Simulazione 4

2) $\int_\gamma \frac{\ln(y)}{y}\, ds$ $\qquad \gamma(t) = (t\ln t - t, t) \quad t\in[1,2]$

$\gamma'(t) = \left(1\cdot\ln t + t\cdot\frac{1}{t} - 1, 1\right) = (\ln t + 1 - 1, 1) = (\ln t, 1)$

$\|\gamma'(t)\| = \sqrt{\ln^2 t + 1}$

$$\int_\gamma \frac{\ln y}{y}\, ds = \int_1^2 \frac{\ln(t)}{t}\cdot\sqrt{\ln^2 t + 1}\, dt$$

$$\left[\left(\ln^2 t + 1\right)^{3/2}\right]' = \frac{3}{2}\cdot\left(\ln^2(t)+1\right)^{\frac{1}{2}}\cdot 2\ln t\cdot\frac{1}{t} = 3\sqrt{\ln^2 t + 1}\cdot\frac{\ln t}{t}$$

$$= \left[\frac{(\ln^2 t + 1)^{3/2}}{2\cdot 3/2}\right]_1^2 =$$

$$= \frac{(\ln^2 2 + 1)^{3/2}}{3} - \frac{1}{3}$$

3 $F(x,y) = (\underbrace{\cos(x+y) - \operatorname{sen}(x-y)}_{F_1}, \underbrace{\cos(x+y) + \operatorname{sen}(x-y)}_{F_2})$

$\frac{\partial F_1}{\partial y} = -\operatorname{sen}(x+y) + \cos(x-y)$ $\qquad \frac{\partial F_2}{\partial x} = -\operatorname{sen}(x+y) + \cos(x-y)$

sono uguali $\Rightarrow$ è irrotazionale (in $\mathbb{R}^2$ s.c.) $\Rightarrow$ conservativo

$\begin{cases} g_x = F_1 \\ g_y = F_2 \end{cases}$ $\quad (\nabla g = F)$

$g = \int F_1\, dx = \int \cos(x+y) - \operatorname{sen}(x-y)\, dx = \operatorname{sen}(x+y) + \cos(x-y) + \phi(y)$

$g_y = \cos(x+y) + \operatorname{sen}(x-y) + \phi'(y) = \cos(x+y) + \operatorname{sen}(x-y)$

$\phi'(y) = 0 \qquad \phi = c$

$g(x,y) = \operatorname{sen}(x+y) + \cos(x-y) + C$

$(1,1) \xrightarrow{\gamma} (2,1)$ segmento $\qquad \int_\gamma F\cdot T_\gamma\, ds = \int_1^2 (\cos(t+1) - \operatorname{sen}(t-1), \cos(t+1) + \operatorname{sen}(t-1))\cdot(1,0)\, dt$

$\gamma = \begin{cases} x = t \\ y = 1 \end{cases} \quad t\in[1,2] \qquad \gamma'(t) = \begin{cases} x' = 1 \\ y' = 0 \end{cases}$

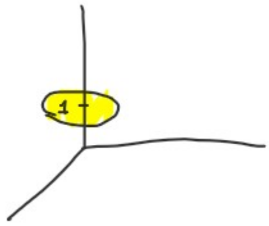
$= \int_1^2 \cos(t+1) - \operatorname{sen}(t-1)\, dt = \left[\operatorname{sen}(t+1) + \cos(t-1)\right]_1^2$

$= \operatorname{sen}(3) + \cos(1) - \operatorname{sen}(2) - \cos(0)$

non c'era bisogno di svolgere l'integrale!

$= g(2,1) - g(1,1) = \operatorname{sen}(3) + \cos(1) - (\operatorname{sen}(2) + \cos(0))$

4) $f(x,y,z) = xy$ $\qquad \Sigma = \{ (x,y,1) \,,\ (x,y) \in D = \{x^2+y^2 \le 1\} \}$



$\varphi(x,y) = (x,y,\overset{g(x,y)}{1})$ $\qquad \varphi_x(x,y) = (1,0,0)$

$\sqrt{1+g_x^2+g_y^2}$ $\qquad \varphi_y(x,y) = (0,1,0)$

$$\varphi_x \wedge \varphi_y = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = i\cdot(0) - j\cdot(0) + k(1) = 1$$

$$= (0,0,1)$$

$\|\varphi_x \wedge \varphi_y\| = \sqrt{1} = 1$

non c'è bisogno di fare il calcolo: $\|\varphi_x \wedge \varphi_y\| = \sqrt{f_x^2+f_y^2+1}$ $\qquad (x,y,f(x,y))$

$$\iint_\Sigma xy \, d\sigma = \iint_D xy \cdot \sqrt{1} \, dx\, dy = \iint_D xy \, dx dy = \int_0^{2\pi} \int_0^1 \rho\cos\theta \cdot \rho\,\text{sen}\theta \cdot \rho \, d\rho\, d\theta$$

