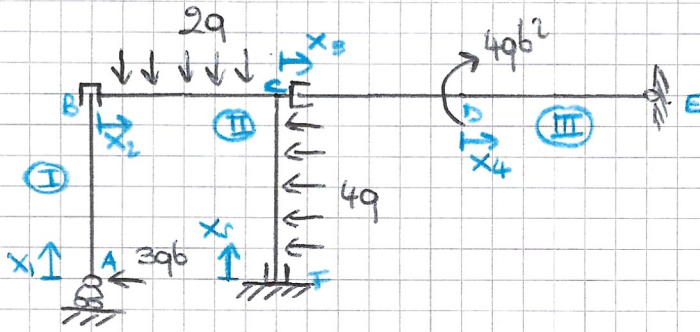


Esercizio 1 - Traccia 1 - Esame 05.09.2013

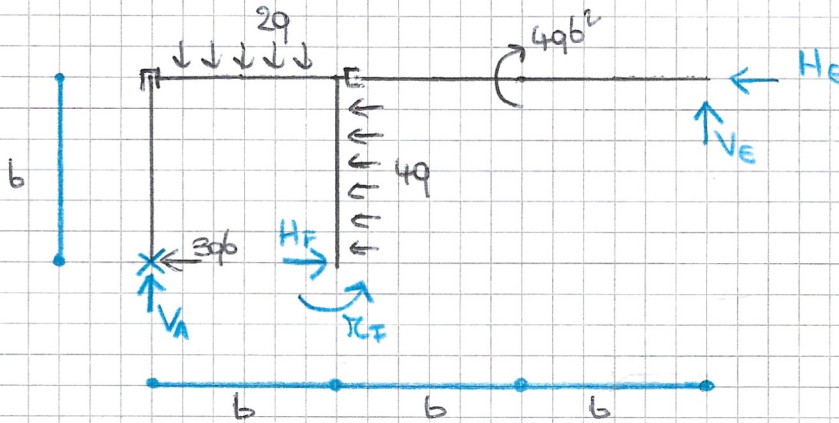


$$GDL = 3(I) + 3(II) + 3(III) = 9$$

$$GDV = 1(A) + 2(B) + 2(C) + 2(E) + 2(F) = 9$$

$$GDL = GDV$$

STRUTTURA ISOSTATICA



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \pi_z(A) = 0 \end{cases} \begin{cases} H_F - 3qb - H_E - 4qb = 0 \quad [1] \\ V_A - 2qb + V_E = 0 \quad [2] \\ \pi_F + 4qb\left(\frac{b}{2}\right) - 2qb\left(\frac{b}{2}\right) - 4qb^2 + V_E 3b + H_E b = 0 \quad [3] \end{cases}$$

eq. aux.

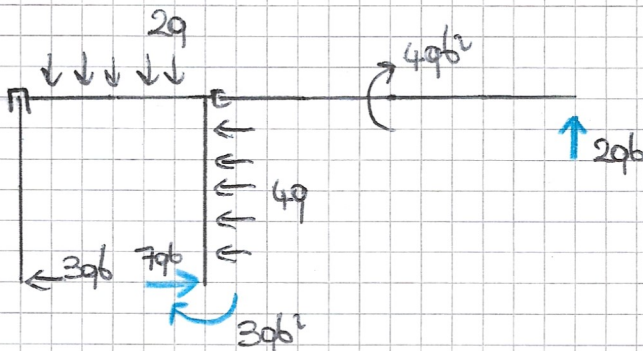
$$\begin{cases} R_y^{(I)} = 0 \\ R_x^{(III)} = 0 \end{cases} \begin{cases} \underline{V_A = 0} \\ \underline{H_E = 0} \end{cases}$$

