12/12/2023

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ESERCIZI SU STUDI DI FUNZIONI.

1) $f(x) = \dfrac{1}{\ln(4-x^2)}$

2) $y = \sqrt{1-x} - \sqrt{x}$

3) $y = \ln\left(\dfrac{-x}{1+x^2}\right)$

[SENZA STUDIARE IL SEGNO DI f'', MA SOLO LA SUA ESPRESSIONE.]

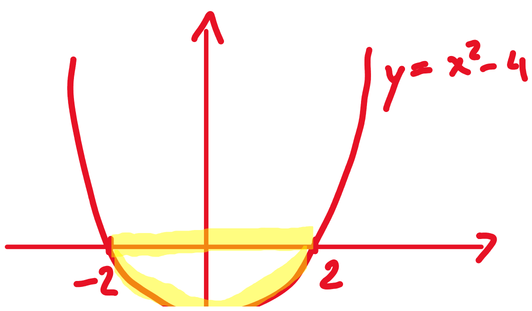
1) $f(x) = \dfrac{1}{\ln(4-x^2)}$ C.E: $\begin{cases} \ln(4-x^2) \neq 0 \\ 4-x^2 > 0 \end{cases}$;

$\begin{cases} \ln(4-x^2) \neq \ln 1 \\ x^2-4 < 0 \end{cases}$; $\begin{cases} 4-x^2 \neq 1 \\ x^2-4 < 0 \end{cases}$; $\begin{cases} x^2 \neq 3 \\ x^2-4 < 0 \end{cases}$

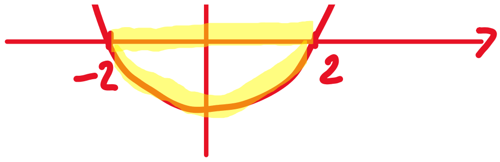
$x^2 \neq 3$; $x \neq \pm\sqrt{3}$ OSSIA $x \neq -\sqrt{3}$, $x \neq +\sqrt{3}$

$x^2-4<0$; $\rightarrow$ PARABOLA CON CONCAVITÁ VERSO L'ALTO E INTERSEZIONI DATE DA

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-0 \pm \sqrt{0-4(1)(-4)}}{2 \cdot 1} = \frac{\pm\sqrt{16}}{2} = \pm 2$$



$\Rightarrow$ $x^2-4<0$ QUANDO $-2 < x < 2$



$\rightarrow$ C.E: $\begin{cases} x \neq \pm\sqrt{3} \\ -2 < x < 2 \end{cases}$ $\qquad D = (-2, 2) \setminus \{+\sqrt{3}, -\sqrt{3}\}$

INTERSEZIONI: $\quad y = 0 \rightsquigarrow 0 = \dfrac{1}{\ln(4-x^2)} \rightarrow$ IMPOS.

$x = 0 \rightsquigarrow y = \dfrac{1}{\ln(4-0^2)} = \dfrac{1}{\ln 4}$

OSSIA IL PUNTO $\left(0, \dfrac{1}{\ln 4}\right)$

SEGNO: $\quad y \geq 0 \iff \dfrac{1}{\ln(4-x^2)} \geq 0$

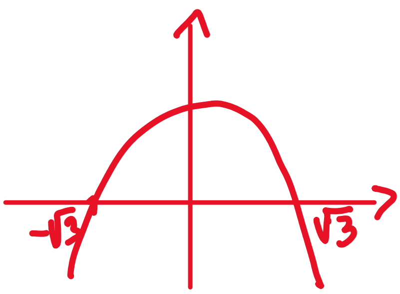
$N \geq 0$: $\quad 1 \geq 0 \quad$ SEMPRE

$D > 0$: $\quad \ln(4-x^2) > 0 \longrightarrow 4 - x^2 > e^0$;

$4 - x^2 > 1$; $\quad 3 - x^2 > 0$;

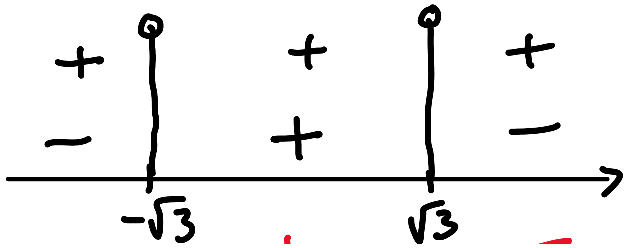
PARABOLA CON CONCAVITÀ VERSO IL BASSO E INTERSEZ.

