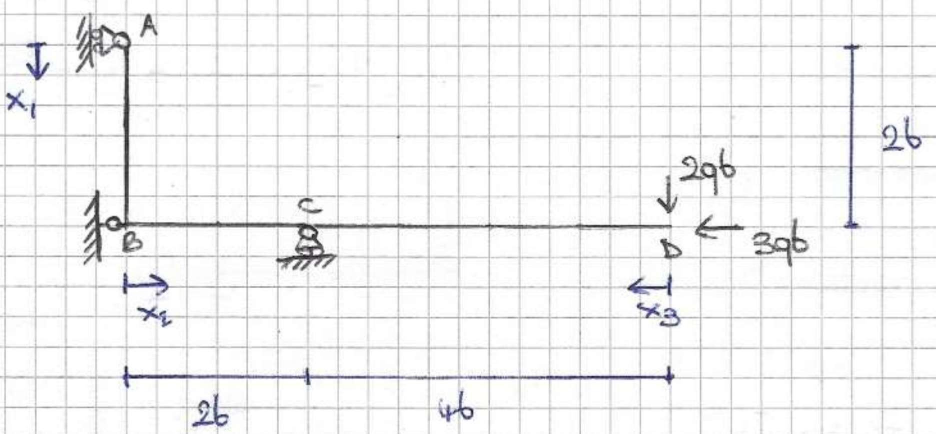
TRACCIA I _ ESERCIZIO I _ ESAME 22.03.24

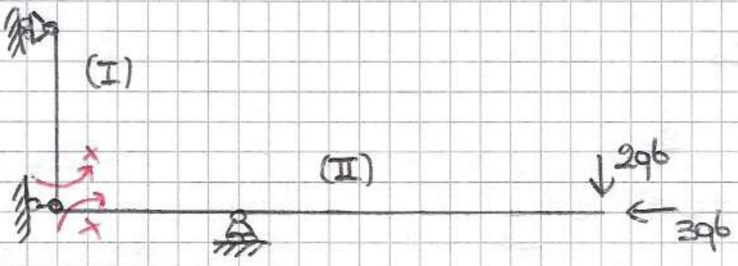


STRUTTURA IPERSTATICA

GDL = 3

GDV = 1(A) + 2(B) + 1(C) = 4

GDL < GDV

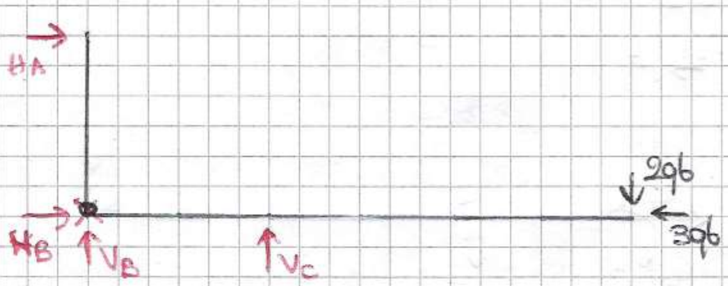


STRUTTURA ISOSTATICA

GDL = 3(I) + 3(II) = 6

GDV = 1(A) + 4(B) + 1(C) = 6

So _ SISTEMA REALE



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \mathcal{M}_{z(B)} = 0 \end{cases} \qquad \begin{cases} H_A + H_B - 3qb = 0 & [1] \\ V_B + V_C - 2qb = 0 & [2] \\ H_A 2b - V_C 2b + 2qb(6b) = 0 & [3] \end{cases}$$

eq. aux

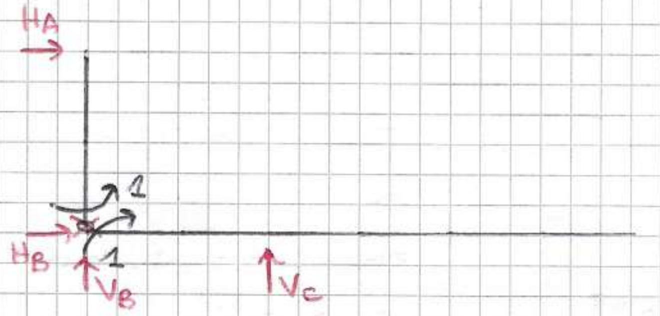
$$\left\{ \mathcal{M}^{(I)}_{z(B)} = 0 \right. \qquad \left\{ H_A 2b = 0 \Rightarrow H_A = 0 \right.$$

