esame 23 novembre Matematica 3

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1)
$f(x,y) = 2x^3+y^2-2x \qquad D=\{2x^2+y^2\leq 1\}$

$f_x = 6x^2-2 \qquad f_y = 2y$

$\nabla f=0 \iff \begin{cases} 3x^2-1=0 \\ y=0 \end{cases} \quad \begin{cases} x^2=\frac{1}{3} \\ y=0 \end{cases} \quad x=-\frac{1}{\sqrt{3}},\ x=\frac{1}{\sqrt{3}}$

punti critici: $\left(-\frac{1}{\sqrt{3}},0\right), \left(\frac{1}{\sqrt{3}},0\right)$

$2\left(-\frac{1}{\sqrt{3}}\right)^2+0^2 = \frac{2}{3}\leq 1; \quad 2\left(\frac{1}{\sqrt{3}}\right)^2+0^2=\frac{2}{3}\leq 1$

i punti stanno nel dominio D.

$f_{xx}=12x \qquad f_{xy}=0 \qquad f_{yy}=2$

$Hf\left(-\frac{1}{\sqrt{3}},0\right) = \begin{pmatrix} -\frac{12}{\sqrt{3}} & 0 \\ 0 & 2 \end{pmatrix}$ matrice indefinita $\Rightarrow$ punto

$Hf\left(\frac{1}{\sqrt{3}},0\right) = \begin{pmatrix} \frac{12}{\sqrt{3}} & 0 \\ 0 & 2 \end{pmatrix}$ matrice definita positiva punto di minimo relativo.

$g=2x^2+y^2-1$

$\nabla g=(4x, 2y)$

$\begin{cases} 6x^2-2 = 4\lambda x \\ 2y=2\lambda y \\ 2x^2+y^2=1 \end{cases}$ II eq., $y\neq 0$ $\lambda=1$

$\begin{cases} 6x^2-2=4x \\ 2x^2+y^2=1 \end{cases} \quad \begin{cases} 3x^2-2x-1=0 \\ 2x^2+y^2=1 \end{cases} \qquad x=\frac{2\pm\sqrt{4+12}}{6} = -\frac{1}{3},\ 1$

III eq. $\quad 2\left(-\frac{1}{3}\right)^2+y^2=1 \longrightarrow \frac{2}{9}+y^2=1 \quad y^2=1-\frac{2}{9}$

$x=-\frac{1}{3} \quad y^2=\frac{7}{9} \quad y = -\frac{\sqrt{7}}{3},\ \frac{\sqrt{7}}{3} \qquad \left(-\frac{1}{3},-\frac{\sqrt{7}}{3}\right), \left(-\frac{1}{3},\frac{\sqrt{7}}{3}\right)$

$x=1 \quad 2(1)^2+y^2=1 \qquad 2+y^2=1 \quad y^2=-1$ impossibile!

II eq.

$\lambda\neq 1,\ y=0$

III eq. $2x^2+0^2=1 \quad 2x^2=1 \quad x^2=\frac{1}{2} \quad x=-\frac{1}{\sqrt{2}},\ x=\frac{1}{\sqrt{2}}$

I eq. $\quad 6x^2-2=4\lambda x$

$6\left(-\frac{1}{\sqrt{2}}\right)^2-2=4\lambda\left(-\frac{1}{\sqrt{2}}\right) \longrightarrow \frac{6}{2}-2=-\frac{4}{\sqrt{2}}\lambda \quad 1=-\frac{4}{\sqrt{2}}\lambda \quad \lambda=-\frac{\sqrt{2}}{4}$

$6\left(\frac{1}{\sqrt{2}}\right)^2-2=4\lambda\left(\frac{1}{\sqrt{2}}\right) \longrightarrow 1=\frac{4}{\sqrt{2}}\lambda \qquad \lambda=\frac{\sqrt{2}}{4}$

$$6\left(\frac{1}{\sqrt2}\right)^2 - 2 = 4\lambda\left(\frac{1}{\sqrt2}\right) \longrightarrow 1 = \frac{4}{\sqrt2}\lambda \qquad \lambda = \frac{\sqrt2}{4}$$

$$\left(-\frac{1}{\sqrt2},0\right) \qquad \left(\frac{1}{\sqrt2},0\right)$$

valutazione di tutti i punti trovati:

$$f\left(\frac{1}{\sqrt3},0\right) = 2\frac{1}{3\sqrt3} - \frac{2}{\sqrt3} = \frac{2-6}{3\sqrt3} = \frac{-4}{3\sqrt3} \sim -0{,}77$$ PUNTO DI MINIMO ASSOLUTO

$$f\left(-\frac{1}{3},-\frac{\sqrt7}{3}\right) = -\frac{2}{27} + \frac{7}{9} + \frac{2}{3} = \frac{2+21+18}{27} = \frac{37}{27} \sim 1{,}37$$

$$f\left(-\frac{1}{3},\frac{\sqrt7}{3}\right) = -\frac{2}{27} + \frac{7}{9} + \frac{2}{3} = \frac{37}{27} \sim 1{,}37$$ PUNTI DI MASSIMO ASSOLUTI

$$f\left(-\frac{1}{\sqrt2},0\right) = -\frac{2}{2\sqrt2} + \frac{2}{\sqrt2} = \frac{1}{\sqrt2} \sim 0{,}7$$

$$f\left(\frac{1}{\sqrt2},0\right) = \frac{2}{2\sqrt2} - \frac{2}{\sqrt2} = -\frac{1}{\sqrt2} \sim -0{,}7$$

