

MHD equilibrium

Plasma model

A plasma model providing the physical understanding of the **macroscopic forces that steer the plasma** need to consider:

- I. *current density* and the
- II. *charge forces*, generated by single-particle motion
- III. external imposed electric and magnetic field.

Two model can be derived

- ✓ Kinetic model
- ✓ fluid model

The kinetic model is very accurate as much as complicate, it includes a wide variety of physical phenomena.

Plasma model

The fluid model provides a macroscopic description of the plasma behaviour. It assumes the *global physical quantities, such as density, temperature, pressure*, etc, as functions of space and time only. Even if it is not detailed as the kinetic model, all the important phenomena can be accurately described. Indeed, The fluid model allows:

- ❑ determining the **transport** of energy, particles, and magnetic flux, across the plasma
- ❑ understanding how **electromagnetic waves propagate** into a plasma, to provide heating and non-inductive current drive
- ❑ learning how small perturbations in current density and charge density can affect the macroscopic (and also microscopic) **stability of the plasma**.

Plasma model

The fluid-dynamics is a subdiscipline of fluid mechanics that describes the flow of fluids, liquids and gases in dynamic equilibrium (uniform motion $v=\text{const}$). The fluid it is characterized by

- ❖ the fluid mass density ρ_m
- ❖ the velocity vector \bar{v}

The foundational axioms of fluid dynamics are the conservation laws, specifically:

- ❑ conservation of mass
- ❑ conservation of linear momentum
- ❑ conservation of energy (I Law of Thermodynamics).

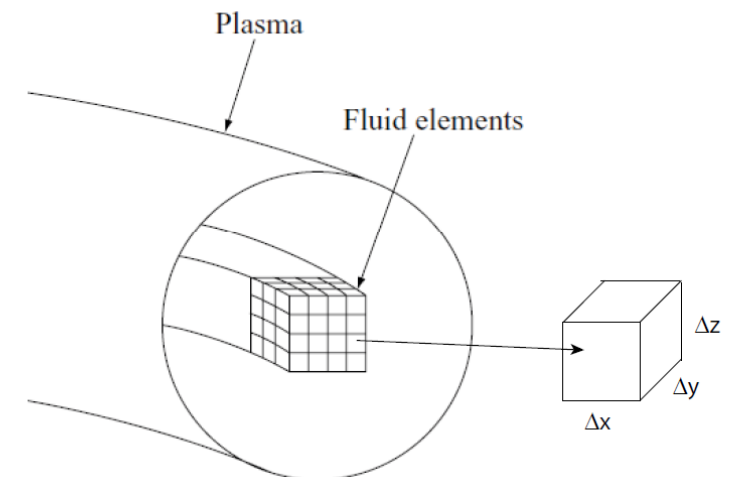
- **Macroscopic averages**

$$n_e(\bar{r}, t) = \frac{\text{number of particles}}{\text{Volume}} = \frac{N_e}{\Delta V}$$

$$\bar{v}_e(\bar{r}, t) = \frac{\sum \text{particle velocities}}{\text{number of particles}} = \frac{\sum_k v_{ek}}{N_e}$$

$$\Delta V = \Delta x \Delta y \Delta z$$

Macroscopic fluid model



Conservation of mass

Conservation of mass (mass continuity) : the rate of change of fluid mass inside a control volume must be equal to the net rate of fluid flow into the volume.

$$\nabla \cdot (\rho_m \bar{v}) - \frac{d\rho_m}{dt} = 0 \quad (1)$$

The mass continuity equation, also called mass continuity equation is similar to the conservation of charge equation $\nabla \cdot \bar{J} + \frac{d\rho}{dt} = 0$ where $\bar{J} = \pm \rho \bar{v}_d$ and $\rho =$ charge density.

