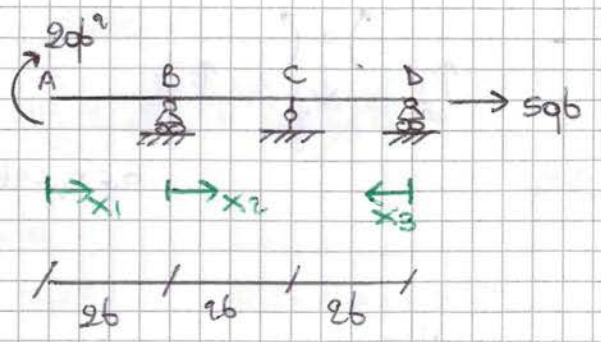
TRACCIA 1 _ ESERCIZIO 1 _ ESAME 27/06/23

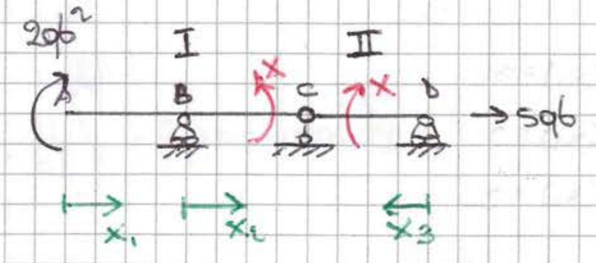


STRUTTURA IPERSTATICA

GDL = 3

GDV = 1(B) + 2(C) + 1(D) = 4

GDL < GDV



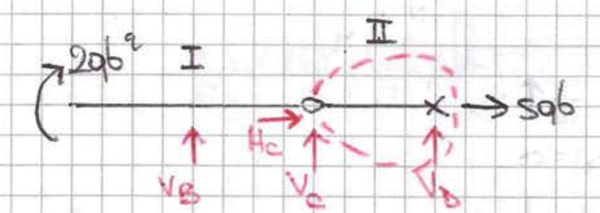
STRUTTURA ISOSTATICA

GDL = 3(I) + 3(II) = 6

GDV = 1(B) + 4(C) + 1(D) = 6

GDL = GDV

S_0 - SISTEMA REALE



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{t(D)} = 0 \end{cases} \quad \begin{cases} H_C + 5qb = 0 \Rightarrow H_C = -5qb \\ V_B + V_C + V_D = 0 \quad V_B = -V_C \\ 2qb^2 + V_B(4b) + V_C(2b) = 0 \quad [*] \end{cases}$$

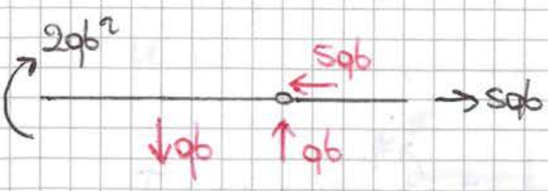
eq. aux

$$\left\{ M^{II}_{t(C)} = 0 \right. \quad \left\{ V_D(2b) = 0 \Rightarrow V_D = 0 \right.$$

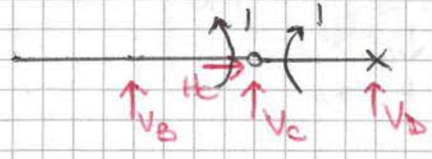
$[*]$ $2qb^2 - 4bV_C + 2bV_C = 0$

$-2bV_C = -2qb^2 \Rightarrow V_C = qb$

$V_B = -qb$



S_1 - SISTEMA EQUILIBRATO



$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{t(D)} = 0 \end{cases} \quad \begin{cases} H_C = 0 \\ V_B + V_C + V_D = 0 \quad [2] \\ V_B(4b) + V_C(2b) = 0 \quad [3] \end{cases}$$

eq. aux

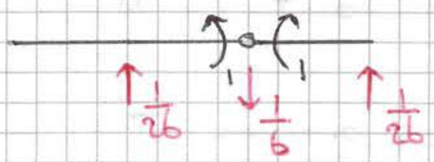
$$\left\{ M^{II}_{t(C)} = 0 \right. \quad \left\{ V_D(2b) - 1 = 0 \Rightarrow V_D = \frac{1}{2b} \right.$$

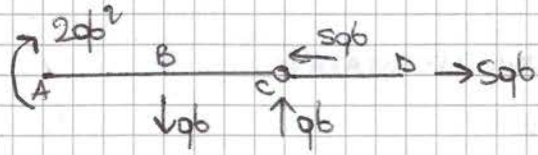
$[2]$ $V_B + V_C + \frac{1}{2b} = 0 \quad V_B = -V_C - \frac{1}{2b}$

