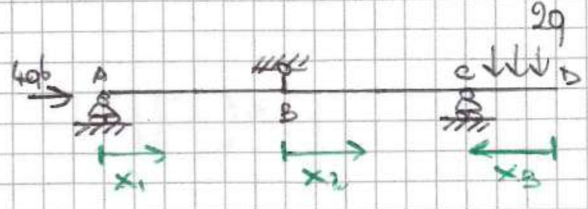
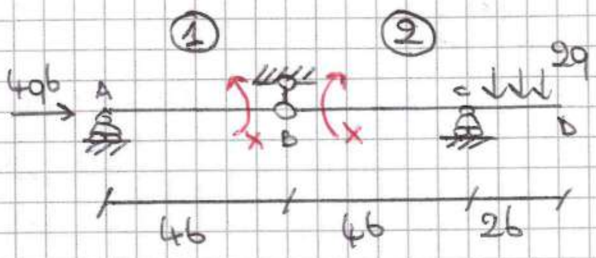
TRACCIA 1 _ ESERCIZIO 1 _ ESAME 06.06.2023



STRUTTURA IPERSTATICA

GDL = 3 GDV = 1(A) + 2(B) + 1(C) = 4

GDL < GDV



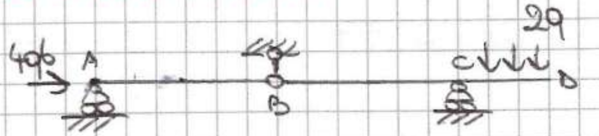
STRUTTURA ISOSTATICA

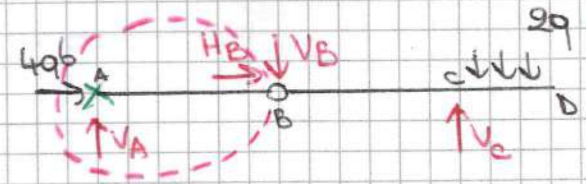
GDL = 3(1) + 3(2) = 6

GDV = 1(A) + 4(B) + 1(C) = 6

GDL = GDV

P_0 _ SISTEMA REALE





$$\begin{cases} H_B + 4qb = 0 \Rightarrow H_B = -4qb \\ V_A - V_B + V_C - 2q(2b) = 0 \quad [2] \\ V_B(4b) - V_C(8b) + 2q(2b)(9b) = 0 \quad [3] \end{cases}$$

eq. aux.

$$\left\{ V_A(4b) = 0 \Rightarrow V_A = 0 \right.$$

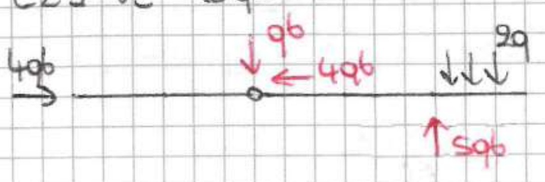
[2] $-V_B + V_C - 4qb = 0 \quad V_C = V_B + 4qb$

[3] $4bV_B - 8b(V_B + 4qb) + 36qb^2 = 0$

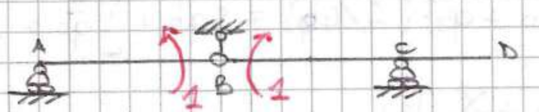
$4bV_B - 8bV_B - 32qb^2 + 36qb^2 = 0$

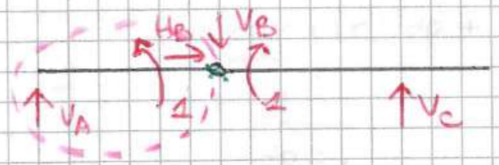
$-4bV_B + 4qb^2 = 0 \Rightarrow V_B = qb$

[2] $V_C = 5qb$



P_1 _ SISTEMA EQUILIBRATO





$$\begin{cases} H_B = 0 \\ V_A - V_B + V_C = 0 \quad [2] \\ V_C(4b) - V_A(4b) = 0 \quad [3] \end{cases}$$

