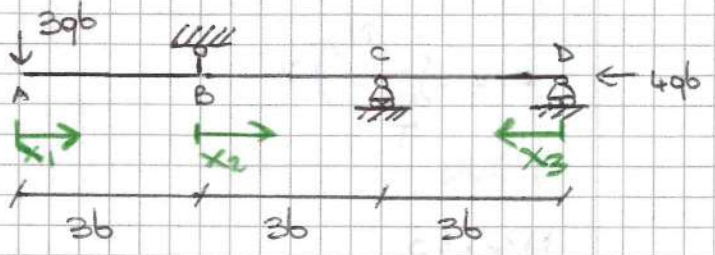
ESERCIZIO N° 1 - TRACCIA 1 - ESAME 18.07.2023

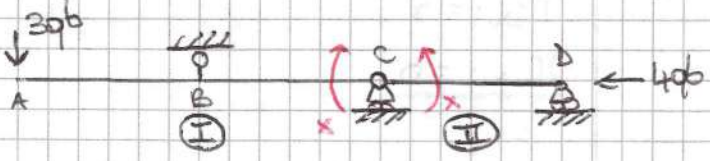


STRUTTURA IPERSTATICA

GDL = 3

GDV = 2(B) + 1(C) + 1(D) = 4

GDL < GDV



STRUTTURA ISOSTATICA

GDL = 6

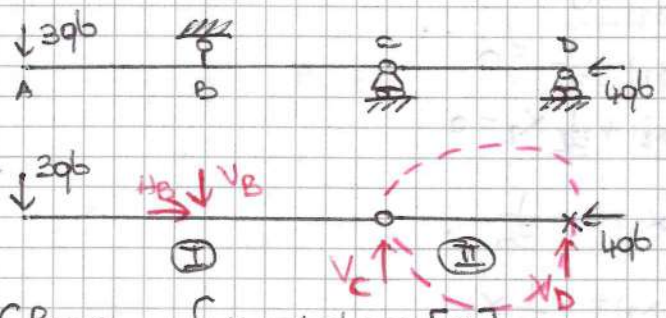
GDV = 2(B) + 3(C) + 1(D) = 6

GDL = GDV

Eq. di congruenza

$\Delta\varphi_C = 0$

S_0 - SISTEMA REALE



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \mathcal{T}_{z(B)} = 0 \end{cases} \quad \begin{cases} H_B - 4qb = 0 \ [1] \\ 3qb + V_B - V_C - V_D = 0 \ [2] \\ 3qb(9b) + V_B(6b) - V_C(3b) = 0 \ [3] \end{cases}$$

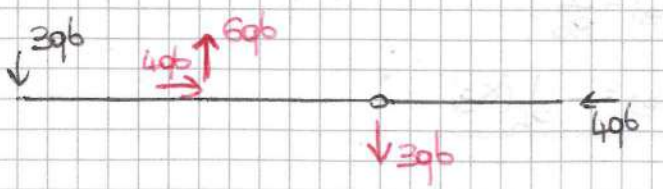
$$\begin{cases} \overset{II}{\mathcal{T}_{z(C)}} = 0 \end{cases} \quad \begin{cases} V_D(3b) = 0 \Rightarrow \underline{V_D = 0} \end{cases}$$

[1] $\underline{H_B = 4qb}$

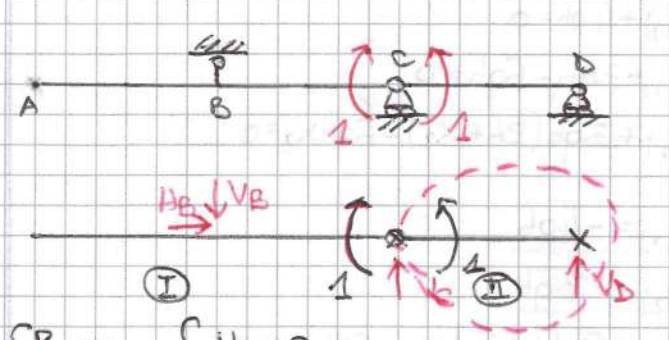
[2] $V_B = V_C - 3qb \Rightarrow \underline{V_B = -6qb}$

