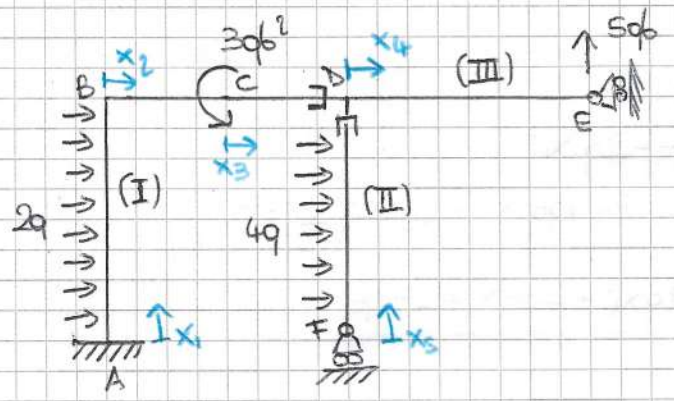
ESERCIZIO 1 – TRACCIA 1 – ESAME 23.01.2024

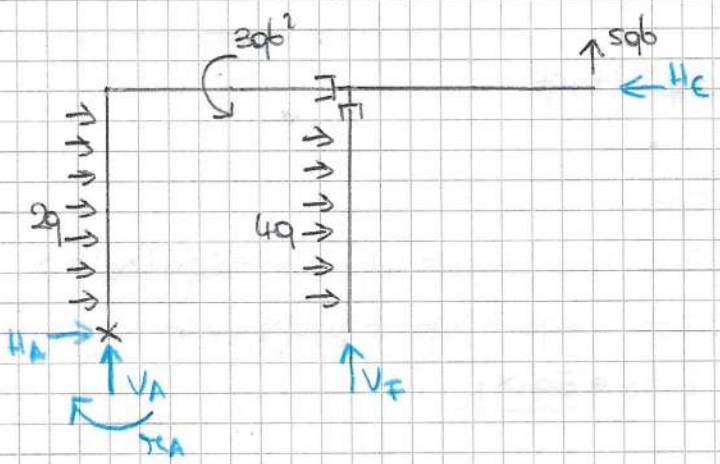


STRUTTURA ISOSTATICA

GDL = 3(I) + 3(II) + 3(III) = 9

GDV = 3(A) + 2(b)$^{(I)}$ + 2(b)$^{(II)}$ + 1(E) + 1(F) = 9

GDL = GDV



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \pi_{z(A)} = 0 \end{cases} \qquad \begin{cases} H_A + 2q(2b) + 4q(2b) - H_E = 0 \Rightarrow -4qb + 4qb + 8qb - H_E = 0 \Rightarrow H_E = 8qb \ [3] \\ V_A + V_F + 5qb = 0 \Rightarrow V_A = -5qb \ [4] \\ 3qb^2 - \pi_A - 2q(2b)(b) - 4q(2b)(b) + V_F(2b) + H_E(2b) + 5qb(4b) = 0 \ [5] \end{cases}$$

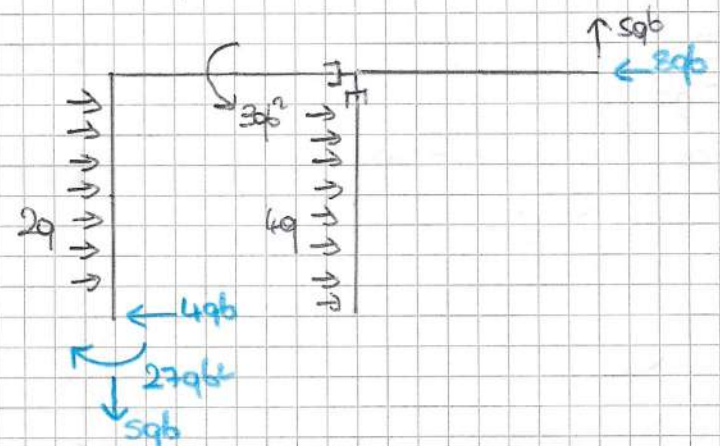
eq. aux.

$$\begin{cases} R_g^{(III)} = 0 \\ R_x^{(I)} = 0 \end{cases} \qquad \begin{cases} V_F = 0 \ [1] \\ H_A + 2q(2b) = 0 \Rightarrow H_A = -4qb \ [2] \end{cases}$$

