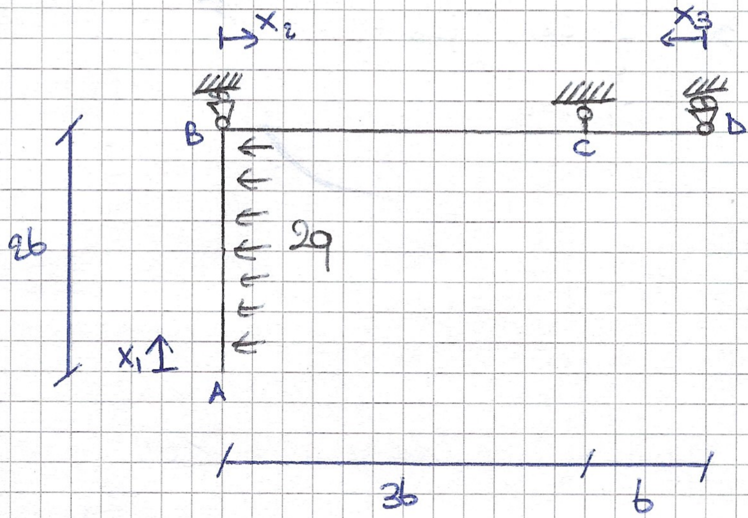
TRACCIA I _ esercizio I _ Esame 17.06.2024



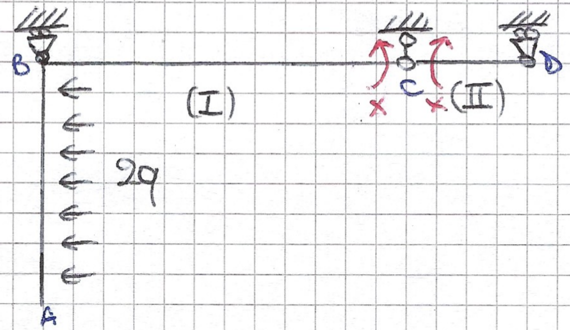
STRUTTURA IPERSTATICA

GDL = 3

GDV = 1(A) + 2(C) + 1(D) = 4

GDL < GDV

eq di congruenza $\Delta\varphi(C) = 0$



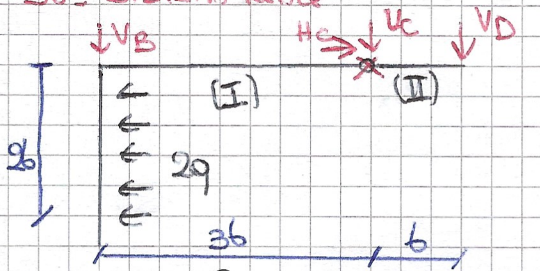
STRUTTURA ISOSTATICA

GDL = 6

GDV = 1(B) + 4(C) + 1(D) = 6

GDL = GDV

S_0 _ Sistema reale



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(C)} = 0 \end{cases} \quad \begin{cases} 2q(2b) - H_C = 0 \; [1] \\ V_B + V_C + V_D = 0 \; [2] \\ 2q(2b)(b) - V_B(3b) + V_D(b) = 0 \; [3] \end{cases}$$

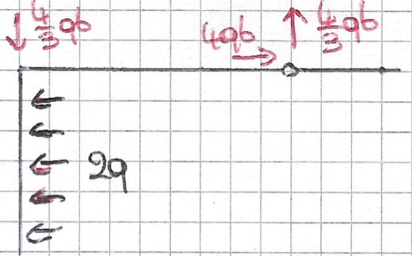
eq aux

$$\left\{ M^{II}_{z(C)} = 0 \right. \quad \left\{ V_D b = 0 \Rightarrow V_D = 0 \right.$$

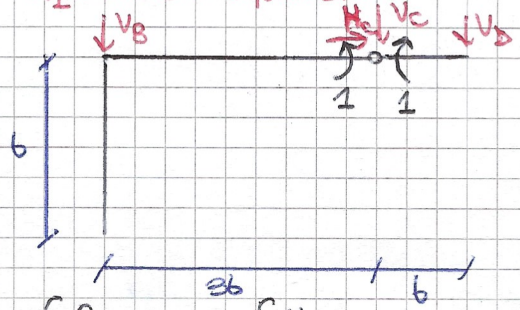
$[3] \quad -V_B 3b = -4qb^2 \Rightarrow V_B = \frac{4}{3}qb$