$$[1] \quad \underline{H_F = 7qb}$$

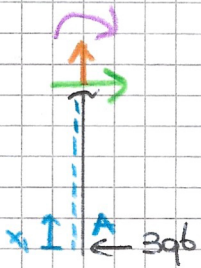
$$[2] \quad \underline{V_E = 2qb}$$

$$[3] \quad \pi_F + 2qb^2 - qb^2 - 4qb^2 + 2qb(3b) = 0$$

$$\Rightarrow \pi_F - 3qb^2 + 6qb^2 = 0 \Rightarrow \underline{\pi_F = -3qb^2}$$



AZIONI INTERNE



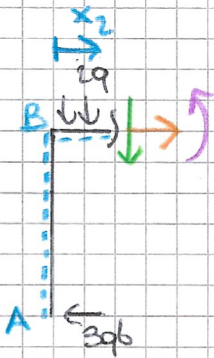
$$0 \leq x_1 \leq b$$

$$N(x_1) = 0$$

$$T(x_1) = 3qb$$

$$\mathcal{M}(x_1) + 3qb x_1 = 0$$

$$\Rightarrow \mathcal{M}(x_1) = -3qb x_1$$



$$0 \leq x_2 \leq b$$

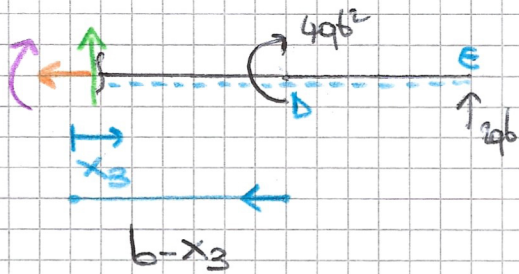
$$N(x_2) = 3qb$$

$$T(x_2) + 2q x_2 = 0$$

$$\Rightarrow T(x_2) = -2q x_2$$

$$\mathcal{M}(x_2) + 2q x_2 \left( \frac{x_2}{2} \right) - 3qb^2 = 0$$

$$\Rightarrow \mathcal{M}(x_2) = 3qb^2 - qx_2^2$$



$$0 \leq x_3 \leq b$$

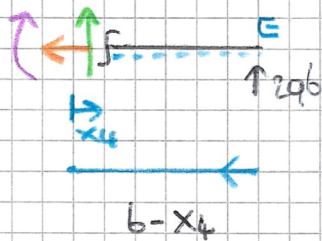
$$N(x_3) = 0$$

$$T(x_3) = -2qb$$

$$\mathcal{M}(x_3) + 4qb^2 - 2qb(b + b - x_3) = 0$$