[1] $\cancel{H_A} + H_B - 3qb = 0 \Rightarrow H_B = 3qb$

[3] $\cancel{H_A 2b} - V_C 2b + 12qb^2 = 0 \Rightarrow V_C = 6qb$

[2] $V_B + 6qb - 2qb = 0 \Rightarrow V_B = -4qb$

S_1 - SISTEMA EQUILIBRATO



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(B)} = 0 \end{cases} \quad \begin{cases} H_A + H_B = 0 \quad [1] \\ V_B + V_C = 0 \quad [2] \\ H_A 2b - V_C 2b - 1 + 1 = 0 \quad [3] \end{cases}$$

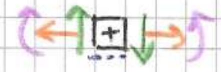
eq. aux

$$\left\{ M_{z(B)}^{(I)} = 0 \right. \quad \left\{ H_A 2b - 1 = 0 \Rightarrow H_A = \frac{1}{2b} \right.$$

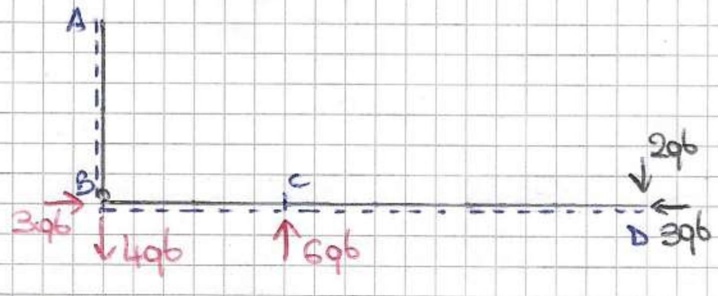
$[1]\ H_A + H_B = 0 \Rightarrow \frac{1}{2b} + H_B = 0 \Rightarrow H_B = -\frac{1}{2b}$

$[3]\ \frac{1}{\cancel{2b}}(\cancel{2b}) - V_C 2b = 0 \Rightarrow -V_C 2b = -1 \Rightarrow V_C = \frac{1}{2b}$

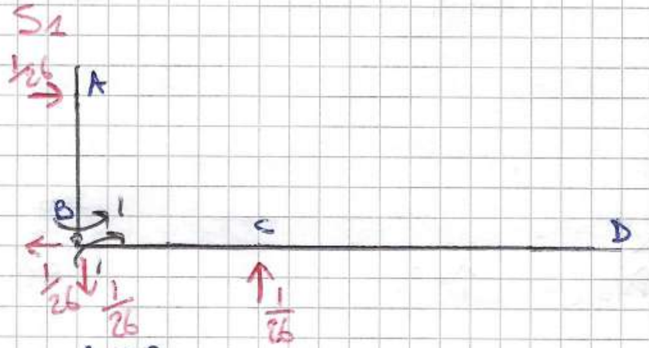
$[2]\ V_B = -V_C \Rightarrow V_B = -\frac{1}{2b}$