$[2] \quad \frac{4}{3}qb + V_C = 0 \Rightarrow V_C = -\frac{4}{3}qb$

$[1] \quad H_C = 4qb$



S_1 _ Sistema equilibrato



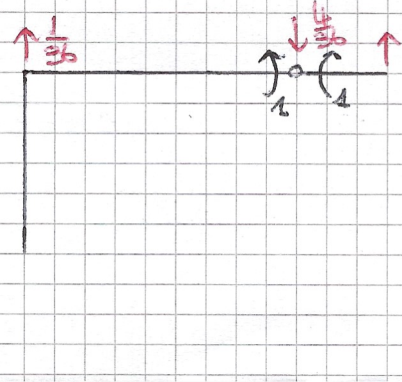
$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(C)} = 0 \end{cases} \quad \begin{cases} H_C = 0 \\ V_B + V_C + V_D = 0 \; [1] \\ V_D b - V_B(3b) = 0 \; [2] \end{cases}$$

$$\left\{ M^{II}_{z(C)} = 0 \right. \quad \left\{ V_D b + 1 = 0 \; [3] \right.$$

$[3] \quad V_D = -\frac{1}{b}$

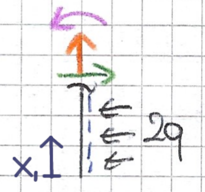
$[2] \quad -1 - 3b V_B = 0 \Rightarrow V_B = -\frac{1}{3b}$

$[1] \quad -\frac{1}{3b} + V_C - \frac{1}{b} = 0 \Rightarrow V_C = +\frac{4}{3b}$



Azioni Interne _ S_0

$A \rightarrow B \quad 0 \le x_1 \le 2b$

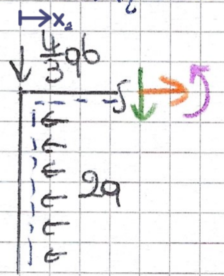


$N_{(x_1)} = 0$

$T_{(x_1)} = 2q x_1$

$M_{(x_1)} = 2q \frac{x_1^2}{2} \Rightarrow M_{(x_1)} = q x_1^2$

$B \rightarrow C \quad 0 \le x_2 \le 3b$



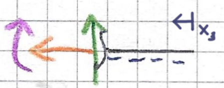
$N_{(x_2)} = 2q(2b) \Rightarrow N_{(x_2)} = 4qb$

