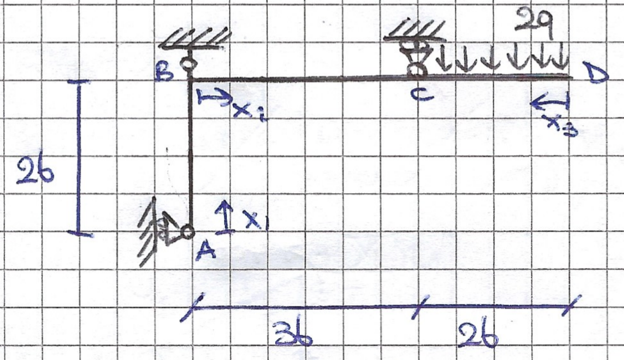
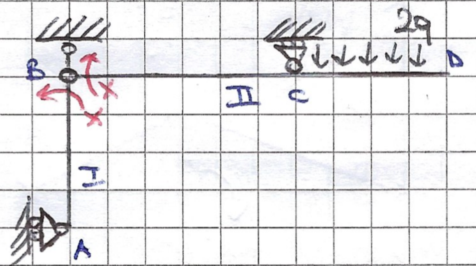
Esame Scienza delle Costruzioni – 01/07/2024



STRUTTURA IPERSTATICA

$GDL = 3$
$GDV = 1(A) + 2(B) + 1(C) = 4$
$GDL < GDV$

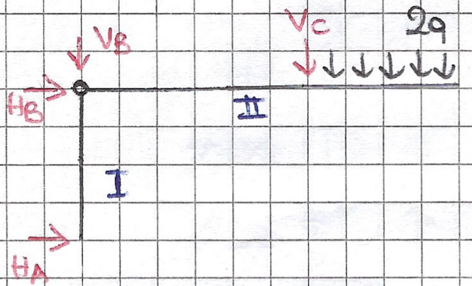
eq. di congruenza $\Delta\varphi(B) = 0$



STRUTTURA ISOSTATICA

$GDL = 6$
$GDV = 1(A) + 4(B) + 1(C) = 6$
$GDL < GDV$

S_0 – Sistema reale



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(B)} = 0 \end{cases} \quad \begin{cases} H_B + H_A = 0 \Rightarrow H_B = 0 \\ V_B + V_C + 2q(2b) = 0 \; [1] \\ H_A(2b) - V_C(3b) - 2q(2b)(4b) = 0 \; [2] \end{cases}$$

eq. aux

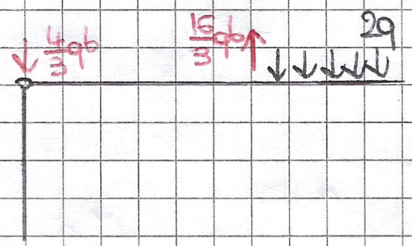
$$\left\{ M^{I}_{z(B)} = 0 \right. \quad \left\{ H_A 2b = 0 \Rightarrow H_A = 0 \right.$$

[2] $3bV_C + 16qb^2 = 0$

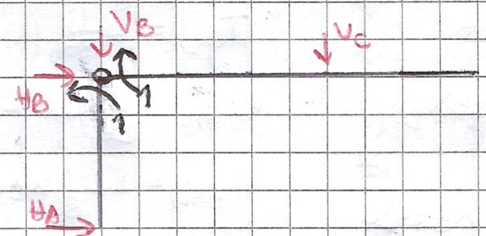
$$V_C = -\frac{16}{3}qb$$

[1] $V_B - \frac{16}{3}qb + 4qb = 0$

$$V_B = \frac{4}{3}qb$$



S_1 – Sistema equilibrato



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(B)} = 0 \end{cases} \quad \begin{cases} H_A + H_B = 0 \Rightarrow H_B = +\frac{1}{2b} \\ V_B + V_C = 0 \; [1] \\ H_A(2b) - V_C(3b) = 0 \; [2] \end{cases}$$