The equation (1) can be derive for each species (electrons, ions)

$$\nabla \cdot (\rho_{me} \bar{v}) - \frac{d\rho_{me}}{dt} = 0 \left[\frac{A}{m^3} \right] \quad (1')$$

$$\nabla \cdot (\rho_{mi} \bar{v}) - \frac{d\rho_{mi}}{dt} = 0 \left[\frac{A}{m^3} \right] \quad (1')$$

The expression of the conservation of mass for a fully ionized gas (ionization and recombination are neglected) is the equation 1 where:

$$\rho_m = \rho_{me} + \rho_{mi} \qquad \bar{v} = \frac{\rho_{me} \bar{v}_e + \rho_{mi} \bar{v}_i}{\rho_{me} + \rho_{mi}}$$

Conservation of linear momentum

Conservation of linear momentum (Newton's II law) any change in the momentum of a control volume is due to net flow of momentum into the volume and the action of external forces acting on the fluid within the volume

$$M = m n \Delta V:$$

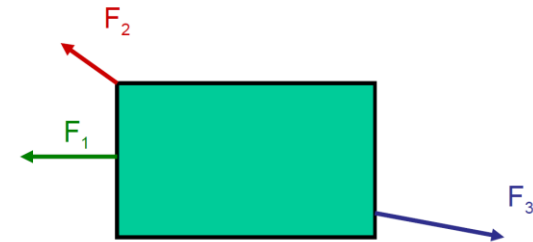
m is the mass of a single particle (m_i for ions and m_e for electrons)

n is the particle density (n_i for ions and n_e for electrons)

ΔV fluid control volume

$\rho_m = m n$ mass density

$$M \frac{d\bar{v}}{dt} = \sum_i \bar{F}_i$$



$$M \frac{d\bar{v}}{dt} = m n \Delta V \frac{d\bar{v}}{dt} = [\rho_m \frac{d\bar{v}}{dt}] \Delta V$$

In our model $\sum_i \bar{F}_i$ includes the force on the fluid elements:

a) \bar{F}_L : Lorentz force due to electric and magnetic fields

b) \bar{F}_p : pressure gradient force (sul volume elementare c'è uno sbilanciamento di forze)

c) \bar{F}_c : collisional friction force (net momentum exchange collisional force)

Gravitational force and viscosity are neglected

Conservation of linear momentum

Lorentz force $\bar{F}_L = \bar{F}_E + \bar{F}_B = Q (\bar{E} + \bar{v} \times \bar{B}) = n e (\bar{E} + \bar{v} \times \bar{B}) \Delta V = \rho (\bar{E}_h + \bar{v} \times \bar{B}) \Delta V$

Where ρ = is the charge density

$$\bar{F}_L = (\rho \bar{E}_h + \bar{j} \times \bar{B}) \Delta V$$

Pressure gradient force $\bar{F}_p = -\nabla p \Delta V$ $p=nT$

Collisional friction force: If electrons have a fluid velocity larger than ions, then Coulomb collisions with the ions produce a drag force on the electrons $\bar{F}_{c,e}$

$$\bar{F}_{c,e} = -m_e n_e f_{ei} (\bar{v}_e - \bar{v}_i) \Delta V \qquad \bar{F}_{c,i} = m_i n_i f_{ei} (\bar{v}_i - \bar{v}_e) \Delta V$$

$f_{e,i}$ is the collision frequency. An opposite drag force is produced on the ions from electrons $\bar{F}_{c,e} = -\bar{F}_{c,i}$.

Thus for a macroscopic neutral plasma the $\bar{F}_{c,e}$ is balanced by $\bar{F}_{c,i}$

$$M \frac{d\bar{v}}{dt} = \sum_i \bar{F}_i \quad \longrightarrow \quad \rho_m \left[\frac{d\bar{v}}{dt} \right] \Delta V = (\rho \bar{E} + \bar{j} \times \bar{B}) \Delta V - \nabla p \Delta V \quad (2)$$

Conservation of energy

Assuming an **adiabatic process** $p V^\gamma = \text{const}$

This equation expresses the relationship between pressure and the volume of an ideal gas undergoing an adiabatic transformation, where $\gamma = \frac{c_p}{c_v}$

- ✓ c_p : specific heat for constant pressure
- ✓ c_v : specific heat for constant volume
- ✓ $\rho_m = \frac{M}{V}$

Un a quasistatic process

$$\frac{d}{dt} (p V^\gamma) = \frac{d}{dt} \left(\frac{p}{\rho_m^\gamma} \right) = 0 \quad (3)$$