[3] $27qb^2 + 6bV_C - 18qb^2 - 3bV_C = 0$

$3bV_C = -9qb^2 \Rightarrow \underline{V_C = -3qb}$



S_1 - SISTEMA EQUILIBRATO

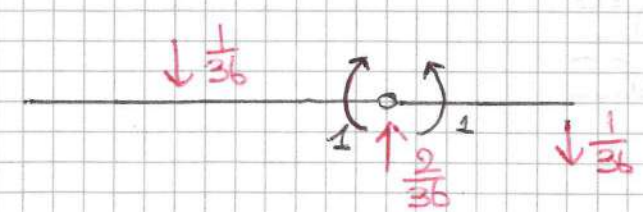


$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \mathcal{T}_{z(B)} = 0 \end{cases} \quad \begin{cases} H_B = 0 \\ V_B - V_C - V_D = 0 \ [2] \\ V_B(6b) - V_C(3b) = 0 \ [3] \end{cases}$$

$$\begin{cases} \overset{II}{\mathcal{T}_{z(C)}} = 0 \end{cases} \quad \begin{cases} V_D(3b) + 1 = 0 \Rightarrow V_D = -\frac{1}{3b} \end{cases}$$

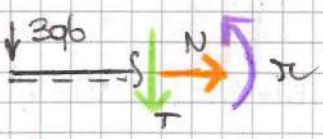
[2] $V_B = V_C - \frac{1}{3b} \Rightarrow V_B = \frac{1}{3b}$

