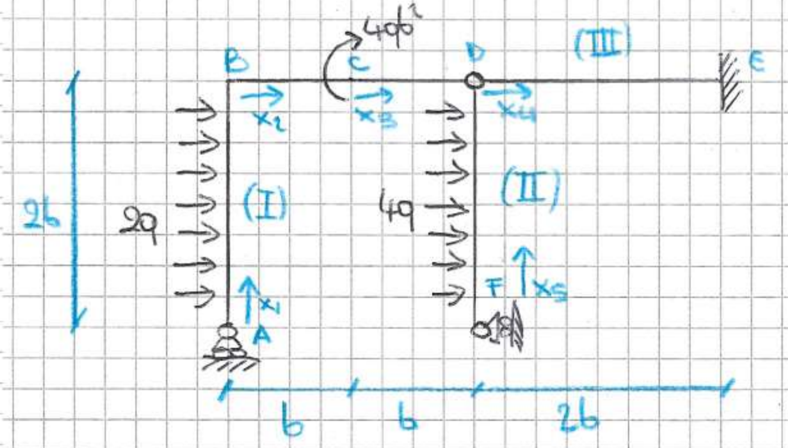
Esercizio 1 _ Traccia 1 _ Esame 09.01.2024

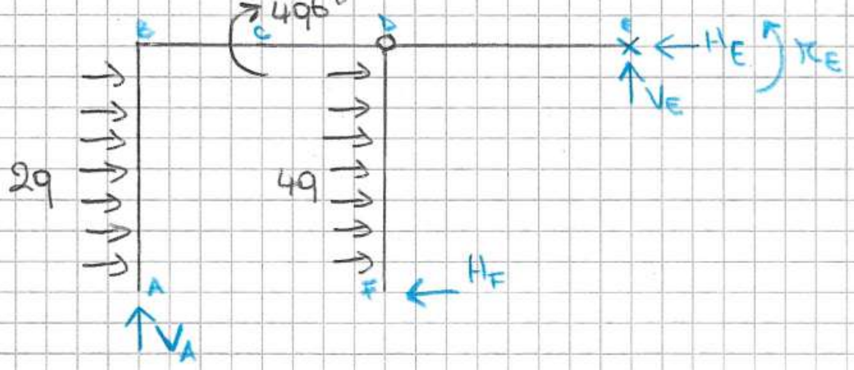


$GDL = 3(I) + 3(II) + 3(III) = 9$

$GDV = 1(A) + 4(D) + 3(E) + 1(F) = 9$

$GDL = GDV$

STRUTTURA ISOSTATICA



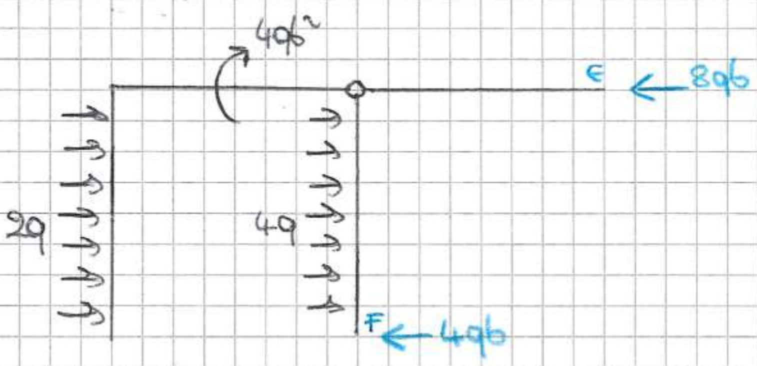
$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \mathcal{M}_{z(E)} = 0 \end{cases} \quad \begin{cases} 2q(2b) + 4q(2b) - H_F - H_E = 0 \Rightarrow \cancel{4qb} + 8qb - \cancel{4qb} - H_E = 0 \Rightarrow \underline{H_E = 8qb} \\ V_A + V_E = 0 \Rightarrow \underline{V_E = 0} \quad [2] \\ \mathcal{M}_E - H_F(2b) + 4q(2b)(b) + 2q(2b)(b) - 4qb^2 - V_A(4b) = 0 \quad [5] \end{cases}$$

