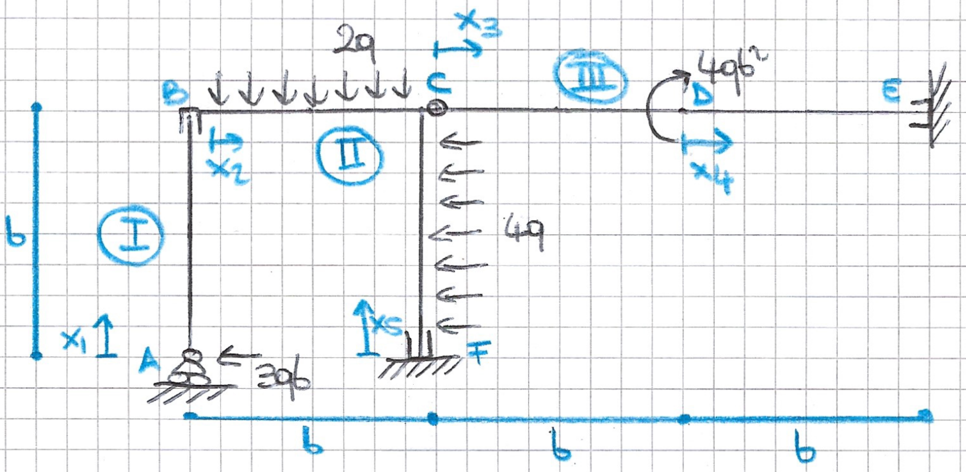
Esercizio 1 _ Traccia 1 _ Esame 20.10.2023