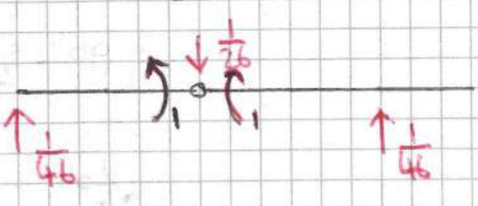
eq. AUX

$$\left\{ V_A(4b) - 1 = 0 \Rightarrow V_A = \frac{1}{4b} \right.$$

[3] $4bV_C - 1 = 0 \Rightarrow V_C = \frac{1}{4b}$

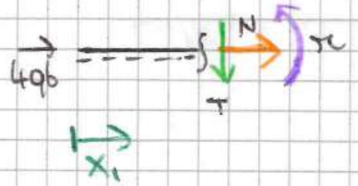
[2] $\frac{1}{4b} - V_B + \frac{1}{4b} = 0 \Rightarrow \frac{2}{4b} - V_B = 0$

$V_B = \frac{1}{2b}$



AZIONI INTERNE

A → B $\quad 0 \le x_1 \le 4b$

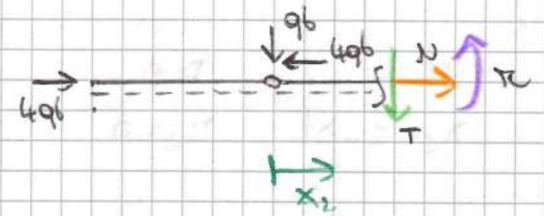


$N(x_1) = -4qb$

$T(x_1) = 0$

$M(x_1) = 0$

B → C $\quad 0 \le x_2 \le 4b$

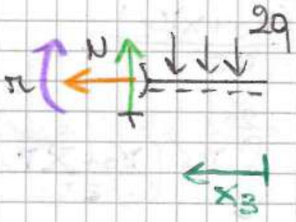


$N(x_2) - 4qb + 4qb = 0 \Rightarrow N(x_2) = 0$

$T(x_1) + qb = 0 \quad T(x_1) = -qb$

$M(x_2) + qb(x_2) = 0 \Rightarrow M(x_2) = -qbx_2$

D → C $\quad 0 \le x_3 \le 2b$



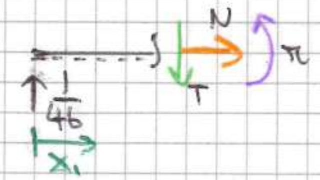
$N(x_3) = 0$

$T(x_3) - 2q(x_3) = 0 \Rightarrow T(x_3) = 2qx_3$

$M(x_3) + \cancel{2}q(x_3)\left(\frac{x_3}{\cancel{2}}\right) = 0$

$M(x_3) + qx_3^2 = 0 \quad M(x_3) = -qx_3^2$

A → B $\quad 0 \le x_1 \le 4b$

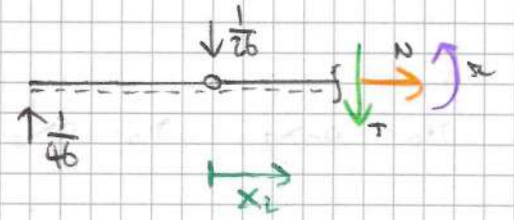


$N(x_1) = 0$

$T(x_1) = \frac{1}{4b}$

$M(x_1) - \frac{1}{4b}(x_1) = 0 \Rightarrow M(x_1) = \frac{x_1}{4b}$

B → C $\quad 0 \le x_2 \le 4b$



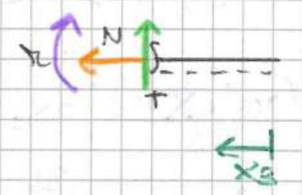
$N(x_2) = 0$

$T(x_2) + \frac{1}{2b} - \frac{1}{4b} = 0 \Rightarrow T(x_2) = -\frac{1}{4b}$

$M(x_2) - \frac{1}{4b}(4b + x_2) + \frac{1}{2b}(x_2) = 0$

$M(x_2) - 1 - \frac{x_2}{4b} + \frac{x_2}{2b} = 0 \Rightarrow M(x_2) = 1 - \frac{x_2}{4b}$