eq. aux

$$\begin{cases} \mathcal{M}^{(I)}_{z(D)} = 0 \\ \mathcal{M}^{(II)}_{z(D)} = 0 \end{cases} \quad \begin{cases} 2q(2b)(b) - V_A 2b - 4qb^2 = 0 \Rightarrow \cancel{4qb^2} - 2bV_A - \cancel{4qb^2} = 0 \Rightarrow \underline{V_A = 0} \quad [1] \\ 4q(2b)(b) - H_F 2b = 0 \Rightarrow 2bH_F = 8qb^2 \Rightarrow \underline{H_F = 4qb} \quad [3] \end{cases}$$

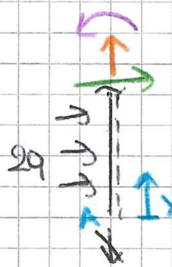
$$[5] \quad \mathcal{M}_E - H_F(2b) + 8qb^2 + \cancel{4qb^2} - \cancel{4qb^2} = 0$$

$$\mathcal{M}_E - 8qb^2 + 8qb^2 = 0 \Rightarrow \underline{\mathcal{M}_E = 0}$$



Azioni interne

A → B

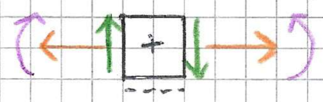


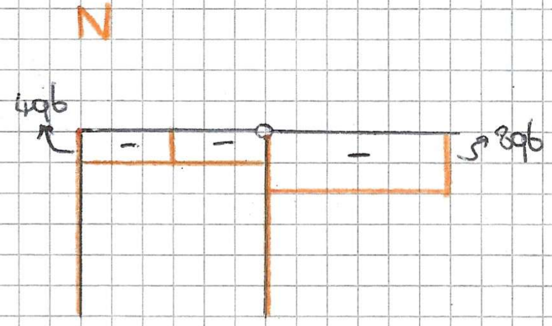
$0 \le x_1 \le 2b$

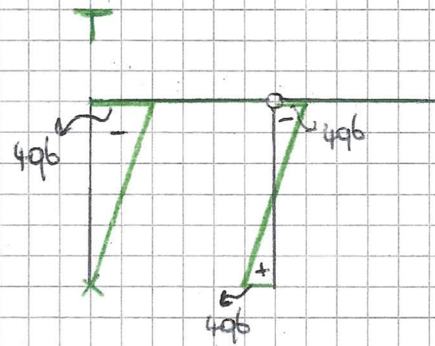