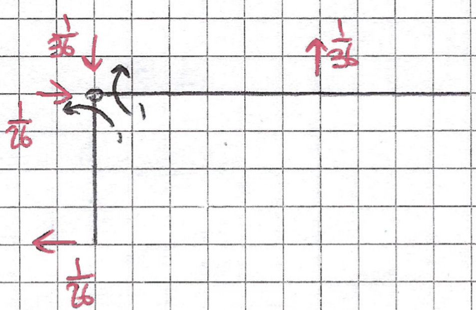
eq. aux.

$$\left\{ M^{I}_{z(B)} = 0 \right. \quad \left\{ 1 + H_A 2b = 0 \Rightarrow H_A = -\frac{1}{2b} \right.$$

[2] $-\frac{1}{2b}(2b) - V_C(3b) = 0$

$$V_C = -\frac{1}{3b}$$

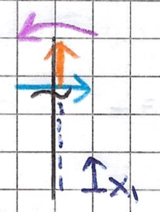
[1] $V_B = +\frac{1}{3b}$



Azioni Interne

Sistema S_0

$A \to B \quad 0 \le x_1 \le 2b$

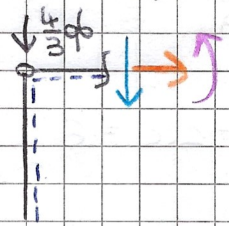


$N_1(x_1) = 0$

$T_1(x_1) = 0$

$M(x_1) = 0$

$B \to C \quad 0 \le x_1 \le 3b$

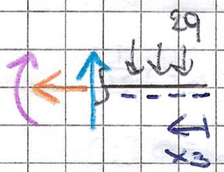


$N(x_2) = 0$

$T(x_2) = -\frac{4}{3}qb$

$M(x_2) = -\frac{4}{3}qb\,x_2$

$D \to C \quad 0 \le x_3 \le 2b$



$N(x_3) = 0$

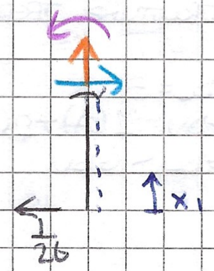
$T(x_3) = 2q\,x_3$

$M(x_3) = -\cancel{2}q\frac{x_3^2}{\cancel{2}}$

$M(x_3) = -q\,x_3^2$

Sistema S_1

$A \to B \quad 0 \le x_1 \le 2b$

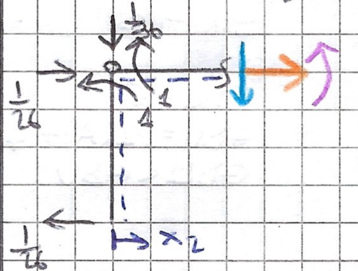


$N(x_1) = 0$

$T(x_1) = \frac{1}{2b}$

$M(x_1) = \frac{1}{2b}x_1$

$B \to C \quad 0 \le x_2 \le 3b$



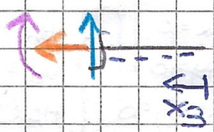
$N(x_2) = 0$

$T(x_2) = -\frac{1}{3b}$

$M(x_2) = \frac{1}{2b}(2b) - \frac{1}{3b}x_2$

$M(x_2) = 1 - \frac{1}{3b}x_2$

$D \to C \quad 0 \le x_3 \le 2b$

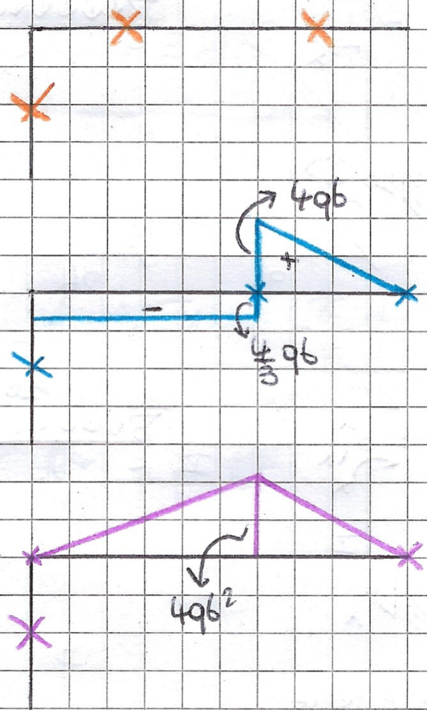


$N(x_3) = 0$

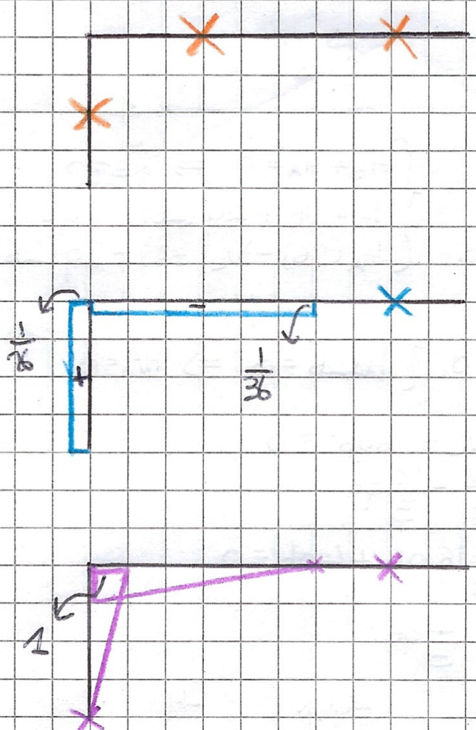
$T(x_3) = 0$

$M(x_3) = 0$

Sistema 0



Sistema 1



P.L.V.

$$\mathcal{L}V_e = \mathcal{L}V_i$$

$$\mathcal{L}V_e = 1\cdot\Delta\varphi_B = 0$$

