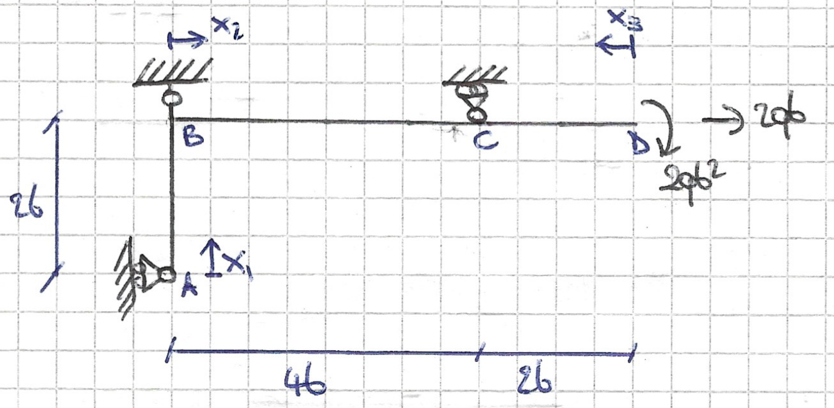
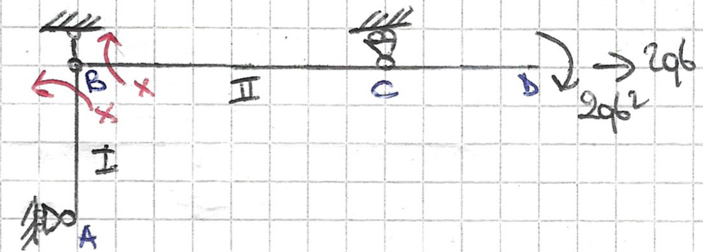
ESAME SCIENZA DELLE COSTRUZIONI 16.07.2024



STRUTTURA IPERSTATICA

GDL = 3
GDV = 1(A) + 2(B) + 1(C) = 4
GDL < GDV

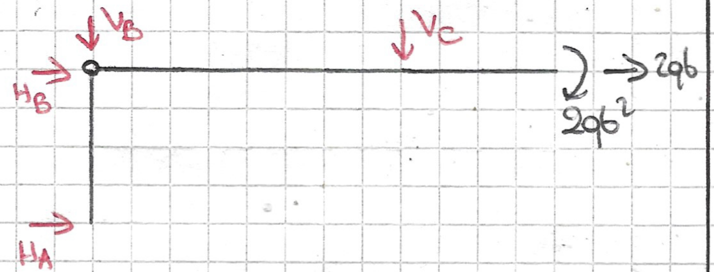
eq. di congruenza $\Delta\varphi(B) = 0$



STRUTTURA ISOSTATICA

GDL = 6
GDV = 1(A) + 4(B) + 1(C) = 6
GDL = GDV

S_0 - SISTEMA REALE

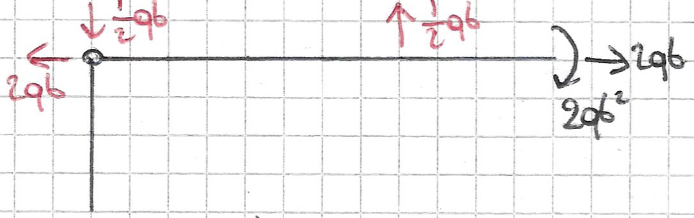


$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \mathcal{M}_{t(B)} = 0 \end{cases} \begin{cases} H_A + H_B + 2qb = 0 \Rightarrow H_B = -2qb \\ V_B + V_C = 0 \quad [1] \\ 2bH_A - V_C 4b - 2qb^2 = 0 \quad [2] \end{cases}$$

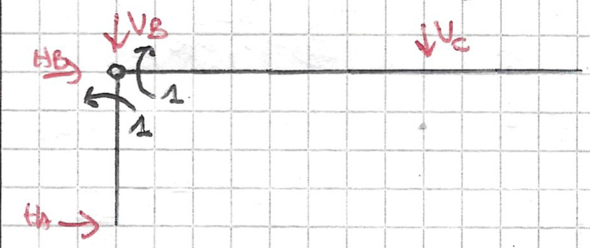
$$\left\{ \mathcal{M}^{I}_{t(B)} = 0 \right. \left\{ H_A 2b = 0 \Rightarrow H_A = 0 \right.$$