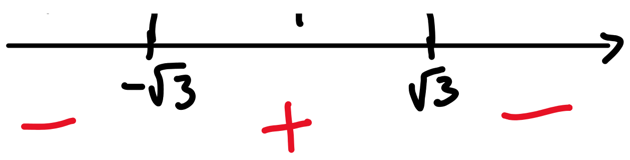
$x = \pm\sqrt{3}$



QUINDI $\quad 3 - x^2 > 0$

QUANDO $\quad -\sqrt{3} < x < \sqrt{3}$





LIMITI AGLI ESTREMI:
$$D = (-2,2)\setminus\{-\sqrt{3},\sqrt{3}\} =$$
$$= (-2,-\sqrt{3}) \cup (-\sqrt{3},\sqrt{3}) \cup (\sqrt{3},2)$$

$$\lim_{x\to -2^+} f(x),\quad \lim_{x\to -\sqrt{3}} f(x),\quad \lim_{x\to \sqrt{3}} f(x),\quad \lim_{x\to 2^-} f(x)$$

$$\lim_{x\to -2^+} \frac{1}{\ln(4-x^2)} = \frac{1}{\ln(4-4)} = \frac{1}{\ln(0)} = \frac{1}{-\infty} = 0$$

$$\left.\begin{aligned} &\lim_{x\to -\sqrt{3}^-} \frac{1}{\ln(4-x^2)} = \frac{1}{\ln(4-3)} = \frac{1}{\ln(1^-)} = \frac{1}{0^-} = -\infty \\ &\lim_{x\to -\sqrt{3}^+} \frac{1}{\ln(4-x^2)} = \frac{1}{\ln(4-3)} = \frac{1}{\ln(1^+)} = \frac{1}{0^+} = +\infty \end{aligned}\right] \bigstar$$

ES: calcolare gli altri 3 limiti e verificare che
$$\lim_{x\to\sqrt{3}^-} f(x) = +\infty,\quad \lim_{x\to\sqrt{3}^+} f(x) = -\infty,\quad \lim_{x\to 2^-} f(x) = 0$$

$\bigstar \Rightarrow x = -\sqrt{3}$ É ASINTOTO VERTICALE
(LO É ANCHE $x = \sqrt{3}$, COME SI VEDE DALL' ESERCIZIO)

MAX - MIN RELATIVI:

$$f'(x) = \frac{0\cdot\ln(4-x^2) - 1\cdot\left[\frac{1}{4-x^2}\cdot(-2x)\right]}{\left[\ln(4-x^2)\right]^2} =$$

$$= \frac{\frac{2x}{4-x^2}}{\ln^2(4-x^2)} = \frac{2x}{4-x^2}\cdot\frac{1}{\ln^2(4-x^2)}$$

$$= \frac{4-x^2}{\ln^2(4-x^2)} = \frac{\ \cdots\ }{4-x^2} \cdot \frac{\ \cdot\ }{\ln^2(4-x^2)}$$

$$= \frac{2x}{(4-x^2)\ln^2(4-x^2)}$$

STUDIAMONE IL SEGNO : $f' \geq 0 \iff \frac{2x}{(4-x^2)\ln^2(4-x^2)} \geq 0$

$N \geq 0$: $2x \geq 0 \rightarrow x \geq 0$

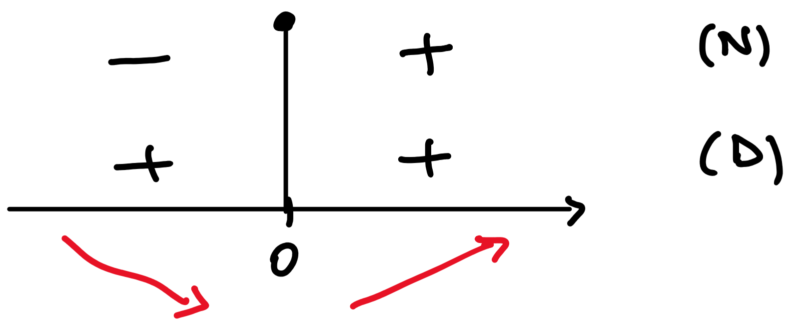
$D > 0$: $\underbrace{(4-x^2)}_{\text{POSITIVO PER LE C.E.}}\underbrace{\ln^2(4-x^2)}_{\text{POSITIVO PERCHÉ É UN QUADRATO E } \ln(4-x^2)\neq 0 \text{ PER LE C.E.}} > 0 \rightarrow \forall x \in D$