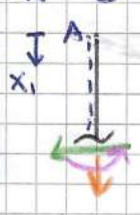
S_0



S_1



A → B

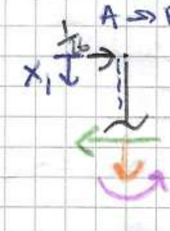


$N(x_1) = 0$

$T(x_1) = 0$

$M(x_1) = 0$

A → B

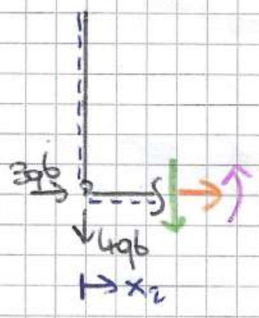


$N(x_1) = 0$

$T(x_1) = \frac{1}{2b}$

$M(x_1) = \frac{1}{2b}(x_1)$

B → C



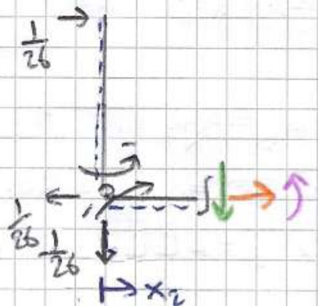
$N(x_2) = -3qb$

$T(x_2) = -4qb$

$M(x_2) + 4qb(x_2) = 0$

$\hookrightarrow M(x_2) = -4qbx_2$

B → C



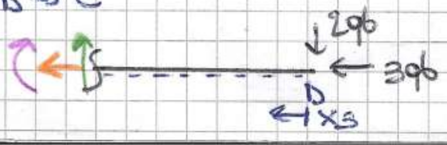
$N(x_2) = 0$

$T(x_2) = -\frac{1}{2b}$

$M(x_2) = \frac{1}{\cancel{2b}}(\cancel{2b}) - \frac{1}{2b}(x_2)$

$\hookrightarrow M(x_2) = 1 - \frac{x_2}{2b}$

D → C



$N(x_3) = -3qb$

$T(x_3) = 2qb$

$M(x_3) = -2qb(x_3)$

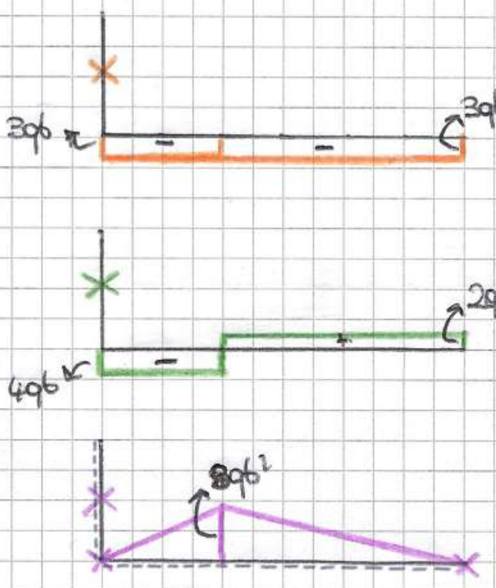
D → C

$N(x_3) = 0$

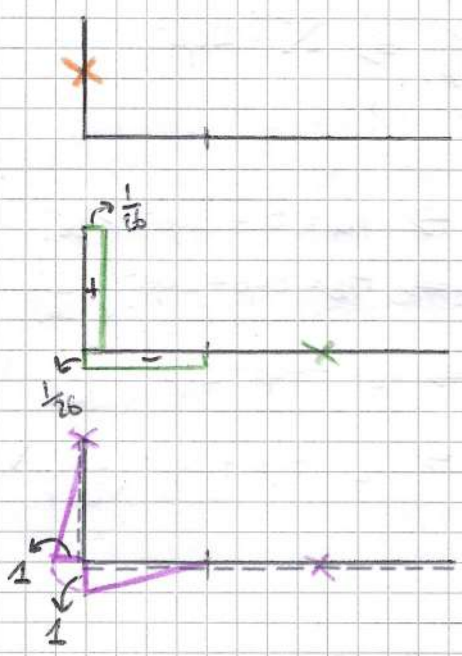
$T(x_3) = 0$

$M(x_3) = 0$

S0



S1



P.L.V

$$\delta V_e = \delta V_i$$

$$\delta V_e = 1 \cdot \Delta\varphi_B \Rightarrow \delta V_e = 0$$

$$\delta V_i = \int_s \cancel{N_1}\,\varepsilon_i + \int_s \cancel{T_1}\,\gamma_1 + \int_s M_1 \chi_i \Rightarrow \delta V_i = \int_s M_1 \left(\frac{M_0 + X M_1}{EI}\right)$$

A → B	B → C	C → D
$M_0 = 0$	$M_0 = -4qbx_2$	$M_0 = -2qbx_3$
$M_1 = \frac{x_1}{2b}$	$M_1 = 1 - \frac{x_2}{2b}$	$M_1 = 0$
$M_1 \cdot M_0 = 0$	$M_1 \cdot M_0 = -4qbx_2 + 2qx_2^2$	$M_1 \cdot M_0 = 0$
$M_1^2 = \frac{x_1^2}{4b^2}$	$M_1^2 = 1 + \frac{x_2^2}{4b^2} - \frac{x_2}{b}$	$M_1^2 = 0$

$$\delta V_i = \frac{1}{EI}\int_0^{2b} X\left(\frac{x_1^2}{4b^2}\right)dx_1 + \frac{1}{EI}\int_0^{2b}\left(-4qbx_2 + 2qx_2^2\right)dx_2 + \frac{1}{EI}\int_0^{2b} X\left(1 + \frac{x_2^2}{4b^2} - \frac{x_2}{b}\right)dx_2$$

$$= \frac{1}{EI}\left[X\left(\frac{x_1^3}{3}\right)\left(\frac{1}{4b^2}\right)\right]_0^{2b} + \frac{1}{EI}\left[-4qb\frac{x_2^2}{2} + 2q\frac{x_2^3}{3}\right]_0^{2b} + \frac{1}{EI}\left[X\left(x_2 + \frac{x_2^3}{12b^2} - \frac{x_2^2}{2b}\right)\right]_0^{2b}$$

$$= \frac{1}{EI}\cdot X\left(\frac{8b^3}{3}\right)\left(\frac{1}{4b^2}\right) + \frac{1}{EI}\left(-2qb(4b^2) + \frac{2}{3}q(8b^3)\right) + \frac{1}{EI}\cdot X\left(2b + \frac{8b^3}{12b^2} - \frac{4b^2}{2b}\right)$$

$$= \frac{1}{EI}X\left(\frac{2}{3}b\right) + \frac{1}{EI}\left(-8qb^3 + \frac{16}{3}qb^3\right) + \frac{1}{EI}\cdot X\left(\frac{2}{3}b\right)$$

$$= \frac{1}{EI}X\left[\frac{2}{3}b + \frac{2b}{3}\right] + \frac{1}{EI}\left(-\frac{8}{3}qb^3\right)$$

$$\delta V_i = \frac{4}{3}bX\left(\frac{1}{EI}\right) + \frac{8}{3}qb^3\left(\frac{1}{EI}\right) \Rightarrow \delta V_i = 0 \Rightarrow \frac{4}{3}bX\left(\cancel{\frac{1}{EI}}\right) - \frac{8}{3}qb^3\left(\cancel{\frac{1}{EI}}\right) = 0$$