[2] $-V_C 4b - 2qb^2 = 0 \Rightarrow V_C = -\frac{1}{2}qb$

[1] $V_B = -V_C \Rightarrow V_B = \frac{1}{2}qb$



S_1 - SISTEMA EQUILIBRATO

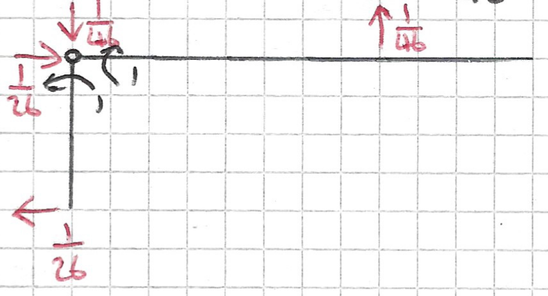


$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \mathcal{M}_{t(B)} = 0 \end{cases} \begin{cases} H_B + H_A = 0 \Rightarrow H_B = +\frac{1}{2b} \\ V_B + V_C = 0 \quad [1] \\ 2bH_A - 4bV_C = 0 \quad [2] \end{cases}$$

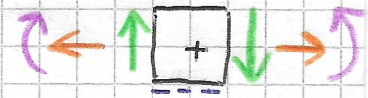
$$\left\{ \mathcal{M}^{I}_{t(B)} = 0 \right. \left\{ 2bH_A + 1 = 0 \Rightarrow H_A = -\frac{1}{2b} \right.$$