2) $$\lim_{(x,y)\to(0,0)} xy^3 \frac{x+y}{\sqrt{x^2+y^2}}$$

$x=0 \quad \to 0$

$y=0 \quad \to 0$

$x=y \quad \displaystyle\lim_{x\to0} \frac{x^4\cdot 2x}{\sqrt{2x^2}} = \lim_{x\to0}\frac{2x^5}{\sqrt2\,|x|} \to 0$

$$\left|xy^3\frac{x+y}{\sqrt{x^2+y^2}}\right| = |xy^3|\left|\frac{x+y}{\sqrt{x^2+y^2}}\right| \leq |xy^3|\left(\frac{|x|}{\sqrt{x^2+y^2}} + \frac{|y|}{\sqrt{x^2+y^2}}\right) =$$

$$|xy^3|\left(\frac{\sqrt{x^2}}{\sqrt{x^2+y^2}} + \frac{\sqrt{y^2}}{\sqrt{x^2+y^2}}\right) = |xy^3|\left(\underbrace{\sqrt{\frac{x^2}{x^2+y^2}}}_{\leq 1} + \underbrace{\sqrt{\frac{y^2}{x^2+y^2}}}_{\leq 1}\right)$$

$$\leq |xy^3|\,(1+1) = |xy^3|\cdot 2 \xrightarrow[(x,y)\to(0,0)]{} 0$$

3) $$\begin{cases} x = \ln(1+v^2) \\ y = \operatorname{sen}(t+v) \end{cases}$$

$x_t = 0 \qquad y_t = \cos(t+v)$

$x_v = \frac{2v}{\ldots} \qquad y_v = \cos(t+v)$

$$X_t = 0 \qquad Y_t = \cos(t+v)$$
$$X_v = \frac{2v}{1+v^2} \qquad Y_v = \cos(t+v)$$

$$H_t = f_x \cdot 0 + f_y \cdot \cos(t+v) = f_y \cos(t+v)$$
$$H_v = f_x \frac{2v}{1+v^2} + f_y \cos(t+v)$$
$$H_{tt} = \left(f_{yx} \cdot 0 + f_{yy} \cdot \cos(t+v)\right)\cos(t+v) + f_y(-\operatorname{sen}(t+v))$$
$$= f_{yy}\cos^2(t+v) - f_y \operatorname{sen}(t+v)$$

$$H_{vt} = \frac{2v}{1+v^2}\left(f_{xx} \cdot 0 + f_{xy} \cdot \cos(t+v)\right) + \left(f_{yx} 0 + f_{yy} \cdot \cos(t+v)\right) \cdot \cos(t+v)$$
$$+ f_y(-\operatorname{sen}(t+v))$$
$$= \frac{2v}{1+v^2}\cos(t+v) f_{xy} + f_{yy}\cos^2(t+v) - f_y \operatorname{sen}(t+v)$$
$$H_{vtt} = \frac{2v}{1+v^2}\left\{-\operatorname{sen}(t+v) \cdot f_{xy} + \cos(t+v)\left[f_{xyx} \cdot 0 + f_{xyy} \cdot \cos(t+v)\right]\right\}$$
$$+ \left[f_{yyx} \cdot 0 + f_{yyy} \cdot \cos(t+v)\right]\cos^2(t+v) + f_{yy} \cdot 2\cos(t+v) \cdot \left[-\operatorname{sen}(t+v)\right]$$
$$- \left[f_{yx} \cdot 0 + f_{yy} \cdot \cos(t+v)\right]\operatorname{sen}(t+v) - f_y \cos(t+v)$$

$$= -\frac{2v}{1+v^2}\operatorname{sen}(t+v) \cdot f_{xy} + \frac{2v}{1+v^2}\cos^2(t+v) f_{xyy} + f_{yyy}\cos^3(t+v)$$
$$- 2\cos(t+v)\operatorname{sen}(t+v) f_{yy} - f_{yy}\cos(t+v)\operatorname{sen}(t+v) - f_y \cos(t+v)$$

4) $$f(x,y) = \begin{cases} xy - y & x \neq 0 \\ -y^2 & x = 0 \end{cases}$$

$(h,0) \rightsquigarrow x \neq 0$ caso 1

$x=0 \rightsquigarrow = 0$

$$\frac{\partial f}{\partial x}(0,0) = f_x(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h \to 0} \frac{(h \cdot 0 - 0) - 0}{h} = 0$$

$(0,h)$ $x=0$ caso 2

$$\frac{\partial f}{\partial y}(0,0) = f_y(0,0) = \lim_{h \to 0} \frac{f(0,h) - f(0,0)}{h} = \lim_{h \to 0} \frac{-h^2 - 0}{h} = \lim_{h \to 0} -h = 0$$

$\nabla f(0,0) = (0,0)$ $\qquad (h,k)$ $x \neq 0 \rightsquigarrow$ caso 1

$$\lim_{(h,k) \to (0,0)} \frac{\left[(0+h)(0+k) - (0+k)\right] - \left[-0^2\right] - (0,0) \cdot (h,k)}{\sqrt{h^2 + k^2}}$$

$$\lim_{(h,k)\to(0,0)} \frac{\qquad}{\sqrt{h^2+k^2}}$$

$$= \lim_{(h,k)\to(0,0)} \frac{hk - k}{\sqrt{h^2+k^2}} = \lim_{(h,k)\to(0,0)} \frac{k(h-1)}{\sqrt{h^2+k^2}}$$

$h=0$ $\quad \lim_{k\to 0} \frac{-k}{\sqrt{k^2}} = \lim_{k\to 0} \frac{-k}{|k|}$ non esiste

il limite non esiste, quindi la funzione non è differenziabile e non esiste il piano tangente nell'origine.