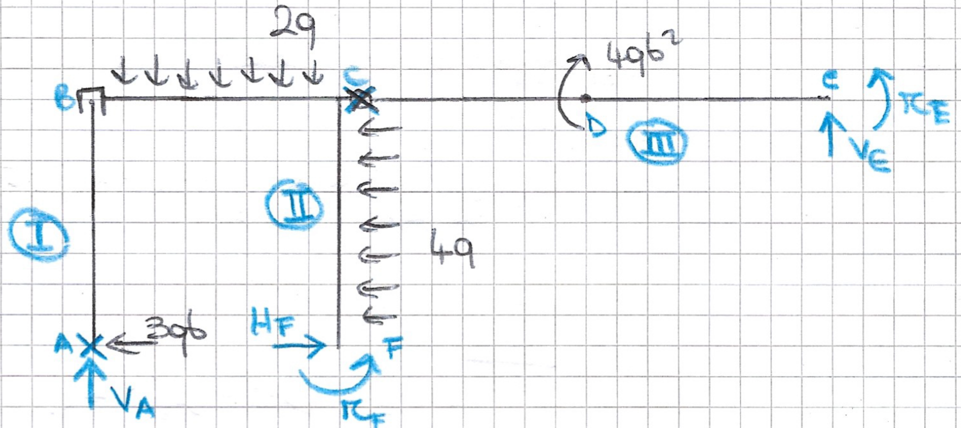


Struttura isostatica

GDL = 3(I) + 3(II) + 3(III) = 9

GDV = 1(A) + 2(B) + 2(C) + 2(E) + 2(F) = 9

GDL = GDV

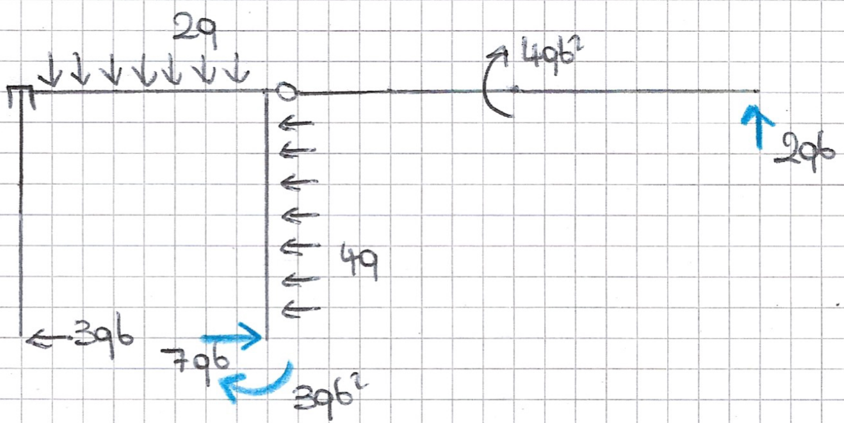


$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \mathcal{M}_{z(A)} = 0 \end{cases} \quad \begin{cases} H_F - 4qb - 3qb = 0 \Rightarrow \underline{H_F = 7qb} \\ V_A - 2qb + V_E = 0 \Rightarrow \underline{V_E = 2qb} \\ \mathcal{M}_F + \mathcal{M}_E + 4qb\left(\frac{b}{2}\right) - 2qb\left(\frac{b}{2}\right) - 4qb^2 + V_E(3b) = 0 \quad [*] \end{cases}$$

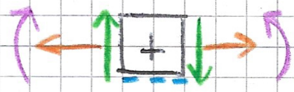
eq. aux.

$$\begin{cases} R_y^{(I)} = 0 \\ \mathcal{M}_{z(E)}^{(III)} = 0 \end{cases} \quad \begin{cases} \underline{V_A = 0} \\ \mathcal{M}_E + V_E(2b) - 4qb^2 = 0 \Rightarrow \mathcal{M}_E + 4qb^2 - 4qb^2 = 0 \Rightarrow \underline{\mathcal{M}_E = 0} \end{cases}$$

[*] $\mathcal{M}_F + 2qb^2 - qb^2 - 4qb^2 + 3bV_E = 0 \Rightarrow \mathcal{M}_F - 3qb^2 + 3b(2qb) = 0 \Rightarrow \underline{\mathcal{M}_F = -3qb^2}$



Azioni interne

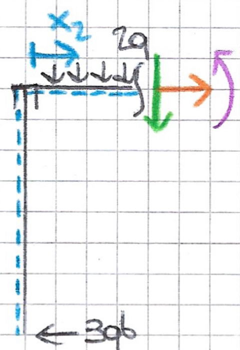


$0 \le x_1 \le b$

$N_{(x_1)} = 0$

$T_{(x_1)} - 3qb = 0 \Rightarrow T_{(x_1)} = 3qb$

$M_{(x_1)} + 3qb\,x_1 = 0 \Rightarrow M_{(x_1)} = -3qb\,x_1$

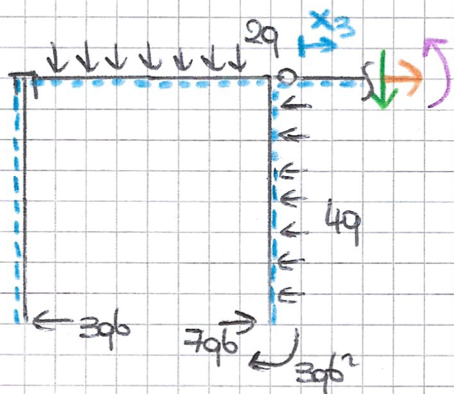


$0 \le x_2 \le b$

$N_{(x_2)} - 3qb = 0 \Rightarrow N_{(x_2)} = 3qb$

$T_{(x_2)} + 2q x_2 = 0 \Rightarrow T_{(x_2)} = -2q x_2$

$M_{(x_2)} + \cancel{2}q x_2 \left(\frac{x_2}{\cancel{2}}\right) - 3qb^2 = 0 \Rightarrow M_{(x_2)} = 3qb^2 - q x_2^2$



$0 \le x_3 \le b$

$N_{(x_3)} - \cancel{4qb} + \cancel{7qb} - \cancel{3qb} = 0$

$T_{(x_3)} + 2qb = 0$

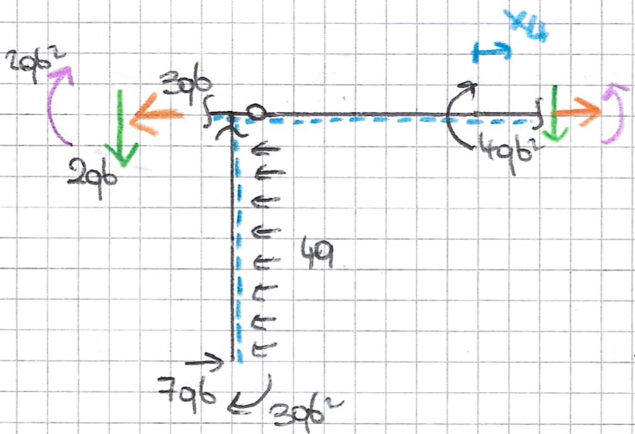
$M_{(x_3)} + 2qb\left(\frac{b}{2} + x_3\right) - \cancel{4}^{2}qb\left(\frac{b}{\cancel{2}}\right) + 7qb^2 - 3qb^2 - 3qb^2 = 0$