[2] $2b\left(-\frac{1}{2b}\right) - 4bV_C = 0 \Rightarrow V_C = -\frac{1}{4b}$

[1] $V_B = -V_C \Rightarrow V_B = \frac{1}{4b}$

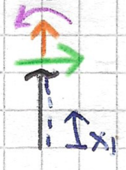


Azioni interne



Sistema S_0

$A \to B \quad 0 \le x_1 \le 2b$

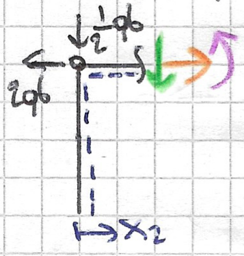


$N(x_1) = 0$

$T(x_1) = 0$

$M(x_1) = 0$

$B \to C \quad 0 \le x_2 \le 4b$

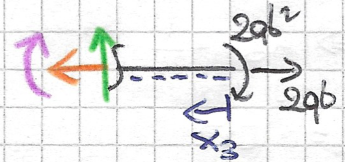


$N(x_2) = 2qb$

$T(x_2) = -\frac{1}{2}qb$

$M(x_2) = -\frac{1}{2}qb\,x_2$

$C \to D \quad 0 \le x_3 \le 2b$



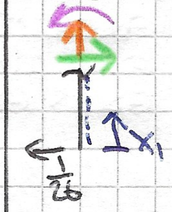
$N(x_3) = 2qb$

$T(x_3) = 0$

$M(x_3) = -2qb^2$

Sistema S_1

$A \to B \quad 0 \le x_1 \le 2b$

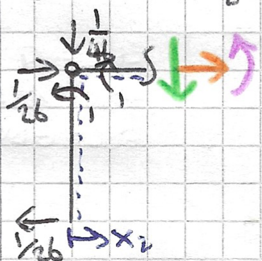


$N(x_1) = 0$

$T(x_1) = \frac{1}{2b}$

$M(x_1) = \frac{1}{2b}x_1$

$B \to C \quad 0 \le x_2 \le 4b$



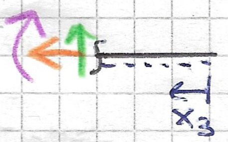
$N(x_2) = 0$

$T(x_2) = -\frac{1}{4b}$

$M(x_2) - \frac{1}{2b}(2b) + \frac{1}{4b}x_2 = 0$

$M(x_2) = 1 - \frac{x_2}{4b}$

$C \to D \quad 0 \le x_3 \le 2b$

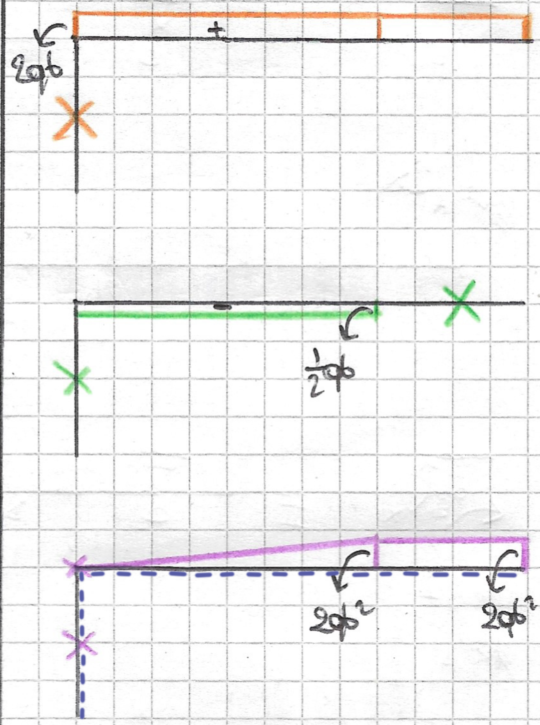


$N(x_3) = 0$

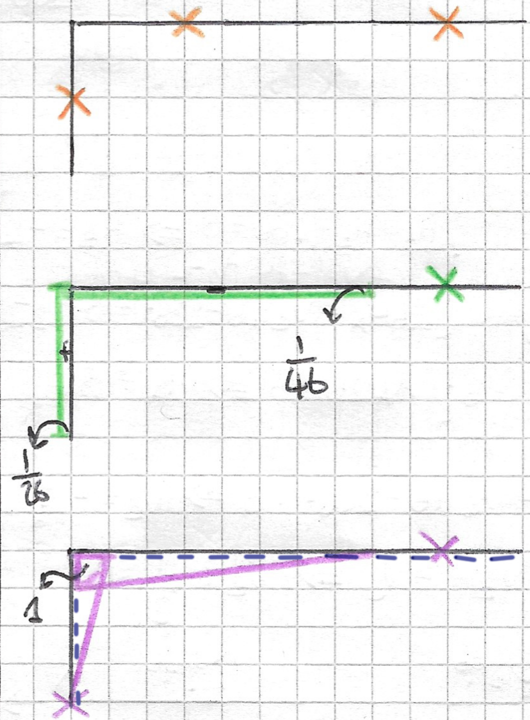
$T(x_3) = 0$

$M(x_3) = 0$

Sistema 0



Sistema 1



P.L.V.

$$\mathcal{L}V_e = \mathcal{L}V_i$$

$$\mathcal{L}V_e = 1 \cdot \Delta\varphi_B = 0$$

$$\mathcal{L}V_i = \int_s M_1 \chi_i$$

$$\mathcal{L}V_i = \int_s M_1 \left(\frac{M_0 + X M_1}{EI}\right) ds \Rightarrow \mathcal{L}V_i = \int_s \left(\frac{M_1 \cdot M_0}{EI} + X\frac{M_1^2}{EI}\right) ds$$

A → B

$M_0 = 0$; $M_1 = \frac{1}{2b} x_1$

$M_1 \cdot M_0 = 0$

$M_1^2 = \frac{x_1^2}{4b^2}$

B → C

$M_0 = -\frac{1}{2} qb x_2$ $M_1 = 1 - \frac{x_2}{4b}$

$M_1 M_0 = -\frac{1}{2} qb x_2 + \frac{1}{8} q x_2^2$

$M_1^2 = 1 + \frac{x_2^2}{16b^2} - \frac{x_2}{2b}$

C → D

$M_0 = -2qb^2$; $M_1 = 0$

$M_1 M_0 = 0$

$M_1^2 = 0$

$$\mathcal{L}V_i = \int_0^{2b} X\left(\frac{x_1^2}{4b^2}\right)\left(\frac{1}{EI}\right) dx_1 + \int_0^{4b} \left(-\frac{1}{2} qb x_2 + \frac{1}{8} q x_2^2\right)\left(\frac{1}{EI}\right) + X\left(1 + \frac{x_2^2}{16b^2} - \frac{x_2}{2b}\right) dx_2$$

$$= \left[X\left(\frac{x_1^3}{12b^2}\right)\left(\frac{1}{EI}\right)\right]_0^{2b} + \left[\left(-\frac{1}{4} qb x_2^2 + \frac{1}{24} q x_2^3\right)\left(\frac{1}{EI}\right) + X\left(x_2 + \frac{x_2^3}{48b^2} - \frac{x_2^2}{4b}\right)\left(\frac{1}{EI}\right)\right]_0^{4b}$$

$$= \frac{1}{EI}\left[X\left(\frac{\cancel{8}^{\,2} b^3}{\cancel{12}_{\,3} b^2}\right) - \frac{16b^3}{4} + \frac{\cancel{64}^{\,8}}{\cancel{24}_{\,3}} qb^3 + X\left(4b + \frac{\cancel{64}^{\,4} b^{\cancel{3}}}{3\cancel{48} b^{\cancel{2}}} - \frac{\cancel{16}^{\,4} b^{\cancel{2}}}{\cancel{4b}}\right)\right]$$

$$= \frac{1}{EI}\left[X\left(\frac{2}{3}b\right) - 4qb^3 + \frac{8}{3}qb^3 + X\left(\frac{4}{3}b\right)\right]$$