$$\mathcal{L}V_i = \int_s \cancel{N_1}\,\varepsilon_i + \int_s \cancel{T_1}\,\gamma_i + \int_s M_1\chi_i = 0$$

$$\mathcal{L}V_i = \int_s M_1\left(\frac{M_0 + X M_1}{EI}\right)$$

A → B

$M_0 = 0$

$M_1 = \frac{1}{2b}x_1$

$M_1M_0 = 0$

$M_1^2 = \frac{x_1^2}{4b^2}$

B → C

$M_0 = -\frac{4}{3}qb\,x_2$

$M_1 = 1-\frac{1}{3b}x_2$

$M_1M_0 = -\frac{4}{3}qbx_2+\frac{4}{9}qx_2^2$

$M_1^2 = 1+\frac{1}{9b^2}x_2^2-\frac{2}{3b}x_2$

C → D

$M_0 = -qx_3^2$

$M_1 = 0$

$M_1M_0 = 0$

$M_1^2 = 0$

$$\mathcal{L}V_i = \int_0^{2b}\frac{x_1^2}{4b^2}\frac{X}{EI}dx_1 + \int_0^{3b}\left(-\frac{4}{3}qbx_2+\frac{4}{9}qx_2^2\right)\frac{1}{EI}dx_2 + \int_0^{3b}\left(1+\frac{x_2^2}{9b^2}-\frac{2}{3b}x_2\right)\frac{X}{EI}dx_2$$

$$= \frac{1}{EI}\left[X\frac{x_1^3}{12b^2}\right]_0^{2b} + \frac{1}{EI}\left[-\frac{4}{6}qbx_2^2+\frac{4}{27}qx_2^3\right]_0^{3b} + \frac{1}{EI}\left[X\left(x_2+\frac{x_2^3}{27b^2}-\frac{2x_2^2}{6b}\right)\right]_0^{3b}$$

$$= \frac{1}{EI}\left(X\cdot\frac{\overset{2}{\cancel{8}}b^{\cancel{3}}}{3\cancel{12}\cancel{b^2}} + -\frac{\overset{6}{\cancel{12}}}{\cancel{2}}qb^3 + 4qb^3 + \cancel{X3b} + bX - \cancel{3bX}\right)$$

$$= \frac{1}{EI}\left(\frac{2b}{3}X - 6qb^3 + 4qb^3 + bX\right)$$

$$\mathcal{L}V_i = \frac{1}{EI}\left(\frac{5}{3}bX - 2qb^3\right) \Rightarrow \mathcal{L}V_i = 0 \Rightarrow \frac{5}{3}bX = 2qb^3 \Rightarrow X = \frac{6}{5}qb^2$$

