1) $f(x,y) = x^3 - y^2 - 2xy$

$\dfrac{\partial f}{\partial x} = 3x^2 - 2y$ $\quad\quad$ $\dfrac{\partial f}{\partial y} = -2y - 2x$

$\begin{cases} 3x^2 - 2y = 0 \\ -2y - 2x = 0 \end{cases}$ $\quad$ $\begin{cases} 3x^2 - 2y = 0 \\ y = -x \end{cases}$ $\quad$ $\begin{cases} 3x^2 + 2x = 0 \\ y = -x \end{cases}$

$\begin{cases} x(3x+2) = 0 \\ y = -x \end{cases}$ $\quad$ $x = 0 \Rightarrow y = 0$; $\quad 3x + 2 = 0$, $\quad x = -\frac{2}{3} \Rightarrow y = \frac{2}{3}$

$P_1 = (0,0) \quad\quad P_2 = \left(-\frac{2}{3}, \frac{2}{3}\right)$

$\dfrac{\partial^2 f}{\partial x^2} = 6x \quad\quad \dfrac{\partial^2 f}{\partial x \partial y} = -2 \quad\quad \dfrac{\partial^2 f}{\partial y^2} = -2$

$Hf(0,0) = \begin{pmatrix} 6 \cdot 0 & -2 \\ -2 & -2 \end{pmatrix} = 0 - 4 = -4 < 0$ punto di sella

$Hf\left(-\frac{2}{3}, \frac{2}{3}\right) = \begin{pmatrix} 6 \cdot \left(-\frac{2}{3}\right) & -2 \\ -2 & -2 \end{pmatrix} = -4 \cdot (-2) - 4 = 8 - 4 > 0$

$f_{xx} < 0$ punto di massimo relativo

2) $\iint_D \frac{2xy}{\sqrt{x^2+y^2}}\,dx\,dy \quad D=\left\{(x,y)\in\mathbb{R}^2 \,\middle|\, x^2+y^2\le 1,\ x,y>0\right\}$

$$\begin{cases} x=\rho\cos\theta \\ y=\rho\,\mathrm{sen}\,\theta \end{cases} \quad \theta\in[0,\tfrac{\pi}{2}] \quad \rho\in[0,1]$$

$$\int_0^1\left(\int_0^{\frac{\pi}{2}} \frac{2\rho^2\cos\theta\,\mathrm{sen}\,\theta\cdot\rho}{\sqrt{\rho^2}}\,d\theta\right)d\rho=$$

$$=\int_0^1\left(\int_0^{\pi/2} 2\rho^2\cos\theta\,\mathrm{sen}\,\theta\,d\theta\right)d\rho=\left(\int_0^1\rho^2\,d\rho\right)\left(\int_0^{\pi/2}2\cos\theta\,\mathrm{sen}\,\theta\,d\theta\right)$$

$$=\left[\frac{\rho^3}{3}\right]_0^1\cdot\left[\mathrm{sen}^2\theta\right]_0^{\pi/2}=\frac{1}{3}\cdot(1-0)=\frac{1}{3}$$

3) $\lim_{(x,y)\to(0,0)}\frac{x+y}{\sqrt{x^2+y^2}}$

$x=0 \quad \lim_{y\to 0}\frac{y}{\sqrt{y^2}}=\lim_{y\to 0}\frac{y}{|y|}$ non esiste! (± 1)

4) $F(x,y)=(6x,-2)$

• irrotazionale: $\frac{\partial F_1}{\partial y}=0$, $\frac{\partial F_2}{\partial x}=0$ uguali $\Rightarrow$ OK

• irrotazionale: $\frac{\partial F_1}{\partial y} = 0$, $\frac{\partial F_2}{\partial x} = 0$ ...

• irrotazionale su $\mathbb{R}^2$ S.C. $\Rightarrow$ conservativo

funzioni potenziale:

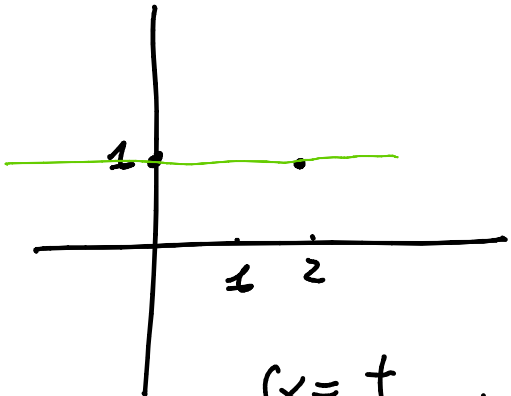
$$f(x,y) = \int F_1 \, dx = \int 6x \, dx = 3x^2 + \phi(y)$$

$$\frac{\partial f}{\partial y}(x,y) = 0 + \phi'(y) = F_2 = -2$$

$$\Rightarrow \phi'(y) = -2 \Rightarrow \phi(y) = -2y + K$$

$$\Rightarrow f(x,y) = 3x^2 - 2y + K.$$

• $(0,1) \rightarrow (2,1)$



$$\int_\gamma F \, ds = \int_0^2 (6t,-2)(1,0) \, dt$$

$$= \int_0^2 6t \, dt = \left[3t^2\right]_0^2 = 12 - 0 = 12.$$

$\begin{cases} x = t \\ y = 1 \end{cases}$ $t \in [0,2]$

oppure:

$$f(2,1) - f(0,1) = 3 \cdot 4 - 2 \cdot 1 + K - (0 - 2 + K)$$

$$= 12 - 2 + K + 2 - K = 12$$

5) $\Sigma = \{(x,y,z) \in \mathbb{R}^3 : z = 3x + \sqrt{6}y, \ x,y \in [0,1]\}$

$$\int_{\Sigma} x^2+z \, d\sigma = \int_0^1\int_0^1 \left(x^2+3x+\sqrt{6}y\right) 4 \, dx\,dy = \quad \boxed{\begin{aligned} d\sigma &= \sqrt{1+3^2+\sqrt{6}^2} = \\ &= \sqrt{1+9+6} = \sqrt{16} \\ &= 4 \end{aligned}}$$

$$= \int_0^1 \left(4\left[\frac{x^3}{3} + \frac{3x^2}{2} + \sqrt{6}xy \right]_0^1 \right) dy =$$

$$= \int_0^1 4\left(\frac{1}{3} + \frac{3}{2} + \sqrt{6}y \right) dy = \left[4\left(\frac{y}{3} + \frac{3}{2}y + \frac{\sqrt{6}}{2}y^2 \right) \right]_0^1 =$$

$$= 4\left(\frac{1}{3} + \frac{3}{2} + \frac{\sqrt{6}}{2} \right) = \frac{4}{3} + 6 + 2\sqrt{6} = \frac{4+18+6\sqrt{6}}{3} =$$

$$= \frac{22+6\sqrt{6}}{3}$$

6) $\begin{cases} y'' - 5y' + 6y = 6x^2+3 \\ y'(0) = 0 \\ y(0) = -\frac{2}{9} \end{cases}$

$\lambda^2 - 5\lambda + 6 = 0 \qquad (\lambda-3)(\lambda-2) = 0 \qquad \begin{aligned} \lambda &= 3, \\ \lambda &= 2 \end{aligned}$

$y_0 = K_1 e^{3x} + K_2 e^{2x}$

$y_s = ax^2 + bx + c \qquad y_s' = 2ax + b \qquad y_s'' = 2a$

$y_s'' - 5y_s' + 6y_s = 6x^2 + 3$

$2a - 5(2ax+b) + 6(ax^2+bx+c) = 6x^2+3$

$2a - 10ax - 5b + 6ax^2 + 6bx + 6c = 6x^2 + 3$

$$2a - 10ax - 5b + 6ax^2 + 6bx + 6c = 6x^2 + 3$$

$$6ax^2 + x(-10a + 6b) + 2a - 5b + 6c = 6x^2 + 3$$

$$\begin{cases} 6a = 6 \\ -10a + 6b = 0 \\ 2a - 5b + 6c = 3 \end{cases} \qquad \begin{cases} a = 1 \\ 6b = 10 \\ 2 - 5b + 6c = 3 \end{cases} \qquad \begin{cases} a = 1 \\ b = \frac{5}{3} \\ 6c = 1 + 5b \end{cases}$$

$$\begin{cases} a = 1 \\ b = \frac{5}{3} \\ 6c = 1 + \frac{25}{3} \end{cases} \qquad 6c = \frac{28}{3} \qquad c = \frac{14}{9}$$

$$y = y_0 + y_s = k_1 e^{3x} + k_2 e^{2x} + x^2 + \frac{5}{3}x + \frac{14}{9}$$

$$y' = 3k_1 e^{3x} + 2k_2 e^{2x} + 2x + \frac{5}{3}$$

$$y'(0) = 1 \Rightarrow \quad 3k_1 + 2k_2 + \frac{5}{3} = \frac{5}{3}$$

$$y(0) = k_1 + k_2 + \frac{14}{9} = -\frac{2}{9}$$

$$\begin{cases} 3k_1 + 2k_2 + \frac{5}{3} = \frac{5}{3} \\ k_1 + k_2 + \frac{14}{9} = -\frac{2}{9} \end{cases} \qquad \begin{cases} 3k_1 + 2k_2 = 0 \\ k_1 + k_2 = -\frac{16}{9} \end{cases} \qquad \begin{cases} k_2 = -\frac{3}{2}k_1 \\ k_1 - \frac{3}{2}k_1 = -\frac{16}{9} \end{cases}$$

$$\begin{cases} k_2 = -\frac{3}{2}k_1 \\ -\frac{1}{2}k_1 = -\frac{16}{9} \end{cases} \qquad \begin{cases} k_2 = -\frac{3}{2} \cdot \frac{32}{9} = -\frac{16}{3} \\ k_1 = \frac{32}{9} \end{cases}$$

$$\bar{y} = \frac{32}{9} e^{3x} - \frac{16}{3} e^{2x} + x^2 + \frac{5}{3}x - \frac{11}{9}$$