$T_{(x_2)} = -\frac{4}{3}qb$

$M_{(x_2)} = 2q(2b^2) - \frac{4}{3}qb x_2$

$M_{(x_2)} = 4qb^2 - \frac{4}{3}qb x_2$

$D \rightarrow C \quad 0 \le x_3 \le b$



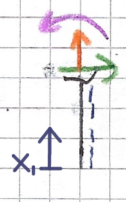
$N_{(x_3)} = 0$

$T_{(x_3)} = 0$

$M_{(x_3)} = 0$

Azioni Interne _ S_1

$A \rightarrow B \quad 0 \le x_1 \le 2b$

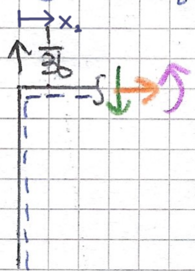


$N_{(x_1)} = 0$

$T_{(x_1)} = 0$

$M_{(x_1)} = 0$

$B \rightarrow C \quad 0 \le x_2 \le 3b$

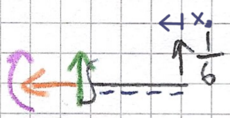


$N_{(x_2)} = 0$

$T_{(x_2)} = \frac{1}{3b}$

$M_{(x_2)} = \frac{1}{3b} x_2$

$D \rightarrow C \quad 0 \le x_3 \le b$

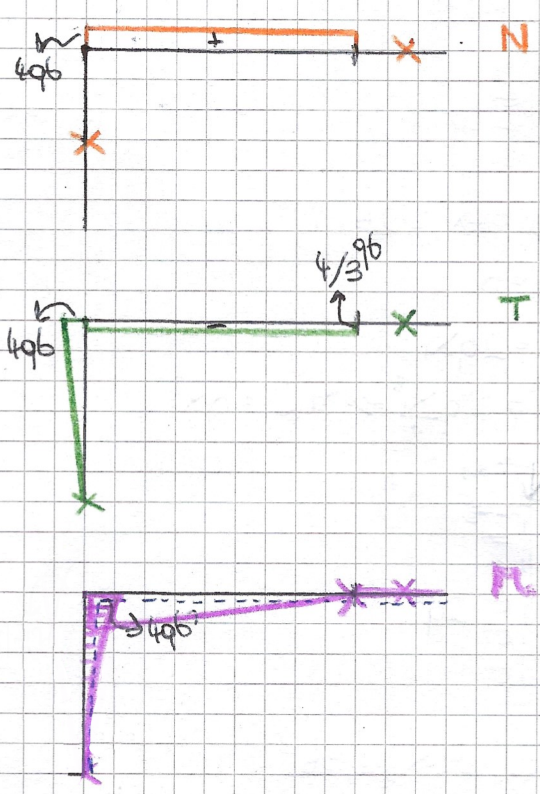


$N_{(x_3)} = 0$

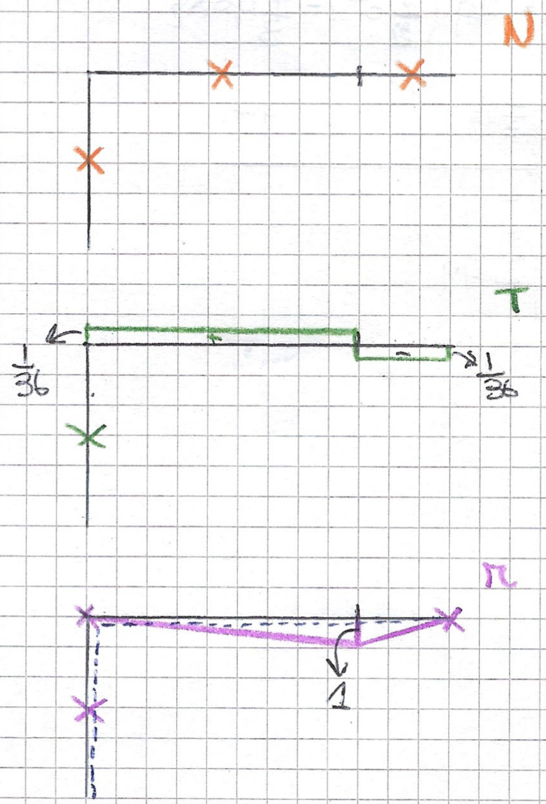
$T_{(x_3)} = -\frac{1}{b}$

$M_{(x_3)} = \frac{1}{b} x_3$

S_0



S_1



PLV

$\mathcal{L}V_e = \mathcal{L}V_i$

- $\mathcal{L}V_e = 1 \cdot \Delta\varphi_c \Rightarrow \mathcal{L}V_e = 0$
- $\mathcal{L}V_i = \int_s \cancel{N_1 \varepsilon_i} + \int_s \cancel{T_1 \gamma_i} + \int_s M_1 \chi_i = 0 \Rightarrow \mathcal{L}V_i = \int_s M_1 \left(\frac{M_0 + X M_1}{EI}\right)$

$A \rightarrow B$	$B \rightarrow C$	$C \rightarrow D$
$M_0 = qx_1^2$	$M_0 = 4qb^2 - \frac{4}{3}qbx_2$	$M_0 = 0$
$M_1 = 0$	$M_1 = \frac{1}{3b}x_2$	$M_1 = \frac{1}{b}x_3$
$M_1M_0 = 0$	$M_1M_0 = \frac{4}{3}qbx_2 - \frac{4}{9}qx_2^2$	$M_0M_1 = 0$
$M_1^2 = 0$	$M_1^2 = \frac{x_2^2}{9b^2}$	$M_1^2 = \frac{x_3^2}{b^2}$

$$\mathcal{L}V_i = \int_0^{3b} \frac{x_2^2}{9b^2}\left(\frac{X}{EI}\right) + \left(\frac{4}{3}qbx_2 - \frac{4}{9}qx_2^2\right)\left(\frac{1}{EI}\right)dx_2 + \int_0^{b} \frac{x_3^2}{b^2}\left(\frac{X}{EI}\right)dx_3$$