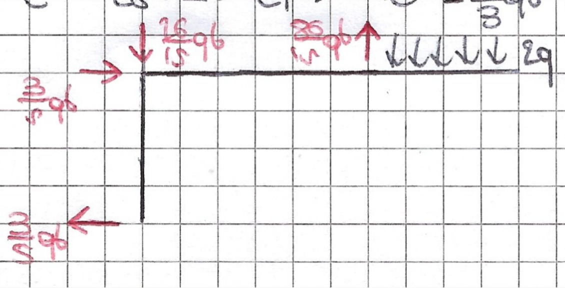
Reazioni Vincolari.

$H_A = H_{A0} + XH_{A1}$ $\quad H_A = \frac{\overset{3}{\cancel{6}}}{5}qb^{\cancel{2}}\left(-\frac{1}{\cancel{2b}}\right)$ $\quad H_A = -\frac{3}{5}qb$

$H_B = H_{B0} + XH_{B1} \Rightarrow H_B = \frac{\cancel{6}}{5}qb^{\cancel{2}}\left(\frac{1}{\cancel{2b}}\right) \Rightarrow H_B = \frac{3}{5}qb$

$V_B = V_{B0} + XV_{B1} \Rightarrow V_B = \frac{4}{3}qb + \frac{\overset{2}{\cancel{6}}}{5}qb^{\cancel{2}}\left(\frac{1}{\cancel{3b}}\right) \Rightarrow V_B = \frac{26}{15}qb$

$V_C = V_{C0} + XV_{C1} \Rightarrow V_C = -\frac{16}{3}qb + \frac{\overset{2}{\cancel{6}}}{5}qb^{\cancel{2}}\left(-\frac{1}{\cancel{3b}}\right) \Rightarrow V_C = -\frac{86}{15}qb$



Azioni Interne

A → B

$N(x_1) = N_0(x_1) + X N_1(x_1) \Rightarrow N(x_1) = 0$

$T(x_1) = T_0(x_1) + X T_1(x_1) \Rightarrow T(x_1) = \frac{1}{\cancel{1{,}96}}\left(\frac{\cancel{8}^{3}}{5} qb^{\cancel{2}}\right) \Rightarrow T(x_1) = \frac{3}{5} qb$

$M(x_1) = M_0(x_1) + X M_1(x_1) \Rightarrow M(x_1) = \frac{1}{\cancel{1{,}96}} x_1 \left(\frac{\cancel{8}^{3}}{5} qb^{\cancel{2}}\right) \Rightarrow M(x_1) = \frac{3}{5} qb x_1$

B → C

$N(x_2) = 0$

$T(x_2) = -\frac{4}{3} qb + \frac{\cancel{8}^{2}}{5} qb^{\cancel{2}} \left(-\frac{1}{\cancel{1{,}36}}\right) \Rightarrow T(x_2) = -\frac{4}{3} qb - \frac{2}{5} qb \Rightarrow T(x_2) = -\frac{26}{15} qb$

$M(x_2) = -\frac{4}{3} qb x_2 + \frac{6}{5} qb^2 \left(1 - \frac{1}{3b} x_2\right) \Rightarrow M(x_2) = -\frac{4}{3} qb x_2 + \frac{6}{5} qb^2 - \frac{2}{5} qb^2 x_2$

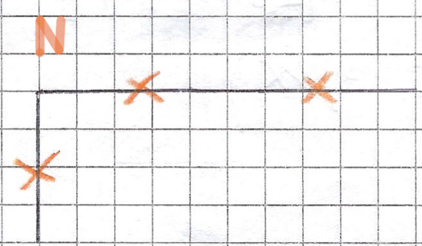
$M(x_2) = -\frac{26}{15} qb x_2 + \frac{6}{5} qb^2$