$D = \{ x^2+y^2 \le 1 \}$ $\qquad \begin{cases} x = \rho\cos\theta \\ y = \rho\,\text{sen}\theta \end{cases}$ $\qquad \theta \in [0,2\pi]$, $\rho \in [0,1]$

$$= \int_0^{2\pi} \left[\frac{\rho^4}{4} \right]_0^1 \cos\theta\,\text{sen}\theta \, d\theta = \int_0^{2\pi} \frac{1}{4}\cos\theta\,\text{sen}\theta \, d\theta = \left[\frac{1}{4}\frac{\text{sen}^2\theta}{2} \right]_0^{2\pi} = \frac{1}{4} \cdot 0 = 0$$

$$[\text{sen}^2(\theta)]' = 2 \cdot \text{sen}^{2-1}\theta \cdot \cos\theta = 2\,\text{sen}x \cdot \cos x$$

5) $\begin{cases} y' = y\cos x \\ y(2\pi) = 3 \end{cases}$ $\qquad \frac{y'}{y} = \cos x$ $\qquad \int \frac{dy}{y} = \int \cos x \, dx$ $\qquad \ln y = \text{sen}\, x + c$

$y = e^{\text{sen}\,x + c}$

$y(2\pi) = e^{\text{sen}\,2\pi + c} = e^{0+c} = e^c = 3$ $\qquad c = \ln 3$

$$\boxed{y = e^{\text{sen}\,x + \ln 3}}$$

6) $y'' - 2y' + y = 0$ $\qquad \lambda^2 - 2\lambda + 1 = 0$

$(\lambda-1)^2 = 0$ $\qquad \lambda = 1$

$y(x) = k_1 \cdot e^x + k_2 \cdot x \cdot e^x$

$y'' - 3y' + 2y = 3x$ $\qquad \lambda^2 - 3\lambda + 2 = 0$ $\qquad$ sol $\lambda = 2$, $\lambda = 1$

$(\lambda-2)(\lambda-1) = 0$

$y(x) = k_1 e^{2x} + k_2 e^x + y_s$

$$y(x) = k_1 e^{2x} + k_2 e^{x} + y_s$$

$y_s = ax + b \qquad y_s' = a \qquad y_s'' = 0$

$y_s'' - 3y_s' + 2y_s = 0 - 3a + 2ax + 2b \overset{?}{=} 3x$

$x(2a) + 2b - 3a = 3x + 0$

$\begin{cases} 2a = 3 \\ 2b - 3a = 0 \end{cases} \qquad a = \frac{3}{2} \qquad 2b - \frac{9}{2} = 0 \qquad 2b = \frac{9}{2} \qquad b = \frac{9}{4}$

$$y(x) = k_1 e^{2x} + k_2 e^{x} + \frac{3}{2}x + \frac{9}{4}$$

Simulazione 5

1) $\iint_D \frac{2x}{1+x^2+y^2}\,dx\,dy \qquad D = \{x^2+y^2 \le 1\} = \{-1 \le y \le 1 \wedge -\sqrt{1-y^2} \le x \le \sqrt{1-y^2}\}$

$$\int_{-1}^{1}\left(\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{2x}{1+x^2+y^2}\,dx\right)dy = \int_{-1}^{1} \left[\ln(1+x^2+y^2)\right]_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dy =$$

$$= \int_{-1}^{1} \ln(1+1-y^2+y^2) - \ln(1+1-y^2+y^2)\,dy = 0$$

oppure:
$$\int_0^{2\pi}\int_0^1 \frac{2\rho\cos\theta}{1+\rho^2}\cdot \rho\,d\rho\,d\theta = \int_0^1 \frac{2\rho^2}{1+\rho^2}\left[\operatorname{sen}\theta\right]_0^{2\pi} d\rho = \int_0^1 \frac{2\rho^2}{1+\rho^2}\cdot 0$$

$$\left(\int \frac{2\rho^2}{1+\rho^2}\,d\rho = 2\int \frac{\rho^2+1-1}{1+\rho^2}\,d\rho = 2\int \frac{\rho^2+1}{1+\rho^2}d\rho + 2\int \frac{-1}{1+\rho^2}\,d\rho = 2\rho - 2\operatorname{arctg}(1+\rho^2)\right)$$

$\iiint_T e^z\,dx\,dy\,dz \qquad T = \{0 \le x \le 1,\ x \le y \le 2x,\ 5-x-y \le z \le 1\}$

$$\int_0^1\left(\int_x^{2x}\left(\int_{5-x-y}^{1} e^z\,dz\right)dy\right)dx = \int_0^1\left(\int_x^{2x}\left[e^z\right]_{5-x-y}^{1} dy\right)dx =$$

$$= \int_0^1\left(\int_x^{2x} e^1 - e^{5-x-y}\,dy\right)dx = \int_0^1 \left[ey + e^{5-x-y}\right]_x^{2x} dx =$$

$$= \int_0^1 \left[e\cdot 2x + e^{5-x-2x} - ex - e^{5-x-x}\right] dx$$

$$\int_0^1 ex + e^{5-3x} - e^{5-2x}\,dx = \left[\frac{ex^2}{2} - \frac{e^{5-3x}}{3} + \frac{e^{5-2x}}{2}\right]_0^1 = \ldots$$

