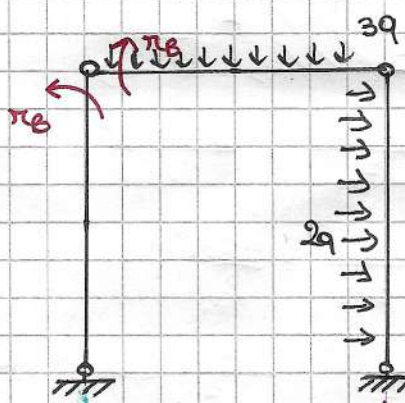


$M_a = b\delta\varphi_2 \quad M_B = M_C = b\delta\varphi_2$

PLV

$H_A \cdot b\delta\varphi_2 + 2qb\left(\frac{b}{2}\delta\varphi_2\right) = 0$

$H_A = -qb$



$C_1 = A = (0;0)$

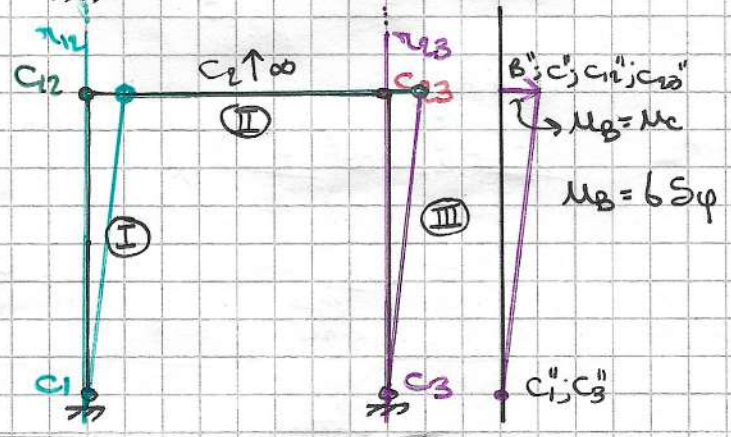
$C_2 \in r_{12}$; $C_2 \in r_{23}$

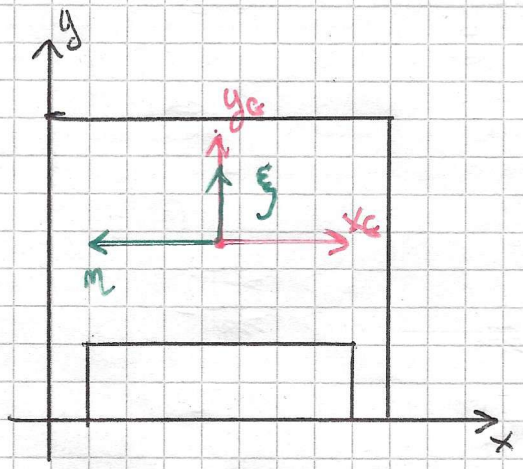
$C_2 = (\infty;\infty)$

$C_3 = (b;0)$

P.L.V. $2q(b)\left(\frac{b}{2}\delta\varphi\right) - r_B\delta\varphi = 0$

$r_B = qb^2$



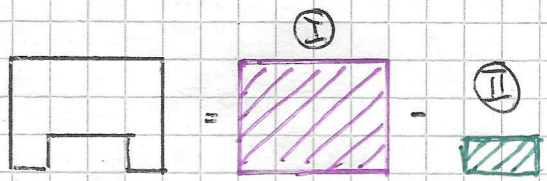


COORDINATE BARICENTRICHE

$$y_G = \frac{S_x}{A} = \frac{274a^3}{58a^2} = \frac{137}{29}a \cong 4{,}7241a$$

$$x_G = \frac{S_y}{A} = \frac{261a^3}{58a^2} = \frac{9}{2}a = 4{,}5a$$

MOMENTO STATICO



$$S_x = S_x^I - S_x^{II}$$

$S_x^I = A^I y_G^I$ $\quad S_x^{II} = A^{II} y_G^{II}$

$= 72a^2 \cdot 4a$ $\quad = 14a^2 \cdot a$

$= 288a^3$ $\quad = 14a^3$

$S_x^{TOT} = (288 - 14)a^3$ $\quad S_x^{TOT} = 274a^3$

$S_y^{TOT} = S_y^I - S_y^{II}$

$S_y^I = A^I x_G^I$ $\quad S_y^{II} = A^{II} x_G^{II}$

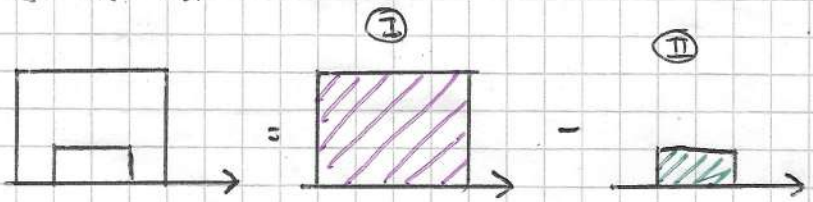
$S_y^I = 72a^2 \cdot \frac{9a}{8}$ $\quad S_y^{II} = 14a^2 \cdot \frac{9a}{8}$

$S_y^I = 324a^3$ $\quad S_y^{II} = 63a^3$