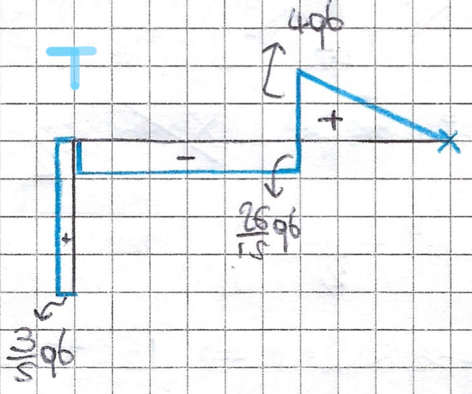
C → D

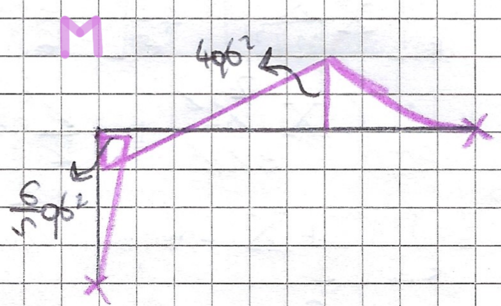
$N(x_3) = 0$

$T(x_3) = 2q x_3$

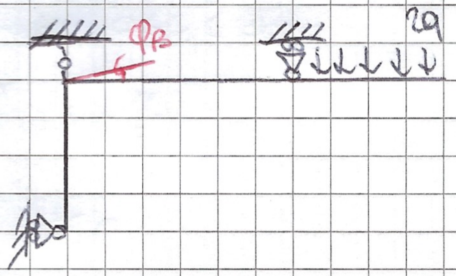
$M(x_3) = -q x_3^2$



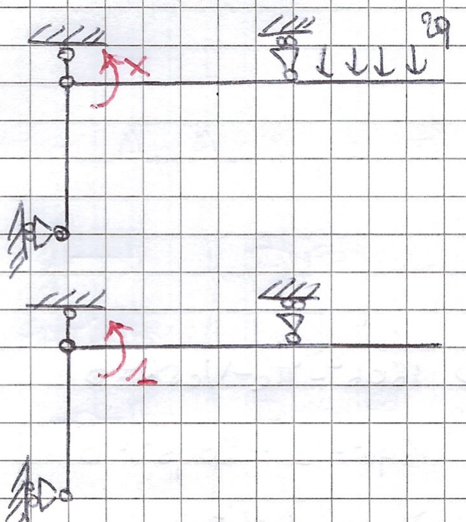




Calcolo rotazione del punto B, φ_B



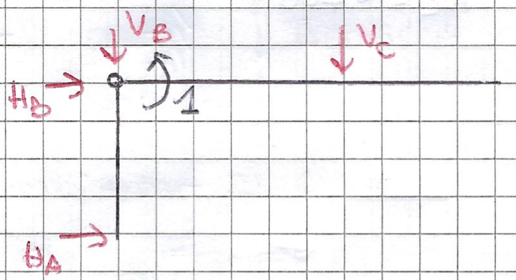
S_2

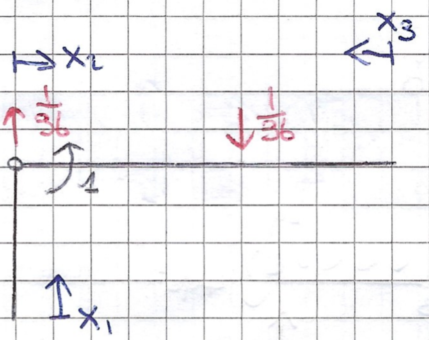


$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(B)} = 0 \end{cases} \quad \begin{cases} H_B + H_A = 0 \Rightarrow H_B = 0 \\ V_B + V_C = 0 \Rightarrow V_B = -\frac{1}{3b} \\ 1 - V_C(3b) + H_A(2b) = 0 \Rightarrow V_C = \frac{1}{3b} \end{cases}$$

eq. aux

$$\left\{ M_{z(B)}^{I} = 0 \right. \quad \left\{ H_A(2b) = 0 \Rightarrow H_A = 0 \right.$$