**I thermodynamics law
in a quasistatic process**

$$dU = \delta Q - \delta L$$



The change in the internal energy of a closed system is equal to net energy added as heat to the system minus the thermodynamic work done by the system

For an adiabatic process is that heat transfer to the system is zero, $\delta Q = 0$

In a quasistatic process the thermodynamic work done by the system on the surroundings is $L = p \cdot V$, if p remain constant

$$dU = -\delta L = -P \cdot dV$$

Maxwell equations

Differential form

$$\nabla \times \bar{\mathbf{E}} = -\frac{\partial \bar{\mathbf{B}}}{\partial t} \quad \text{Faraday' law}$$

$$\nabla \times \bar{\mathbf{H}} = \bar{\mathbf{J}} + \frac{\partial \bar{\mathbf{D}}}{\partial t} \quad \text{Ampere' law}$$

$$\nabla \cdot \bar{\mathbf{B}} = 0 \quad \text{Gauss' law for the magnetic flux density } B$$

$$\nabla \cdot \bar{\mathbf{D}} = \rho \quad \text{Gauss' law for the electric flux density } D$$

Integral form

$$\oint_C \bar{\mathbf{E}} \cdot d\bar{\mathbf{l}} = -\int_S \frac{\partial \bar{\mathbf{B}}}{\partial t} \cdot d\bar{\mathbf{s}}$$

$$\oint_C \bar{\mathbf{H}} \cdot d\bar{\mathbf{l}} = \int_S \left(\bar{\mathbf{J}} + \frac{\partial \bar{\mathbf{D}}}{\partial t} \right) \cdot d\bar{\mathbf{s}}$$

$$\oint_S \bar{\mathbf{D}} \cdot d\bar{\mathbf{s}} = \int_V \rho \, dv$$

$$\oint_S \bar{\mathbf{B}} \cdot d\bar{\mathbf{s}} = 0$$

ρ = is the free charge density

Electromagnetic field equations

Constitutive equations for uniform materials

$$\begin{aligned}\bar{D} &= \epsilon \bar{E} \\ \bar{B} &= \mu \bar{H}\end{aligned}$$

Conservation of the electric charge

$$\nabla \cdot \bar{J} = -\frac{\partial \rho}{\partial t} \quad \text{where} \quad \bar{J} = nq\bar{v}_d = \rho\bar{v}_d$$

Ohm' law for conductive current (no transport, no material motion)

$$\bar{J} = \sigma \bar{E}$$

Faraday-Neumann' Law

$$\oint_C \bar{E}' \cdot d\bar{l} = - \int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s} + \oint_C (\bar{v} \times \bar{B}) \cdot d\bar{l}$$

$$\oint_C \bar{E}' \cdot d\bar{l}$$

Total induced e.m.f in a conducting loop in a non-stationary field B .

$$- \int_S \frac{\partial \bar{B}}{\partial t} \cdot d\bar{s}$$

Induced e.m.f . due to the B variation over time.

$$\oint_C (\bar{u} \times \bar{B}) \cdot d\bar{l}$$

Motional e.m.f due to motion or deformation of the conducting loop in B .

Electromagnetic field equations

By combining $\nabla \cdot \bar{J} = -\frac{\partial \rho}{\partial t}$ With the Ohm' law $\bar{J} = \sigma \bar{E}$ and $\bar{D} = \epsilon \bar{E}$

$$\frac{\sigma}{\epsilon} \nabla \cdot \bar{D} = -\frac{\partial \rho}{\partial t} \quad \xrightarrow{\nabla \cdot \bar{D} = \rho} \quad \frac{\sigma}{\epsilon} \rho + \frac{\partial \rho}{\partial t} = 0 \quad \rightarrow \quad \frac{\partial \rho}{\partial t} + \tau \rho = 0$$

$\rho(t) = \rho(0)e^{-\frac{t}{\tau}}$ The *free charges* inside an isolate (no external fields) conductive fluid will move towards the surface of the conductor and will redistribute themselves on the edge so that in equilibrium conditions $E=0$ and the charge density $\rho = 0$

For plasmas with $T=10\text{keV}$ ($\epsilon = \epsilon_0 = 8.86 \cdot 10^{-12} \text{ F/m}$, $\sigma = 10^9 \text{ S/m}$), $\tau = 10^{-20}$