[3] $V_C = 2V_B \Rightarrow V_C = \frac{2}{3b}$



AZIONI INTERNE

A→B $0 \le X_1 \le 3b$

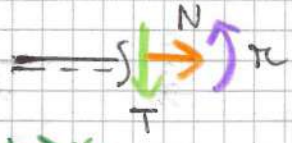


$N(x_1) = 0$

$T(x_1) + 3qb = 0 \Rightarrow T(x_1) = -3qb$

$M(x_1) + 3qb(x_1) = 0 \Rightarrow M(x_1) = -3qbx_1$

A→B $0 \le X_1 \le 3b$

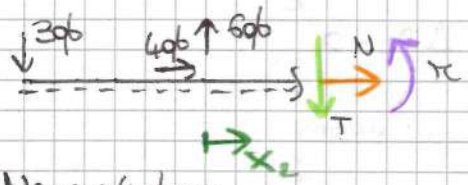


$N(x_1) = 0$

$T(x_1) = 0$

$M(x_1) = 0$

B→C $0 \le X_2 \le 3b$



$N(x_2) + 4qb = 0$

$T(x_2) + 3qb - 6qb = 0$

$M(x_2) + 3qb(3b + X_2) - 6qbx_2 = 0$

$N(x_2) = -4qb$

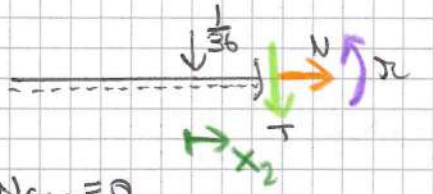
$T(x_2) = 3qb$

$M(x_2) + 9qb^2 + 3qbx_2 - 6qbx_2 = 0$

$M(x_2) + 9qb^2 - 3qbx_2 = 0$

$M(x_2) = 3qbx_2 - 9qb^2$

B→C $0 \le X_2 \le 3b$



$N(x_2) = 0$

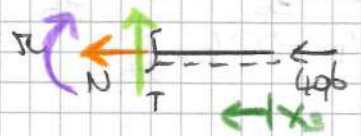
$T(x_2) + \frac{1}{3b} = 0$

$M(x_2) + \frac{1}{3b}X_2 = 0$

$T(x_2) = -\frac{1}{3b}$

$M(x_2) = -\frac{1}{3b}X_2$

C→D $0 \le X_3 \le 3b$

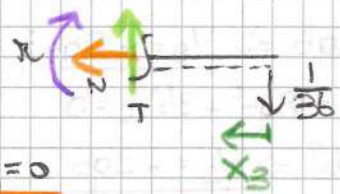


$N(x_3) + 4qb = 0 \Rightarrow N(x_3) = -4qb$

$T(x_3) = 0$

$M(x_3) = 0$

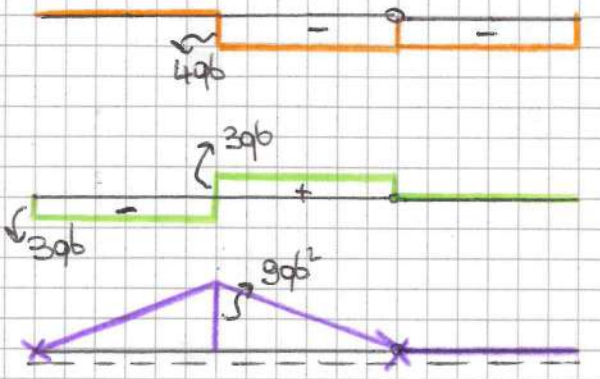
C→D $0 \le X_3 \le 3b$

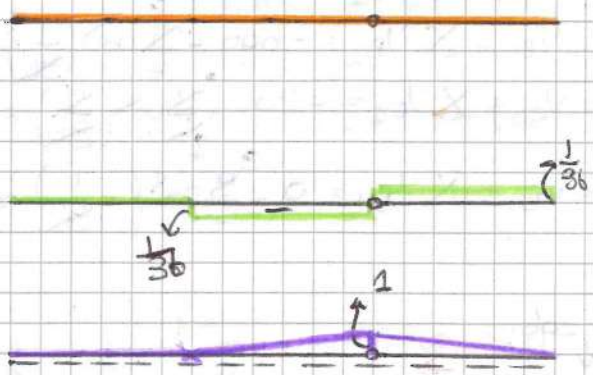


$N(x_3) = 0$

$T(x_3) = \frac{1}{3b}$

$M(x_3) = -\frac{1}{3b}(X_3)$





P.L.V

$\mathcal{L}V_e - \mathcal{L}V_i$

$\mathcal{L}V_e = 1 \cdot \Delta\varphi_c = 0$

$$\mathcal{L}V_i = \int_S N_1 \varepsilon_i + \int_S T_1 \gamma_i + \int_S m_1 \chi_i \Rightarrow \mathcal{L}V_i = \int_S m_1 \left(\frac{m_0 + X m_1}{EI}\right)$$

A → B		B → C		C → D	
$m_0 = -3qbx_1$	$m_1 = 0$	$m_0 = 3qbx_2 - 9qb^2$	$m_1 = -\frac{1}{3b}x_2$	$m_0 = 0$	$m_1 = -\frac{x_3}{3b}$
$m_1 \cdot m_0$	m_1^2	$m_1 \cdot m_0$	m_1^2	$m_1 m_0$	m_1^2
0	0	$-qx_2^2 + 3qbx_2$	$+\frac{1}{9b^2}x_2^2$	0	$\frac{x_3^2}{9b^2}$

$$\mathcal{L}V_i = \int_0^{3b} \frac{-qx_2^2 + 3qbx_2}{EI}\,dx_2 + X\int_0^{3b} \frac{1}{9b^2}x_2^2\left(\frac{1}{EI}\right)dx_2 + X\int_0^{3b} \frac{x_3^2}{9b^2}\left(\frac{1}{EI}\right)dx_3$$

$$= \frac{1}{EI}\left[-q\frac{x_2^3}{3} + 3qb\frac{x_2^2}{2}\right]_0^{3b} + \frac{X}{EI}\left[\frac{1}{9b^2}\left(\frac{x_2^3}{3}\right)\right]_0^{3b} + \frac{X}{EI}\left[\frac{1}{9b^2}\left(\frac{x_3^3}{3}\right)\right]_0^{3b}$$

$$= \left[-q\left(\frac{27b^3}{3}\right) + 3qb\left(\frac{9b^2}{2}\right)\right] + X\left[\frac{1}{9b^2}\left(\frac{27b^3}{3}\right)\right] + X\left[\frac{1}{9b^2}\left(\frac{27b^3}{3}\right)\right]$$

$$= -9qb^3 + \frac{27}{2}qb^3 + X(b) + X(b)$$

$$\mathcal{L}V_i = \frac{9}{2}qb^3 + 2bX \Rightarrow 2b\bar{X} + \frac{9}{2}qb^3 = 0 \Rightarrow \bar{X} = -\frac{9}{2}qb^2\left(\frac{1}{2b}\right)$$