Azioni interne (momento)

$M(x_1) = 0$

$M(x_2) = -1 + \frac{1}{3b} x_2$

$M(x_3) = 0$

PLV

$$L_{Ve} = L_{Vi} \; ; \; L_{Ve} = 1\varphi_B \; ; \; L_{Vi} = \int_0^{3b} \left(-1 + \frac{1}{3b}x_2\right)\left(-\frac{26}{15}qbx_2 + \frac{6}{5}qb^2\right)\left(\frac{1}{EI}\right) dx_2$$

$$\varphi_B = \int_0^{3b} \left(\frac{26}{15}qbx_2 - \frac{6}{5}qb^2 - \frac{26}{45}qx_2^2 + \frac{2}{5}x_2qb\right)\left(\frac{1}{EI}\right) dx_2$$

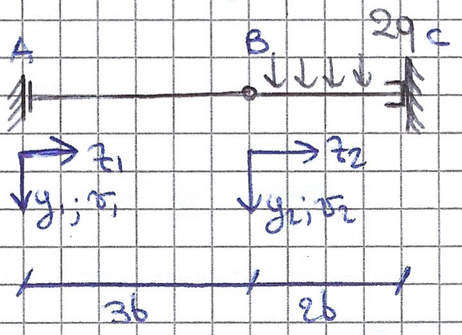
$$= \frac{1}{EI}\left[\frac{\cancel{32}^{16}}{15}qb \cdot \frac{x_2^2}{\cancel{2}_1} - \frac{6}{5}qb^2x_2 - \frac{26}{45}q\frac{x_2^3}{3}\right]_0^{3b}$$

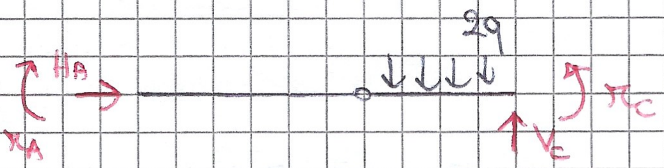
$$= \frac{1}{EI}\left(\frac{16}{\cancel{15}_5}qb \cdot \cancel{9}^3 b^2 - \frac{6}{5}qb^2 \cdot 3b - \frac{26}{\cancel{45}_5}q \cdot \cancel{27}^{1} b^3\right)$$

$$= \frac{1}{EI}\left(\frac{48}{5}qb^3 - \frac{18}{5}qb^3 - \frac{26}{5}qb^3\right)$$

$$\varphi_B = \frac{4}{5}\frac{qb^3}{EI}$$

Esercizio II _ Travia 1





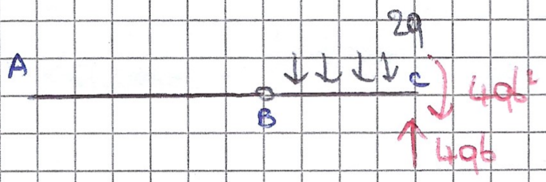
$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(A)} = 0 \end{cases} \qquad \begin{cases} H_A = 0 \\ V_c - 2q(2b) = 0 \Rightarrow V_c = 4qb \\ M_A + 2q(2b)(4b) - M_c - V_c 5b = 0 \Rightarrow 16qb^2 - M_c - V_c 5b = 0 \end{cases}$$