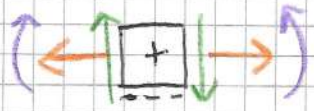
[5] $3qb^2 - \pi_A - 4qb^2 - 8qb^2 + 8qb(2b) + 20qb^2 = 0$

$-\pi_A - 9qb^2 + 16qb^2 + 20qb^2 = 0$

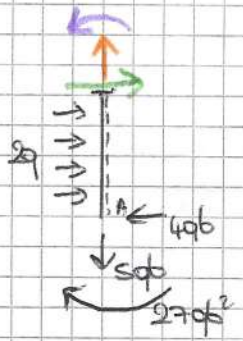
$\pi_A = 27qb^2$



AZIONI INTERNE



A → B $0 \le x_1 \le 2b$



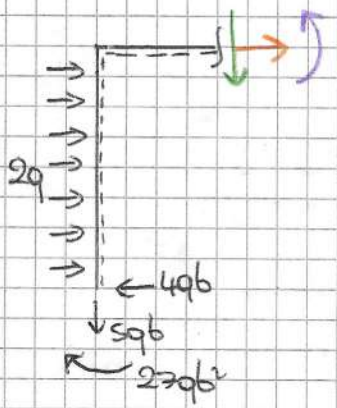
$N_{(x_1)} = 5qb$

$T_{(x_1)} = 4qb - 2q(x_1)$

$\pi_{(x_1)} - 27qb^2 - 4qb(x_1) + 2q(x_1)\left(\frac{x_1}{2}\right) = 0$

$\pi_{(x_1)} = -qx_1^2 + 4qbx_1 + 27qb^2$

B → C $0 \le x_2 \le b$



$N_{(x_2)} = -2q(2b) + 4qb$

$N_{(x_2)} = 0$

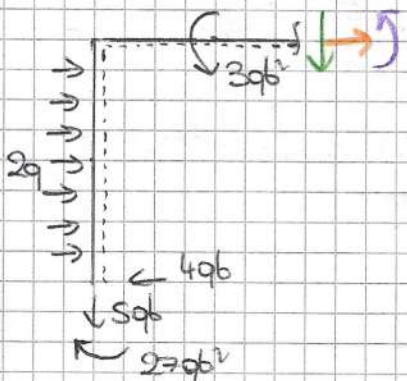
$T_{(x_2)} = -5qb$

$\pi_{(x_2)} - 27qb^2 + 2q(2b)(b) - 4qb(2b) + 5qb(x_2) = 0$

$\pi_{(x_2)} - 27qb^2 + 4qb^2 - 8qb^2 + 5qbx_2 = 0$

$\pi_{(x_2)} = 31qb^2 - 5qbx_2$

C → D $0 \le x_3 \le b$



$N_{(x_3)} + 2q(2b) - 4qb = 0$

$N_{(x_3)} = 0$

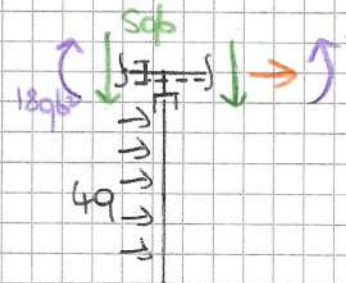
$T_{(x_3)} = -5qb$