$[3]$ $-4bV_C - 2 + 2bV_C = 0 \Rightarrow -2bV_C = 2$

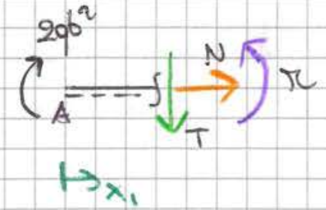
$V_C = -\frac{1}{b}$

$[2]$ $V_B = +\frac{1}{b} - \frac{1}{2b} = \frac{1}{2b}$





A → B $\quad 0 \leq x_1 \leq 2b$

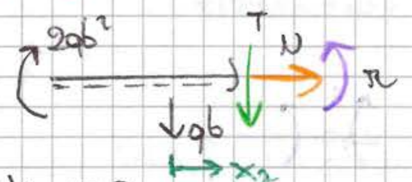


$N(x_1) = 0$

$T(x_1) = 0$

$\mathcal{M}(x_1) = 2qb^2$

B → C $\quad 0 \leq x_2 \leq 2b$

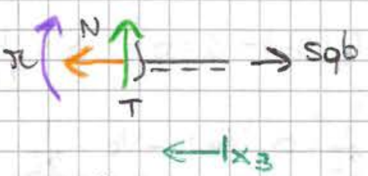


$N(x_2) = 0$

$T(x_2) = -qb$

$\mathcal{M}(x_2) = -qbx_2 + 2qb^2$

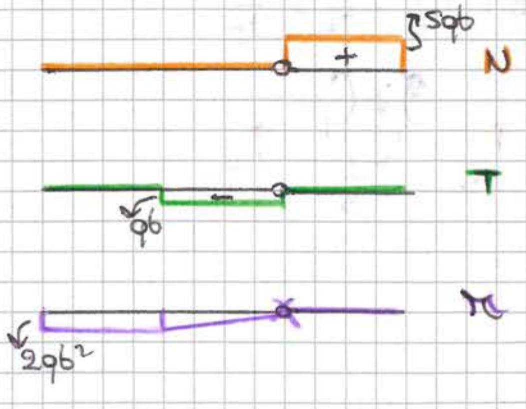
C → D $\quad 0 \leq x_3 \leq 2b$

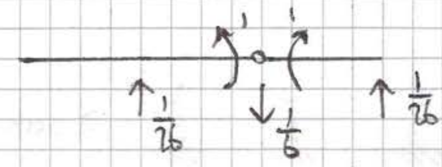


$N(x_3) = 5qb$

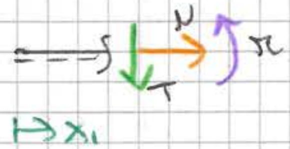
$T(x_3) = 0$

$\mathcal{M}(x_3) = 0$





A → B $\quad 0 \leq x_1 \leq 2b$

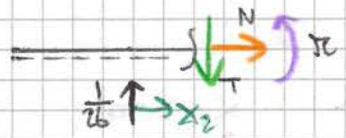


$N(x_1) = 0$

$T(x_1) = 0$

$\mathcal{M}(x_1) = 0$

B → C $\quad 0 \leq x_2 \leq 2b$

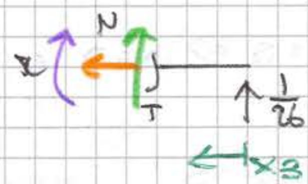


$N(x_2) = 0$

$T(x_2) = \frac{1}{2b}$

$\mathcal{M}(x_2) = \frac{1}{2b} x_2$

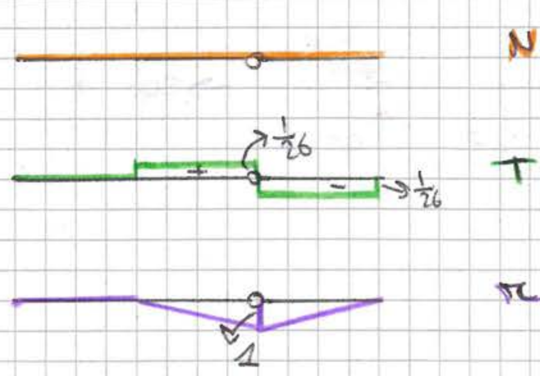
C → D $\quad 0 \leq x_3 \leq 2b$



$N(x_3) = 0$

$T(x_3) = -\frac{1}{2b}$

$\mathcal{M}(x_3) = \frac{1}{2b} x_3$



P.L.V.