D → C $\quad 0 \le x_3 \le 2b$



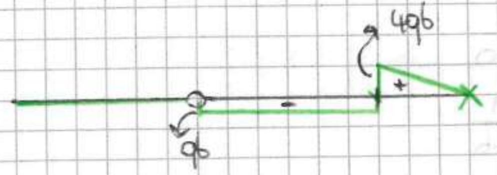
$N(x_3) = 0$

$T(x_3) = 0$

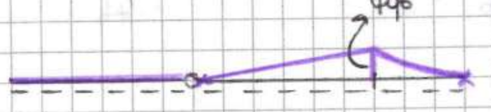
$M(x_3) = 0$

N

T

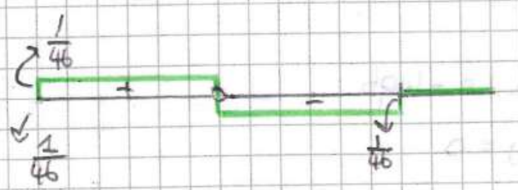


M

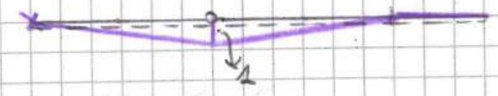


N

T



M



A→B $M_0 = 0$

B→C $M_0 = -qbx_2$

C→D $M_0 = -qx_3^2$

A→B $M_1 = \frac{x_1}{4b}$

B→C $M_1 = 1 - \frac{x_2}{4b}$

D→C $M_1 = 0$

P.L.V.

$\mathcal{L}V_i = \mathcal{L}V_e$

$\mathcal{L}V_e = 1 \cdot \Delta\varphi_B = 0$

$$\mathcal{L}V_i = \int_S \cancel{N_1 \varepsilon_i} + \int_S \cancel{T_1 \gamma_i} + \int_S M_1 \chi_i \Rightarrow \mathcal{L}V_i = \int_S M_1\left(\frac{M_0 + XM_1}{EJ}\right)$$

$$\mathcal{L}V_i = \int_0^{4b} M_1\left(\frac{\cancel{M_0} + XM_1}{EJ}\right)dx_1 + \int_0^{4b} M_1\left(\frac{M_0 + XM_1}{EJ}\right)dx_2 + \int_0^{2b} \cancel{M_1}\left(\cancel{\frac{M_0 + XM_1}{EJ}}\right)dx_3$$

$$\mathcal{L}V_i = \int_0^{4b} X\frac{M_1^2}{EJ}dx_1 + \int_0^{4b} M_1\left(\frac{M_0 + XM_1}{EJ}\right)dx_2$$

$$= \int_0^{4b} X\frac{x_1^2}{16b^2}\frac{1}{EJ}dx_1 + \int_0^{4b}\left(1 - \frac{x_2}{4b}\right)\left[-qbx_2 + X\left(1 - \frac{x_2}{4b}\right)\right]\left(\frac{1}{EJ}\right)dx_2$$

$$= \frac{1}{EJ}\left[X\frac{x^3}{48b^2}\right]_0^{4b} + \int_0^{4b}\left(1 - \frac{x_2}{4b}\right)\left(-qbx_2 + X - X\frac{x_2}{4b}\right)\left(\frac{1}{EJ}\right)dx_2$$

$$= \frac{1}{EJ}\left(X\frac{64b^3}{48b^2}\right) + \int_0^{4b}\left(-qbx_2 + X - X\left(\frac{x_2}{4b}\right) + \frac{x_2^2}{4}q - X\frac{x_2}{4b} + X\frac{x_2^2}{16b^2}\right)\left(\frac{1}{EJ}\right)dx_2$$

$$= \frac{4}{3}bX + \int_0^{4b}\left(-qbx_2 + X - X\frac{x_2}{2b} + \frac{x_2^2}{4}q + X\frac{x_2^2}{16b^2}\right)dx_2$$

$$= \frac{4}{3}bX + \left[-qb\frac{x_2^2}{2} + Xx_2 - X\frac{x_2^2}{4b} + \frac{x_2^3}{12}q + X\frac{x_2^3}{48b^2}\right]_0^{4b}$$

$$= \frac{4}{3}bX - qb\frac{\cancel{16}^{8}b^2}{\cancel{2}} + X(4b) - X\frac{\cancel{16}^{4}b^2}{\cancel{4b}} + \frac{64b^3}{12}q + X\frac{64b^3}{48b^2}$$

$$\delta V_i = \frac{4}{3} b \bar{X} - 8 q b^3 + \cancel{4 b \bar{X}} - \cancel{4 b \bar{X}} + \frac{16}{3} q b^3 + \bar{X} \frac{4}{3} b$$