$\pi_{(x_3)} = 27qb^2 - 2q(2b)(b) + 4qb(2b) - 5qb(b + x_3) - 3qb^2$

$\pi_{(x_3)} = 31qb^2 - 5qb^2 - 5qbx_3 - 3qb^2$

$\pi_{(x_3)} = 23qb^2 - 5qbx_3$

D → E $0 \le x_4 \le 2b$



$N_{(x_4)} = -4q(2b)$

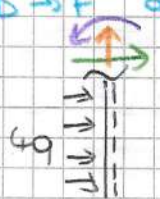
$N_{(x_4)} = -8qb$

$T_{(x_4)} = -5qb$

$\pi_{(x_4)} = 18qb^2 - 4q(2b)(b) - 5qb(x_4)$

$\pi_{(x_4)} = 10qb^2 - 5qbx_4$

D → F $0 \le x_5 \le 2b$



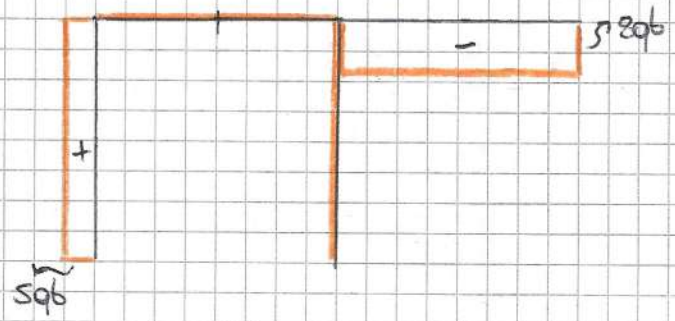
$N_{(x_5)} = 0$

$T_{(x_5)} = -4q(x_5)$

$\pi_{(x_5)} = -4q(x_5)\left(\frac{x_5}{2}\right)$

$\pi_{(x_5)} = -2qx_5^2$

N



$N_{(x_1)} = 5qb$

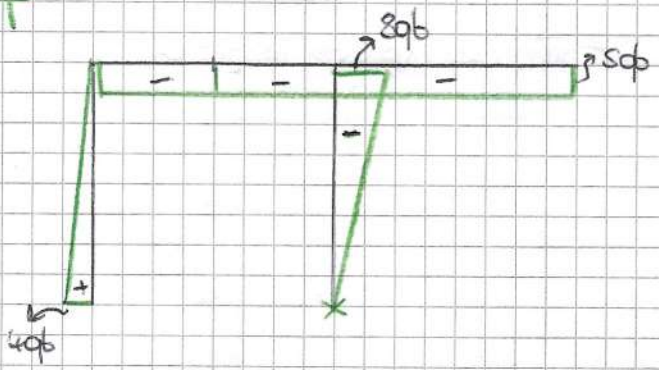
$N_{(x_2)} = 0$

$N_{(x_3)} = 0$

$N_{(x_4)} = -8qb$

$N_{(x_5)} = 0$

T



$T_{(x_1)} = 4qb - 2qx_1$

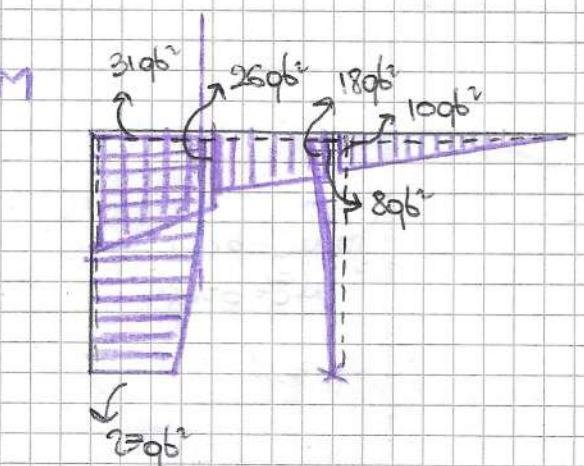
$T_{(x_2)} = -5qb$

$T_{(x_3)} = -5qb$

$T_{(x_4)} = -5qb$

$T_{(x_5)} = -4q(x_5)$

M



$M_{(x_1)} = -qx_1^2 + 4qbx_1 + 27qb^2$