$\mathcal{L}V_e = \mathcal{L}V_i$

$\mathcal{L}V_e = 1 \cdot \Delta\varphi_c = 0$

$\mathcal{L}V_i = \int_S N_1 \varepsilon_i + \int_S T_1 \gamma_1 + \int_S M_1 \chi_i \Rightarrow \mathcal{L}V_i = \int_S M_1 \left(\frac{M_0 + X M_1}{EI} \right)$

A → B

$M_0 = 2qb^2$

$M_1 = 0$

$M_1 \cdot M_0 = 0$

$M_1^2 = 0$

B → C

$M_0 = -qbx_2 + 2qb^2$

$M_1 = \frac{1}{2b} x_2$

$M_1 \cdot M_0 = -\frac{1}{2} q x_2^2 + qb x_2$

$M_1^2 = \frac{1}{4b^2} x_2^2$

C → D

$M_0 = 0$

$M_1 = \frac{1}{2b} x_3$

$M_1 \cdot M_0 = 0$

$M_1^2 = \frac{1}{4b^2} x_3^2$

$$\mathcal{L}V_i = \frac{1}{EI} \int_0^{2b} -\frac{1}{2} q x_2 + qbx_2 \, dx_2 + \int_0^{2b} \frac{X}{EI} \left(\frac{1}{4b^2} x_2^2 \right) dx_2 + \int_0^{2b} \frac{X}{EI} \left(\frac{1}{4b^2} x_3^2 \right) dx_3$$

$$= \frac{1}{EI} \left[-\frac{1}{2} q \frac{x_2^3}{3} + qb \frac{x_2^2}{2} \right]_0^{2b} + \frac{X}{EI} \left[\frac{1}{4b^2} \frac{x_2^3}{3} \right]_0^{2b} + \left[\frac{X}{EI} \left(\frac{1}{4b^2} \frac{x_3^3}{3} \right) \right]_0^{2b}$$

$$= \frac{1}{EI} \left[-\frac{1}{6} q (8b^3) + \frac{qb}{2} (4b^2) \right] + \frac{X}{EI} \left[\frac{1}{12 b^2} (8b^3) \right] + \frac{X}{EI} \left[\frac{1}{12 b^2} (8b^3) \right]$$

$$= \frac{1}{EI} \left[-\frac{4}{3} qb^3 + 2qb^3 \right] + \frac{X}{EI} \left[\frac{2}{3} b \right] + \frac{X}{EI} \left[\frac{2}{3} b \right]$$

$\mathcal{L}V_i = \frac{1}{EI} \left[\frac{2}{3} qb^3 + \frac{4}{3} b X \right] \Rightarrow \mathcal{L}V_i = 0 \quad \frac{1}{EI} \left[\frac{2}{3} qb^3 + \frac{4}{3} b X \right] = 0$

$\frac{4}{3} b X = -\frac{2}{3} qb^3 \quad X = -\frac{1}{2} qb^2$

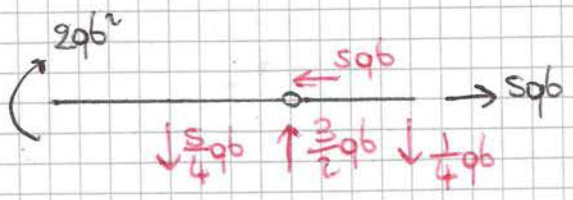
REAZIONI VINCOLARI

$H_C = H_{C0} + X H_{C1} \Rightarrow H_C = -5qb$

$V_C = V_{C0} + X V_{C1} \Rightarrow V_C = qb + \left(-\frac{1}{2} qb^2 \right) \left(-\frac{1}{b} \right) = qb + \frac{1}{2} qb = \frac{3}{2} qb$

$V_B = V_{B0} + X V_{B1} \Rightarrow V_B = -qb + \left(-\frac{1}{2} qb^2 \right) \left(\frac{1}{2b} \right) = -qb - \frac{1}{4} qb = -\frac{5}{4} qb$

$V_D = V_{D0} + X V_{D1} \Rightarrow V_D = 0 + \left(-\frac{1}{2} qb^2 \right) \left(\frac{1}{2b} \right) = -\frac{1}{4} qb$