2) $\int_\gamma z \, ds \qquad \gamma = (t\cos t, t\sin t, t) \qquad t \in [1,2]$

$\gamma' = (\cos t - t\sin t, \sin t + t\cos t, 1)$

$\|\gamma'\| = \sqrt{1 + t^2 + 1} \to \sqrt{2+t^2}$

$\int_1^2 t\sqrt{2+t^2}\, dt = \left[\frac{(1+t^2)^{3/2}}{3/2 \cdot 2}\right]_1^2$

3) $F = (x(5x^3 + 8y^3), 3y^2(4x^2+1))$ (verificare che è irrotazionale)

$5x^4 + 8xy^3 \qquad 12x^2y^2 + 3y^2$

$f_x = F_1 \qquad f = \int 5x^4 + 8xy^3 dx = x^5 + 4x^2y^3 + \phi(y)$

$f_y = 0 + 12x^2y^2 + \phi'(y) = 12x^2y^2 + 3y^2$

$\phi'(y) = 3y^2 \Rightarrow \phi(y) = \int 3y^2 dy = y^3 + c$

$f = x^5 + 4x^2y^3 + y^3 + c$

lavoro lungo γ circonferenza ecc... curva chiusa $\Rightarrow = 0$

4) $\iint_\Sigma \frac{(z - \frac{x^2}{2})x}{\sqrt{x^2+y^2}} d\sigma \qquad \Sigma = \{(x, y, \overset{g}{\frac{x^2}{2} - y}) \quad \overbrace{x^2+y^2 \leq 1}^{D}\}$

$\|\varphi_x \wedge \varphi_y\| = \sqrt{1 + g_x^2 + g_y^2} = \sqrt{1 + x^2 + 1} = \sqrt{2+x^2}$

$\iint_D \frac{(\frac{x^2}{2} - y - \frac{x^2}{2})x}{\sqrt{x^2+y^2}} dx\, dy = \iint_D \frac{-xy}{\sqrt{x^2+y^2}}\sqrt{2+x^2}\, dx\, dy$

uso coordinate polari $\int_0^{2\pi}\int_0^1 \frac{-\rho^2 \cos\sigma \sin\sigma}{\sqrt{\rho^2}} \sqrt{2+\rho^2\cos^2\sigma}\,\rho\, d\rho d\sigma =$

$\int_0^{2\pi}\int_0^1 \frac{-\rho^2 \cdot \cancel{\rho}}{\cancel{\rho}} \sqrt{2 + \rho^2\cos^2\sigma} \cos\sigma\sin\sigma \, d\rho d\sigma = \int_0^1 \left[\frac{(2+\rho^2\cos^2\sigma)^{3/2}}{3/2 \cdot 2}\right]_0^{2\pi} d\rho =$

$(2 + \rho^2\cos^2\sigma)^{\frac{3}{2}}_\sigma = \frac{3}{2}(2 + \rho^2\cos^2\sigma)^{\frac{1}{2}} \cdot 2\rho^2\cos\sigma \cdot (-\sin\sigma)$

$= \int_0^1 \frac{(2+\rho^2)^{3/2}}{3} - \frac{(2+\rho^2)^{3/2}}{3} = 0$

5) $\begin{cases} y' = y(x-3) \\ y(6) = 1 \end{cases} \qquad \frac{y'}{y} = x - 3 \qquad \int \frac{dy}{y} = \int x - 3 \, dx$

$\ln y = \frac{x^2}{2} - 3x + c$

$y = e^{\frac{x^2}{2} - 3x + c}$

$y(6) = e^{\frac{6^2}{2} - 3 \cdot 6 + c} = 1$

c

$y(0) = e^{2}$ $= 1$ $y = e^{\frac{x^2}{2} - 3x + c}$

$e^{0+c} = 1$ $e^{c} = 1$ $c = 0$

$$\boxed{y = e^{\frac{x^2}{2} - 3x}}$$

6) $y'' - 3y' - 4y = 0$ $\lambda^2 - 3\lambda - 4 = 0$ $\Delta = 9 + 16 = 25$

$\lambda_{1,2} = \frac{3 \pm 5}{2} = 4, -1$

$y = k_1 e^{4x} + k_2 e^{-x}$

non omogenea $y'' - 3y' - 4y = 2x$

$y_s = ax + b$ $y_s' = a$ $y_s'' = 0$

$y_s'' - 3y_s' - 4y_s = 0 - 3(a) - 4(ax + b) \overset{?}{=} 2x$

$-3a - \underline{4ax} - 4b = \underline{2x}$

$\begin{cases} -4a = 2 \\ -3a - 4b = 0 \end{cases}$ $a = -\frac{1}{2}$

$+\frac{3}{2} - 4b = 0$ $4b = \frac{3}{2}$ $b = \frac{3}{8}$

$y_s = -\frac{x}{2} + \frac{3}{8}$ $\boxed{y = k_1 e^{4x} + k_2 e^{-x} - \frac{x}{2} + \frac{3}{8}}$