$$\Rightarrow \mathcal{M}(x_3) + 4qb^2 - 4qb^2 + 2qb x_3 = 0$$

$$\Rightarrow \mathcal{M}(x_3) = -2qb x_3$$



$$0 \leq x_4 \leq b$$

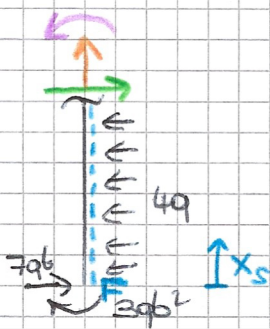
$$N(x_4) = 0$$

$$T(x_4) = -2qb$$

$$\mathcal{M}(x_4) - 2qb(b - x_4) = 0$$

$$\Rightarrow \mathcal{M}(x_4) - 2qb^2 + 2qb x_4 = 0$$

$$\Rightarrow \mathcal{M}(x_4) = 2qb^2 - 2qb x_4$$



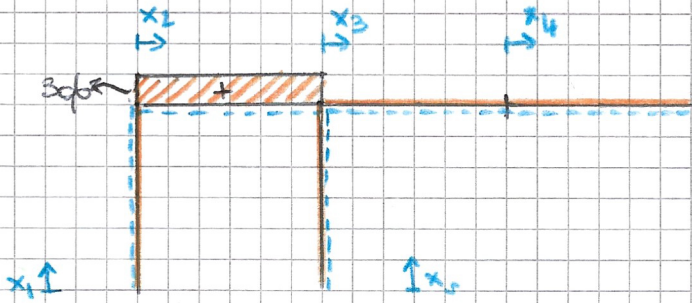
$$0 \leq x_5 \leq b$$

$$N(x_5) = 0$$

$$T(x_5) = 4q x_5 - 7qb$$

$$\mathcal{M}(x_5) - 3qb^2 - 4q x_5 \left( \frac{x_5}{2} \right) + 7qb x_5 = 0$$

$$\Rightarrow \mathcal{M}(x_5) = 3qb^2 - 7qb x_5 + 2q x_5^2$$



N

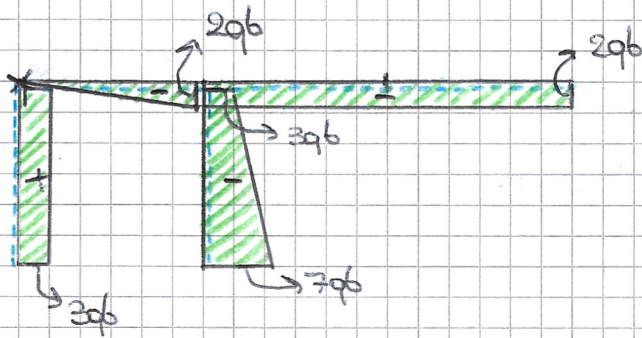
$$N(x_1) = 0$$

$$N(x_2) = 3qb$$

$$N(x_3) = 0$$

$$N(x_4) = 0$$

$$N(x_5) = 0$$



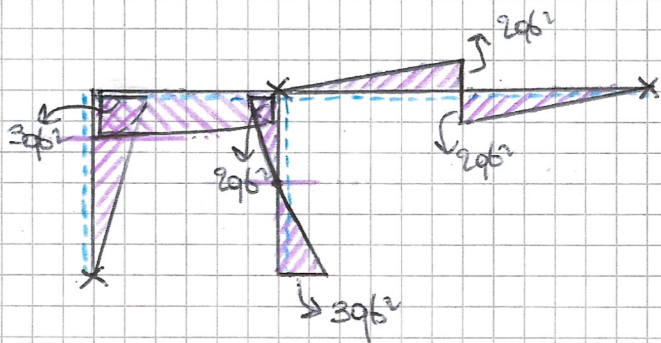
$$T(x_1) = 3qb$$

$$T(x_2) = -2qx_2$$

$$T(x_3) = -2qb$$

$$T(x_4) = -2qb$$

$$T(x_5) = -7qb + 4qx_5$$