AZIONI INTERNE

A → B

$N(x_1) = N_0(x_1) + \bar{X} N_1(x_1)$

$N(x_1) = 0$

$T(x_1) = T_0(x_1) + \bar{X} T_1(x_1)$

$T(x_1) = 0$

$M(x_1) = M_0(x_1) + \bar{X} M_1(x_1)$

$M(x_1) = 2qb^2$

B → C

$N(x_2) = N_0(x_2) + \bar{X} N_1(x_2)$

$N(x_2) = 0$

$T(x_2) = T_0(x_2) + \bar{X} T_1(x_2)$

$T(x_2) = -qb + \left(-\frac{1}{2}qb^2\right)\left(\frac{1}{2b}\right)$

$T(x_2) = -qb - \frac{1}{4}qb = -\frac{5}{4}qb$

$M(x_2) = M_0(x_2) + \bar{X} M_1(x_2)$

$M(x_2) = -qbx_2 + 2qb^2 + \left(-\frac{1}{2}qb^2\right)\left(\frac{1}{2b}x_2\right)$

$= -qbx_2 + 2qb^2 - \frac{1}{4}qbx_2$

$= -\frac{5}{4}qbx_2 + 2qb^2$

C → D

$N(x_3) = N_0(x_3) + \bar{X} N_1(x_3)$

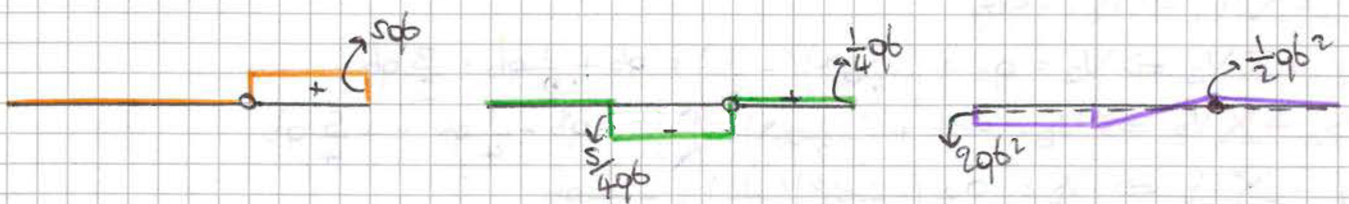
$N(x_3) = 5qb$

$T(x_3) = T_0(x_3) + \bar{X} T_1(x_3)$

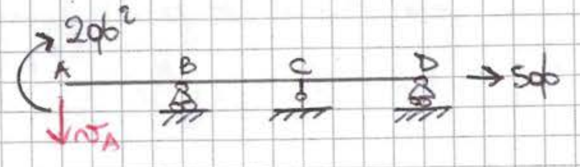
$T(x_3) = 0 - \frac{1}{2}qb^2\left(-\frac{1}{2b}\right) = +\frac{1}{4}qb$

$M(x_3) = M_0(x_3) + \bar{X} M_1(x_3)$

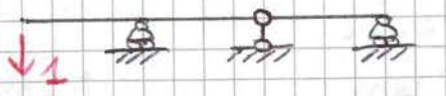
$M(x_3) = 0 - \frac{1}{2}qb^2\left(\frac{1}{2b}x_3\right) = -\frac{1}{4}qbx_3$



• CALCOLO COMPONENTE DI SPOSTAMENTO v_A



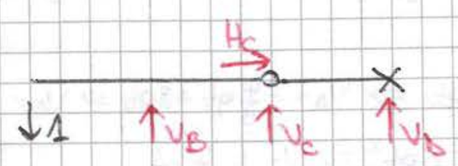
S_2



$$\begin{cases} R_x=0 \\ R_y=0 \\ M_{z(D)}=0 \end{cases} \quad \begin{cases} H_C=0 \\ V_B+V_C+\cancel{V_D}-1=0 \; [2] \\ V_C(2b)+V_B(4b)-1(6b)=0 \; [3] \end{cases}$$

eq aux

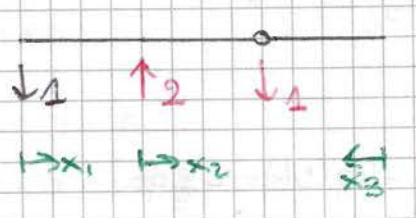
$$\begin{cases} II \\ M_{z(C)}=0 \end{cases} \quad \{ V_D(2b)=0 \Rightarrow V_D=0$$