$\frac{4}{3}b\bar{X} - \frac{8}{3}qb^3 = 0$

$\bar{X} = \frac{\cancel{8}^{2}}{\cancel{3}}qb^{\cancel{3}^{2}}\left(\frac{\cancel{3}}{\cancel{4b}_{1}}\right) \Rightarrow \bar{X} = 2qb^2$

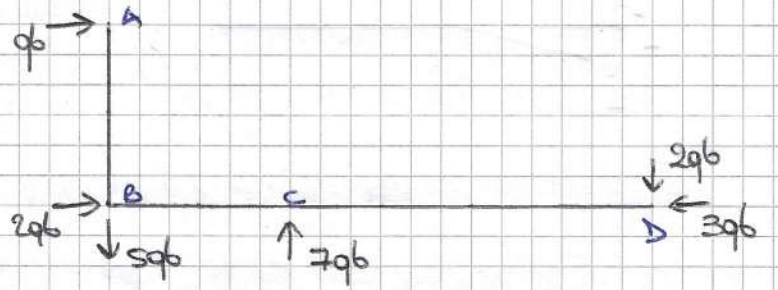
Reazioni Vincolari

$H_A = H_{A_0} + \bar{X}H_{A_1} \Rightarrow H_A = 0 + 2qb^2\left(\frac{1}{2b}\right) \quad H_A = qb$

$H_B = H_{B_0} + \bar{X}H_{B_1} \Rightarrow H_B = 3qb + 2qb^2\left(-\frac{1}{2b}\right) \quad H_B = 2qb$

$V_B = V_{B_0} + \bar{X}V_{B_1} \Rightarrow V_B = -4qb + 2qb^2\left(-\frac{1}{2b}\right) \quad V_B = -5qb$

$V_C = V_{C_0} + \bar{X}V_{C_1} \Rightarrow V_C = 6qb + 2qb^2\left(\frac{1}{2b}\right) \quad V_C = 7qb$