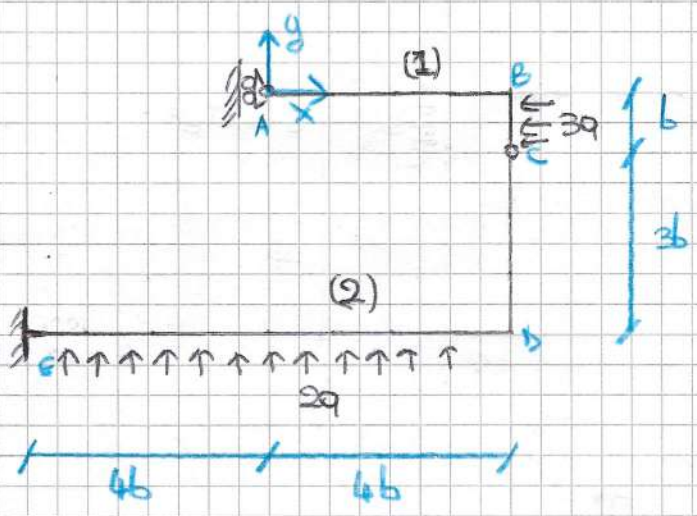
$M_{(x_2)} = 31qb^2 - 5qbx_2$

$M_{(x_3)} = 23qb^2 - 5qbx_3$

$M_{(x_4)} = 10qb^2 - 5qbx_4$

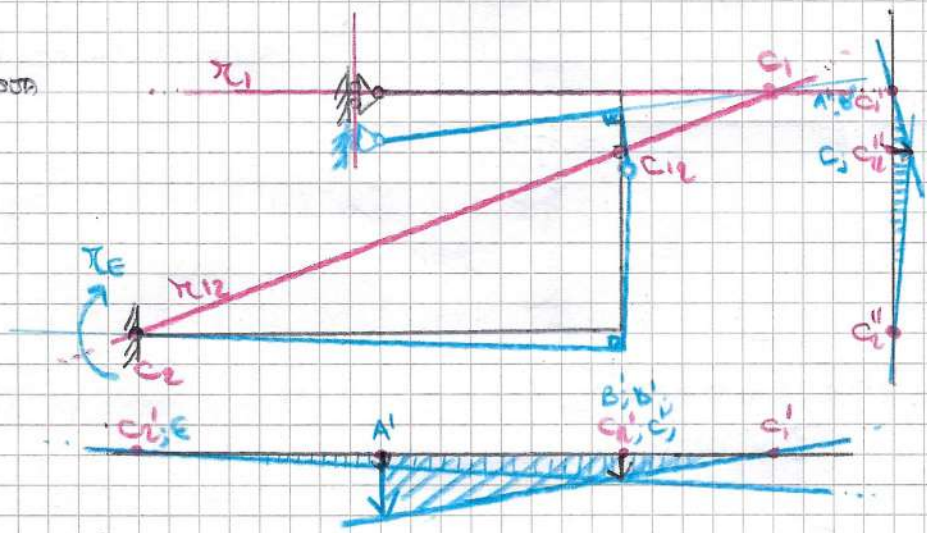
$M_{(x_5)} = -2qx_4^2$

ESERCIZIO 2



$\pi_E = ?$

STRUTTURA UNA VOLTA ISOSTATICA



$u_C^{(1)} = b\,\delta\varphi_1$

$u_C^{(2)} = 3b\,\delta\varphi_2$

$u_C^{(1)} = u_C^{(2)}$

$\delta\varphi_1 = 3\,\delta\varphi_2$

CIR

$C_1 \in \pi_1$

$C_{12} = C = (4b; -b)$

$C_2 = E = (-4b; -4b)$

$\vec{v}_A = -\frac{2}{3} b\,\delta\varphi_1$

$C_1 \Leftrightarrow C_{12} \Leftrightarrow C_2$

$C_2 \in \pi_{12}$

$C_2 = \left(\frac{2}{3}b; 0\right)$

Principio dei lavori virtuali

$\delta\mathcal{L} = 0 \quad \forall \varphi \quad \delta\varphi_1 = 3\,\delta\varphi_2$

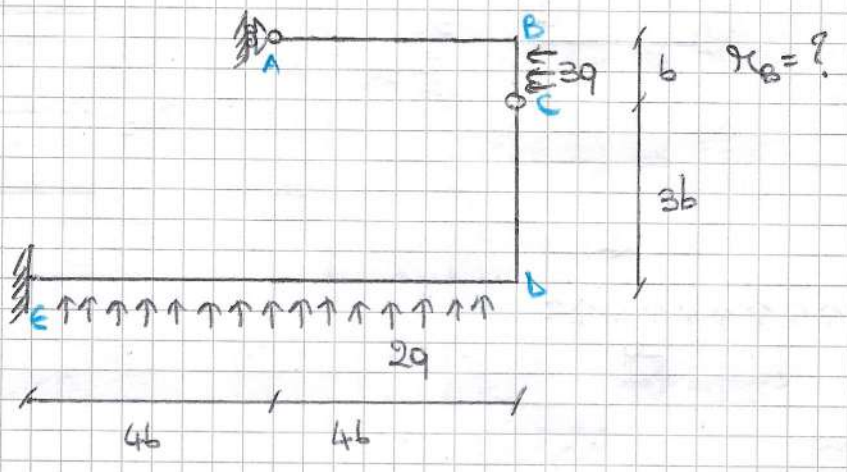
$\pi_E \delta\varphi_2 - 2q(8b)(4b)\delta\varphi_2 - 3q(b)(b/2)\delta\varphi_1 = 0$