$$= \frac{1}{EI}\left[\frac{x_2^3(X)}{27b^2} + \frac{4}{3}qb\frac{x_2^2}{2} - \frac{4}{9}q\frac{x_2^3}{3}\right]_0^{3b} + \frac{1}{EI}\left[\frac{x_3^3}{3b^2} \cdot X\right]_0^{b}$$

$$= \frac{1}{EI}\left(\frac{27b^3}{27b^2} \cdot X + \frac{4}{3}qb \cdot \frac{9b^2}{2} - \frac{4}{9}q\frac{27b^3}{3}\right) + \frac{1}{EI}\left(\frac{b^3}{3b^2}X\right)$$

$$= \frac{1}{EI}\left(bX + 6qb^3 - 4qb^3\right) + \frac{1}{EI}\left(\frac{1}{3}bX\right)$$

$$\mathcal{L}V_i = \frac{1}{EI}\left(\frac{4}{3}bX + 2qb^3\right) \Rightarrow \mathcal{L}V_i = 0 \Rightarrow \frac{1}{EI}\left(\frac{4}{3}bX + 2qb^3\right) = 0 \Rightarrow X = -\frac{3}{2}qb^2$$

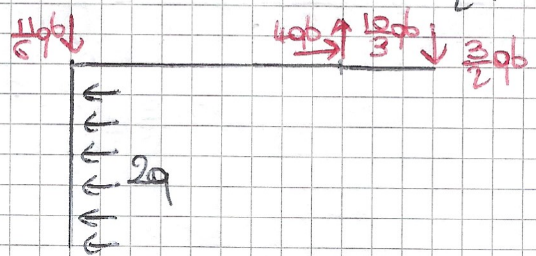
REAZIONI VINCOLARI

$V_B = V_{B_0} + XV_{B_1} \Rightarrow V_B = \frac{4}{3}qb + \frac{3}{2}qb^2\left(\frac{1}{3b}\right) \Rightarrow V_B = \frac{11}{6}qb$

$H_C = H_{C_0} + XH_{C_1} \Rightarrow H_C = 4qb$

$V_C = V_{C_0} + XV_{C_1} \Rightarrow V_C = -\frac{4}{3}qb - \frac{3}{2}qb^2\left(\frac{x}{3b}\right) \Rightarrow V_C = -\frac{10}{3}qb$

$V_D = V_{D_0} + XV_{D_1} \Rightarrow V_D = -\frac{3}{2}qb^2\left(-\frac{1}{b}\right) \Rightarrow V_D = \frac{3}{2}qb$



Azioni Interne

$N(x_1) = N_0(x_1) + X N_1(x_1) \Rightarrow \underline{N_{x_1} = 0}$

$T(x_1) = T_0(x_1) + X T_1(x_1) \Rightarrow \underline{T(x_1) = 2qx_1}$

$M(x_1) = M_0(x_1) + X M_1(x_1) \Rightarrow \underline{M(x_1) = qx_1^2}$

$N(x_2) = N_0(x_2) + X N_1(x_2) \Rightarrow \underline{N(x_2) = 4qb}$

$T(x_2) = T_0(x_2) + X T_1(x_2) \Rightarrow T(x_2) = -\frac{4}{3}qb - \frac{3}{2}qb^2\left(\frac{1}{\cancel{3b}}\right)$

$= \underline{-\frac{11}{6}qb}$

$M(x_2) = M_0(x_2) + X M_1(x_2) \Rightarrow M(x_2) = 4qb^2 - \frac{4}{3}qbx_2 - \frac{3}{2}qb^2\left(\frac{1}{3b}x_2\right)$

$= \underline{4qb^2 - \frac{11}{6}qbx_2}$

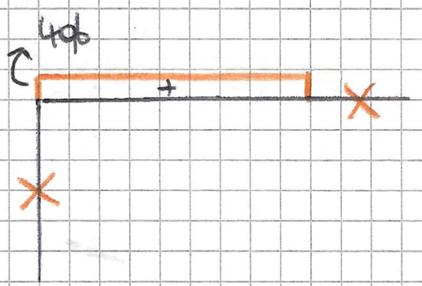
$N(x_3) = N_0(x_3) + X N_1(x_3) \Rightarrow \underline{N(x_3) = 0}$