Azioni Interne

A → B

$N_{(x_1)} = N_{0(x_1)} + \bar{X}N_{1(x_1)}$

$N_{(x_1)} = 0$

$T_{(x_1)} = T_{0(x_1)} + \bar{X}T_{1(x_1)}$

$T_{(x_1)} = 2qb^2\left(\frac{1}{2b}\right) = qb$

$M_{(x_1)} = M_{0(x_1)} + \bar{X}M_{1(x_1)}$

$M_{(x_1)} = 2qb^2\left(\frac{x_1}{2b}\right) \Rightarrow M_{(x_1)} = qx_1$

B → C

$N_{(x_2)} = -3qb + 2qb^2(0)$

$N_{(x_2)} = -3qb$

$T_{(x_2)} = -4qb + 2qb^2\left(-\frac{1}{2b}\right)$

$T_{(x_2)} = -5qb$

$M_{(x_2)} = -4qbx_2 + 2qb^2\left(1 - \frac{x_2}{2b}\right)$

$M_{(x_2)} = -5qbx_2 + 2qb^2$

C → D

$N_{(x_3)} = -3qb + 2qb^2(0)$

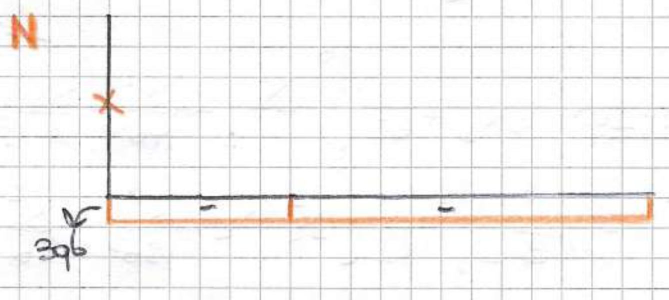
$N_{(x_3)} = -3qb$

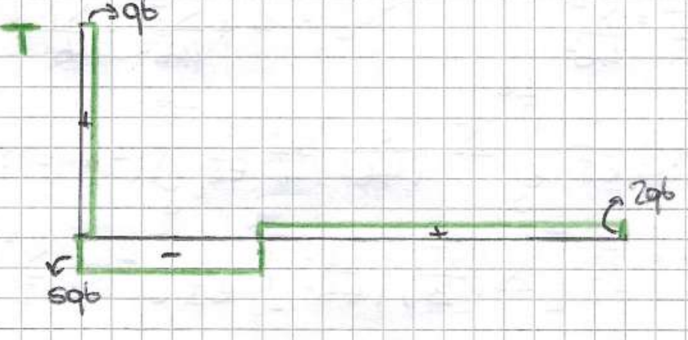
$T_{(x_3)} = 2qb + 2qb^2(0)$

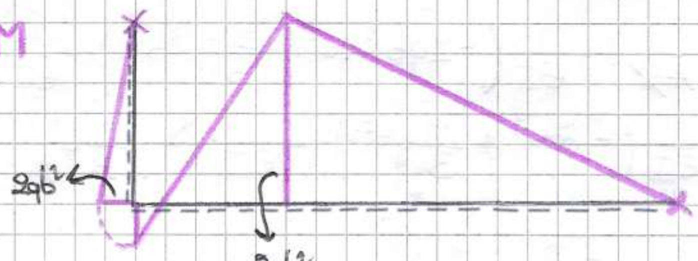
$T_{(x_3)} = 2qb$

$M_{(x_3)} = -2qbx_3 + 2qb^2(0)$

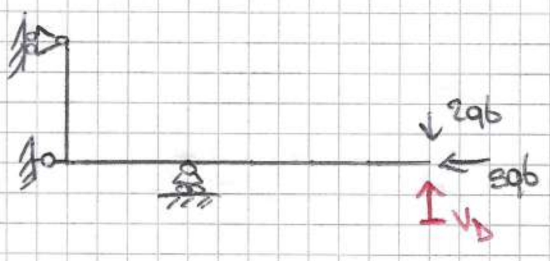
$M_{(x_3)} = -2qbx_3$

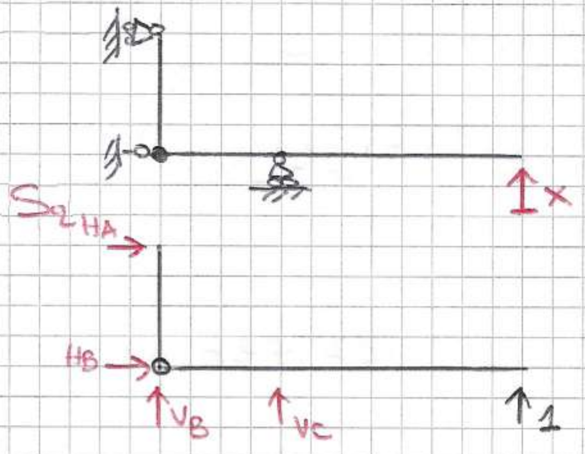






COMPONENTE DI SPOSTAMENTO VERTICALE DEL PUNTO D, V_D





$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{A(B)} = 0 \end{cases} \quad \begin{cases} H_A + H_B = 0 \Rightarrow H_B = 0 \\ V_B + V_C + 1 = 0 \Rightarrow V_B = 2 \\ H_A(2b) - V_C(2b) - 1(6b) = 0 \Rightarrow V_C = -3 \end{cases}$$

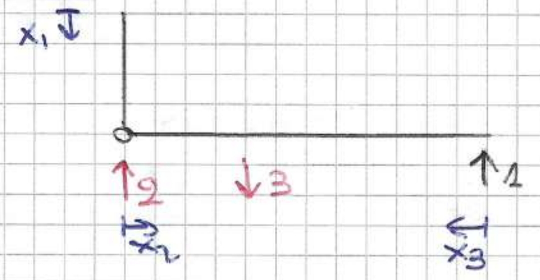
$$\begin{cases} M^{(I)}_{A(B)} = 0 \end{cases} \quad \begin{cases} H_A = 0 \end{cases}$$

Azioni Interne (Momento)

A→B $M(x_1) = 0$

B→C $M(x_2) = 2(x_2)$

D→C $M(x_3) = x_3$



P.L.V

$\mathcal{L}V_e = \mathcal{L}V_i$

$\mathcal{L}V_e = 1 \cdot v_D = v_D$

$\mathcal{L}V_i = \int_S M_q \chi_i \, dx$

$$= \int_0^{2b} 2x_2 \left(-5qbx_2 + 2qb^2\right)\left(\frac{1}{EI}\right) dx_2 + \int_0^{4b} x_3 \left(-2qbx_3\right)\left(\frac{1}{EI}\right) dx_3$$