$S_y^{TOT} = (324 - 63)a^3$ $\quad S_y^{TOT} = 261a^3$

MOMENTO DI INERTIA

$J_x = J_x' - J_x^{II}$



$J_x^I = \frac{b_1 h_1^3}{3} - \frac{9a \cdot (8a)^3}{3} = \frac{4.608\, a^4}{3}$

$J_x^{II} = \frac{b_2 h_2^3}{3} = \frac{7a(2a)^3}{3} = \frac{56a^4}{3}$

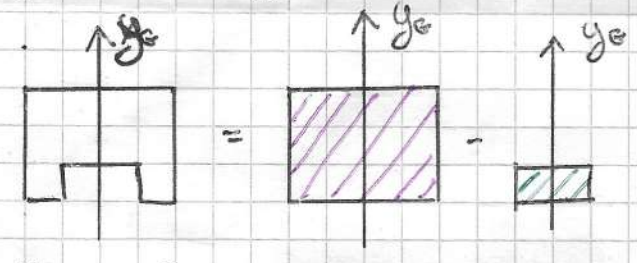
$J_x^{TOT} = \frac{4608 - 56}{3} a^4 = \frac{4552}{3}$

$J_{x_G} = J_x - A y_G^2$

$= \frac{4552}{3} - 58a^2 \left(\frac{18769}{841 \cdot 29}\right)$

$= \frac{132008 - 112614}{87} = \frac{19394}{87} a^4 \cong 222.9195 a^4$

$J_{y_G} = J_{y_G}^I - J_{y_G}^{II}$



$J_{y_G}^I = \frac{h_I b_I^3}{12} = \frac{8a \cdot 729a^3}{12} = \frac{5832}{12} a^4$

$J_{y_G}^{II} = \frac{h_{II} b_{II}^3}{12} = \frac{2a(343a^3)}{12} = \frac{686}{12} a^4$

$J_{y_G} = \frac{5832 - 686}{12} a^4 = \frac{5146}{12} \frac{2573}{6} a^4$

$J_{y_G} = \frac{2573}{6} a^4 \cong 428.8333 a^4$

MOMENTO CENTRIFUGO

$J_{x_G y_G} = 0 \Rightarrow y_G$ = ASSE DI SIMMETRIA

$tg 2\vartheta = \frac{-2 J_{x_G y_G}}{J_{x_G} - J_{y_G}} \Rightarrow tg 2\vartheta = 0$

$J_{y_G} > J_{x_G} \qquad \vartheta = \frac{\pi}{2} = 90°$

$J_{max} = J_{y_G} = J_\xi$

$J_{min} = J_{x_G} = J_\eta$