In a quasi-neutral plasma free charge can exist, but in $\tau = 10^{-20} \text{ s}$ it is stabilized to a negligible value. *Thus, After 10^{-20} the Ampere' law and the conservation of linear momentum can be simplified as:*

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

$$\rho_m \left[\frac{d\bar{v}}{dt} \right] = \cancel{\rho \bar{E}} + (\bar{J} \times \bar{B}) - \nabla p \quad (1)$$

Since $\nabla \cdot \bar{D} = \rho$, if $\rho = 0$ the D variation in time can be neglected

MHD model

Ohm's law in a conductive fluids

In a conducting plasma surrounded by a magnetic field, the Ohm's law must take into account also the contribute \bar{E}' due to motion of the charge.

$$\bar{E} = \eta \bar{J} \quad \longrightarrow \quad \bar{E} + (\bar{v} \times \bar{B}) = \eta \bar{J}$$

The same equation can be derived by the II Newton's Law written for the electron motion in a conductive plasma:

$$m_e n_e \left[\frac{d\bar{v}_e}{dt} \right] = -n_e e (\bar{E} + \bar{v}_e \times \bar{B}) + m_e n_e f_{ei} (\bar{v}_e - \bar{v}_i)$$

\bar{v}_i and $\frac{d\bar{v}_e}{dt}$ Are negligible, thus $n_e e (\bar{E} + \bar{v}_e \times \bar{B}) + m_e n_e f_{ei} \bar{v}_e = 0$

$$\left(\bar{E} + \bar{v}_e \times \bar{B} \right) = \frac{m_e f_{ei}}{n_e e^2} e n_e \bar{v}_e \quad \longrightarrow \quad \begin{array}{l} \frac{m_e f_{ei}}{n_e e^2} = \eta \\ e n_e \bar{v}_e = \bar{J} \end{array} \quad \left(\bar{E} + \bar{v}_e \times \bar{B} \right) = \eta \bar{J}$$

MHD equations

Single-fluid equations

$$\nabla \cdot (\rho_m \bar{v}) - \frac{d\rho_m}{dt} = 0 \quad (1)$$

$$\rho_m \left[\frac{d\bar{v}}{dt} \right] = (\bar{J} \times \bar{B}) - \nabla p \quad (2)$$

$$\frac{d}{dt} \left(\frac{p}{\rho_m^\gamma} \right) = 0 \quad (3)$$

Electromagnetic field equations

$$\bar{E} + (\bar{v} \times \bar{B}) = \eta \bar{J} \quad (4)$$

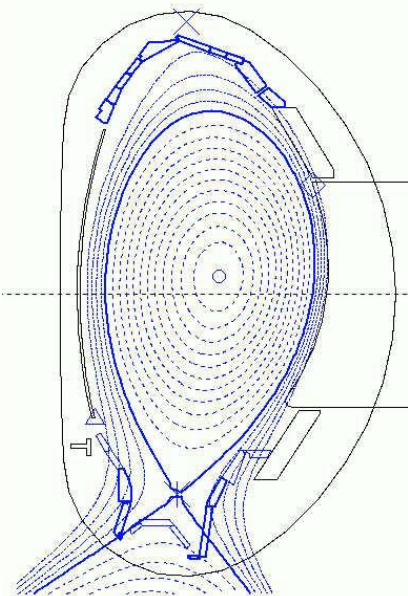
$$\nabla \times \bar{B} = \mu_0 \bar{J} \quad (5)$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad (6)$$

Linked through

the term $\bar{J} \times \bar{B}$

magnetic equilibrium



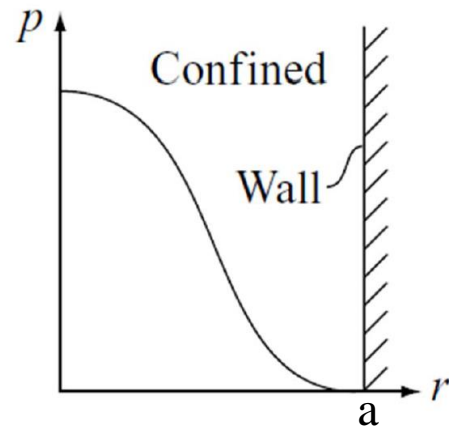
Use of the MHD equations

- ❑ $\partial/\partial t = 0, \bar{v} = 0, \eta = 0 \rightarrow$ magnetic equilibrium
- ❑ $\partial/\partial t \neq 0, \eta = 0$ stability of ideal modes (i.e. kink, ballooning)
- ❑ $\partial/\partial t \neq 0, \eta \neq 0$ stability of resistive modes (i.e. tearing modes w. island structure)