$\pi_E \delta\varphi_2 - 64qb^2\delta\varphi_2 - \frac{3}{2}qb^2(3\delta\varphi_2) = 0$

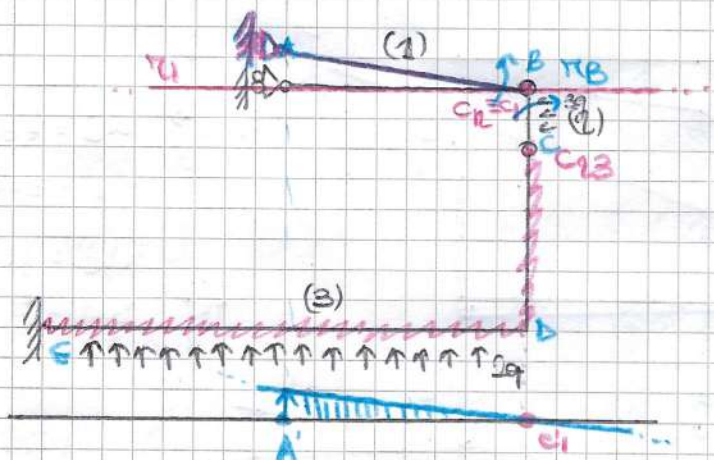
$\pi_E - 64qb^2 - \frac{9}{2}qb^2 = 0$

$\pi_E = \frac{128+9}{2} qb^2 \Rightarrow \pi_E = \frac{137}{2} qb^2$

ESERCIZIO 2



STRUTTURA UNA VOLTA IPOSTATICA



CIR

$c_1 \in r_1$

$c_3 \nexists$

$c_1 = B = (4b; 0)$

$r_A = 4b\,\delta\varphi$

$M_C = 0$

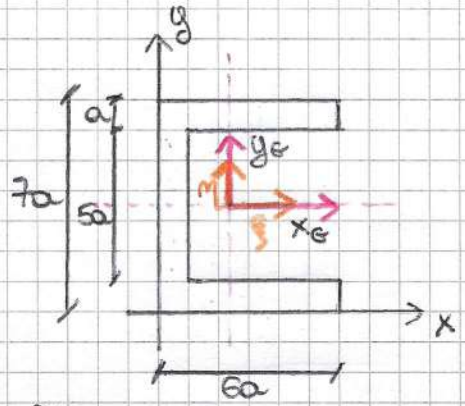
Principio dei lavori virtuali

$\delta\mathcal{L} = 0 \quad \forall \delta\varphi$

$-r_B \delta\varphi_1 = 0$

$r_B = 0$

Esercizio 3



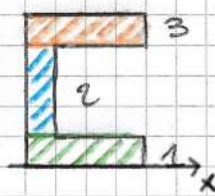
$S_x = \frac{119}{2}a^3 \quad S_y = \frac{77}{2}a^3$

$y_G = \frac{S_x}{A} \Rightarrow \frac{119}{2}a^3\left(\frac{1}{17a^2}\right) = \frac{7}{2}a = 3{,}5\,a$

$x_G = \frac{S_y}{A} \Rightarrow \frac{77}{2}a^3\left(\frac{1}{17a^2}\right) = \frac{77}{34}a = 2{,}264\,a$

Momento statico

$S_x^{TOT} = S_x^{(1)} + S_x^{(2)} + S_x^{(3)}$



$S_x^{(1)} = A^{(1)} y_G^{(1)} = 6a^2\left(\frac{a}{2}\right) = 3a^3$

$S_x^{(2)} = A^{(2)} y_G^{(2)} = 5a^2\left(\frac{7}{2}a\right) = \frac{35a^3}{2}$