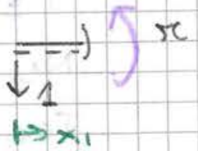
$[2]\ V_B = 1 - V_C \Rightarrow V_B = 1+1 = 2$

$[3]\ 2bV_C + 4b - 4bV_C - 6b = 0$

$-2bV_C - 2b = 0 \Rightarrow V_C = -1$

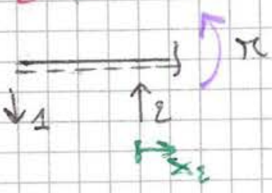


A → B



$M_{(x_1)} = -x_1$

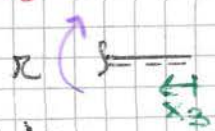
B → C



$M_{(x_2)} = -(2b+x_2) + 2x_2$

$M_{(x_2)} = -2b + x_2$

C → D



$M_{(x_3)} = 0$

P.L.V

$L_{ve} = L_{vi}$

$L_{ve} = 1 \cdot v_A = v_A$

$L_{vi} = \int_s M_2 \chi$

$$= \int_0^{2b} -x_1\left(\frac{2qb^2}{EI}\right)dx_1 + \int_0^{2b}(x_2-2b)\left(-\frac{5}{4}x_2 qb + 2qb^2\right)\left(\frac{1}{EI}\right)dx_2$$

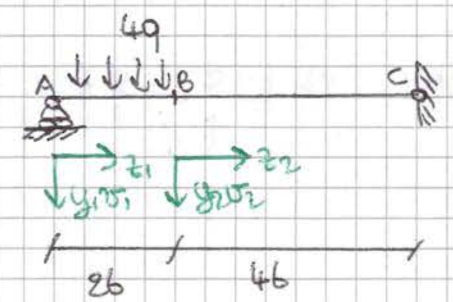
$$= \left[-\frac{x_1^2}{2}\left(\frac{2qb^2}{EI}\right)\right]_0^{2b} + \int_0^{2b}\left(-\frac{5}{4}x_2^2 qb + 2qb^2 x_2 + \frac{5}{2}x_2 qb^2 - 4qb^3\right)\frac{1}{EI}dx_2$$

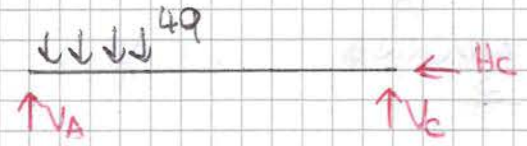
$$= \left[-4b^2\left(\frac{qb^2}{EI}\right)\right] + \left[-\frac{5}{4}\frac{x_2^3}{3}qb + \frac{x_2^2}{2}(2qb^2) + \frac{x_2^2}{2}\left(\frac{5}{2}qb^2\right) - 4qb^3 x_2\right]_0^{2b}\left(\frac{1}{EI}\right)$$

$$= -\frac{4qb^4}{EI} + \left(-\frac{5}{4}\left(\frac{8b^3}{3}\right)(qb) + \frac{4b^2}{2}(2qb^2) + \frac{4b^2}{8}\left(\frac{5}{2}qb^2\right) - 8qb^4\right)\left(\frac{1}{EI}\right)$$

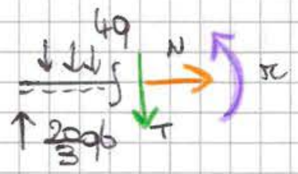
$$= -\frac{4qb^4}{EI} + \left(4qb^4 - \frac{10}{3}qb^4 + 5qb^4 - 8qb^4\right)\left(\frac{1}{EI}\right) = -\frac{10}{3}\frac{qb^4}{EI}$$

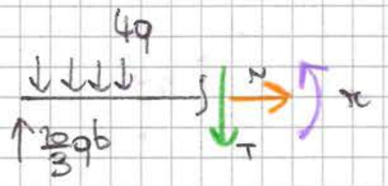
TRACCIA 1 _ ESERCIZIO 2 _ ESAME 27/06/2023





$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(A)} = 0 \end{cases} \quad \begin{cases} H_C = 0 \\ V_A + V_C - 4q(2b) = 0 \Rightarrow V_A = -V_C + 8qb \Rightarrow V_A = -\frac{4}{3}qb + 8qb \Rightarrow V_A = \frac{20}{3}qb \\ 4q(2b)(b) - V_C(6b) = 0 \Rightarrow V_C(6b) = 8qb^2 \Rightarrow V_C = \frac{4}{3}qb \end{cases}$$