$N_{(x_3)} = 0$

$T_{(x_3)} = -2qb$

$M_{(x_3)} + \cancel{qb^2} + 2qb\,x_3 - \cancel{2qb^2} + \cancel{7qb^2} - \cancel{6qb^2} = 0$

$\Rightarrow M_{(x_3)} = -2qb\,x_3$



$0 \le x_4 \le b$

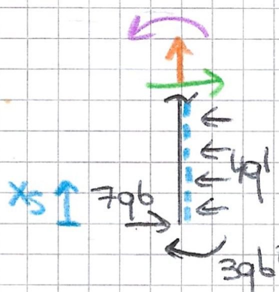
$N_{(x_4)} - 3qb - 4qb + 7qb = 0 \quad N_{(x_4)} = 0$

$T_{(x_4)} + 2qb = 0 \Rightarrow T_{(x_4)} = -2qb$

$M_{(x_4)} - 3qb^2 - \cancel{4}^{2}qb\left(\frac{b}{\cancel{2}}\right) - 2qb^2 + 2qb\,(b + x_4) + 7qb^2 - 4qb^2 = 0$

$\Rightarrow M_{(x_4)} - \cancel{7qb^2} + 2qb^2 + 2qb\,x_4 + \cancel{7qb^2} - 4qb^2 = 0$