$S_x^{(3)} = A^{(3)} y_G^{(3)} = 6a^2\left(\frac{13}{2}a\right) = \frac{39}{2}a^3$

$S_x^{TOT} = 3a^3 + \frac{35}{2}a^3 + 39a^3$

$= 42a^3 + \frac{35}{2}a^3$

$= \frac{84a^3 + 35a^3}{2} =$

$S_x^{TOT} = \frac{119}{2}a^3 = 59{,}500\,a^3$

$S_y^{TOT} = S_y^{(2)} + 2S_y^{(1)}$

$S_y^{(2)} = 5a^2\left(\frac{a}{2}\right) = \frac{5}{2}a^3$

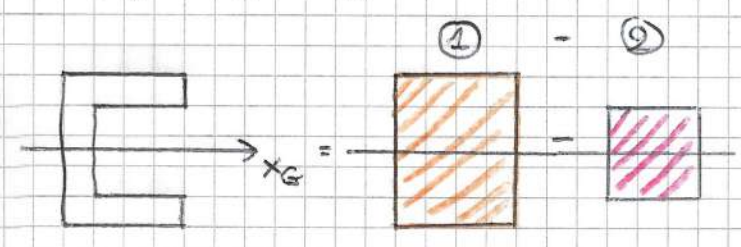
$S_y^{(1)} = A^{(1)}\left(x_G^{(1)}\right) = 6a^2(3a) = 18a^3$

$S_y^{TOT} = \frac{5}{2}a^3 + 2(18a^3) = \frac{5}{2}a^3 + 36a^3$

$S_y^{TOT} = \frac{5a^3 + 72a^3}{2} = \frac{77}{2}a^3 = 38{,}500\,a^3$

Momento di inerzia

$J_{x_G} = J_{x_G}^{④} - J_{x_G}^{①}$



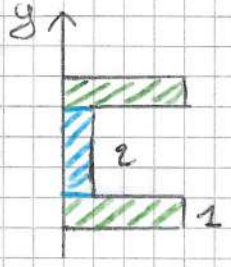
$J_{x_G}^{①} = \frac{b \cdot h^3}{12} = \frac{6a(7a)^3}{12} = \frac{6a(343a^3)}{12}$

$J_{x_G}^{①} = \frac{343}{2}a^3$

$J_{x_G}^{②} = \frac{5a(5a)^3}{12} = \frac{5a(125a^3)}{12} = \frac{625}{12}a^3$

$J_{x_G} = \frac{343}{2}a^3 - \frac{625}{12}a^3 = \frac{2058 - 625}{12}a^3 = \frac{1433}{12}a^4 = 119{,}416\,a^4$

$J_{y_G} = J_y - A x_G^2 \qquad J_y^{TOT} = 2 J_y^{(1)} + J_y^{(2)}$



$$J_y^{①} = \frac{8 \cdot b^3}{3} = \frac{a(6a)^3}{3} = \frac{216}{3} a^4 = 72a^4$$

$$J_y^{②} = \frac{8 \cdot b^3}{3} = \frac{5a(a^3)}{3} = \frac{5}{3} a^4$$

$$J_y^{TOT} = 2(72a^4) + \frac{5}{3} a^4$$

$$= 144 a^4 + \frac{5}{3} a^4 = \frac{437}{3} a^4$$

$$J_{y_G} = \frac{437}{3} a^4 - 17a^2 \left(\frac{77}{34} a\right)^2$$

$$= \frac{437}{3} a^4 - \frac{100793}{1156} a^4$$

$$= \frac{505.172 - 302379}{3468} = \frac{202793}{3468} \overset{11929}{204} = \frac{11929}{204} a^4 = 58{,}475\, a^4$$

MOMENTO CENTRIFUGO

$J_{x_G y_G} = 0 \Rightarrow X_G$ È ASSE DI SIMMETRIA

$$tg 2\theta = \frac{-2 J_{x_G y_G}}{J_{x_G} - J_{y_G}} = 0 \qquad tg 2\theta = 0 \qquad J_{x_G} > J_{y_G} \Rightarrow \theta = 0$$

$$J_\xi = J_{max} = J_{x_G} = \frac{1433}{12} a^4 \qquad J_\eta = J_{min} = J_{y_G} = \frac{11924}{204} a^4$$