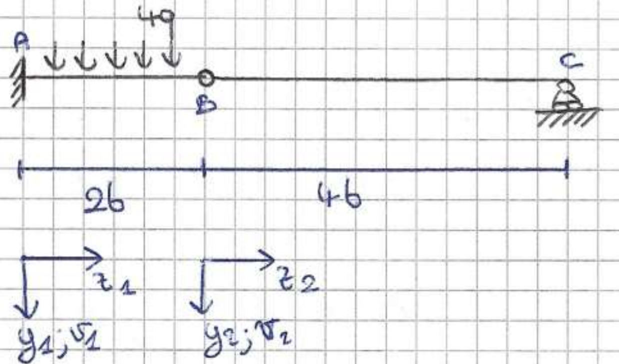
$$= \frac{1}{EI}\left[-10qb\frac{x_2^3}{3} + 4qb^2\frac{x_2^2}{2}\right]_0^{2b} + \frac{1}{EI}\left[-2qb\frac{x_3^3}{3}\right]_0^{4b}$$

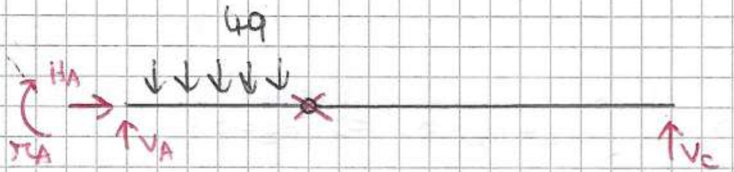
$$= \frac{1}{EI}\left[-10qb\left(\frac{8b^3}{3}\right) + 4qb^2\left(\frac{4b^2}{2}\right) - 2qb\left(\frac{64b^3}{3}\right)\right]$$

$$= \frac{1}{EI}\left(-\frac{80}{3}qb^4 + 8qb^4 - \frac{128}{3}qb^4\right)$$

$$= \frac{1}{EI}\left(\frac{-80+24-128}{3}qb^4\right) = -\frac{184}{3}\frac{qb^4}{EI} \quad (\downarrow)$$

TRACCIA I _ ESERCIZIO II _ ESAME 22.03.24

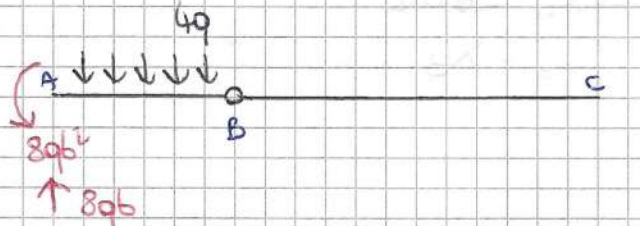




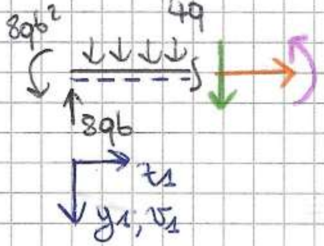
$$\begin{cases} R_x=0 \\ R_y=0 \\ M_{z(B)}=0 \end{cases} \quad \begin{cases} \underline{H_A=0} \\ V_A+V_C-4q(2b)=0 \Rightarrow \underline{V_A=8qb} \\ M_A+V_A(2b)-4q(2b)(b)-V_C(4b)=0 \Rightarrow M_A=-16qb^2+8qb^2 \Rightarrow \underline{M_A=-8qb^2} \end{cases}$$

eq. aux

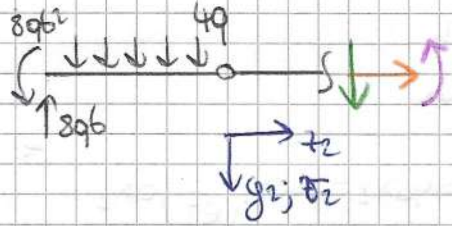
$$\left\{ M^{II}_{z(B)}=0 \right. \quad \left\{ 4bV_C=0 \Rightarrow \underline{V_C=0} \right.$$



A ⇒ B



B ⇒ C



$N_{z_1}=0$

$T_{z_1}+4q(z_1)-8qb=0 \Rightarrow T_{z_1}=-4qz_1+8qb$

$M_{z_1}+8qb^2-8qb(z_1)+4q(z_1)\left(\frac{z_1}{2}\right)=0$

$M_{z_1}=8qbz_1-2qz_1^2-8qb^2$

$N_{z_2}=0$

$T_{z_2}-8qb+4q(2b)=0 \Rightarrow T_{z_2}=0$

$M_{z_2}-8qb(2b+z_2)+8qb^2+4q(2b)(b+z_2)=0$

$M_{z_2}-\cancel{16qb^2}-\cancel{8qbz_2}+\cancel{8qb^2}+\cancel{8qb^2}+\cancel{8qbz_2}=0$