$N_{(x_1)} = 0$

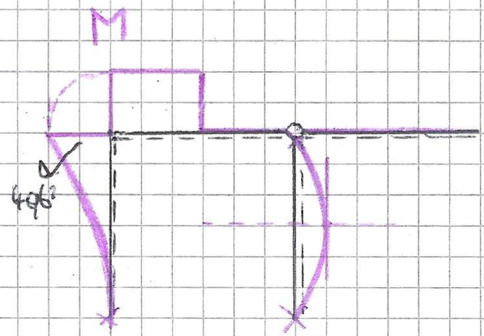
$T_{(x_1)} = -2q x_1$

$M_{(x_1)} = -\cancel{2}q \frac{x_1^2}{\cancel{2}}$

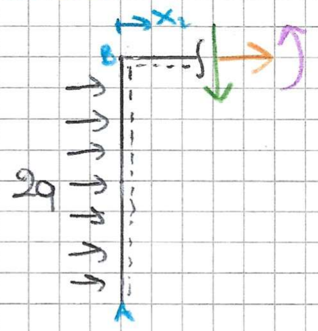








B → C



$0 \le x_2 \le b$

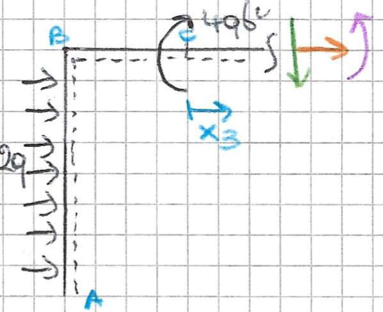
$N_{(x_2)} = -4qb$

$T_{(x_2)} = 0$

$M_{(x_2)} = -2q(2b)(b)$

$M_{(x_2)} = -4qb^2$

C → D



$0 \le x_3 \le b$

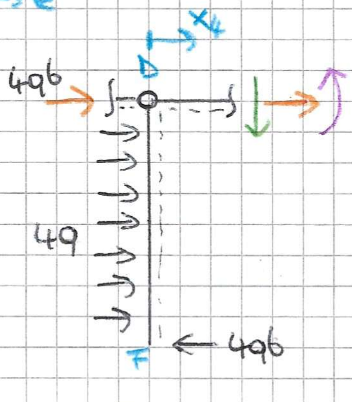
$N_{(x_3)} = -4qb$

$T_{(x_3)} = 0$

$M_{(x_3)} = -4qb^2 + 4qb^2$

$M_{(x_3)} = 0$

D → E



$0 \le x_4 \le 2b$

$N_{(x_4)} = -8qb + 4qb - 4qb$

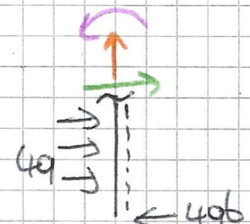
$N_{(x_4)} = -8qb$

$T_{(x_4)} = 0$

$M_{(x_4)} + 4q(2b)(b) - 4qb(2b) = 0$

$M_{(x_4)} = 0$

F → D



$0 \le x_5 \le 2b$

$N_{(x_5)} = 0$