$\Rightarrow M_{(x_4)} = -2qb\,x_4 + 2qb^2$



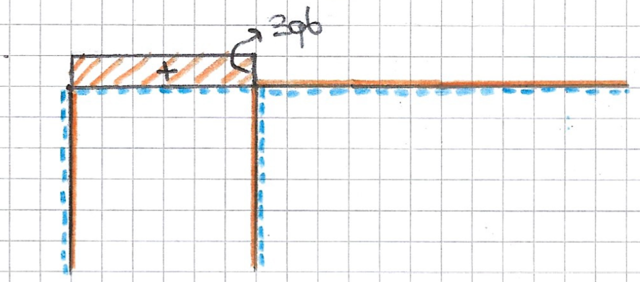
$0 \le x_5 \le b$

$N_{(x_5)} = 0$

$T_{(x_5)} + 7qb - 4qx_5 = 0 \Rightarrow T_{(x_5)} = 4qx_5 - 7qb$

$M_{(x_5)} - 3qb^2 + 7qbx_5 - \cancel{4}^{2}qx_5\left(\frac{x_5}{2}\right) = 0$

$\Rightarrow M_{(x_5)} = 3qb^2 - 7qbx_5 + 2qx_5^2$



N

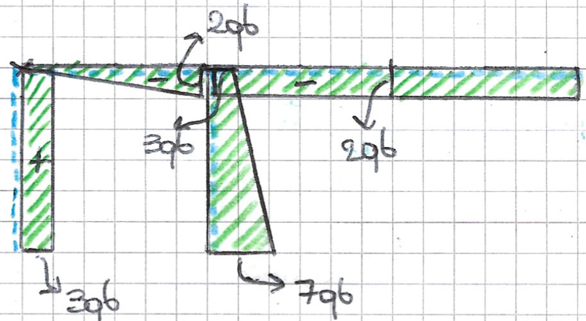
$N_{(x_1)} = 0$

$N_{(x_2)} = 3qb$

$N_{(x_3)} = 0$

$N_{(x_4)} = 0$

$N_{(x_5)} = 0$



T

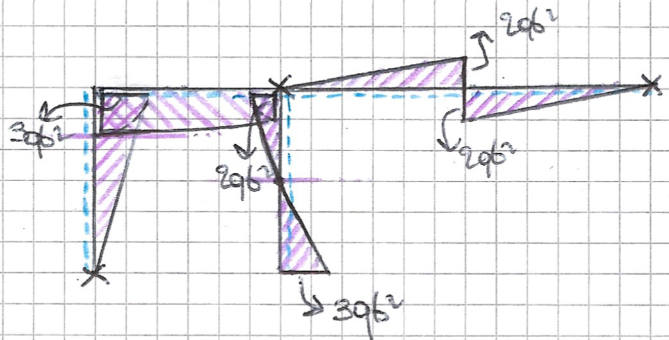
$T_{(x_1)} = 3qb$

$T_{(x_2)} = -2qx_2$

$T_{(x_3)} = -2qb$

$T_{(x_4)} = -2qb$

$T_{(x_5)} = 4qx_5 - 7qb$



M

$M_{(x_1)} = -3qbx_1$

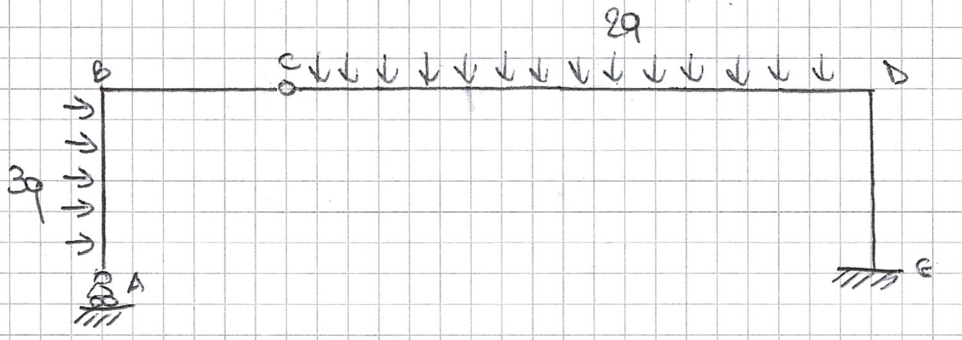
$M_{(x_2)} = 3qb^2 - qx_2^2$

$M_{(x_3)} = -2qbx_3$

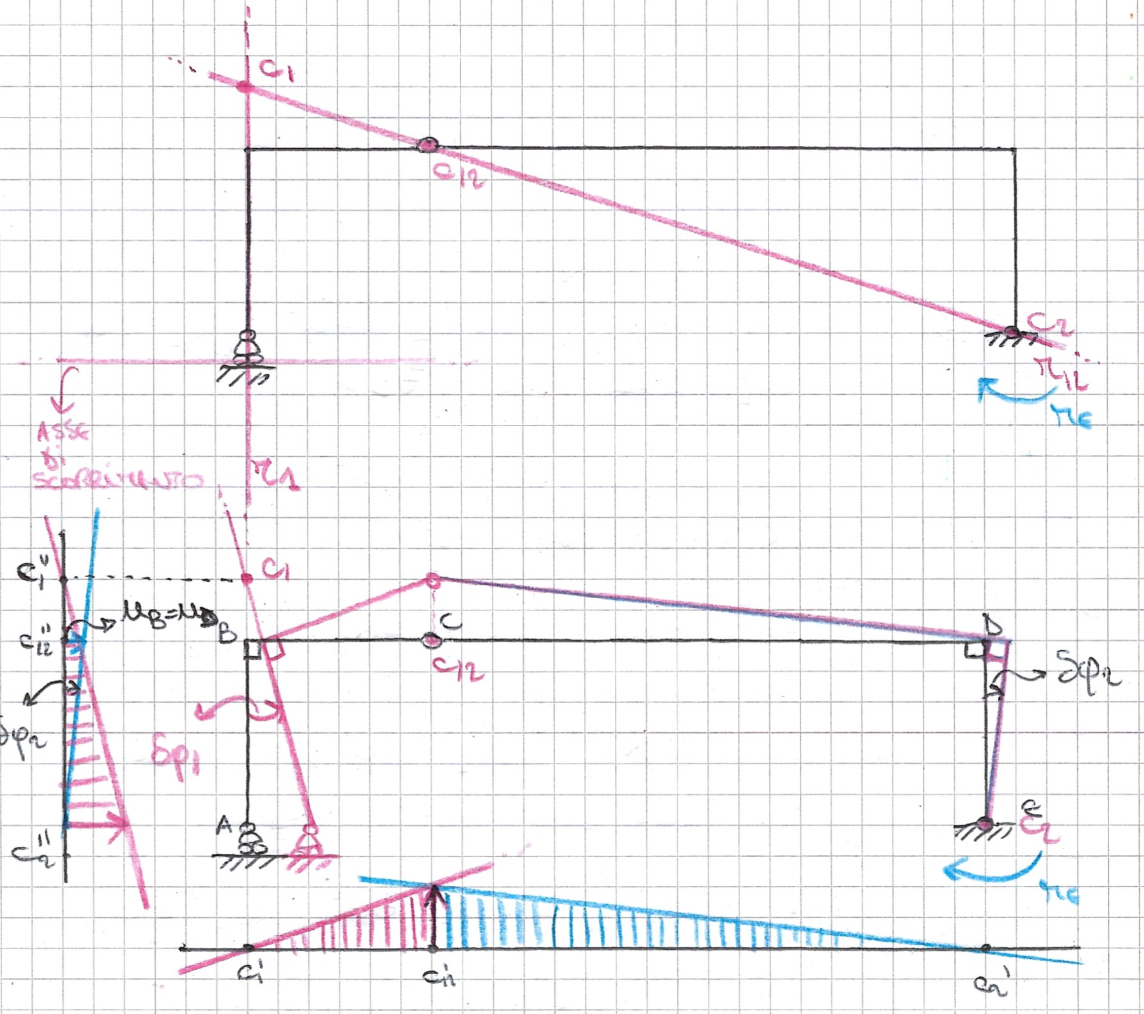
$M_{(x_4)} = -2qbx_4 + 2qb^2$

$M_{(x_5)} = 3qb^2 - 7qbx_5 + 2qx_5^2$

Esercizio 2 _ Traccia 1 - 20.10.2023



π_E ?



STRUTTURA UNA VOLTA IPOSTATICA

C.I.R.

$C_1 \in r_1$