$$\mathcal{L}V_i = \frac{1}{EI}\left[2bX - \frac{4}{3}qb^3\right] \Rightarrow \mathcal{L}V_i = 0 \Rightarrow \frac{1}{EI}\left(2bX - \frac{4}{3}qb^3\right) = 0$$

$$2b X = \frac{4}{3} qb^3$$

$$\boxed{X = \frac{2}{3} qb^2}$$

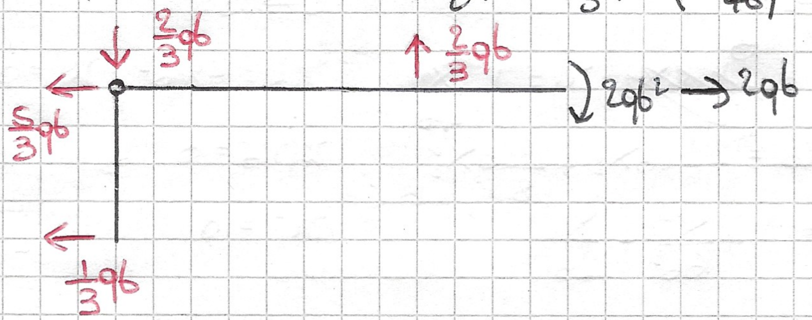
REAZIONI VINCOLARI

$$H_A = H_{A_0} + X H_{A_1} \Rightarrow H_A = 0 + \frac{2}{3}qb^2\left(-\frac{1}{2b}\right) \Rightarrow H_A = -\frac{1}{3}qb$$

$$H_B = H_{B_0} + X H_{B_1} \Rightarrow H_B = -2qb + \frac{2}{3}qb^2\left(\frac{1}{2b}\right) \Rightarrow H_B = -\frac{5}{3}qb$$

$$V_B = V_{B_0} + X V_{B_1} \Rightarrow V_B = \frac{1}{2}qb + \frac{2}{3}qb^2\left(\frac{1}{4b}\right) \Rightarrow V_B = \frac{2}{3}qb$$

$$V_C = V_{C_0} + X V_{C_1} \Rightarrow V_C = -\frac{1}{2}qb + \frac{2}{3}qb^2\left(-\frac{1}{4b}\right) \Rightarrow V_C = -\frac{2}{3}qb$$



AZIONI INTERNE

A → B

$$N(x_1) = N_0(x_1) + X N_1(x_1) \Rightarrow N(x_1) = 0$$

$$T(x_1) = T_0(x_1) + X T_1(x_1) \Rightarrow T(x_1) = \frac{2}{3}qb^2\left(\frac{1}{2b}\right) \Rightarrow T(x_1) = \frac{1}{3}qb$$

$$M(x_1) = M_0(x_1) + X M_1(x_1) \Rightarrow M(x_1) = \frac{2}{3}qb^2\left(\frac{1}{2b}x_1\right) \Rightarrow M(x_1) = \frac{1}{3}qb x_1$$

B → C

$$N(x_2) = 2qb$$

$$T(x_2) = -\frac{1}{2}qb + \frac{2}{3}qb^2\left(-\frac{1}{\cancel{4b}_{2}}\right) \Rightarrow T(x_1) = -\frac{2}{3}qb$$

$$M(x_2) = -\frac{1}{2}qb x_2 + \frac{2}{3}qb^2\left(1 - \frac{x_2}{4b}\right) \Rightarrow M(x_2) = -\frac{1}{2}qb x_2 + \frac{2}{3}qb^2 - \frac{1}{6}x_2 qb$$