$T_{(x_5)} = -4q x_5 + 4qb$

$M_{(x_5)} = -\cancel{4}^{2} q \frac{x_5^2}{\cancel{2}} + 4qb x_5$

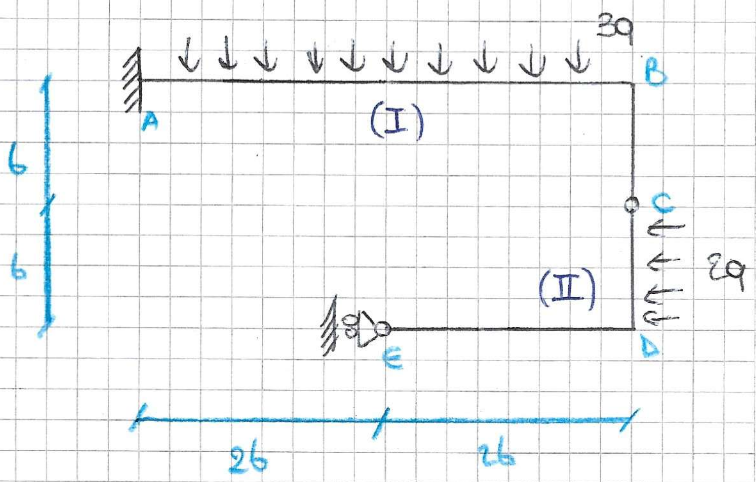
$M_{(x_5)} = -2q x_5^2 + 4qb x_5$

per $x_5 = b$ punto di minimo

$M_{(x_5=b)} = -2qb^2 + 4qb^2$

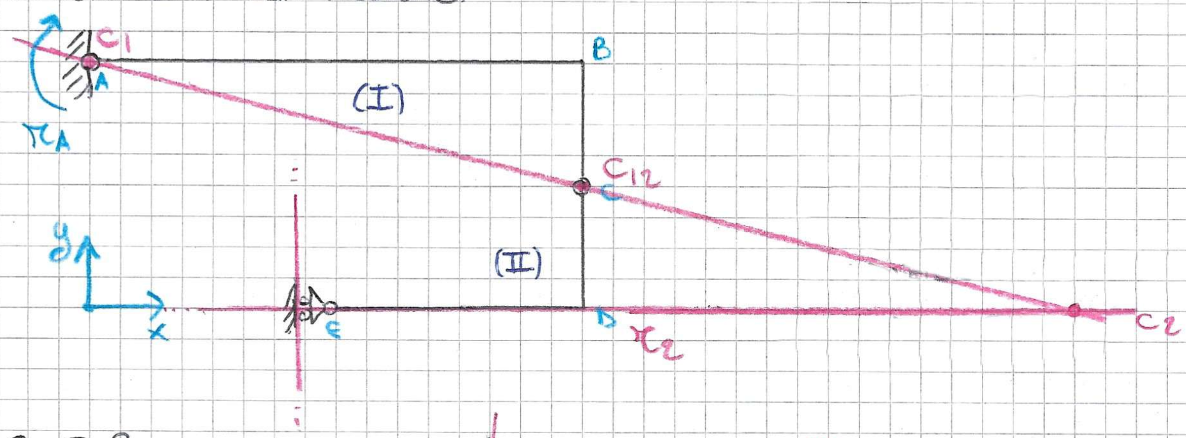
$= 2qb^2$

Esercizio 2



$r_A = ?$

Struttura una volta ipostatica



C.I.R

$C_1 = A = (0;2b)$

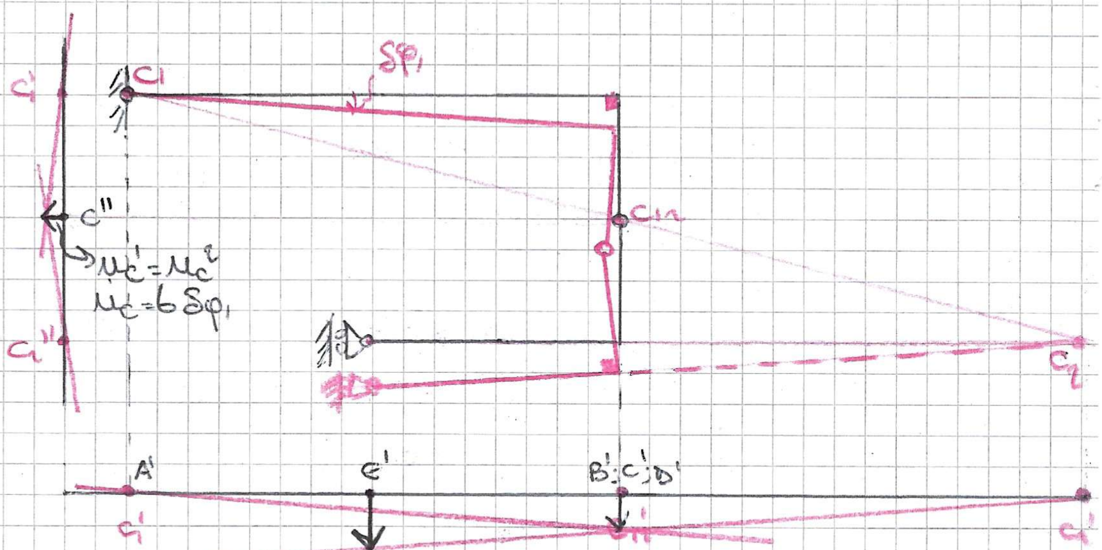
$C_{12} = C = (4b;b)$

$C_2 \in r_2$

Condizione cinematica

$C_1 \Leftrightarrow C_{12} \Leftrightarrow C_2$

$C_2 = (8b;0)$



$u_C = -b\delta\varphi_1$

$v_B = -4b\delta\varphi_1$

$v_E = 6b\delta\varphi_2$ $\quad v_B = 4b\delta\varphi_1$

$v_D = 4b\delta\varphi_2$; $\delta\varphi_1 = \delta\varphi_2$

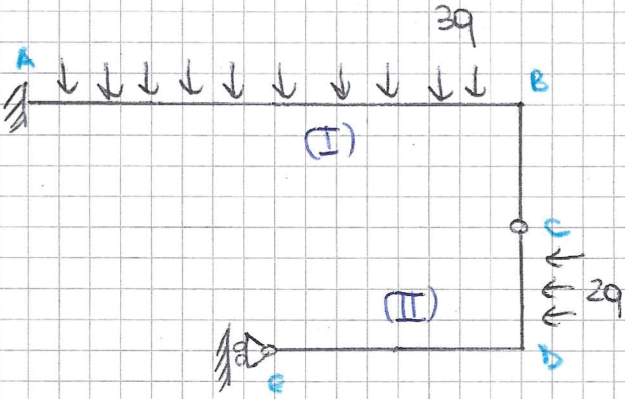