M

$$M(x_1) = -3qb x_1$$

$$M(x_2) = 3qb^2 - qx_2^2$$

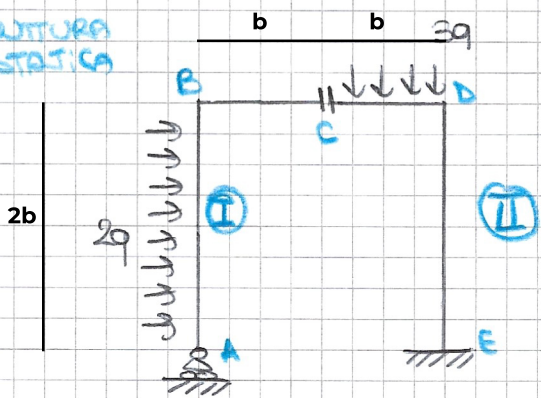
$$M(x_3) = -2qb x_3$$

$$M(x_4) = 2qb^2 - 2qb x_4$$

$$M(x_5) = 3qb^2 - 7qb x_5 + 2qx_5^2$$

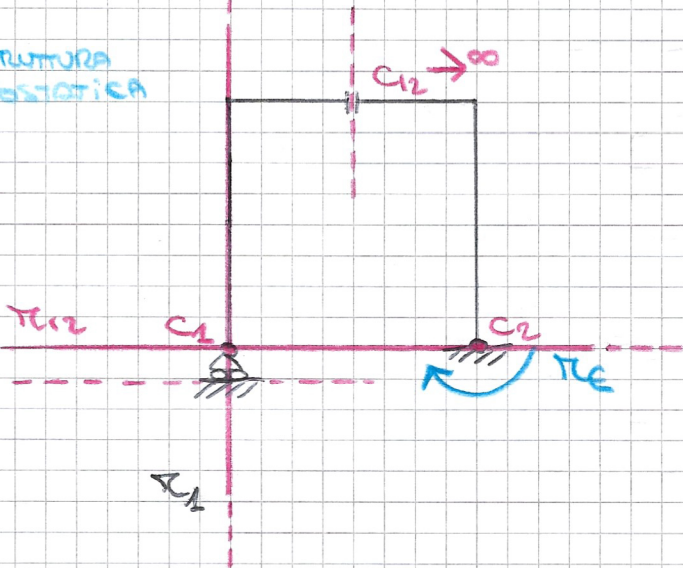
Esercizio 2 - Traccia 1

STRUTTURA ISOSTATICA



$\pi_C?$

STRUTTURA IPOTATICA



CIR

$$C_1 \in \pi_1$$

$$C_2 = E = (2b; 0)$$

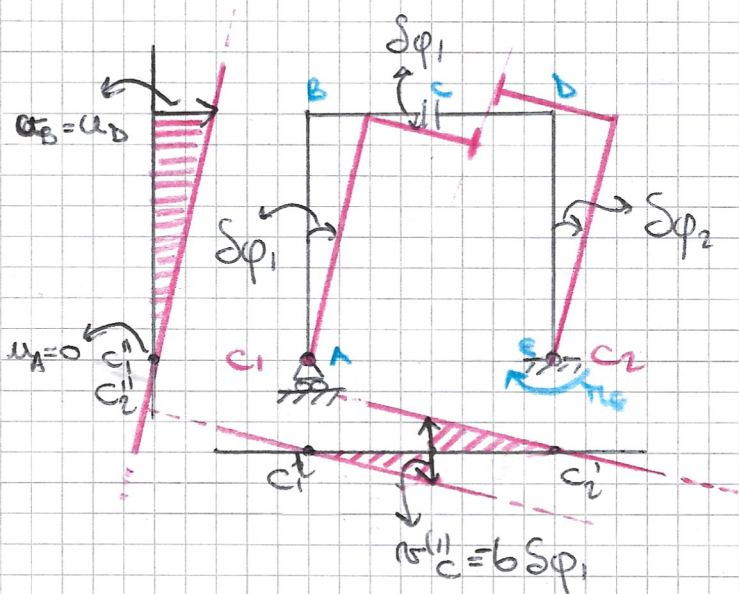
$$C_{12} = (\infty; 0)$$

CONDIZIONE CINEMATICA

$$C_1 \leftrightarrow C_{12} \leftrightarrow C_2$$

$$\begin{cases} C_1 \in \pi_{12} \\ C_1 \in \pi_1 \end{cases}$$

$$C_1 = A = (0; 0)$$



$$\delta\varphi_1 = \delta\varphi_2$$

$$u_A = 0$$

$$v_C^{(1)} = -b\delta\varphi_1$$

Principio dei lavori virtuali

$$\delta L = 0$$

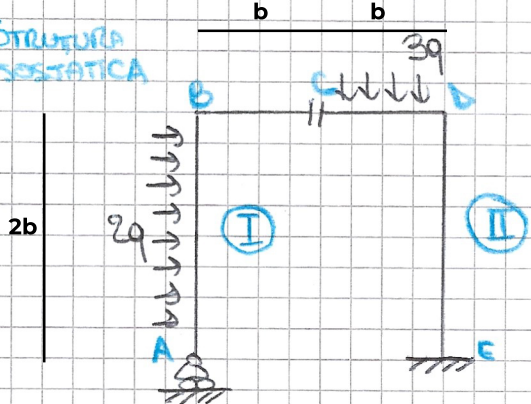
$$\pi_C \delta\varphi_2 + 2q(2b)(b\delta\varphi_1) - 3q(b)\left(\frac{b}{2}\delta\varphi_2\right) = 0$$