$$\Rightarrow M(x_2) = \frac{2}{3}qb^2 - \frac{2}{3}qb x_2$$

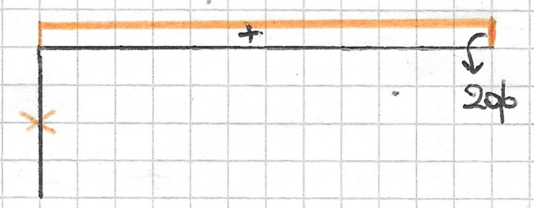
C → D

$$N(x_3) = 2qb$$

$$T(x_3) = 0$$

$$M(x_3) = -2qb^2$$

N

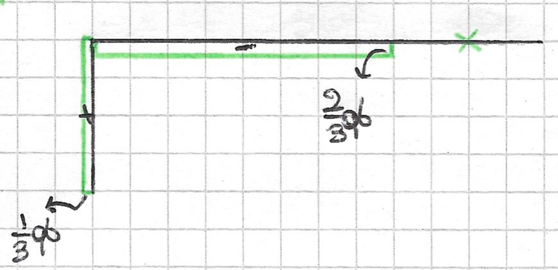


$N(x_1) = 0$

$N(x_2) = 2qb$

$N(x_3) = 2qb$

T

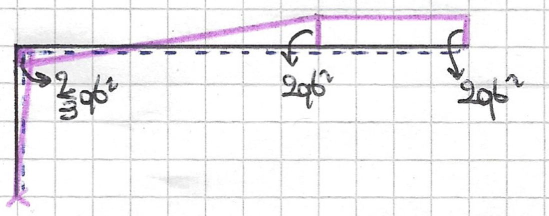


$T(x_1) = \frac{1}{3}qb$

$T(x_2) = -\frac{2}{3}qb$

$T(x_3) = 0$

M

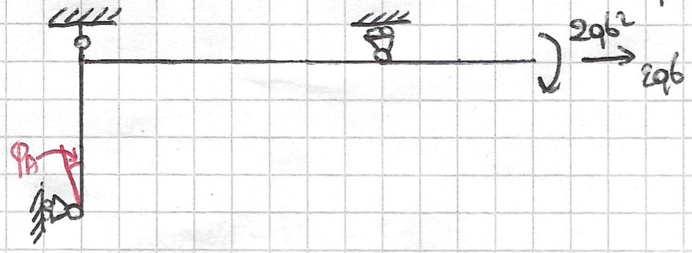


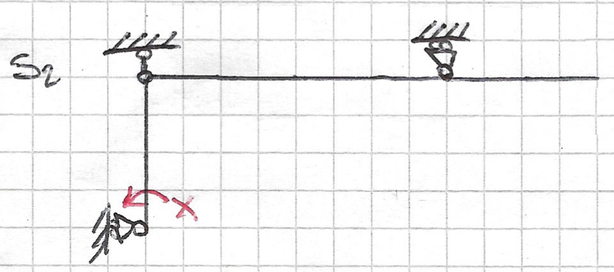
$M(x_1) = \frac{1}{3}qbx_1$

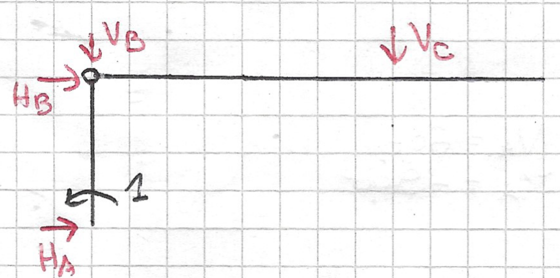
$M(x_2) = \frac{2}{3}qb^2 - \frac{2}{3}qbx_2$

$M(x_3) = -2qb^2$

Calcolo della rotazione nel punto A ; φ_A





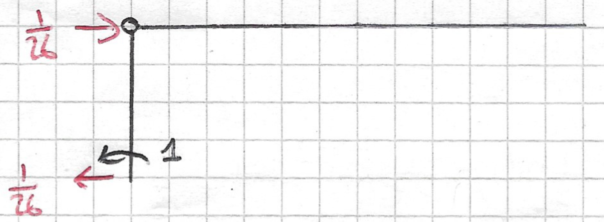


$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(B)} = 0 \end{cases} \quad \begin{cases} H_B + H_A = 0 \Rightarrow H_B = \frac{1}{2b} \\ V_B + V_C = 0 \; [1] \\ H_A 2b + 1 - V_C 4b = 0 \; [2] \end{cases}$$

$$\left\{ M^{I}_{z(B)} = 0 \right. \quad \left\{ H_A 2b + 1 = 0 \Rightarrow H_A = -\frac{1}{2b} \right.$$

[2] $-1 + 1 - 4bV_C = 0 \Rightarrow V_C = 0$

[1] $V_B = 0$



Azioni interne (momento)

$M(x_1) = -1 + \frac{1}{2b}x_1$

$M(x_2) = -1 + 1 \Rightarrow M(x_2) = 0$

$M(x_3) = 0$

P.L.V.

$$\mathcal{L}V_e = \mathcal{L}V_i$$

$$\mathcal{L}V_e = 1 \cdot \varphi_A$$

$$\mathcal{L}V_i = \int_0^{2b} \left(\frac{1}{2b}x_1 - 1\right)\left(\frac{1}{3}qbx_1\right)\left(\frac{1}{EI}\right)dx_1$$

$$= \int_0^{2b}\left(\frac{1}{6}qx_1^2 - \frac{1}{3}qbx_1\right)\left(\frac{1}{EI}\right)dx_1$$