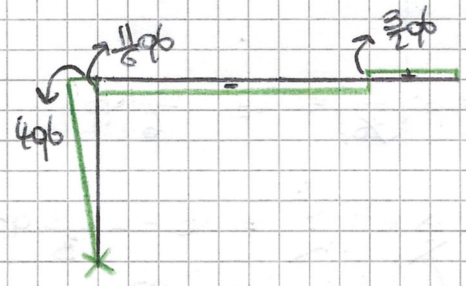
$T(x_3) = T_0(x_3) + X T_1(x_3) \Rightarrow T(x_3) = -\frac{3}{2}qb^2\left(-\frac{1}{b}\right)$

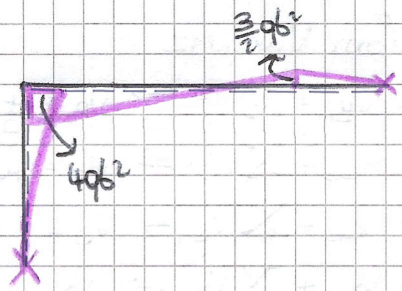
$= \underline{\frac{3}{2}qb}$

$M(x_3) = M_0(x_3) + X M_1(x_3) \Rightarrow M(x_3) = -\frac{3}{2}qb^2\left(\frac{1}{b}x_3\right)$

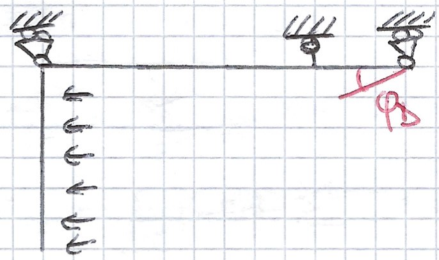
$= \underline{-\frac{3}{2}qbx_3}$

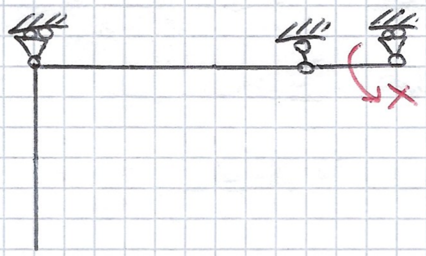


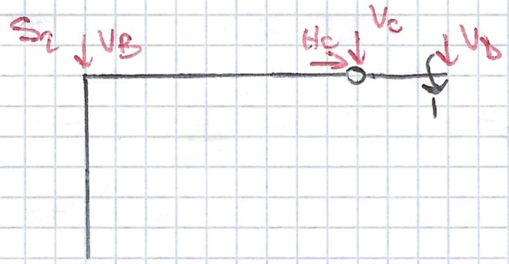




CALCOLO DELLA ROTAZIONE NEL PUNTO D, φ_D

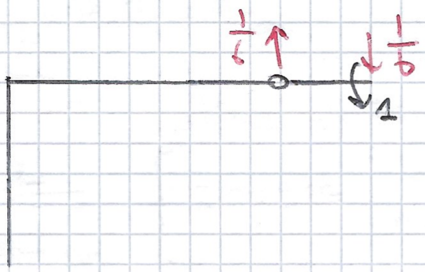






$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{1(C)} = 0 \end{cases} \begin{cases} H_C = 0 \\ V_B + V_C + V_D = 0 \Rightarrow V_C = -\frac{1}{6} \\ V_B(36) + 1 - V_D(6) = 0 \Rightarrow 36 V_B = 0 \Rightarrow V_B = 0 \end{cases}$$

$$\left\{ M^{II}_{2(C)} = 0 \right. \left\{ 1 - V_D(6) = 0 \Rightarrow V_D = \frac{1}{6} \right.$$

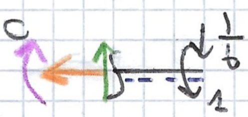


Azioni Interne (momento)

A→B $M(x_1) = 0$

B→C $M(x_2) = 0$

D→C



$$M(x_3) = -\frac{1}{6}x_3 + 1$$

PLV

$$LVe = LVi$$

$$LVe = 1 \cdot \varphi_D$$

$$LVi = \int_0^6 \left(-\frac{1}{6}x_3 + 1\right)\left(-\frac{3}{2}q6x_3\right)\left(\frac{1}{EI}\right)dx_3$$