$$\delta\varphi_1 = \delta\varphi_2$$

$$\pi_C \delta\varphi_1 + 4qb^2\delta\varphi_1 - \frac{3}{2}qb^2\delta\varphi_1 = 0$$

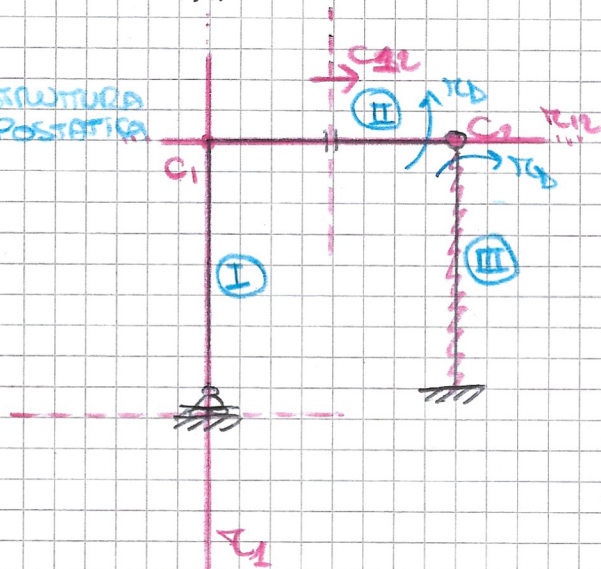
$$\pi_C = -\frac{5}{2}qb^2$$

STRUTTURA ISOSTATICA



$\pi_b$ ?

STRUTTURA IPOTATICA



CIR

$$C_1 \in \pi_1$$

$$C_2 = D = (2b; 2b)$$

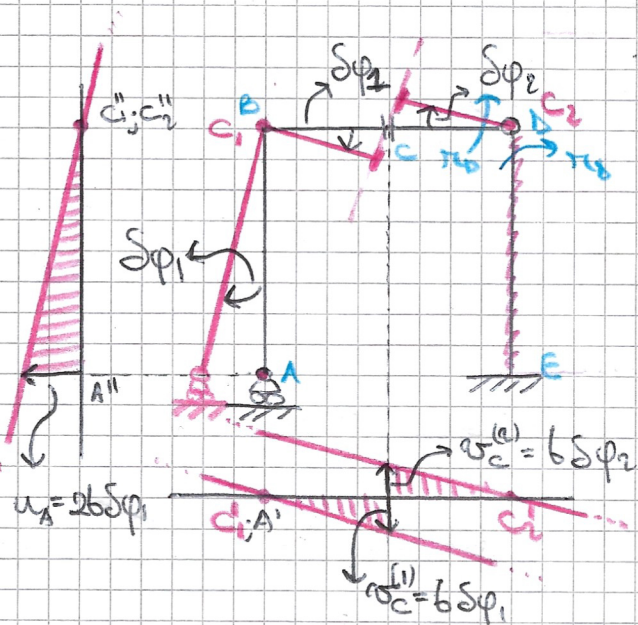
$$C_3 \notin$$

$$C_{12} = (\infty; 0)$$

CONDIZIONI CINEMATICHE

$$C_1 \leftrightarrow C_2 \leftrightarrow C_3$$

$$\begin{cases} C_2 \in \pi_{12} \\ C_1 \in \pi_1 \end{cases}$$



$$\delta \varphi_1 = \delta \varphi_2$$

$$u_A = 2b \delta \varphi_1$$

$$r_C^{(1)} = -b \delta \varphi_1$$

Principio dei lavori virtuali

$$\delta \mathcal{L} = 0$$

$$-\pi_b \delta \varphi_2 - 2q(2b)(b \delta \varphi_1) - 3qb \left(\frac{b}{2} \delta \varphi_2\right) = 0$$