Principio dei lavori virtuali

$\delta L = 0 \quad \forall \delta\varphi$

$r_A \cancel{\delta\varphi_1} + 3q(4b)(2b\cancel{\delta\varphi_1}) + 2q(b)\left(\frac{b}{2}\cancel{\delta\varphi_2}\right) = 0$

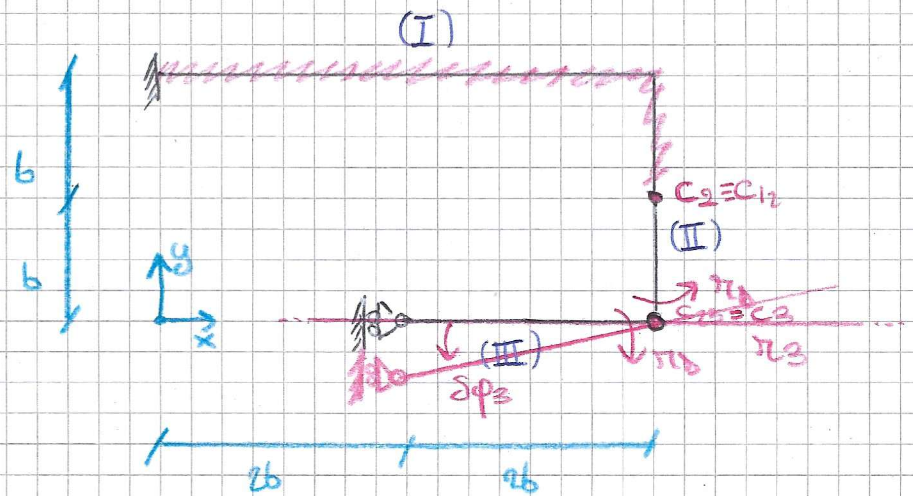
$r_A + 24qb^2 + qb^2 = 0$

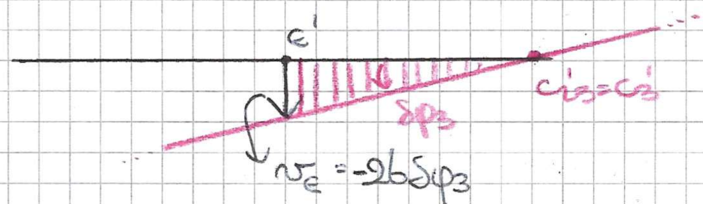
$r_A = -25qb^2$



$r_D = ?$

STRUTTURA ISOSTATICA





CIR

$C_1 \nexists$ CORPO I RESTA FERMO

$C_2 = C = (4b; b) = C_{12}$

$C_{23} = D = (4b; 0)$

$C_3 \in r_3$

$C_3 \leftrightarrow C_{23} \leftrightarrow C_2$

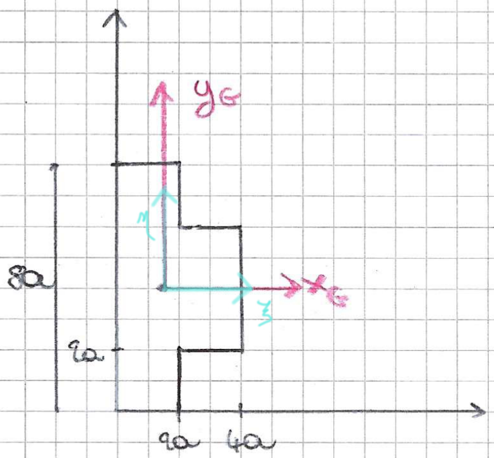
$C_3 = D = (4b; 0)$

Principio dei lavori virtuali

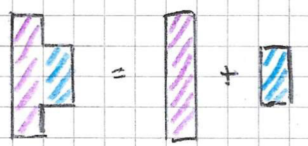
$\delta \mathcal{L} = 0 \quad \forall \delta\varphi$