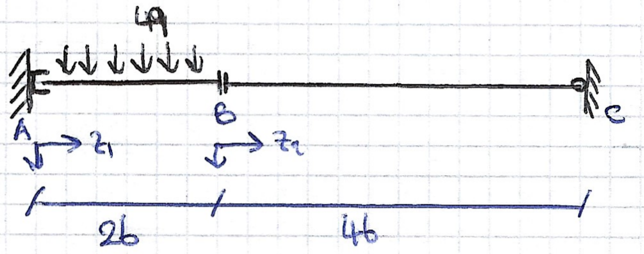
$$= \int_0^6 \left(+\frac{3}{2}qx_3^2 - \frac{3}{2}q6x_3\right)\left(\frac{1}{EI}\right)dx_3$$

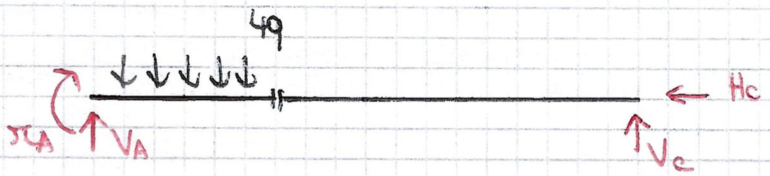
$$= \frac{1}{EI}\left[\frac{3}{2}q\frac{x_3^3}{3} - \frac{3}{2}q6\frac{x_3^2}{2}\right]_0^6$$

$$= \frac{1}{EI}\left(\frac{1}{2}q6^3 - \frac{3}{4}q6^3\right)$$

$$\varphi_D = \frac{1}{EI}\left(-\frac{q6^3}{4}\right)$$

Esercizio 2_Traccia I

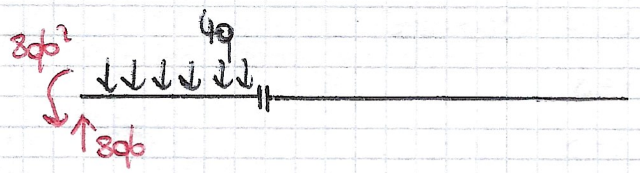




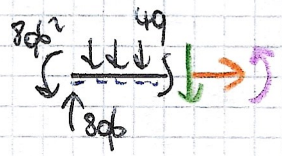
$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(A)} = 0 \end{cases} \quad \begin{cases} H_c = 0 \\ V_A + V_c - 4q(2b) = 0 \quad \Rightarrow \quad V_A = 8qb \\ M_A + 4q(2b)(b) - V_c(6b) = 0 \quad \Rightarrow \quad M_A = -8qb^2 \end{cases}$$

eq. aux

$$\{ R_y^{II} = 0 \quad \{ V_c = 0$$



A→B $0 \leq z_1 \leq 2b$



$N_{z_1} = 0$

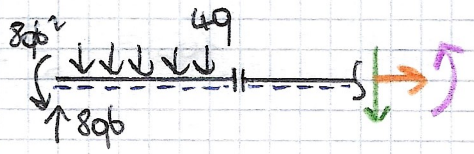
$T_{z_1} + 4qz_1 - 8qb = 0$

$T_{z_1} = 8qb - 4qz_1$

$M_{z_1} + 8qb^2 - 8qbz_1 + 4qz_1\left(\frac{z_1}{2}\right) = 0$

$M_{z_1} = 8qbz_1 - 8qb^2 - 2qz_1^2$

B→C $0 \leq z_2 \leq 4b$



$N_{z_2} = 0$

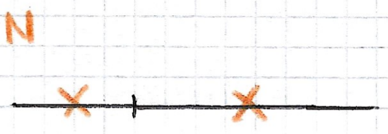
$T_{z_2} + 4q(2b) - 8qb = 0$

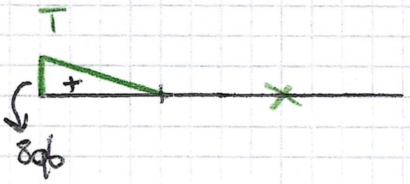
$T_{z_2} = 0$

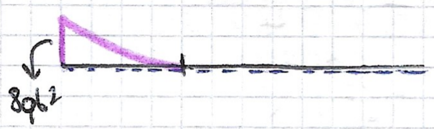
$M_{z_2} + 8qb^2 - 8qb(2b + z_2) + 4q(2b)(b + z_2) = 0$

$M_{z_2} + 8qb^2 - \cancel{16qb^2} - \cancel{8qbz_2} + \cancel{8qb^2} + \cancel{8qbz_2} = 0$