$C_2 = E = (4b;0)$

$C_{12} = C = (b;b)$

CONDIZIONI CINEMATICHE

$C_1 \leftrightarrow C_{12} \leftrightarrow C_2$

$$\begin{cases} C_1 \in r_{12} \\ C_2 \in r_1 \end{cases}$$

$C_1 = (0; \frac{4}{3}b)$

$u_B = \frac{1}{3}b\,\delta\varphi_1$

$v_C^{(1)} = b\,\delta\varphi_1 = v_C^{(2)} = 3b\,\delta\varphi_2$

$\Rightarrow b\,\delta\varphi_1 = 3b\,\delta\varphi_2$

$\delta\varphi_1 = 3\,\delta\varphi_2$

Principio dei lavori virtuali $\quad \delta\mathcal{L} = 0 \ \forall \delta\varphi$

$$3qb\left(\frac{b}{2}+\frac{b}{3}\right)\delta\varphi_1 - 2q(3b)\left(\frac{3}{2}b\right)\delta\varphi_2 + \pi_E\,\delta\varphi_2 = 0$$

$$3qb\left(\frac{5}{6}b\right)\delta\varphi_1 - 9qb^2\,\delta\varphi_2 + \pi_E\,\delta\varphi_2 = 0$$

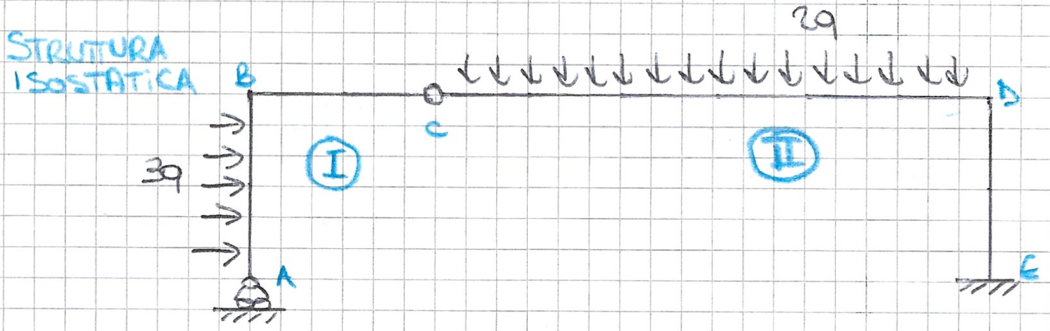
$$\frac{5}{2}qb^2\,\delta\varphi_1 - 9qb^2\,\delta\varphi_2 + \pi_E\,\delta\varphi_2 = 0$$

$$\frac{5}{2}qb^2(3\,\delta\varphi_2) - 9qb^2\,\delta\varphi_2 + \pi_E\,\delta\varphi_2 = 0$$