$-r_D\, \delta\varphi_3 = 0 \Rightarrow r_D = 0$

Esercizio 3 _ Traccia 1 -



MOMENTO STATICO



$$S_x^{TOT} = S_x^{(1)} + S_x^{(2)}$$

$$S_x^{(1)} = A_1 y_{G_1} = 16a^2(4a) = 64a^3$$

$$S_x^{2} = A_2 y_{G_2} = 8a^2(4a) = 32a^3$$

$$S_x^{TOT} = 96a^3$$

$$S_y^{TOT} = S_y^{(1)} + S_x^{(2)}$$

$$S_y^{(1)} = A_1 x_{G_1} = 16a^2(a) = 16a^3$$

$$S_y^{(2)} = A_2 x_{G_2} = 8a^2(3a) = 24a^3$$

$$S_y^{TOT} = 40a^3$$

COORDINATE BARICENTRICHE

$$S_x = A y_G \Rightarrow y_G = \frac{S_x}{A}$$

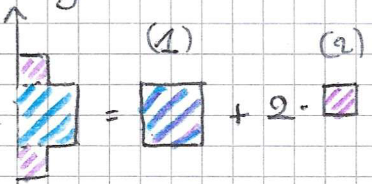
$$y_G = \frac{96a^3}{24a^2} = 4a$$

$$S_y = A x_G \Rightarrow x_G = \frac{S_y}{A}$$

$$x_G = \frac{40a^3}{24a^2} = \frac{5}{3}a = 1{,}6667a$$

MOMENTO DI INERZIA

J_{y_G}

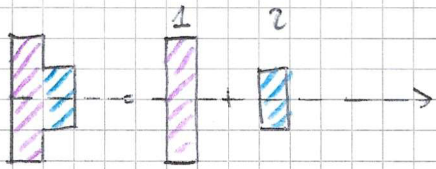


$$J_y = J_y^{(1)} + 2(J_y^{(2)}) = \frac{4a(64a^3)}{3} + 2\left[\frac{2a(8a^3)}{3}\right] = \frac{256}{3}a^4 + \frac{32}{3}a^4 = \frac{288}{3}a^4$$

$$J_{y_G} = J_y - A x_G^2 = \frac{288}{3}a^4 - \frac{200}{3}a^4 = \frac{88}{3}a^4$$

MOMENTO DI INERZIA

J_{x_G}



$$J_{x_G} = J_{x_G}^{(1)} + J_{x_G}^{(1)}$$

$$J_{x_G}^{(1)} = \frac{b_1 h_1^3}{12} = \frac{\cancel{2}^{1}a(512a^3)}{\cancel{12}_{6}} = \frac{512}{6}a^4 = \frac{256}{3}a^4$$

$$J_{x_G}^{2} = \frac{b_1 h_1^3}{12} = \frac{2a(64a^3)}{12} = \frac{64}{6}a^4 = \frac{32}{3}a^4$$

$$J_{x_G} = \frac{256}{3}a^4 + \frac{32}{3}a^4 = \frac{288}{3}a^4 = 96a^4$$

MOMENTO CENTRIFUGO

$J_{x_G y_G} = 0 \Rightarrow X_G$ ASSE DI SIMMETRIA

$$tg2\theta = \frac{-2J_{x_G y_G}}{J_{x_G} - J_{y_G}} = 0 \qquad tg2\theta = 0 \qquad J_{x_G} > J_{y_G} \Rightarrow \theta = 0$$

$$J_\xi = J_{max} = J_{x_G} = 96a^4$$

$$J_\eta = J_{min} = J_{y_G} = \frac{88}{3}a^4$$