[ALTERNATIVAMENTE STUDIEREI IL SEGNO DEL PRODOTTO

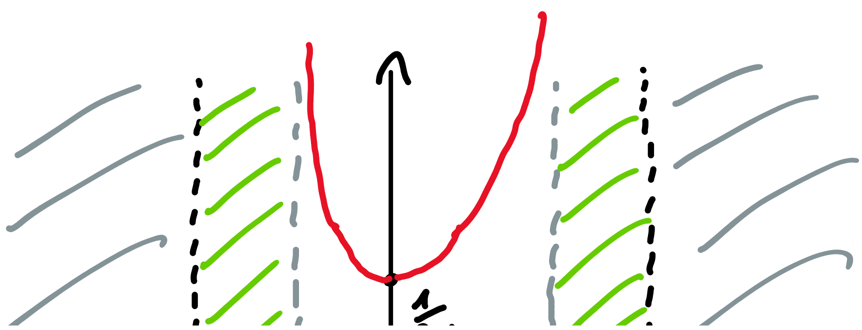
F_1 : $(4-x^2) > 0 \longrightarrow -2 < x < 2$

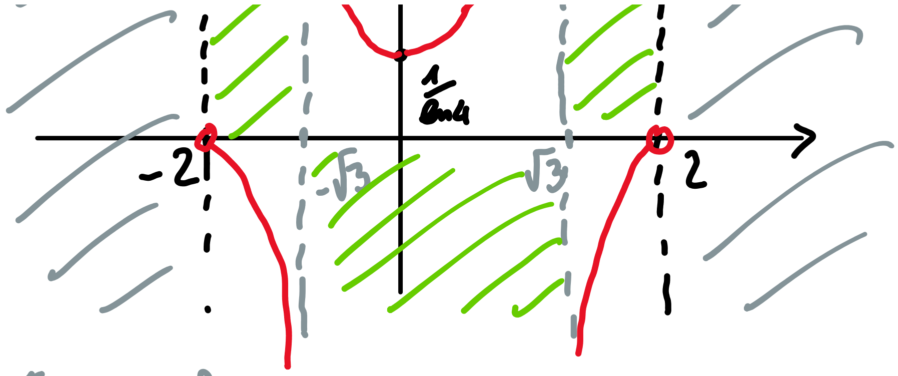
F_2 : $\ln^2(4-x^2) > 0 \longrightarrow \ln(4-x^2) \neq 0 \longrightarrow$

$\forall x \in (-2,2) \setminus \{-\sqrt{3}, +\sqrt{3}\}$]



$\rightarrow x = 0$ É PUNTO DI MIN RELATIVO





→ PUNTI ESCLUSI DAL DOMINIO

→ PUNTI ESCLUSI DAL SEGNO

$$f''(x) = \frac{2\cdot\left[(4-x^2)\ln^2(4-x^2)\right] - 2x\left[-2x\cdot\ln^2(4-x^2) + \cancel{(4-x^2)}\cdot 2\ln(4-x^2)\cdot\frac{1}{\cancel{4-x^2}}\cdot(-2x)\right]}{\left[(4-x^2)\ln^2(4-x^2)\right]^2}$$

$$= \frac{2(4-x^2)\ln^2(4-x^2) + 4x^2\ln^2(4-x^2) + 8x^2\ln(4-x^2)}{\left[(4-x^2)\ln^2(4-x^2)\right]^2}$$

$$= \frac{8\ln^2(4-x^2) - 2x^2\ln^2(4-x^2) + 4x^2\ln^2(4-x^2) + 8x^2\ln(4-x^2)}{\left[(4-x^2)\ln^2(4-x^2)\right]^2}$$

$$= \frac{8\ln^2(4-x^2) + 2x^2\ln^2(4-x^2) + 8x^2\ln(4-x^2)}{\left[(4-x^2)\ln^2(4-x^2)\right]^2}$$

2) $y = \sqrt{1-x} - \sqrt{x}$ C.E: $\begin{cases} 1-x \geq 0 \\ x \geq 0 \end{cases}$