$N_{(z_1)} = 0$

$T_{(z_1)} = \frac{20}{3}qb - 4qz_1$

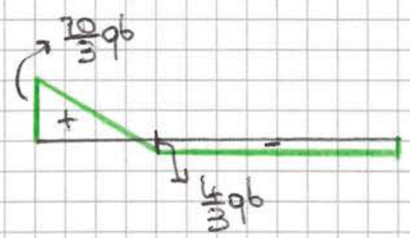
$M_{(z_1)} = -4q\frac{z_1^2}{2} + \frac{20}{3}qbz_1 = -2qz_1^2 + \frac{20}{3}qbz_1$

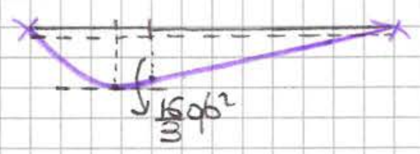
$N_{(z_2)} = 0$

$T_{(z_2)} = \frac{20}{3}qb - 4q(2b) = -\frac{4}{3}qb$

$M_{(z_2)} = -4q(2b)(b+z_2) + \frac{20}{3}qb(2b+z_2)$

$M_{(z_2)} = -8qb^2 - 8qbz_2 + \frac{40}{3}qb^2 + \frac{20}{3}qbz_2 = -8qb^2 + \frac{40}{3}qb^2 - \frac{4}{3}qbz_2 = \frac{16}{3}qb^2 - \frac{4}{3}qbz_2$





$M'_{(z_1)} = -4qz_1 + \frac{20}{3}qb$

$M'_{(z_1)} = 0 \quad z_1 = \frac{5}{3}b$

per $z_1 = \frac{5}{3}b$ PUNTO DI MINIMO

Eq. LINEA ELASTICA

$A \to B$

$v''_{(z_1)} = -\frac{M}{EI} \Rightarrow v''_{(z_1)} = \frac{1}{EI}\left(2qz_1^2 - \frac{20}{3}qbz_1\right)$

$v'_{(z_1)} = \frac{1}{EI}\left(2q\frac{z_1^3}{3} - \frac{20}{3}qb\frac{z_1^2}{2}\right) + A_1 \Rightarrow v'_{(z_1)} = \frac{1}{EI}\left(\frac{2}{3}qz_1^3 - \frac{10}{3}qbz_1^2\right) + A_1$

$v_{(z_1)} = \frac{1}{EI}\left(\frac{2}{3}q\frac{z_1^4}{4} - \frac{10}{3}qb\frac{z_1^3}{3}\right) + A_1z_1 + A_2 \Rightarrow v_{(z_1)} = \frac{1}{EI}\left(\frac{1}{6}qz_1^4 - \frac{10}{9}qbz_1^3\right) + A_1z_1 + A_2$

B → C

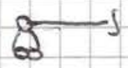
$$v''(z_2) = -\frac{M}{EI} \Rightarrow v''(z_2) = \frac{1}{EI}\left(-\frac{16}{3}qb^2 + \frac{4}{3}qb z_2\right)$$

$$v'(z_2) = \frac{1}{EI}\left(-\frac{16}{3}qb^2 z_2 + \frac{4}{3}qb\frac{z_2^2}{2}\right) + B_1 \Rightarrow v'(z_2) = \frac{1}{EI}\left(-\frac{16}{3}qb^2 z_2 + \frac{2}{3}qb z_2^2\right) + B_1$$

$$v(z_2) = \frac{1}{EI}\left(-\frac{16}{3}qb^2\frac{z_2^2}{2} + \frac{2}{3}qb\frac{z_2^3}{3}\right) + B_1 z_2 + B_2 \Rightarrow v(z_2) = \frac{1}{EI}\left(-\frac{8}{3}qb^2 z_2^2 + \frac{2}{9}qb z_2^3\right) + B_1 z_2 + B_2$$

COSTANTI $A_1; A_2; B_1; B_2$

IN A



IMPEDISCE { SPOSTAMENTO ↕ ⇒ $v(z_1=0) = 0$

CARRELLO

IN B

IMPONE { UGUALE ROTAZIONE ↻ $v'(z_1=2b) = v'(z_2=0)$
UGUALE SPOSTAMENTO ↕ $v(z_1=2b) = v(z_2=0)$