$M_{z_2}=0$

Eq. LINEA ELASTICA

$A \to B \quad 0 \le z_1 \le 2b$

$$v''_{1(z_1)} = -\frac{M}{EI} \Rightarrow v''_{1(z_1)} = \frac{1}{EI}\left(-8qbz_1 + 2qz_1^2 + 8qb^2\right)$$

$$v'_{1(z_1)} = \frac{1}{EI}\left(-\cancel{8}^{4}qb \cdot \frac{z_1^2}{\cancel{2}} + 2q\frac{z_1^3}{3} + 8qb^2 z_1\right) + A_1$$

$$= \frac{1}{EI}\left(-4qbz_1^2 + \frac{2}{3}qz_1^3 + 8qb^2 z_1\right) + A_1$$

$$v_{1(z_1)} = \frac{1}{EI}\left(-4qb\frac{z_1^3}{3} + \frac{2}{3}q\frac{z_1^4}{4} + 8qb^2\frac{z_1^2}{2}\right) + A_1 z_1 + A_2$$

$$= \frac{1}{EI}\left(-\frac{4}{3}qbz_1^3 + \frac{1}{6}qz_1^4 + 4qb^2 z_1^2\right) + A_1 z_1 + A_2$$

$B \to C \quad 0 \le z_1 \le 4b$

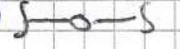
$$v''_{2(z_2)} = -\frac{M}{EI} \Rightarrow v''_{2(z_2)} = 0$$

$$v'_{2(z_2)} = B_1$$

$$v_{2(z_2)} = B_1 z_2 + B_2$$

COSTANTI $A_1; A_2; B_1; B_2 \Rightarrow$ CONDIZIONI A CONTORNO

A)  INCASTRO — IMPEDISCE: SPOSTAMENTO $\Rightarrow v_{z_1}(A) = 0$; ROTAZIONE $\Rightarrow v'_{(z_1)}(A) = 0$

B)  CERNIERA — IMPONE: UGUALE ABBASSAMENTO IN B_1 E B_2 $\Rightarrow v_{z_1}(B) = v_{z_2}(B)$

C)  CARRELLO — IMPEDISCE: SPOSTAMENTO $\Rightarrow v_{z_2}(C) = 0$

IN A CON $z_1 = 0$

$v'_{(z_1)}(A) = 0 \Rightarrow A_1 = 0$

$v_{z_1}(A) = 0 \Rightarrow A_2 = 0$

IN B CON $z_1 = 2b$ E $z_2 = 0$

$$v_{z_1}(B) = v_{z_2}(B)$$

$$\frac{1}{EI}\left(-\frac{4}{3}qbz_1^3 + \frac{1}{6}qz_1^4 + 4qb^2z_1^2\right) = B_2$$

$$B_2 = \frac{1}{EI}\left[-\frac{4}{3}qb(8b^3) + \frac{1}{\cancel{6}_3}q(\cancel{16}^{8}b^4) + 4qb^2(4b^2)\right]$$

$$B_2 = \frac{1}{EI}\left(-\frac{32}{3}qb^4 + \frac{8}{3}qb^4 + 16qb^4\right)$$

$$B_2 = \frac{8qb^4}{EI}$$

IN C CON $z_2 = 4b$

$v_{z_2}(C) = 0$

$$v_{2(z_2=4b)} = B_1(4b) + \frac{8qb^4}{EI} \Rightarrow 4bB_1 = -\frac{8qb^4}{EI} \Rightarrow B_1 = -\frac{2qb^3}{EI}$$

DEFORMATA DELLA LINEA D'ASSE

$$v_{1(z_1)} = \frac{1}{EI}\left(-\frac{4}{3}qbz_1^3 + \frac{1}{6}qz_1^4 + 4qb^2z_1^2\right)$$

$$v_2(z_2) = \frac{1}{EI}\left(-2qb^3z_2 + 8qb^4\right)$$

DERIVATA PRIMA

$$v'_{1(z_1)} = \frac{1}{EI}\left(-4qbz_1^2 + \frac{2}{3}qz_1^3 + 8qb^2z_1\right)$$