$M_{z_2} = 0$







Eq. LINEA ELASTICA

A → B $\quad 0 \le z_1 \le 2b$

$$v''_{1(z_1)} = -\frac{M}{EI} \Rightarrow v''_{1(z_1)} = \frac{1}{EI}\left(-8qbz_1 + 8qb^2 + 2qz_1^2\right)$$

$$v'_{1(z_1)} = \frac{1}{EI}\left(-8qb\cdot\frac{z_1^2}{2} + 8qb^2 z_1 + 2q\cdot\frac{z_1^3}{3}\right) + A_1$$

$$= \frac{1}{EI}\left(-4qbz_1^2 + 8qb^2 z_1 + \frac{2}{3}qz_1^3\right) + A_1$$

$$v_{(z_1)} = \frac{1}{EI}\left(-4qb\frac{z_1^3}{3} + 8qb^2\frac{z_1^2}{2} + \frac{2}{3}q\frac{z_1^4}{4}\right) + A_1 z_1 + A_2$$

$$= \frac{1}{EI}\left(-\frac{4}{3}qbz_1^3 + 4qb^2 z_1^2 + \frac{1}{6}qz_1^4\right) + A_1 z_1 + A_2$$

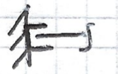
B → C $\quad 0 \le z_2 \le 4b$

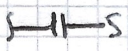
$$v''_{z_2} = -\frac{M}{EI} \Rightarrow z''_{z_2} = 0$$

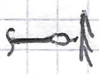
$$v'_{z_2} = B_1$$

$$v_{z_2} = B_1 z_2 + B_2$$

COSTANTI $A_1; A_2; B_1; B_2$. CONDIZIONI A CONTORNO

A. 
MANICOTTO — IMPEDISCE
- • SPOSTAMENTO ↕ $\Rightarrow v_{z_1}(A) = 0$
- • ROTAZIONE $\Rightarrow v'_{(z_1)}(A) = 0$

B. 
PATTINO — IMPONE
- • UGUALE ROTAZIONE IN B_1 e B_2 $\Rightarrow v'_{(z_1)}(B) = v'_{(z_2)}(B)$

C. 
CERNIERA — IMPEDISCE
- • SPOSTAMENTO ↕ $\Rightarrow v_{z_2}(C) = 0$

IN A CON $z_1 = 0$

$$v_{z_1}(z_1 = 0) = 0 \Rightarrow A_2 = 0$$

$$v'_{z_1}(z_1 = 0) = 0 \Rightarrow A_1 = 0$$

IN B CON $z_1 = 2b;\ z_2 = 0$

$$v'_{(z_1 = 2b)} = v'_{(z_2 = 0)}$$

$$\frac{1}{EI}\left(-4qb\cdot 4b^2 + 8qb^2\cdot 2b + \frac{2}{3}q\cdot 8b^3\right) = B_1$$

$$B_1 = \frac{1}{EI}\left(-16\cancel{qb^3} + 16\cancel{qb^3} + \frac{16}{3}qb^3\right)$$

$$B_1 = \frac{16}{3}\frac{qb^3}{EI}$$

IN C CON $z_2 = 4b$

$v_{(z_2=4b)} = 0$

$\frac{16}{3}\frac{qb^3}{EI}(4b) + B_2 = 0$

$B_2 = -\frac{64}{3}\frac{qb^4}{EI}$

DEFORMATA DELLA LINEA D'ASSE

$v_1(z_1) = \frac{1}{EI}\left(-\frac{4}{3}qb\,z_1^3 + 4qb^2 z_1^2 + \frac{1}{6}q z_1^4\right)$

$v_2(z_2) = \frac{1}{EI}\left(\frac{16}{3}qb^3 z_2 - \frac{64}{3}qb^4\right)$

DERIVATA PRIMA

$v'_{1(z_1)} = \frac{1}{EI}\left(-4qb z_1^2 + 8qb^2 z_1 + \frac{2}{3}q z_1^3\right)$