$$\bar{X} = -\frac{9}{4}qb^2$$

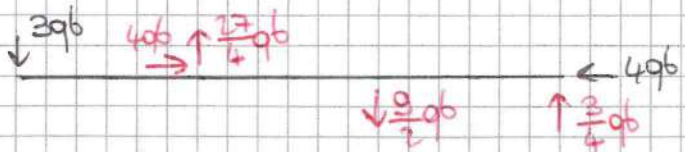
REAZIONI VINCOLARI

$$H_B = H_{B_0} + X H_{B_1} = 4qb$$

$$V_B = V_{B_0} + X V_{B_1} = -6qb - \frac{9}{4}qb^2\left(\frac{1}{3b}\right) = -\frac{27}{4}qb$$

$$V_C = V_{C_0} + X V_{C_1} = -3qb - \frac{9}{4}qb^2\left(\frac{2}{3b}\right) = -\frac{9}{2}qb$$

$$V_D = V_{D_0} + X V_{D_1} = 0 - \frac{9}{4}qb^2\left(-\frac{1}{3b}\right) = \frac{3}{4}qb$$



AZIONI INTERNE

A → B

$$N(x_1) = N_0(x_1) + X N_1(x_1) = 0$$

$$T(x_1) = T_0(x_1) + X T_1(x_1) = -3qb$$

$$\pi(x_1) = \pi_0(x_1) + X \pi_1(x_1) = -3qbx_1$$

B → C

$$N(x_2) = -4qb$$

$$T(x_2) = 3qb - \frac{9}{4}qb^2\left(-\frac{1}{3b}\right)$$

$$= 3qb + \frac{3}{4}qb = \frac{15}{4}qb$$

$$\pi(x_2) = 3qbx_2 - 9qb^2 - \frac{9}{4}qb^2\left(-\frac{1}{3b}x_2\right)$$

$$= 3qbx_2 - 9qb^2 + \frac{3}{4}qbx_2$$

$$= \frac{15}{4}qbx_2 - 9qb^2$$

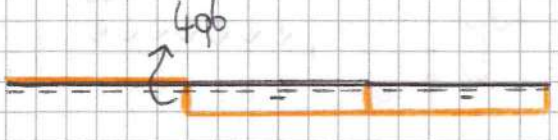
C → D

$$N(x_3) = -4qb$$

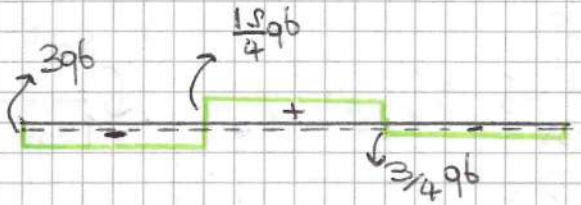
$$T(x_3) = -\frac{9}{4}qb^2\left(\frac{1}{3b}\right) = -\frac{3}{4}qb$$