$$16qb^2 - M_c - 20qb^2 = 0$$

$$M_c = -4qb^2$$

eq. aux

$$\left\{ M^{I}_{z(B)} = 0 \right. \qquad \left\{ M_A = 0 \right.$$



Azioni interne

A → B $\quad 0 \le z_1 \le 3b$

$N(z_1) = 0$

$T(z_1) = 0$

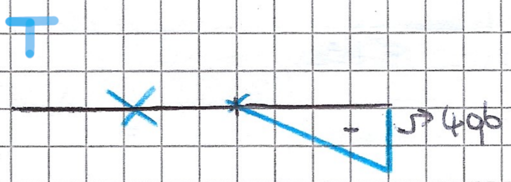
$M(z_1) = 0$

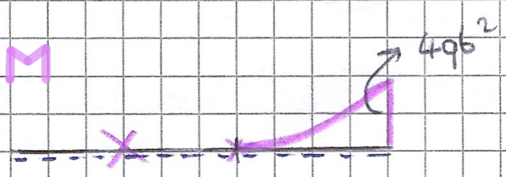
B → C $\quad 0 \le z_2 \le 2b$

$N(z_2) = 0$

$T(z_2) = -2q z_2$

$M(z_2) = -\cancel{2}q \cdot \frac{z_2^2}{\cancel{2}} \Rightarrow M_{z_2} = -q z_2^2$





Eq. linea elastica

$A \to B \quad 0 \le z_1 \le 3b$

$v''(z_1) = -\dfrac{M}{EI}$

$v''(z_1) = 0$

$v'(z_1) = A_1$

$v(z_1) = A_1 z_1 + A_2$

$B \to C \quad 0 \le z_2 \le 2b$

$v''(z_2) = -\dfrac{M}{EI}$

$v''(z_2) = +\dfrac{q z_2^2}{EI}$

$v'(z_2) = \dfrac{q z_2^3}{3EI} + B_1$

$v(z_2) = \dfrac{q z_2^4}{12EI} + B_1 z_2 + B_2$

Costanti A_1; A_2; B_1; B_2 ⇒ Condizioni a contorno

A) 

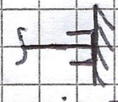
Pattino

{ Impedisce • Rotazione ⇒ $v'(z_1 = A) = 0$

B) 

Cerniera

{ Impone • uguale abbassamento in B_1 e B_2 ⇒ $v(z_1 = B) = v(z_2 = B)$

C) 

Manicotto

{ Impedisce
- Spostamento ⇒ $v(z_2 = C) = 0$
- Rotazione ⇒ $v'(z_2 = C) = 0$

Condizione in A

$v'(z_1 = 0) = 0 \Rightarrow A_1 = 0$

Condizione in B

$v(z_1 = 3b) = v(z_2 = 0)$

$A_2 = B_2 \Rightarrow A_2 = +4\dfrac{qb^4}{EI}$

Condizione in C

$v'(z_2 = 2b) = 0 \Rightarrow \dfrac{8}{3}\dfrac{qb^3}{EI} + B_1 = 0 \Rightarrow B_1 = -\dfrac{8}{3}\dfrac{qb^3}{EI}$

$v(z_2 = 2b) = 0 \Rightarrow \dfrac{4}{3}\dfrac{qb^4}{EI} - \dfrac{16}{3}\dfrac{qb^4}{EI} + B_2 = 0 \Rightarrow B_2 = +4\dfrac{qb^4}{EI}$

Deformata della linea d'asse

- $v(z_1) = +4\dfrac{qb^4}{EI}$
- $v(z_2) = \dfrac{1}{12}\dfrac{q z_2^4}{EI} - \dfrac{8}{3}\dfrac{qb^3 z_2}{EI} + 4\dfrac{qb^4}{EI}$

Derivata prima

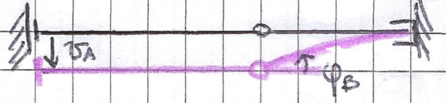
- $v'(z_1) = 0$
- $v'(z_2) = \dfrac{q}{3}\dfrac{z_2^3}{EI} - \dfrac{8}{3}\dfrac{qb^3}{EI}$