$$= \frac{8}{3} b \bar{X} - \frac{8}{3} q b^3 \qquad \bar{X} = q b^2$$

$V_A = V_{A0} + \bar{X} V_{A1}$

$V_A = 0 + q b^2 (1/4b) = \frac{1}{4} q b$

$V_B = V_{B0} + \bar{X} V_{B1}$

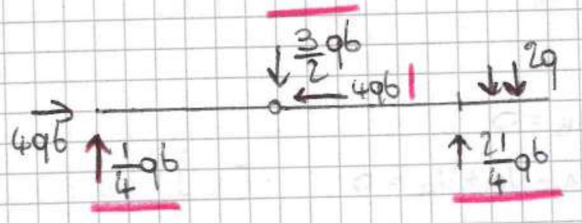
$V_B = q b + q b^2 (1/2b) = q b + \frac{1}{2} q b = \frac{3}{2} q b$

$V_C = V_{C0} + \bar{X} V_{C1}$

$V_C = 5 q b + q b^2 (1/4b) = 5 q b + \frac{1}{4} q b = \frac{21}{4} q b$

$H_B = H_{B0} + \bar{X} H_{B1}$

$H_B = 4 q b + q b^2 (0) = 4 q b$



$A \to B$

$N = N_0 + X N_1$

$N = -4 q b + q b^2 (0) = -4 q b$

$T = T_0 + X T_1$

$T = 0 + q b^2 \left(\frac{1}{4b}\right) = \frac{q b}{4}$

$M = M_0 + X M_1$

$M = 0 + q b^2 \left(\frac{x_1}{4b}\right) = x_1 \frac{q b}{4}$

$B \to C$

$N = N_0 + X N_1$

$N = 0 + q b^2 (0) = 0$

$T = T_0 + X T_1$

$T = -q b + q b^2 \left(-\frac{1}{4b}\right) = -\frac{5}{4} q b$

$M = M_0 + X M_1$

$M = -q b x_2 + q b^2 \left(1 - \frac{x_2}{4b}\right)$

$= -q b x_2 + q b^2 - \frac{1}{4} q b x_2$

$= q b^2 - \frac{5}{4} q b x_2$

$D \to C$

$N = N_0 + X N_1$

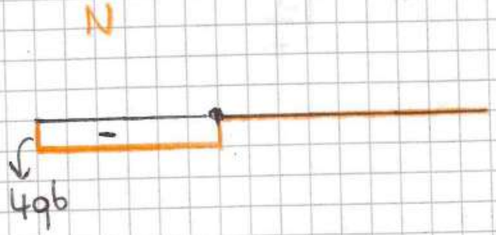
$N = 0 + q b^2 (0) = 0$

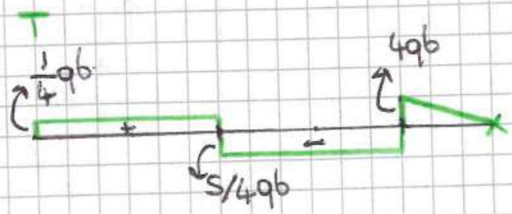
$T = T_0 + X T_1$

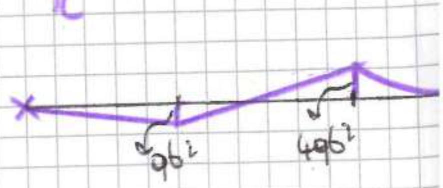
$T = 2 q x_3 + q b^2 (0) = 2 q x_3$

$M = M_0 + X M_1$

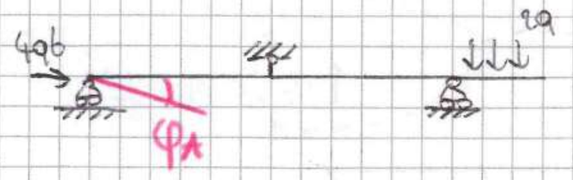
$M = -q x_3^2$



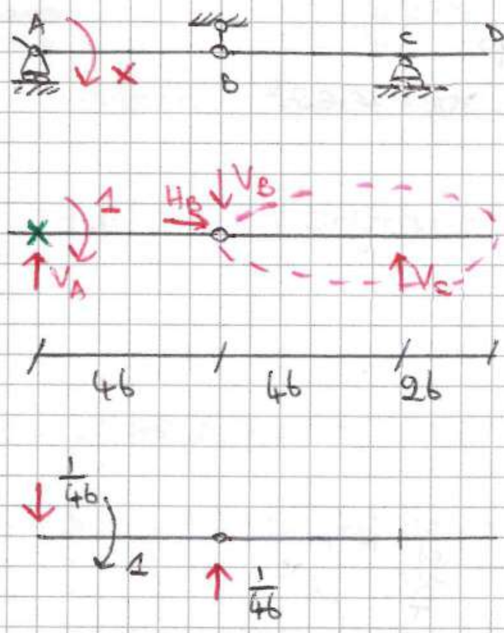




• CALCOLO COMPONENTE DI SPOSTAMENTO φ_A



P_2

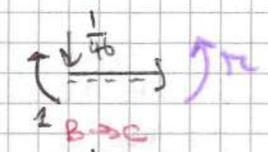