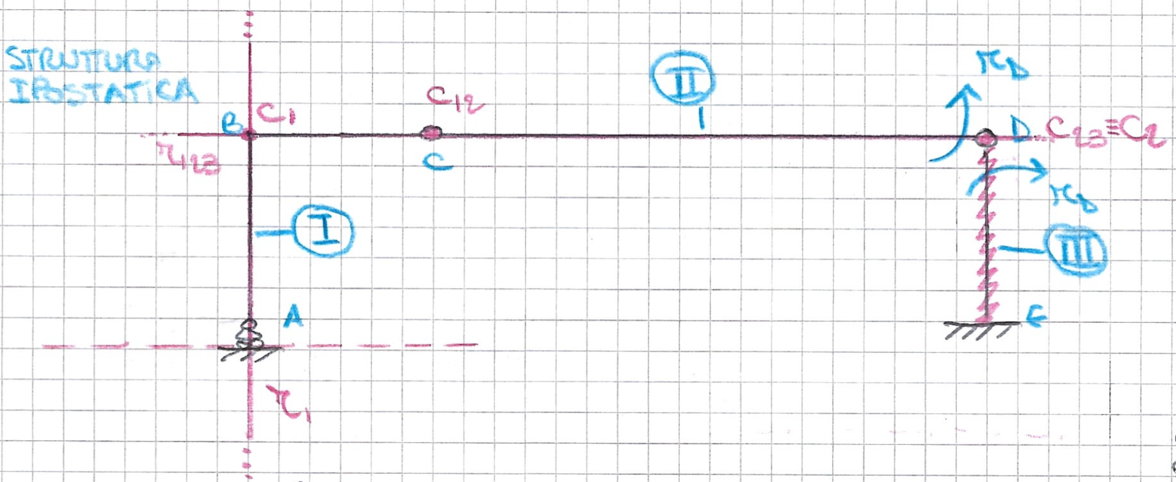
$$\frac{15}{2}qb^2\,\cancel{\delta\varphi_2} - 9qb^2\,\cancel{\delta\varphi_2} + \pi_E\,\cancel{\delta\varphi_2} = 0$$