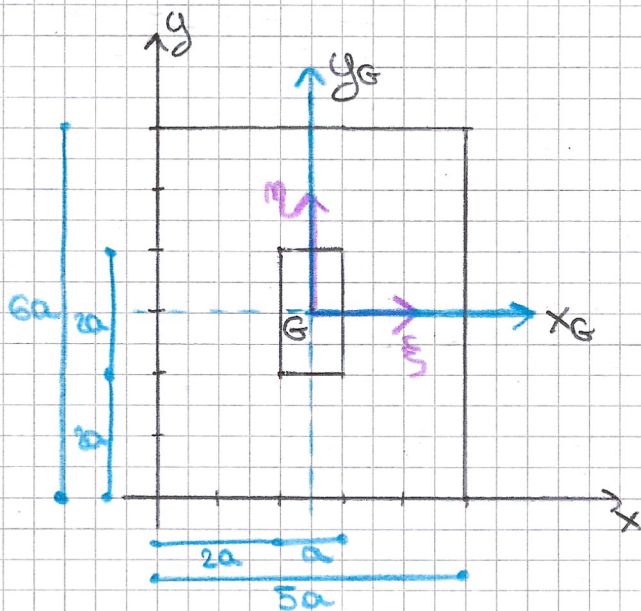
$$\delta \varphi_2 = \delta \varphi_1$$

$$-\pi_b \delta \varphi_1 - 4qb^2 \delta \varphi_1 - \frac{3}{2} qb^2 \delta \varphi_1 = 0$$

$$\pi_b + \frac{11}{2} qb = 0$$

$$\pi_b = -\frac{11}{2} qb$$

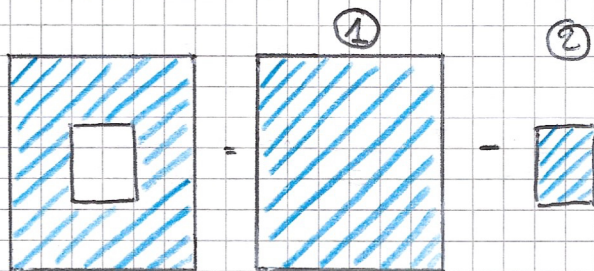
# Esercizio 3 Traccia 1 - Esame 05.09.2013



$$S_x^? ; S_y^? ; G(x_G; y_G)^? ;$$

$$J_{x_G}; J_{y_G}; J_{x_G y_G}; J_G; J_G$$

MOMENTO STATICO



$$S_x = (30a^2)(3a) - (2a^2)(3a)$$

$$= 90a^3 - 6a^3 = \underline{84a^3}$$

$$S_y = (30a^2)\left(\frac{5a}{2}\right) - (2a^2)\left(\frac{5a}{2}\right)$$

$$= 75a^3 - 5a^3 = \underline{70a^3}$$

$$y_G = \frac{84a^3}{30a^2} = \underline{3a} \quad x_G = \frac{70a^3}{30a^2} = \underline{\frac{5a}{2}}$$

$$S_x^{TOT} = S_x^{(1)} - S_x^{(2)}$$

$$S_x^{(1)} = A_1 y_{G1}^2 \quad \text{con } A_1 = 30a^2; y_{G1} = 3a$$

$$S_x^{(2)} = A_2 y_{G2}^2 \quad \text{con } A_2 = 2a^2; y_{G2} = 3a$$

$$S_x^{TOT} = S_x^{(1)} - S_x^{(2)}$$

$$S_y^{(1)} = A_1 x_{G1}^2 \quad \text{con } A_1 = 30a^2; x_{G1} = \frac{5a}{2} \approx 2,5a$$