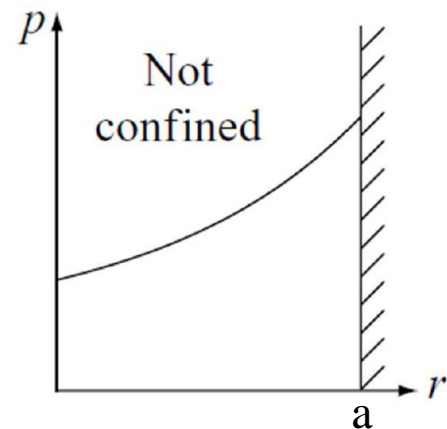
plasma equilibrium

In a MHD fluid isolated the vacuum chamber, *externally applied and internally induced magnetic fields* act to provide an equilibrium force balance that holds the plasma together at the desired location. The *Externally* applied magnetic field is *to isolate the plasma from the first wall* vacuum chamber.

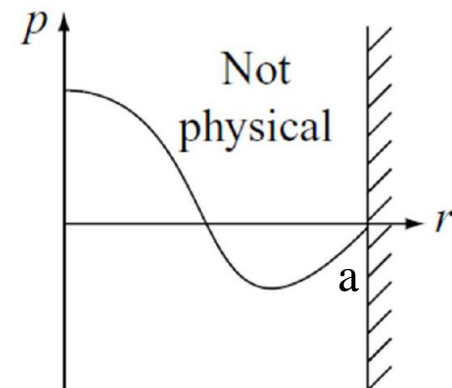
The generate *magnetic forces* necessary to maintain a static equilibrium are *solution of the MHD model*. Note that, MHD equations allow a wide variety of different types of mathematical solutions, not each physically acceptable or corresponding a confined plasma.



Confined plasma $p(a)=0$



Confined plasma $p(a)>0$



Confined plasma $p(r)>0$

MHD model

By combining the Electromagnetic field equations (4, 5 and 6 of the MHD model)

By combining equation (4) and (5) $\bar{E} + (\bar{v} \times \bar{B}) = \eta \bar{J} = \frac{\eta}{\mu_0} \nabla \times \bar{B}$

$\nabla \times (\bar{E} + (\bar{v} \times \bar{B})) = \nabla \times \left(\frac{\eta}{\mu_0} \nabla \times \bar{B} \right)$ By combining with equation (6) $-\frac{\partial \bar{B}}{\partial t} + \nabla \times (\bar{v} \times \bar{B}) = \frac{\eta}{\mu_0} \nabla \times \nabla \times \bar{B}$

Since $\nabla \times (\nabla \times \bar{B}) = \nabla (\nabla \cdot \bar{B}) - \nabla^2 \bar{B}$ and $\nabla \cdot \bar{B} = 0$

$$\frac{\partial \bar{B}}{\partial t} = \frac{\eta}{\mu_0} \nabla (\nabla \cdot \bar{B}) + \frac{\eta}{\mu_0} \nabla^2 \bar{B} + \nabla \times (\bar{v} \times \bar{B})$$

the time rate of change of B is controlled by the following two terms

- $\nabla \times (\bar{v} \times \bar{B}) =$ convection term
- $\frac{\eta}{\mu_0} \nabla^2 \bar{B} =$ diffusion term

magnetic Reynolds number $R_m = \frac{\nabla \times (\bar{v} \times \bar{B})}{\frac{\eta}{\mu_0} \nabla^2 \bar{B}} = \frac{\text{convection}}{\text{diffusion}} = \frac{\mu_0}{\eta} vL$