SALDATURA

IN C

IMPEDISCE { SPOSTAMENTO ↕ $v(z_2=4b) = 0$

CERNIERA

CONDIZIONE IN A

- $v(z_1=0) = 0 \Rightarrow A_2 = 0$

CONDIZIONI IN B

- $v'(z_1=2b) = v'(z_2=0)$

$$\frac{1}{EI}\left(\frac{2}{3}q\cdot 8b^3 - \frac{10}{3}qb\cdot 4b^2\right) + A_1 = B_1 \Rightarrow B_1 = \frac{1}{EI}\left(\frac{16}{3}qb^3 - \frac{40}{3}qb^3\right) + A_1 \Rightarrow B_1 = \frac{1}{EI}\left(-\frac{24}{3}qb^3\right) + A_1$$

$$B_1 = -\frac{1}{EI}\cdot 8qb^3 + A_1$$

- $v(z_1=2b) = v(z_2=0)$

$$\frac{1}{EI}\left(\frac{1}{3}q\cdot 8b^4 - \frac{10}{9}qb\cdot 8b^3\right) + A_1(2b) = B_2 \Rightarrow B_2 = \frac{1}{EI}\left(\frac{8}{3}qb^4 - \frac{80}{9}qb^4\right) + 2b(A_1)$$

$$B_2 = \frac{1}{EI}\left(-\frac{56}{9}\right)qb^4 + 2b(A_1)$$

CONDIZIONE IN C

- $v(z_2=4b) = 0$

$$\frac{1}{EI}\left(-\frac{8}{3}qb^2\cdot 16b^2 + \frac{2}{9}qb\cdot 64b^3\right) + B_1(4b) + B_2 = 0$$

$$\frac{1}{EI}\left(-\frac{128}{3}qb^4 + \frac{128}{9}qb^4\right) - 32\frac{qb^4}{EI} + 4b(A_1) - \frac{56}{9}\frac{qb^4}{EI} + 2b(A_1) = 0$$

$$\frac{1}{EI}\left(-\frac{256}{9}qb^4 - 32qb^4 - \frac{56}{9}qb^4\right) + 6bA_1 = 0 \Rightarrow 6bA_1 = \frac{600}{9}\frac{qb^4}{EI} \Rightarrow A_1 = \frac{100}{9}\frac{qb^3}{EI}$$

$A_1 = \frac{100}{9}\frac{qb^3}{EI}$; $A_2 = 0$; $B_1 = \frac{qb^3}{EI}\left(-8+\frac{100}{9}\right) = \frac{28}{9}\frac{qb^3}{EI}$; $B_2 = \frac{qb^4}{EI}\left(-\frac{56}{9}+\frac{200}{9}\right) = +16\frac{qb^4}{EI}$

$v'_{(z_1)} = \frac{1}{EI}\left(\frac{2}{3}qz_1^3 - \frac{10}{3}qbz_1^2\right) + \frac{100}{9}\frac{qb^4}{EI}$; $v'_{(z_2)} = \frac{1}{EI}\left(-\frac{16}{3}qb^2z_2 + \frac{2}{3}qbz_2^2 + \frac{28}{9}qb^3\right)$

$v_{(z_1)} = \frac{1}{EI}\left(\frac{1}{6}qz_1^4 - \frac{10}{9}qbz_1^3 + \frac{100}{9}qb^3z_1\right)$; $v_{(z_2)} = \frac{1}{EI}\left(-\frac{8}{3}qb^2z_2^2 + \frac{2}{9}qbz_2^3 + \frac{28}{9}qb^3z_2 + 16qb^4\right)$

- ROTAZIONE DEL PUNTO C ; φ_C

$\varphi_C = v'_{(z_2=4b)}$

$$v'_{(z_2=4b)} = \frac{1}{EI}\left(-\frac{16}{3}qb^2\cdot 4b + \frac{2}{3}qb(4b)^2 + \frac{28}{9}qb^3\right)$$

$$= \frac{1}{EI}\left(-\frac{64}{3}qb^3 + \frac{32}{3}qb^3 + \frac{28}{9}qb^3\right)$$

$$= \frac{1}{EI}\left(-\frac{32}{3}qb^3 + \frac{28}{9}qb^3\right)$$

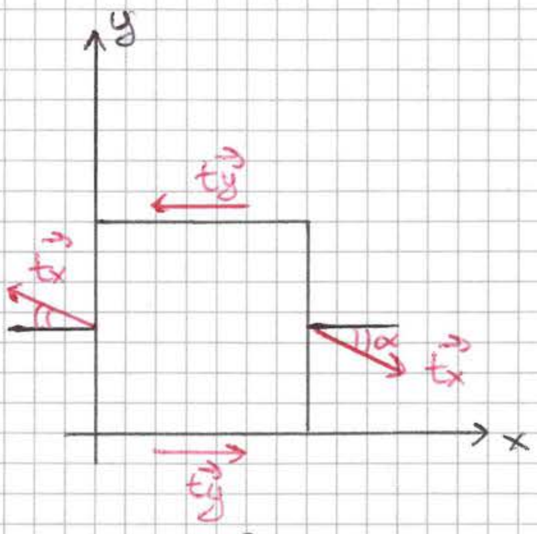
$$\varphi_C = \frac{1}{EI}\left(-\frac{68}{9}qb^3\right)$$