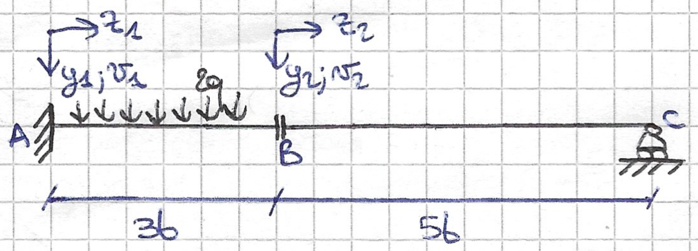
$$= \frac{1}{EI}\left[\frac{1}{18}qx_1^3 - \frac{1}{6}qbx_1^2\right]_0^{2b}$$

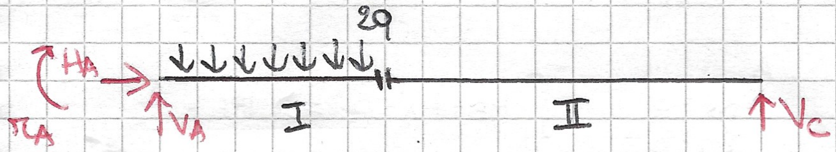
$$= \frac{1}{EI}\left(\frac{\cancel{8}^{4}}{\cancel{18}_{9}}qb^3 - \frac{\cancel{4}^{2}}{\cancel{6}_{3}}qb^3\right)$$

$$= \frac{1}{EI}\left(\frac{4}{9}qb^3 - \frac{2}{3}qb^3\right)$$

$$\mathcal{L}V_i = -\frac{2}{9}\frac{qb^3}{EI} \Longrightarrow \varphi_A = -\frac{2}{9}\frac{qb^3}{EI}$$

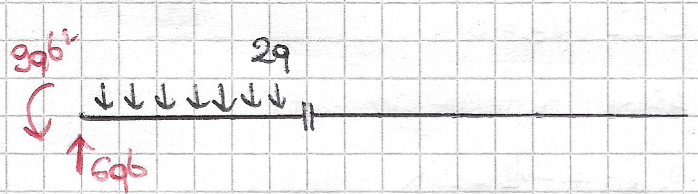
Esercizio II _ Traccia I





$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \mathcal{M}_{z(A)} = 0 \end{cases} \quad \begin{cases} H_A = 0 \\ V_A + V_C - 2q(3b) = 0 \quad \Longrightarrow \quad \Longrightarrow \quad V_A = 6qb \\ \mathcal{M}_A + 2q(3b)(3/2\,b) - V_C(8b) = 0 \quad \Longrightarrow \quad \mathcal{M}_A = -9qb^2 \end{cases}$$

$$\left\{ R_y^{II} = 0 \right. \quad \left\{ V_C = 0 \right.$$



AZIONI INTERNE

$A \to B \quad 0 \le z_1 \le 3b$

$N(z_1) = 0$

$T(z_1) = -2qz_1 + 6qb$

$M(z_1) = -9qb^2 - 2qz_1\left(\frac{z_1}{2}\right) + 6qbz_1$

$\stackrel{!}{=} 6qbz_1 - qz_1^2 - 9qb^2$

$B \to C \quad 0 \le z_2 \le 5b$

$N(z_2) = 0$

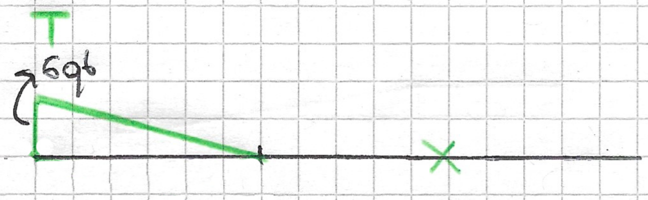
$T(z_2) = -2q(3b) + 6qb$

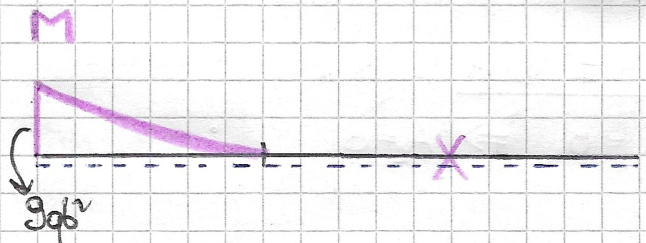
$T(z_2) \stackrel{!}{=} 0$

$M(z_2) = -9qb^2 - 2q(3b)\left(\frac{3}{2}b + z_2\right) + 6qb(3b+z_2)$

$\stackrel{!}{=} -9qb^2 - 9qb^2 - 6qbz_2 + 18qb^2 + 6qbz_2$

$M(z_2) \stackrel{!}{=} 0$





Eq. linea elastica

$A \to B \quad 0 \le z_1 \le 3b$

$v''(z_1) = -\frac{M}{EI}$

$v''(z_1) = -\frac{(6qbz_1 - qz_1^2 - 9qb^2)}{EI}$

$v'(z_1) = \left(-\cancel{6}^{3}qb\frac{z_1^2}{\cancel{2}} + q\frac{z_1^3}{3} + 9qb^2z_1\right)\left(\frac{1}{EI}\right) + A_1$