$$\pi(x_3) = -\frac{9}{4}qb^2\left(-\frac{x_3}{3b}\right) = +\frac{3}{4}qbx_3$$

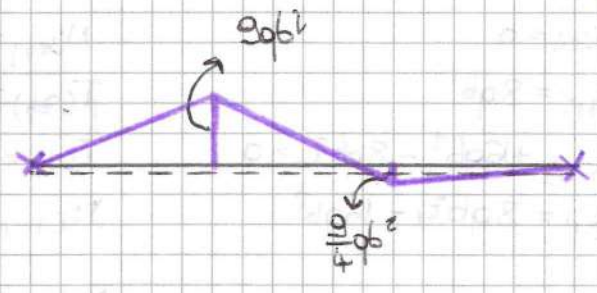
N



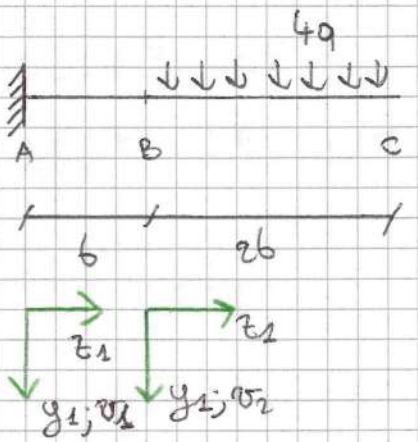
T



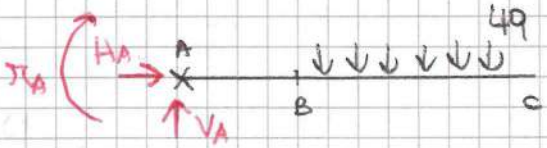
π

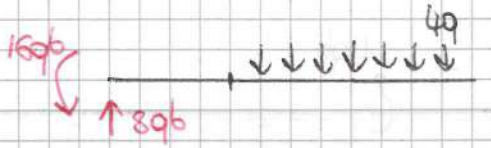


TRACCIA 1 - ESERCIZIO 2 - ESAME 18.07.2023



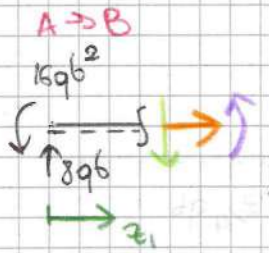
$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \mathcal{M}_{z(A)} = 0 \end{cases} \quad \begin{cases} H_A = 0 \\ V_A - 4q(2b) = 0 \Rightarrow V_A = 8qb \\ \mathcal{M}_A + 4q(2b)(2b) = 0 \quad \mathcal{M}_A = -16qb^2 \end{cases}$$





AZIONI INTERNE

A → B



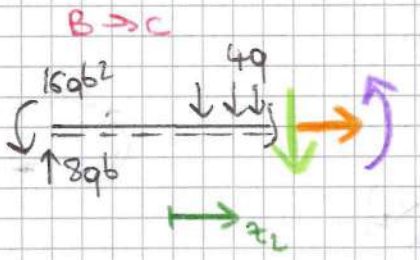
$N(z_1) = 0$

$T(z_1) = 8qb$

$\mathcal{M}(z_1) + 16qb^2 - 8qbz_1 = 0$

$\mathcal{M}(z_1) = 8qbz_1 - 16qb^2$

B → C



$N(z_2) = 0$

$T(z_2) - 8qb + 4qz_2 = 0$

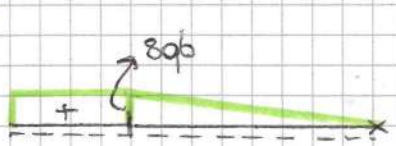
$T(z_2) = 8qb - 4qz_2$

$\mathcal{M}(z_2) + 16qb^2 - 8qb(b + z_2) + 4q\left(\frac{z_2^2}{2}\right) = 0$

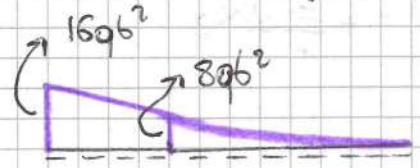
$\mathcal{M}(z_2) + 16qb^2 - 8qb^2 - 8qbz_2 + 2qz_2^2 = 0$

$\mathcal{M}(z_2) = 8qbz_2 - 2qz_2^2 - 8qb^2$

T



$\mathcal{M}$



Eq. LINEA ELASTICA

A → B

$$v''_{(z_1)} = -\frac{M}{EI} \Rightarrow v''_{(z_1)} = \frac{1}{EI}\left(-8qbz_1 + 16qb^2\right)$$

$$v'_{(z_1)} = \frac{1}{EI}\left(-8qb\frac{z_1^2}{2} + 16qb^2 z_1\right) + A_1 \Rightarrow v'_{(z_1)} = \frac{1}{EI}\left(-4qbz_1^2 + 16qb^2 z_1\right) + A_1$$

$$v_{(z_1)} = \frac{1}{EI}\left(-4qb\frac{z_1^3}{3} + 16qb^2\frac{z_1^2}{2}\right) + A_1 z_1 + A_2 \Rightarrow v_{(z_1)} = \frac{1}{EI}\left(-\frac{4}{3}qbz_1^3 + 8qb^2 z_1^2\right) + A_1 z_1 + A_2$$