$\begin{cases} x \leq 1 \\ x \geq 0 \end{cases} \rightarrow D = [0, 1]$

INTERSEZIONI : $x = 0 \rightarrow y = \underline{\sqrt{1-0}} - \sqrt{0} =$

$$= \sqrt{1} - 0 = 1$$

OSSIA IL PUNTO $(0,1)$

$$y=0 \longrightarrow 0 = \sqrt{1-x} - \sqrt{x}$$

$$\sqrt{x} = \sqrt{1-x}$$

$$x = 1-x \quad \text{OSSIA} \quad x = \frac{1}{2}$$

QUINDI IL PUNTO $\left(\frac{1}{2}, 0\right)$

SEGNO: $y \geq 0 \iff \sqrt{1-x} - \sqrt{x} \geq 0$

$$\sqrt{1-x} \geq \sqrt{x}$$

$$1-x \geq x$$

OSSIA $x \leq \frac{1}{2}$

LIMITI AGLI ESTREMI: $D = [0,1]$

$$\lim_{x \to 0^+} f(x), \quad \lim_{x \to 1^-} f(x)$$

$$\lim_{x \to 0^+} \sqrt{1-x} - \sqrt{x} = \sqrt{1-0} - \sqrt{0} = \sqrt{1} = 1$$

$$\lim_{x \to 1^-} \sqrt{1-x} - \sqrt{x} = \sqrt{1-1} - \sqrt{1} = -\sqrt{1} = -1$$

DERIVATE: $f(x) = \sqrt{1-x} - \sqrt{x} = (1-x)^{\frac{1}{2}} - (x)^{\frac{1}{2}}$

$$f'(x) = \frac{1}{2}(1-x)^{\frac{1}{2}-1} \cdot (-1) - \frac{1}{2} x^{\frac{1}{2}-1}$$

$$= -\frac{1}{2}(1-x)^{-\frac{1}{2}} - \frac{1}{2} x^{-\frac{1}{2}} = -\frac{1}{2} \cdot \frac{1}{\sqrt{1-x}} - \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$$

$$= -\frac{1}{2\sqrt{1-x}} - \frac{1}{2\sqrt{x}} = \frac{-1\cdot\sqrt{x} - \sqrt{1-x}}{2\sqrt{x}\,\sqrt{1-x}}$$

$$f'(x) \geq 0 \iff \frac{-\sqrt{x} - \sqrt{1-x}}{2\sqrt{x}\,\sqrt{1-x}} \geq 0$$

$N \geq 0$: $\underbrace{-\sqrt{x}}_{\leq 0} \underbrace{-\sqrt{1-x}}_{\leq 0} \geq 0$ $\to$ MAI PERCHÉ SOMMA DI 2 TERMINI NEGATIVI

$D > 0$: $2\sqrt{x}\cdot\sqrt{1-x} > 0$ $\to$ SEMPRE POSITIVO, TRANNE QUANDO

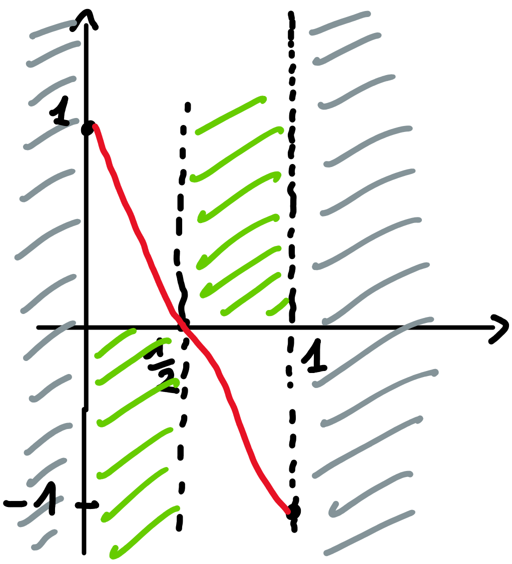
$\sqrt{x} = 0$ o $\sqrt{1-x} = 0$

$\downarrow$ $\qquad\qquad$ $\downarrow$

$x = 0$ $\qquad\qquad$ $x = 1$

(N)	$-$	o	$-$	o	$-$
(D)	$+$	|	$+$	|	$+$
	$\searrow$	0	$\searrow$	1	$\searrow$

$\to$ f É SEMPRE DECRESCENTE E NON HA PUNTI DI ESTREMO RELATIVO.



$$y' = -\frac{1}{2\sqrt{1-x}} - \frac{1}{2\sqrt{x}} = -\frac{1}{2}(1-x)^{-\frac{1}{2}} - \frac{1}{2}(x)^{-\frac{1}{2}}$$

$$y'' = -\frac{1}{2}\cdot\left(-\frac{1}{2}\right)(1-x)^{-\frac{1}{2}-1}\cdot(-1) - \frac{1}{2}\cdot\left(-\frac{1}{2}\right)(x)^{-\frac{1}{2}-1}$$