- SPOSTAMENTO VERTICALE DEL PUNTO B, v_B

$v_B = v_{(z_2=0)}$

$v_{(z_2=0)} = 16\frac{qb^4}{EI}$

ESERCIZIO 3 _ TRACCIA 1 _ ESAME 27/06/2023



$\alpha = -30°$

$sen\alpha = -\frac{1}{2}$

$\cos\alpha = \frac{\sqrt{3}}{2}$

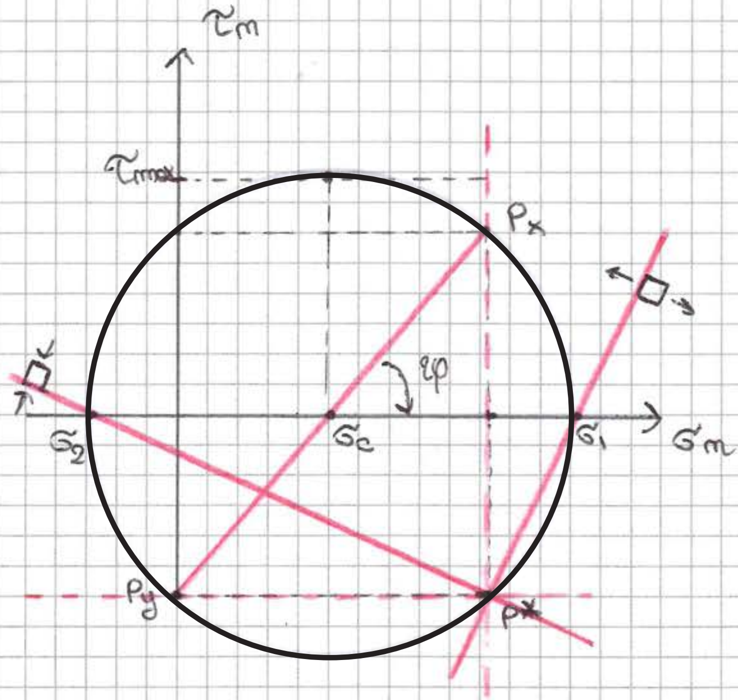
$|t_x| = 60\ MPa$

$\sigma_x = |t_x| \cos\alpha = 60 \cdot \frac{\sqrt{3}}{2} = 30\sqrt{3}\ MPa = 51,961\ MPa$

$\tau_{xy} = |t_x| sen\alpha = 60\left(-\frac{1}{2}\right) = -30\ MPa$

$\tau_{yx} = \tau_{xy} = -30\ MPa$

$$\underline{\underline{\sigma}} = \begin{bmatrix} 51,961 & -30 & 0 \\ -30 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



CERCHIO DI MOHR

$P_x(\sigma_x ; \tau_{xy}) \Rightarrow \tau_{xy}$ ↻

$P_y(\sigma_y ; -\tau_{yx}) \Rightarrow \tau_{yx}$ ↺

$P_x = (51,961 ; 30) ; P_y(0 ; -30)$

$C\left(\frac{\sigma_x + \sigma_y}{2} ; 0\right) = (25,980 ; 0)$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}} = \sqrt{(25,980)^2 + (30)^2}$

$= \sqrt{674,960 + 900} = \sqrt{1574,96} = 39,685$

$\tau_{max} = R = 39,685\ MPa$

$\sigma_1 = \sigma_c + R = 25,980 + 39,685 = 65,665\ MPa$

$\sigma_2 = \sigma_c - R = 25,980 - 39,685 = -13,705$

$\varphi = ?$

$\tan 2\varphi = \frac{\tau_{xy}}{\sigma_x - \sigma_c} = \frac{-30}{25,981} = -1,154$

$arctg(-1,154) = -49,089 = 2\varphi \Rightarrow \varphi = -24,544°$