$$\pi_E = 9qb^2 - \frac{15}{2}qb^2 \Rightarrow \pi_E = \frac{3}{2}qb^2$$

Esercizio 2_Traccia 1_Esame 20.10.2023



$r_D = ?$



CIR

$C_1 \in r_1$

$C_{12} = C = (b;b)$

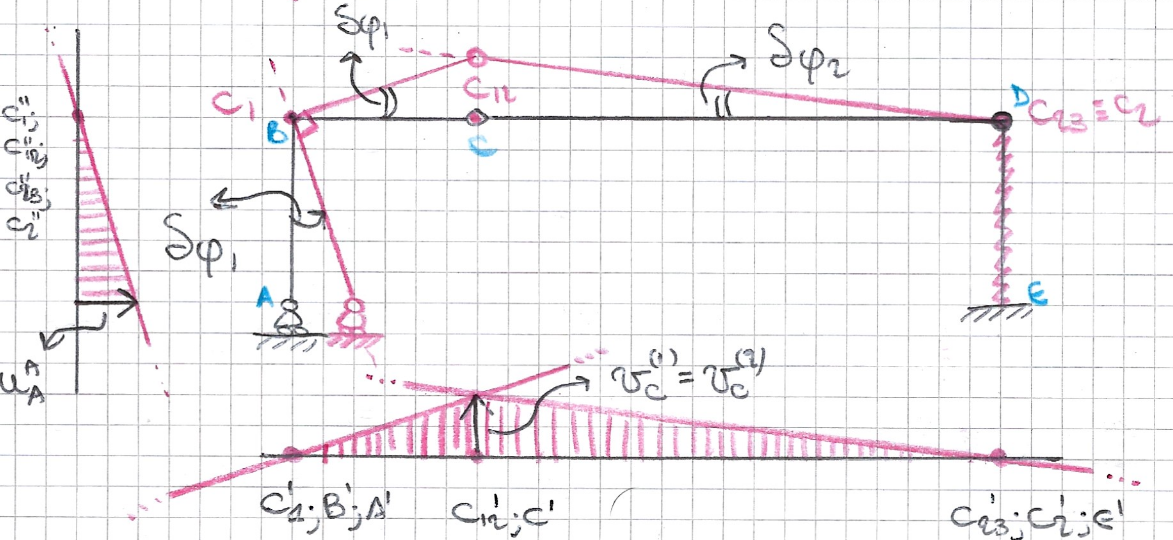
$C_{23} = D = (4b;b) = C_2$

$C_3 \nexists$

CONDIZIONI CINEMATICHE

$\begin{cases} C_1 \leftrightarrow C_{12} \leftrightarrow C_{23} \\ C_2 \leftrightarrow C_{23} \leftrightarrow C_{12} \end{cases}$

$\begin{cases} C_1 \in r_1 \\ C_1 \in r_{123} \end{cases}$



$C_1 = B = (0;b)$

$u_B = 0$

$v_C^{(1)} = b\,\delta\varphi_1$

$v_C^{(2)} = 3b\,\delta\varphi_2$

$v_C^{(1)} = v_C^{(2)}$

$b\,\delta\varphi_1 = 3b\,\delta\varphi_2$

$\delta\varphi_1 = 3\,\delta\varphi_2$

Pricipio dei lavori virtuali $\delta\mathcal{L} = 0$

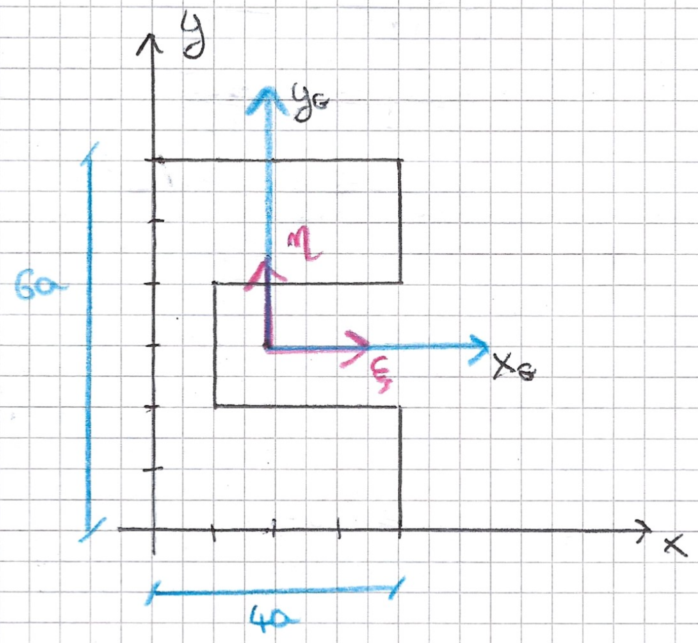
$$3qb\left(\frac{b}{2}\delta\varphi_1\right) - 2q(3b)\left(\frac{3b}{2}\delta\varphi_2\right) - r_D\,\delta\varphi_2 = 0$$

$$\frac{3}{2}qb^2\left(3\cancel{\delta\varphi_2}\right) - 9qb^2\cancel{\delta\varphi_2} - r_D\cancel{\delta\varphi_2} = 0$$

$$\frac{9}{2}qb^2 - 9qb^2 - r_D = 0$$

$$r_D = -9qb^2 + \frac{9}{2}qb^2 \Rightarrow r_D = -\frac{9}{2}qb^2$$

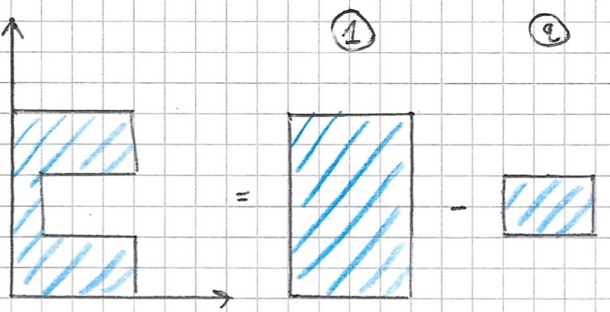
Esercizio 3 _ Traccia 1 _ Esame 20.10.2023