• ROTAZIONE DEL PUNTO B, $\varphi_B^{(2)}$

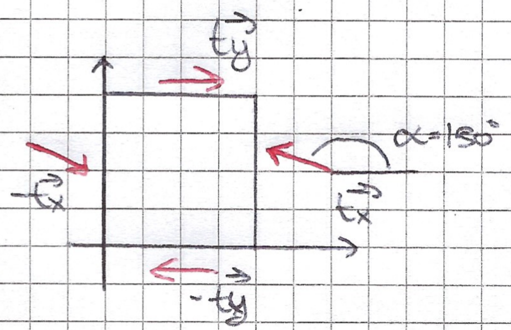
$$v'_{(z_2=0)} = -\frac{8qb^3}{3EI} \qquad \varphi_B = -\frac{8qb^3}{3EI}$$

• SPOSTAMENTO VERTICALE DEL PUNTO A, v_A

$$v_{(z_1=0)} = +\frac{4qb^4}{EI}$$



ESERCIZIO III



$$\alpha = 150° \begin{cases} \operatorname{sen}\alpha = \frac{1}{2} \\ \cos\alpha = -\frac{\sqrt{3}}{2} \end{cases}$$

$$|t_x| = 55 \text{ MPa}$$

$$\sigma_x = |t_x|\cos\alpha = 55\left(-\frac{\sqrt{3}}{2}\right) = -\frac{55\sqrt{3}}{2} \text{ MPa} \cong -47{,}631 \text{ MPa}$$

$$\sigma_y = 0$$

$$\tau_{xy} = |t_x|\operatorname{sen}\alpha = 55\left(\frac{1}{2}\right) = 27{,}5 \text{ MPa}$$

$$\underline{\underline{\sigma}} = \begin{pmatrix} -55\sqrt{3}/2 & 55/2 & 0 \\ 55/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

CERCHIO DI MOHR

$$P_x: (-\sigma_x ; \tau_{xy}) \Rightarrow P_x: (-47{,}631 ; -27{,}500)$$
$$P_y: (\sigma_y ; \tau_{yx}) \Rightarrow P_y: (0 ; 27{,}500)$$

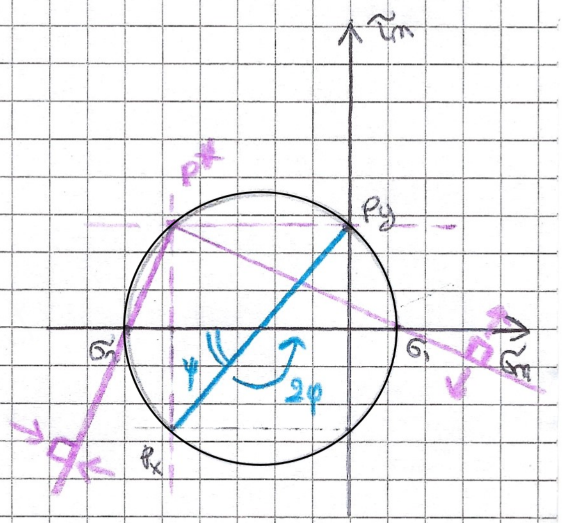
$$C = \left(\frac{\sigma_x + \sigma_y}{2} ; 0\right) \Rightarrow C: (-23{,}815 ; 0)$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{567{,}154 + 756{,}25} = \sqrt{1.323{,}404}$$

$$R = 36{,}378 \Rightarrow \tau_{MAX} = 36{,}378 \text{ MPa}$$

$$\sigma_1 = \sigma_c + R \Rightarrow \sigma_1 = -23{,}815 + 36{,}378 = 12{,}563$$

$$\sigma_2 = \sigma_c - R \Rightarrow \sigma_2 = -23{,}815 - 36{,}378 = -60{,}193$$



$$2\varphi = 180° - \psi$$

$$tg\psi = \frac{+\tau_{xy}}{\sigma_x - \sigma_c} = \frac{+27{,}500}{-47{,}631 + 23{,}815} = \frac{27{,}500}{-23{,}815} = -1{,}154$$

$$\operatorname{arctg}(-1{,}154) = -49{,}089°$$

$$2\varphi = 180° - 49{,}089° = 130{,}911°$$

$$\varphi = 65{,}45°$$