- $R_m \gg 1$ «frozen field»
- $R_m \ll 1$ «magnetic diffusion»

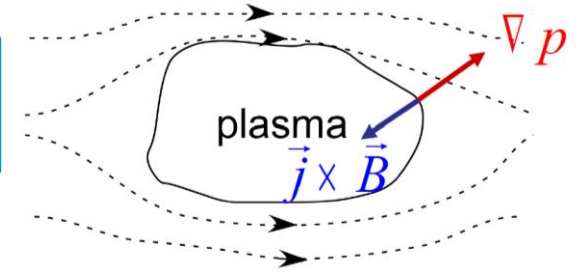
for a time dependent field convection dominates over diffusion.

Static Plasma equilibrium

conditions for a static equilibrium in a plasma: $\frac{\partial}{\partial t} = 0, v=0, \eta=0 \rightarrow$ magnetic equilibrium

Momentum conservation: $\rho_m \left[\frac{d\bar{v}}{dt} \right] = (\bar{J} \times \bar{B}) - \nabla p \implies \boxed{(\bar{J} \times \bar{B}) - \nabla p = 0}$

Calculating the *plasma equilibrium* means finding $B(r)$, sum of the magnetic fields created by the external coils and by the plasma, in which plasma pressure and $\bar{J} \times \bar{B}$ forces balance.



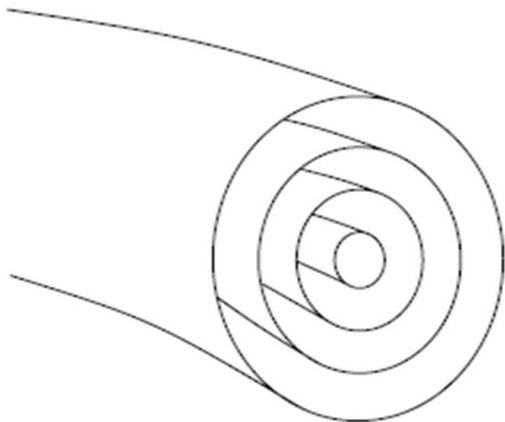
By taking the «dot» product of with \bar{J} and \bar{B}

$$\bar{J} \cdot (\bar{J} \times \bar{B}) - \bar{J} \cdot \nabla p = 0 \implies \bar{J} \cdot \nabla p = 0$$

$$\bar{B} \cdot (\bar{J} \times \bar{B}) - \bar{B} \cdot \nabla p = 0 \implies \bar{B} \cdot \nabla p = 0$$

Since ∇p is perpendicular to a surface of constant pressure, both \bar{J} and \bar{B} lie on surfaces of constant pressure

Constant pressure surfaces in cylindrical geometry form a set of closed and nested surfaces

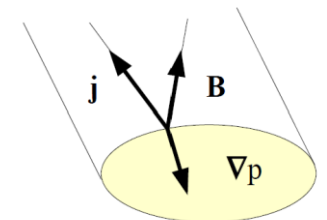


If \bar{B} lie on surfaces of constant pressure, this means the pressure and flux surface coincide.

Depending on the pressure behaviour:

$$\nabla p = 0 \rightarrow (\bar{J} \times \bar{B}) = 0 \quad \text{Force free equilibrium}$$

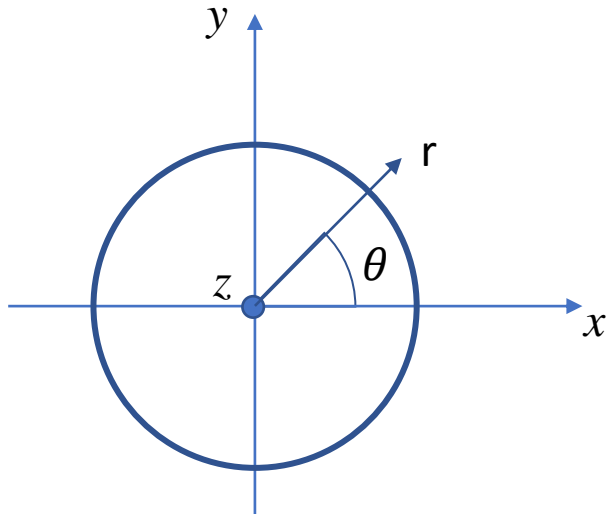
$$\nabla p \neq 0 \rightarrow (\bar{J} \times \bar{B}) = \nabla p \quad \text{Force balanced equilibrium}$$