$S_x = A y_G \qquad S_y = A x_G$

$S_x = 54a^3 \qquad S_y = 33a^3$

MOMENTO STATICO



$S_x = S_{x_1} - S_{x_2}$

$S_{x_1} = A_1 y_{G_1}$ con $A_1 = 24a^2$; $y_{G_1} = 3a$

$S_{x_1} = (24a^2)(3a) = 72a^3$

$S_{x_2} = A_2 y_{G_2}$ con $A_2 = 6a^2$; $y_{G_2} = 3a$

$S_{x_2} = (6a^2)(3a) = 18a^3$

$S_x^{TOT} = 72a^3 - 18a^3 = 54a^3$

$S_y = S_{y_1} - S_{y_2}$

$S_{y_1} = A_1 x_{G_1}$ con $A_1 = 24a^2$; $x_{G_1} = 2a$

$S_{y_1} = (24a^2)(2a) = 48a^3$

$S_{y_2} = A_2 x_{G_2}$ con $A_2 = 6a^2$; $x_{G_2} = \frac{5}{2}a$

$S_{y_2} = \cancel{6}^{3} a^2 \left(\frac{5}{\cancel{2}}a\right) = 15a^3$

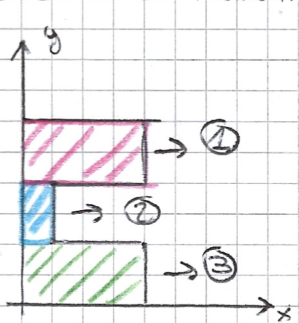
$S_y^{TOT} = 48a^3 - 15a^3 = 33a^3$

COORDINATE BARICENTRICHE

$y_G = \frac{S_x}{A} \Rightarrow y_G = \frac{54a^3}{18a^2} = 3a$

$x_G = \frac{S_y}{A} \Rightarrow x_G = \frac{33a^3}{18a^2} = \frac{11}{6}a \cong 1{,}833\,a$

MOMENTO DI INERZIA $\quad J_y = J_{y_1} + J_{y_2} + J_{y_3}$



$J_y^{TOT} = \frac{h_1 b_1^3}{3} + \frac{h_2 b_2^3}{3} + \frac{h_3 b_3^3}{3}$

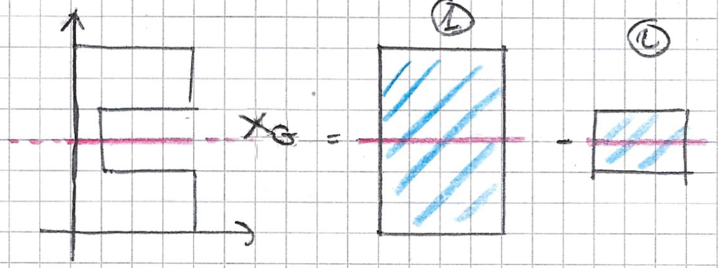
$= \frac{(2a)(64a^3)}{3} + \frac{(2a)(a^3)}{3} + \frac{(2a)(64a^3)}{3}$

$= \frac{128}{3}a^4 + \frac{2}{3}a^4 + \frac{128}{3}a^4 = 86a^4$

$J_{y_G} = J_y - A x_G^2 = 86a^4 - 18a^2\left(\frac{11}{6}a\right)^2$

$= 86a^4 - \cancel{18}^{1}a^2\left(\frac{121}{\cancel{36}_{2}}a^2\right) = \frac{51}{2}a^4 = 25{,}50a^4$

MOMENTO DI INERZIA $\quad J_{x_G} = J_{x_{G1}} - J_{x_{G2}}$



$$J_{x_G} = \frac{b_1 h_1^3}{12} - \frac{b_2 h_2^3}{12}$$

$$= \frac{4a(6a)^3}{12} - \frac{3a(8a^3)}{12}$$

$$= \frac{4a(216a^3)}{12} - \frac{24a^4}{12}$$

$$= \frac{864}{12} - \frac{24a^4}{12} = 70a^4$$

MOMENTO CENTRIFUGO

$J_{x_G y_G} = 0 \implies x_G$ ASSE DI SIMMETRIA

$$tg2\theta = \frac{-2\cancel{J_{x_G y_G}}}{J_{x_G} - J_{y_G}} = 0 \qquad tg2\theta = 0 \implies J_{x_G} > J_{y_G} \implies \theta = 0$$

$J_\xi = J_{max} = J_{x_G} = 70a^4$

$J_\eta = J_{min} = J_{y_G} = 25{,}500\ a^4$