$= \left(-3qbz_1^2 + \frac{1}{3}qz_1^3 + 9qb^2z_1\right)\left(\frac{1}{EI}\right) + A_1$

$v(z_1) = \left(-\frac{3}{3}qbz^3 + \frac{1}{12}qz_1^4 + \frac{9}{2}qb^2z_1^2\right)\left(\frac{1}{EI}\right) + A_1z_1 + A_2$

$= \left(-qbz^3 + \frac{1}{12}qz_1^4 + \frac{9}{2}qb^2z_1^2\right)\left(\frac{1}{EI}\right) + A_1z_1 + A_2$

$B \to C \quad 0 \le z_2 \le 5b$

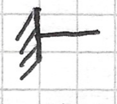
$v''(z_2) = -\frac{M}{EI}$

$v''(z_2) = 0$

$v'(z_2) = B_1$

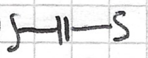
$v(z_2) = B_1z_2 + B_2$

COSTANTI $A_1; A_2; B_1; B_2$ $\Rightarrow$ CONDIZIONI AL CONTORNO

A) 

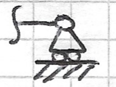
INCASTRO

IMPEDISCE
- ROTAZIONE $\Rightarrow$ $v'_{(z_1=A)}=0$
- SPOSTAMENTO $\Rightarrow$ $v_{(z_1=0)}=0$

B) 

PATTINO

IMPONE
- UGUALE ROTAZIONE in B_1 e B_2 $\Rightarrow$ $v'_{(z_1=B)}=v'_{(z_2=B)}$

C) 

CARRELLO

IMPEDISCE
- SPOSTAMENTO $\Rightarrow$ $v_{(z_2=C)}=0$

CONDIZIONE in A

- $v'_{(z_1=0)}=0 \Rightarrow A_1=0$
- $v_{(z_1=0)}=0 \Rightarrow A_2=0$

CONDIZIONE IN B

$$v'_{(z_1=3b)}=v'_{(z_2=3b)}$$

$$\left(-3qbz_1^2+\frac{1}{3}qz_1^3+9qb^2z_1\right)\left(\frac{1}{EI}\right)=B_1$$

$$\left(-\cancel{27qb^3}+9qb^3+\cancel{27qb^3}\right)\left(\frac{1}{EI}\right)=B_1$$

$$B_1=9\frac{qb^3}{EI}$$

CONDIZIONE IN C

$$v_{(z_2=5b)}=0$$

$$B_1z_2+B_2=0$$

$$B_2=-\frac{9qb^3}{EI}(5b)=-45\frac{qb^4}{EI}$$

DEFORMATA DELLA LINEA D'ASSE

- $v_{(z_1)}=\frac{1}{12}\frac{qz_1^4}{EI}-\frac{qbz_1^3}{EI}+\frac{9}{2}\frac{qb^2z_1^2}{EI}$
- $v_{(z_2)}=\frac{9qb^3z_2}{EI}-45\frac{qb^4}{EI}$

DERIVATA PRIMA

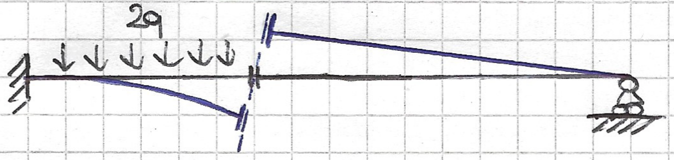
- $v'_{(z_1)}=\frac{1}{3}\frac{qz_1^3}{EI}+\frac{9qb^2z_1}{EI}-\frac{3qbz_1^2}{EI}$
- $v'_{(z_2)}=9\frac{qb^3}{EI}$

• ROTAZIONE DEL PUNTO B φ_B

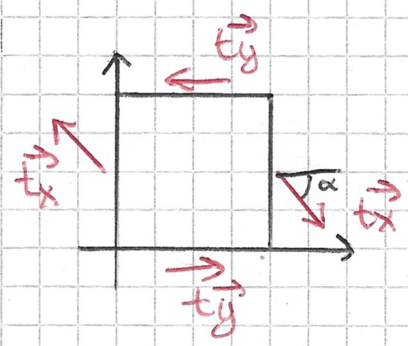
$$\varphi_B = \frac{9qb^3}{EI} \curvearrowright$$