$v'_{2(z_2)} = \frac{16}{3}\frac{qb^3}{EI}$

ROTAZIONE DEL PUNTO C, φ_C

$\varphi_C = \frac{16}{3}\frac{qb^3}{EI}$

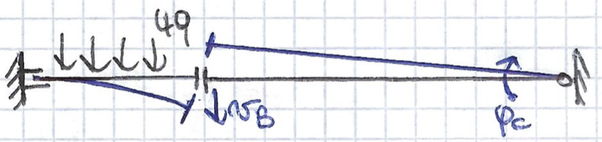
SPOSTAMENTO VERTICALE RELATIVO AL CORPO 1 DEL PUNTO B ; $v_B^{(1)}$

$v_B^{(1)} = v^{(1)}_{(z_1=2b)}$

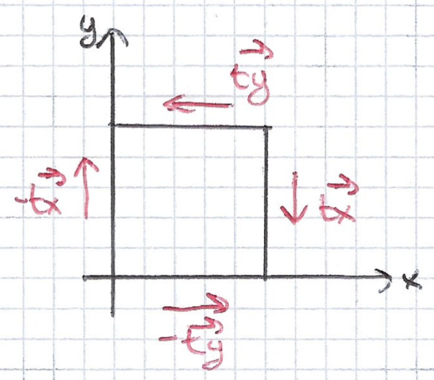
$v_B^{(1)} = \frac{1}{EI}\left(-\frac{4}{3}qb\cdot 8b^3 + 4qb^2\cdot 4b^2 + \frac{1}{\cancel{6}_3}q\cdot \cancel{16}^{8}b^4\right)$

$= \frac{1}{EI}\left(-\frac{32}{3}qb^4 + 16qb^4 + \frac{8}{3}qb^4\right)$

$= 8\frac{qb^4}{EI}$



Esercizio 3 _ Traccia I



$\alpha = -90°$

$|t_x| = 75\ MPa$

$\sin\alpha = -1$

$\cos\alpha = 0$

$\sigma_x = |t_x| \cos\alpha = 0$

$\sigma_y = 0$

$\tau_{xy} = |t_x| \sin\alpha = -75\ MPa$

$\tau_{xy} = \tau_{yx} = -75\ MPa$

$$\underline{\underline{\sigma}} = \begin{pmatrix} 0 & -75 & 0 \\ -75 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

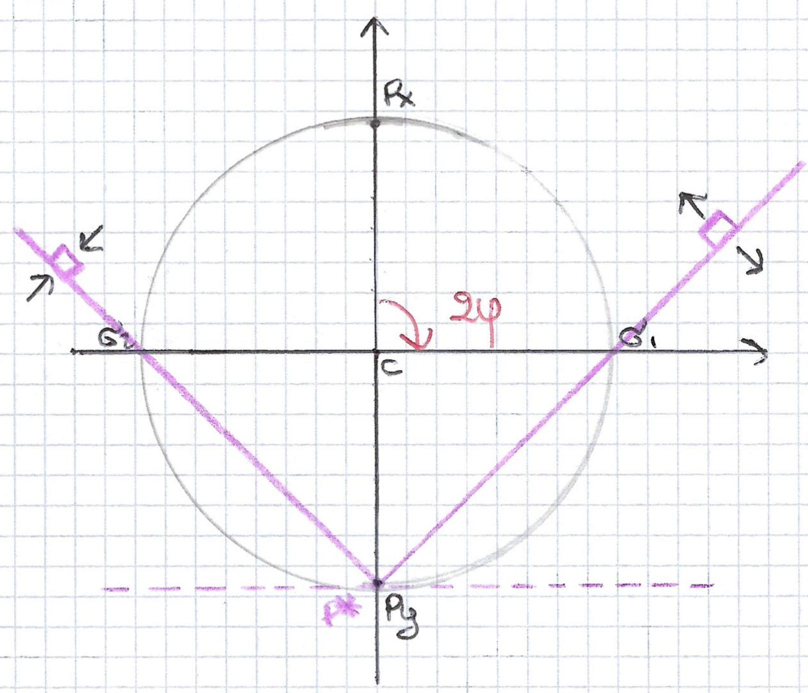
Cerchio di Mohr - Piano (σ_n; τ_n)

$P_x(\sigma_x; \tau_{xy}) \Rightarrow \tau_{xy}$ ↻

$P_x(0; 75)$

$P_y(\sigma_y; -\tau_{yx}) \Rightarrow \tau_{yx}$ ↺

$P_y(0; -75)$



$C(0; 0)$

$R = 75$

$\tau_{max} = 75\ MPa$

$\sigma_1 = 75\ MPa$

$\sigma_2 = -75\ MPa$

Angolo φ

$2\varphi = -90°$

$\varphi = -45°$