$$\begin{cases} H_B = 0 \\ V_A + \cancel{V_C} - V_B = 0 \qquad V_A = -\frac{1}{4b} \\ 1 + V_B(4b) - \cancel{V_C(8b)} = 0 \Rightarrow V_B = -\frac{1}{4b} \end{cases}$$

eq. aux

$$\{ V_C\, 4b = 0 \Rightarrow V_C = 0$$

A → B



$$M(x_1) - 1 + \frac{1}{4b} x_1 = 0$$

$$M(x_1) = 1 - \frac{1}{4b} x_1$$

B → C

$$M(x_2) - \frac{1}{4b} x_2 - 1 + \frac{1}{4b}(4b + x_2) = 0$$

$$M(x_2) = 0$$

D → C

$$M(x_3) = 0$$

P.L.V.

$\mathcal{L}V_e = \mathcal{L}V_i$

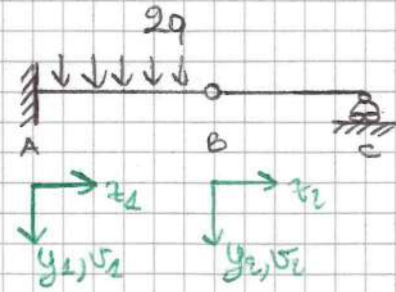
$\mathcal{L}V_e = 1 \cdot \varphi_B = \varphi_B$

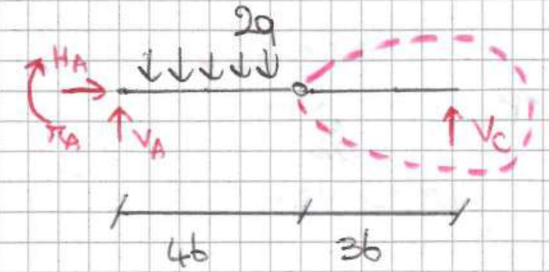
$$\mathcal{L}V_i = \int_S M_2(x)\, \chi_x\, dx$$

$$= \int_0^{4b} \left(1 - \frac{1}{4b} x_1\right)\left(x_1 \frac{qb}{4}\right)\left(\frac{1}{EJ}\right) dx_1 = \frac{1}{EJ}\int_0^{4b} x_1 \frac{qb}{4} - \frac{1}{16} q x_1^2 \, dx_1 =$$

$$= \frac{1}{EJ}\left[\frac{x^2}{2}\frac{qb}{4} - \frac{x^3}{3}\frac{1}{16} q\right]_0^{4b} = \frac{1}{EJ}\left[\cancel{\frac{16}{8}}^{2} qb^3 - \cancel{\frac{64}{48}}^{\frac{4}{3}} qb^3\right] = \frac{6-4}{3}\frac{qb^3}{EJ} = \frac{2}{3}\frac{qb^3}{EJ}$$

TRACCIA 1 _ ESERCIZIO 2 _ ESAME 06.06.2023

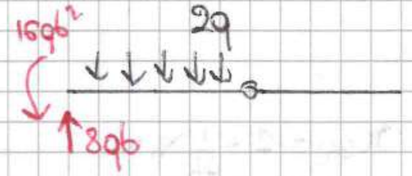




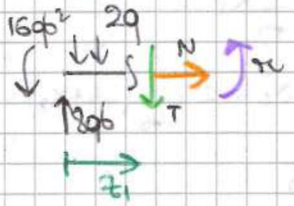
$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(A)} = 0 \end{cases} \qquad \begin{cases} H_A = 0 \\ V_A + \cancel{V_C} - 2q(4b) = 0 \implies V_A = 8qb \\ M_A + 2q(4b)(2b) - \cancel{V_C}(7b) = 0 \implies M_A = -16qb^2 \end{cases}$$

eq. AUX

$$\left\{ M^{I}_{z(B)} \right. \qquad \left\{ V_C(3b) = 0 \rightarrow V_C = 0 \right.$$