• SPOSTAMENTO VERTICALE DEL PUNTO B RELATIVO AL CORPO 2; $v_B^{(2)}$

$$v_{(z_2=0)} = -45\frac{qb^4}{EI} \uparrow$$



Esercizio III _ Teoria I



$\alpha = -60°$ | $\operatorname{sen}\alpha = -\frac{\sqrt{3}}{2}$

$\cos\alpha = +\frac{1}{2}$

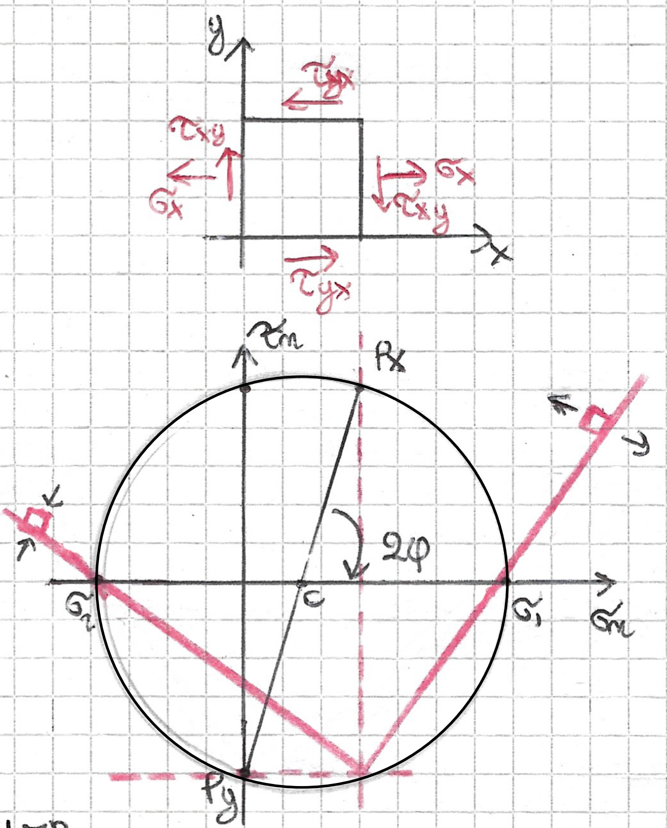
$|t_x| = 30\ MPa$

$\sigma_x = |\vec{t_x}| \cos\alpha$
$= 30 \cdot \frac{1}{2} = 15\ MPa$

$\tau_{xy} = |\vec{t_x}| \operatorname{sen}\alpha$
$= 30\left(-\frac{\sqrt{3}}{2}\right) = -15\sqrt{3}\ MPa \doteq -25{,}981\ MPa$

$\sigma_y = 0$

$$\underline{\underline{\sigma}} = \begin{pmatrix} 15 & -25{,}981 & 0 \\ -25{,}981 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Cerchio di Mohr.

$P_x : (\sigma_x ; \tau_{xy}) \Rightarrow P_x : (15 ; 25{,}981)$

$P_y : (\sigma_y ; \tau_{yx}) \Rightarrow P_y : (0 ; -25{,}981)$

$c = \left(\frac{\sigma_x + \sigma_y}{2} ; 0\right) \Rightarrow \left(\frac{15}{2} ; 0\right)$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7{,}5)^2 + (25{,}981)^2}$
$= \sqrt{(56{,}25) + 675{,}012} = \sqrt{731{,}262}$
$= 27{,}041 \qquad \tau_{MAX} = 27{,}041\ MPa$

$\sigma_1 = \sigma_c + R = 7{,}5 + 27{,}041 = 34{,}541\ MPa$

$\sigma_2 = \sigma_c - R \quad \sigma_2 = 7{,}5 - 27{,}041 \Rightarrow \sigma_2 = -19{,}541$

$\operatorname{sen} 2\varphi = \tau_{xy} \ ; \ \cos 2\varphi = \sigma_x - \sigma_c$

$\operatorname{tg} 2\varphi = \frac{\operatorname{sen} 2\varphi}{\cos 2\varphi} = \frac{-\tau_{xy}}{\sigma_x - \sigma_c} = \frac{-25{,}981}{7{,}5} = 3{,}46$

$2\varphi = \operatorname{arctg}(3{,}46) = 73{,}879 \Rightarrow \varphi = -36{,}95°$