B → C

$$v''_{(z_2)} = -\frac{M}{EI} \Rightarrow v''_{(z_2)} = \frac{1}{EI}\left(-8qbz_2 + 2qz_2^2 + 8qb^2\right)$$

$$v'_{(z_2)} = \frac{1}{EI}\left(-8qb\frac{z_2^2}{2} + 2q\frac{z_2^3}{3} + 8qb^2 z_2\right) + B_1$$

$$= \frac{1}{EI}\left(-4qbz_2^2 + \frac{1}{3}qz_2^3 + 8qb^2 z_2\right) + B_1$$

$$v_{(z_2)} = \frac{1}{EI}\left(-4qb\frac{z_2^3}{3} + \frac{1}{3}q\frac{z_2^4}{2} + 8qb^2\frac{z_2^2}{2}\right) + B_1 z_2 + B_2$$

$$v_{(z_2)} = \frac{1}{EI}\left(-\frac{4}{3}qbz_2^3 + \frac{1}{6}qz_2^4 + 4qb^2 z_2^2\right) + B_1 z_2 + B_2$$

COSTANTI A_1; A_2; B_1; B_2

A) INCASTRO — IMPEDISCE
- SPOSTAMENTO ↕ $\Rightarrow v_{(z_1)}(A) = 0$
- ROTAZIONE ↻ $\Rightarrow v'_{(z_1)}(A) = 0$

B) SALDATURA — IMPONE
- SPOSTAMENTO ↕ $\Rightarrow v_{(z_1)}(B) = v_{(z_2)}(B)$
- ROTAZIONE ↻ $\Rightarrow v'_{(z_1)}(B) = v'_{(z_2)}(B)$

CONDIZIONI IN A

$$v'_{(z_1)}(A) = 0 \Rightarrow A_1 = 0$$

$$v_{(z_1)}(A) = 0 \Rightarrow A_2 = 0$$

CONDIZIONI IN B

$$v'_{(z_1)}(B) = v'_{(z_2)}(B)$$

$$\frac{1}{EI}\left[-4qb(b^2) + 16qb^2(b)\right] = \frac{1}{EI}(0) + B_1 \Rightarrow B_1 = \frac{12qb^3}{EI}$$

$$v_{(z_1)}(B) = v_{(z_2)}(B)$$

$$\frac{1}{EI}\left[-\frac{4}{3}qb(b)^3 + 8qb^2(b^2)\right] = B_2 \Rightarrow B_2 = \frac{20}{3}\frac{qb^4}{EI}$$

$$v'_{(z_1)} = \frac{1}{EI}\left(16qb^2 z_1 - 4qbz_1^2\right)$$

$$v_{(z_1)} = \frac{1}{EI}\left(8qb^2 z_1^2 - \frac{4}{3}qbz_1^3\right)$$

$$v'_{(z_2)} = \frac{1}{EI}\left(\frac{2}{3}qz_2^3 + 8qb^2 z_2 - 4qbz_2^2 + 12qb^3\right)$$

$$v_{(z_2)} = \frac{1}{EI}\left(\frac{1}{6}qz_2^4 + 4qb^2 z_2^2 - \frac{4}{3}qbz_2^3 + 12qb^3 z_2 + \frac{20}{3}qb^4\right)$$

ROTAZIONE NEL PUNTO C, φ_C

$$\varphi_C = v'_{(z_2=2b)}$$

$$\varphi_C = \frac{1}{EI}\left[\frac{2}{3}q(8b^3) + 8qb^2(2b) - 4qb(4b^2) + 12qb^3\right]$$

$$= \frac{1}{EI}\left[\frac{16}{3}qb^3 + \cancel{16qb^3} - \cancel{16qb^3} + 12qb^3\right]$$