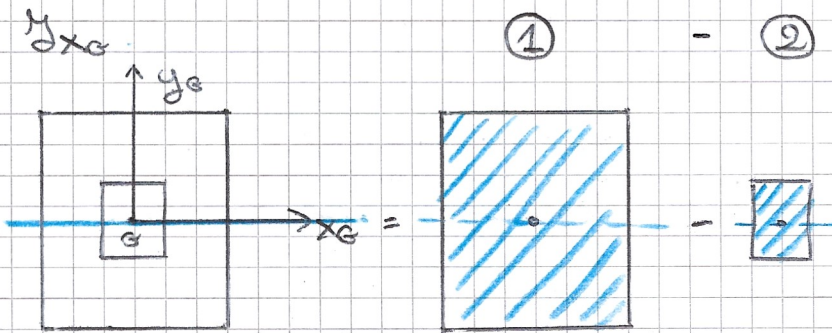
$$S_y^{(2)} = A_2 x_{G2}^2 \quad \text{con } A_2 = 2a^2; x_{G2} = 2,5a$$

COORDINATE BARICENTRICHE

$$S_x = A y_G \Rightarrow y_G = \frac{S_x}{A}$$

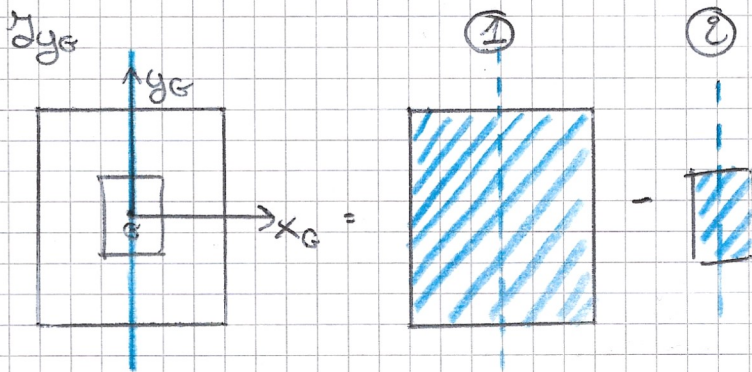
$$S_y = A x_G \Rightarrow x_G = \frac{S_y}{A}$$

## TORCENTI DI INERZIA



$$J_{x_G} = \frac{b_1 R_1^3}{12} - \frac{b_2 R_2^3}{12}$$

$$J_{x_G} = \frac{(5a)(216a^3)}{12} - \frac{(a)(8a^3)}{12} = \frac{1072}{12} a^4 = \frac{268}{3} a^4 \approx 89,333 a^4$$



$$J_{y_G} = \frac{R_1 b_1^3}{12} - \frac{R_2 b_2^3}{12} = \frac{(6a)(12sa^3)}{12} - \frac{(2a)(a^3)}{12} = \frac{748}{12} a^4 = \frac{187}{3} a^4 \approx 62,333 a^4$$

## TORCENTO CENTRIFUGO

$$J_{x_G y_G} = 0 \Rightarrow y_G \text{ e } x_G \text{ SONO ASSI DI SINFONIA}$$

$$\operatorname{tg} 2\theta = \frac{-2 J_{x_G y_G}}{J_{x_G} - J_{y_G}} = 0 \quad \operatorname{tg} 2\theta = 0 \Rightarrow J_{x_G} > J_{y_G} \Rightarrow \theta = 0$$

$$J_{\xi} = J_{\max} = J_{x_G} = \frac{268}{3} a^4 \approx 89,333 a^4$$

$$J_{\eta} = J_{\min} = J_{y_G} = \frac{187}{3} a^4 \approx 62,333 a^4$$