A → B



$N_{z_1} = 0$

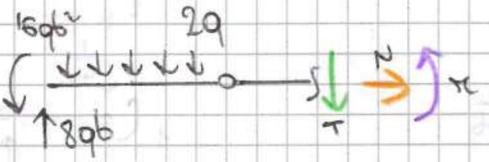
$T_{z_1} + 2qz_1 - 8qb = 0$

$T_{z_1} = 8qb - 2qz_1$

$M_{z_1} + 2q\frac{z_1^2}{2} - 8qbz_1 + 16qb^2 = 0$

$M_{z_1} = 8qbz_1 - 16qb^2 - qz_1^2$

B → C



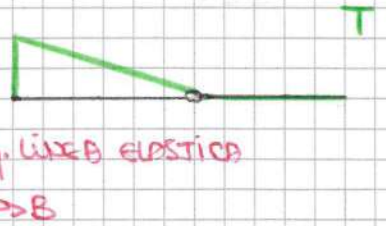
$N_{z_2} = 0$

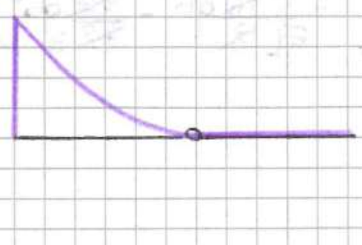
$T_{z_2} + 2q(4b) - 8qb = 0 \quad T_{z_2} = 0$

$M_{z_2} + 2q(4b)(2b + z_2) - 8qb(4b + z_2) + 16qb^2 = 0$

$M_{z_2} + \cancel{16qb^2} + \cancel{8qbz_2} - \cancel{32qb^2} - \cancel{8qbz_2} + \cancel{16qb^2} = 0$

$M_{z_2} = 0$





Eq. LINEA ELASTICA

A → B

$$v''_{1(z_1)} = -\frac{M}{EJ} \implies v''_{(z_1)} = \frac{1}{EJ}\left(-8qbz_1 + 16qb^2 + qz_1^2\right)$$

$$v'_{1(z_1)} = \frac{1}{EJ}\left(-8qb\cdot\frac{z_1^2}{2} + 16qb^2z_1 + q\frac{z_1^3}{3}\right) + A_1 = \frac{1}{EJ}\left(-4qbz_1^2 + 16qb^2z_1 + \frac{1}{3}qz_1^3\right) + A_1$$

$$v_{1}(z_1) = \frac{1}{EJ}\left(-4qb\frac{z_1^3}{3} + 16qb^2\frac{z_1^2}{2} + \frac{1}{3}q\frac{z_1^4}{4}\right) + A_1z_1 + A_2 = \frac{1}{EJ}\left(-\frac{4}{3}qbz_1^3 + 8qb^2z_1^2 + \frac{1}{12}qz_1^4\right) +$$

$$+ A_1z_1 + A_2$$

B → C

$v''_{2(z_2)} = -\frac{M}{EJ} \Rightarrow v''_{z(z_2)} = 0$

$v'_{2(z_2)} = B_1$

$v_{2(z_2)} = B_1 z_2 + B_2$

COSTANTI $A_1; A_2; B_1; B_2$

A)  IMPEDISCE { SPOSTAMENTO $\Rightarrow v_{z_1}(A) = 0$
ROTAZIONE $\Rightarrow v'_{z_1}(A) = 0$

INCASTRO

B)  IMPONE { UGUALE ABBASSAMENTO IN B_1 E B_2 $\Rightarrow v_{(z_1)}(B) = v_{z_2}(B) \Rightarrow v_1(z_1 = 4b) = v_2(z_2 = 0)$

CERNIERA

C)  IMPEDISCE { SPOSTAMENTO $\Rightarrow v_2(C) = 0$

CARRELLO

IN A

$v'_{z_1}(A=0) = 0 \Rightarrow A_1 = 0 \Rightarrow v'_{z_1} = \frac{1}{EJ}\left(-4qbz_1^2 + 16qb^2z_1 + \frac{1}{3}qz_1^3\right)$