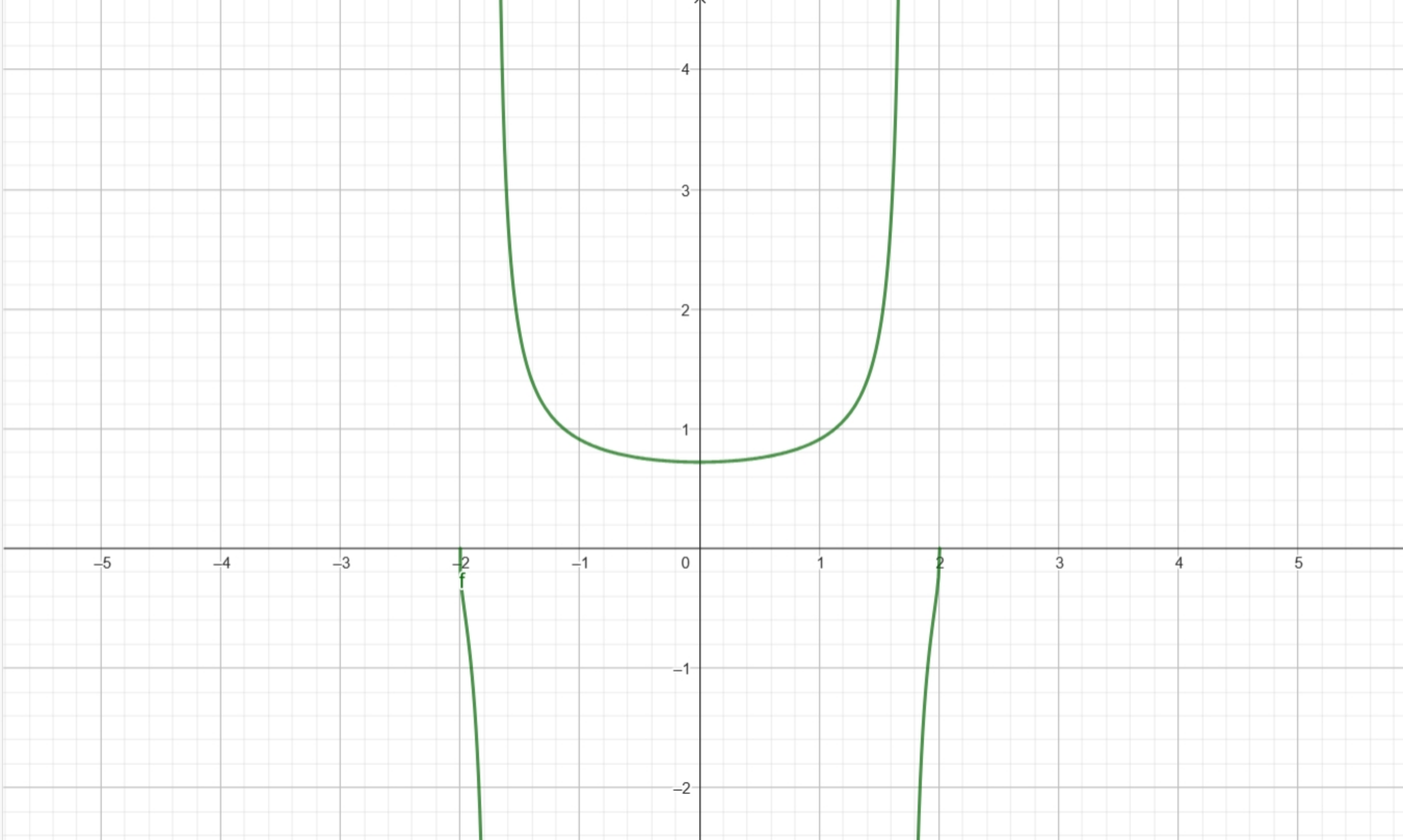
$$y'' = -\frac{1}{2} \cdot \left(-\frac{1}{2}\right)(1-x) \cdot (-1) - \frac{1}{2} \cdot \left(-\frac{1}{2}\right)(x)$$

$$= -\frac{1}{4}(1-x)^{-\frac{3}{2}} - \frac{1}{4}\, x^{-\frac{3}{2}} = -\frac{1}{4\sqrt{(1-x)^3}} - \frac{1}{4\sqrt{x^3}} \;.$$

ESERCIZIO: STUDIARE LA FUNZIONE (3) (VEDI INIZIO LEZIONE).

$f : y = \dfrac{1}{\ln(4 - x^2)}$

Inserimento...



$f : y = \sqrt{1 - x} - \sqrt{x}$

Inserimento...