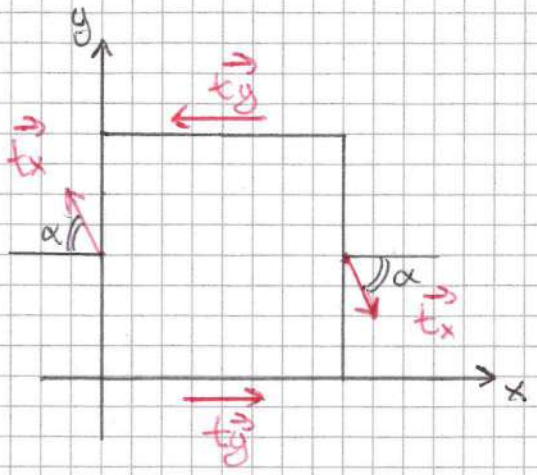
$$= \frac{1}{EI}\left(52qb^3\right)$$

SPOSTAMENTO VERTICALE NEL PUNTO B, v_B

$$v_B = v_{(z_2=0)}$$

$$v_B = \frac{1}{EI}\left(\frac{2}{3}qb^4\right)$$

ESERCIZIO 3 - TRACCIA 1 - ESAME 18.07.2023



$\alpha = -60°$

$\sin\alpha = -\frac{\sqrt{3}}{2}$

$\cos\alpha = \frac{1}{2}$

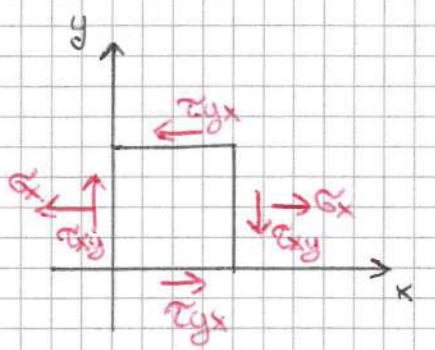
$|t_x| = 30$ MPa

$\sigma_x = |t_x|\cos\alpha = 30 \cdot \frac{1}{2} = 15$ MPa

$\tau_{xy} = |t_x|\sin\alpha = 30\left(-\frac{\sqrt{3}}{2}\right) = -15\sqrt{3}$ MPa $= -25{,}981$ MPa

$\tau_{xy} = \tau_{yx} = -25{,}981$ MPa

$$\underline{\underline{\sigma}} = \begin{bmatrix} 15 & -25{,}981 & 0 \\ -25{,}981 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



CERCHIO DI MOHR

$P_x(\sigma_x ; \tau_{xy}) \Rightarrow \tau_{xy}$ ↓

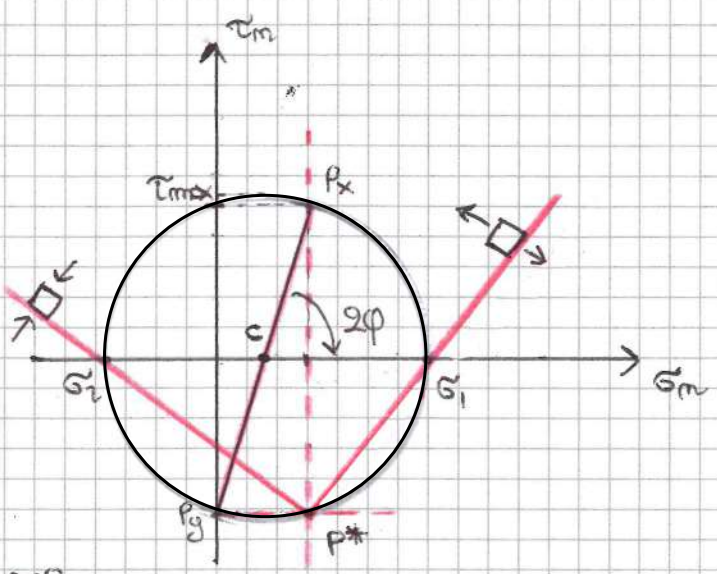
$P_y(\sigma_y ; -\tau_{yx}) \Rightarrow \tau_{yx}$ ↑

$P_x(15 ; 25{,}981)$; $P_y(0 ; -25{,}981)$

$c\left(\frac{\sigma_x+\sigma_y}{2} ; 0\right) = (7{,}5 ; 0)$

$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(7{,}5)^2 + (25{,}981)^2}$

$= \sqrt{56{,}25 + 675{,}012} = \sqrt{731{,}262} = 27{,}042$ MPa



$\tau_{max} = R = 27{,}042$ MPa

$\sigma_1 = \sigma_c + R = 7{,}5 + 27{,}042 = 34{,}542$ MPa

$\sigma_2 = \sigma_c - R = 7{,}5 - 27{,}042 = -19{,}542$ MPa

$\varphi = ?$

$\tan 2\varphi = \frac{\tau_{xy}}{\sigma_x - \sigma_c} = \frac{-25{,}981}{7{,}5} = -3{,}464$

$\arctan(-3{,}464) = -73{,}897 = 2\varphi \Rightarrow \varphi = -36{,}95°$