$v_{z_1}(A=0) = 0 \Rightarrow A_2 = 0 \Rightarrow v_{z_1} = \frac{1}{EJ}\left(-\frac{4}{3}qbz_1^3 + 8qb^2z_1^2 + \frac{1}{12}qz_1^4\right)$

IN B

$v_{z_1}(z_1 = 4b) = v_{z_2}(z_2 = 0)$

$\frac{1}{EJ}\left(-\frac{4}{3}qb \cdot 64b^3 + 8qb^2 \cdot 16b^2 + \frac{1}{3\cancel{12}} q \cdot \overset{64}{\cancel{256}}b^4\right) = B_2$

$\frac{1}{EJ}\left(-\frac{256}{3}qb^4 + 128qb^4 + \frac{64}{3}qb^4\right) = B_2$

$B_2 = \frac{1}{EJ}\left(\frac{192}{3}qb^4\right) = 64\frac{qb^4}{EJ}$

IN C

$v_{z_2}(z_2 = 3b) = 0$

$B_1(3b) + \frac{64qb^4}{EJ} = 0 \Rightarrow B_1 = -\frac{64}{EJ}qb^{\cancel{4}\,3} \cdot \frac{1}{3b} = -\frac{64}{3}\frac{qb^3}{EJ} \Rightarrow v_{z_2} = -\frac{64}{3}\frac{qb^3}{EJ}z_2 + 64\frac{qb^4}{EJ}$

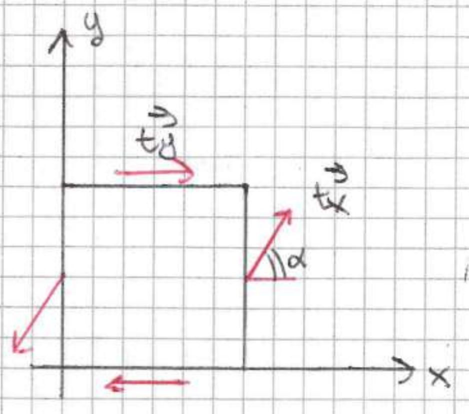
$v'_{z_2} = -\frac{64}{3}\frac{qb^3}{EJ}$

φ_C? v_B?

$v'_{z_2}(z_2 = 3b) = -\frac{64qb^4}{3EJ} \Rightarrow \varphi_C = -\frac{64qb^4}{3EJ}$

$v_{z_2}(z_2 = 0) = \frac{64qb^4}{EJ} \Rightarrow v_B = 64\frac{qb^4}{EJ}$

ESERCIZIO 3 _ TEORIA 1 _ ESAME 05.05.2023



$\alpha = 60°$

$\cos\alpha = \frac{1}{2}$ $\quad \operatorname{sen}\alpha = \frac{\sqrt{3}}{2}$

$|t_x| = 45\,MPa$

$\sigma_x = |t_x| \cos\alpha = 45\,MPa \cdot \frac{1}{2} = 22,5\,MPa$

$\tau_{xy} = |t_x| \operatorname{sen}\alpha = 45 \cdot \frac{\sqrt{3}}{2} = 38,971\,MPa$

$\tau_{yx} = \tau_{xy} = 38,971\,MPa$

$$\underline{\underline{\sigma}} = \begin{bmatrix} 22,5 & 38,971 & 0 \\ 38,971 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

CERCHIO DI MOHR

$P_x(\sigma_x ; -\tau_{xy}) \rightarrow \tau_{xy}$

$P_y(\sigma_y ; \tau_{yx}) \rightarrow \tau_{yx}$

$P_x = (22,5 ; -38,971)$

$P_y = (0 ; 38,971)$

$C = (11,25 ; 0) \Rightarrow \left(\frac{\sigma_x + \sigma_y}{2} ; 0\right)$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(11,25)^2 + (38,971)^2}$

$= \sqrt{126,562 + 1518,738} = \sqrt{1645,3} = 40,562$

$\tau_{max} = R \Rightarrow 40,562\,MPa$

$\sigma_1 = \sigma_c + R = 11,25 + 40,562 = 51,812$

$\sigma_2 = \sigma_c - R = 11,25 - 40,562 = -29,312$

$\varphi = ?$

$\tan 2\varphi = \frac{\tau_{xy}}{\sigma_x - \sigma_c} = \frac{38,971}{22,5 - 11,25} = \frac{38,971}{11,25} = 3,464$

$\operatorname{arctg}(3,464) = 73,897 = 2\varphi \Rightarrow \varphi = 36,95°$