$$v'_{2(z_2)} = -\frac{2qb^3}{EI}$$

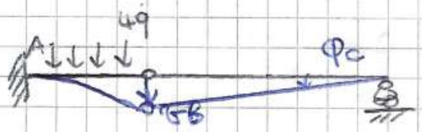
ROTAZIONE DEL PUNTO C; φ_C

$$\varphi_C = -\frac{2qb^3}{EI}$$

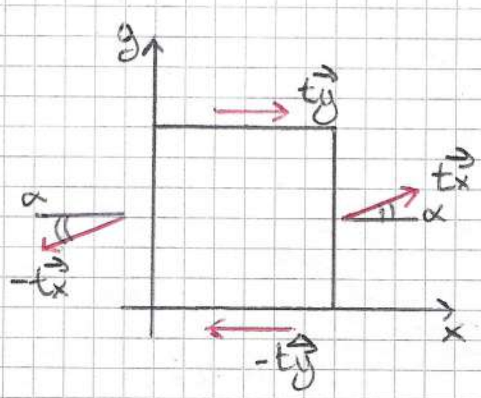
SPOSTAMENTO VERTICALE DEL PUNTO B; v_B

$$v_B = v_2(z_2=0)$$

$$v_B = \frac{8qb^4}{EI} \downarrow$$



TRACCIA I – ESERCIZIO III



$\alpha = 30°$

$|t_x| = 50\ MPa$

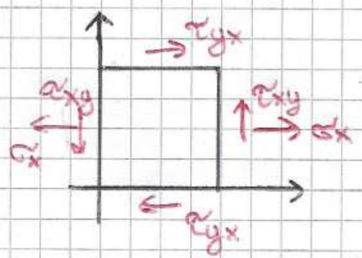
$\sigma_x = |t_x| \cos\alpha = 50 \cdot \frac{\sqrt{3}}{2}\ MPa = 25\sqrt{3}\ MPa \cong 43{,}301\ MPa$

$\sigma_y = 0$

$\tau_{xy} = |t_x| \operatorname{sen}\alpha = 50 \cdot \frac{1}{2}\ MPa = 25\ MPa$

$\tau_{xy} = \tau_{yx} = 25\ MPa$

$$\underline{\underline{\sigma}} = \begin{pmatrix} 25\sqrt{3} & 25 & 0 \\ 25 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



CERCHIO DI MOHR – PIANO (σ_n; τ_n)

$P_x: (\sigma_x ; -\tau_{xy}) \Rightarrow \tau_{xy}$ ↻

$P_x = (43{,}301 ; -25)$

$P_y = (\sigma_y ; \tau_{yx}) \Rightarrow \tau_{yx}$ ↺

$P_y = (0 ; 25)$

$C = \left(\frac{\sigma_x + \sigma_y}{2} ; 0\right) = \left(\frac{25\sqrt{3}}{2} ; 0\right) = (21{,}650 ; 0)$

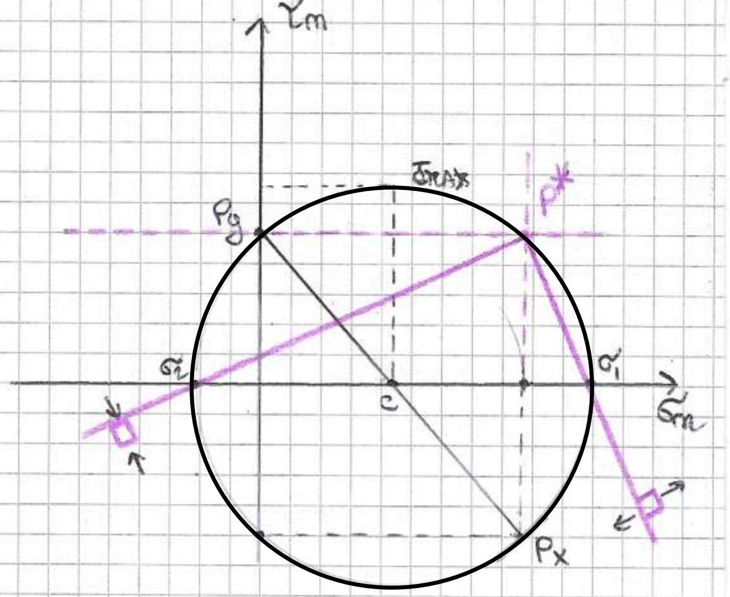
$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(21{,}650)^2 + (25)^2}$

$R = \sqrt{468{,}722 + 625} = 33{,}072$

$\tau_{max} = 33{,}072\ MPa$

$\sigma_1 = \sigma_c + R = 21{,}650 + 33{,}072 = 54{,}722\ MPa$

$\sigma_2 = \sigma_c - R = 21{,}650 - 33{,}072 = -11{,}422\ MPa$



$\operatorname{sen} 2\varphi = \tau_{xy}$; $\cos 2\varphi = \sigma_x - \sigma_c$

$tg 2\varphi = \frac{\operatorname{sen} 2\varphi}{\cos 2\varphi} = \frac{\tau_{xy}}{\sigma_x - \sigma_c} = \frac{25}{43{,}301 - 21{,}650}$

$tg 2\varphi = 1{,}154$

$2\varphi = arctg(1{,}154) = 48{,}867 \Rightarrow \varphi = 24{,}55°$