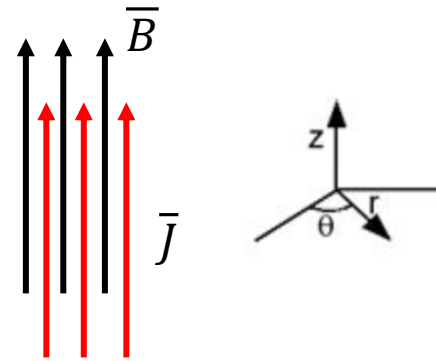
Static Plasma equilibrium

Force free equilibrium ($\nabla p = 0$)

in a cylindrical symmetry ($\bar{\mathbf{B}} = B_\theta \vec{a}_\theta + B_z \vec{a}_z$)



$$(\bar{\mathbf{J}} \times \bar{\mathbf{B}}) = \mathbf{0} \implies \bar{\mathbf{J}} // \bar{\mathbf{B}}$$



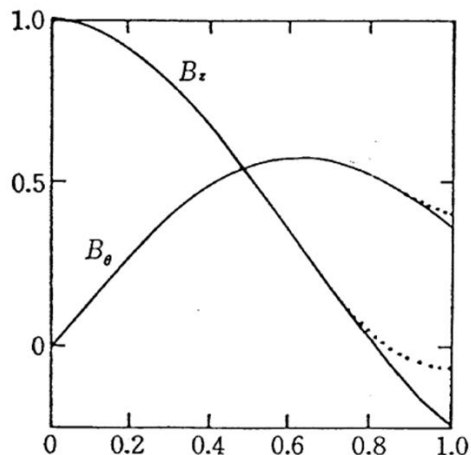
For this condition the solution of the Amper's law are

$$\nabla \times \bar{\mathbf{B}} = \mu_0 \bar{\mathbf{J}}$$

two class of solution are:

$$B_\theta = B_0 J_1(\alpha, r) \quad \text{where } r \text{ is the radius, } (\alpha, B_0) \text{ are positive constants}$$

$$B_z = B_0 J_0(\alpha, r) \quad \text{and } (J_0, J_1) \text{ are the Bessel functions (order 0 or 1)}$$



Static Plasma equilibrium

Force balanced equilibrium $(\vec{J} \times \vec{B}) \neq 0$ \vec{J} and \vec{B} lie on the surface with $\nabla p = \text{const}$

By combining

$$\begin{cases} (\vec{J} \times \vec{B}) - \nabla p = 0 \\ \nabla \times \vec{B} = \mu_0 \vec{J} \end{cases}$$

We get a new momentum equation: $\nabla \left(p + \frac{B^2}{2\mu_0} \right) = \frac{1}{\mu_0} (\vec{B} \cdot \nabla) \vec{B}$

kinetic pressure

magnetic pressure

Magnetic tension force

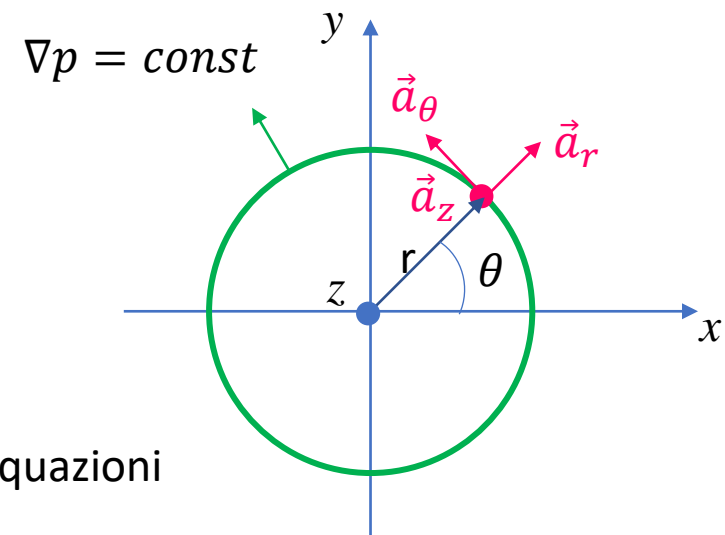
the momentum equation for cylindrical symmetry ($\vec{B} = B_\theta \vec{a}_\theta + B_z \vec{a}_z$) became:

$$\frac{d}{dr} \left(p + \frac{B_\theta^2}{2\mu_0} + \frac{B_z^2}{2\mu_0} \right) = -\frac{B_\theta^2}{\mu_0 r}$$

the Ohm'law equation $\nabla \times \vec{B} = \mu_0 \vec{J}$ for cylindrical symmetry *allows to evaluate* \vec{J} as *function of* \vec{B}

$$-\frac{1}{\mu_0} \frac{\partial B_z}{\partial r} \vec{a}_\theta + \frac{1}{r\mu_0} \frac{\partial (rB_\theta)}{\partial r} \vec{a}_z = J_\theta \vec{a}_\theta + J_z \vec{a}_z$$

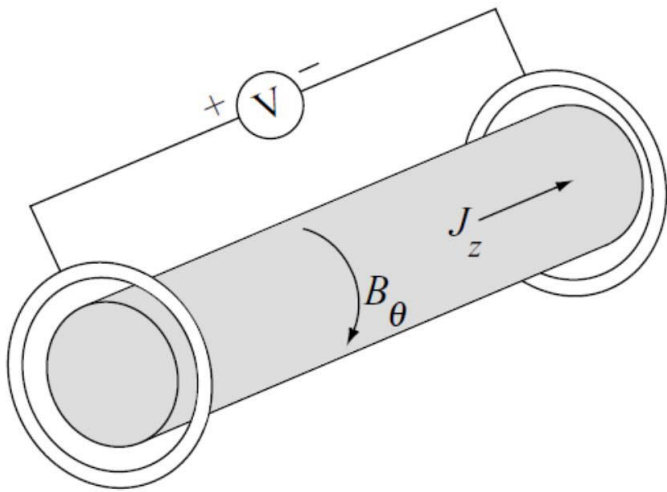
Fissi le componenti di \vec{J} con la legge di ampere e la conservazione del momento, hai 2 equazioni in 2 incognite che permette di determinare di B_θ e la pressione



Static Plasma equilibrium: Z-pinch

The momentum equation allows an infinite number of possible solutions, each solution can be a representative of a confinement configuration. The most interesting ones will be shown in the following

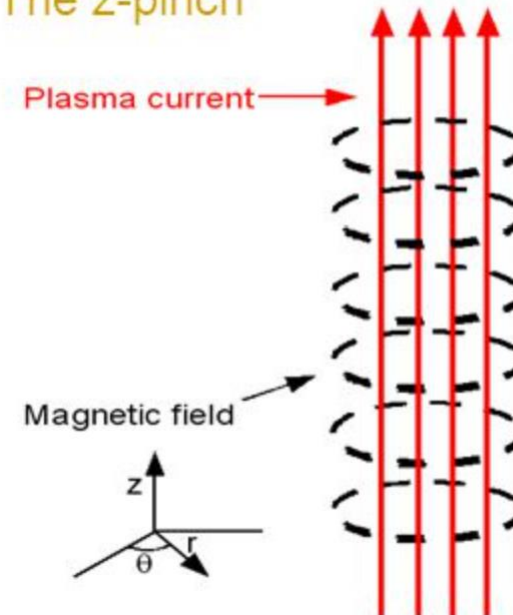
Mmm **the Z-pinch** configuration: A cylindrically symmetric plasma is confined by a **purely azimuthal magnetic field** $B_\theta(r)$ generated **by the axial current density**



$$\vec{B} = B_\theta(r)\vec{a}_\theta$$

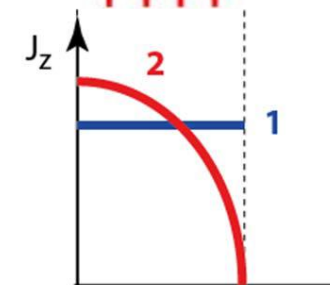
$$\vec{J} = J_z(r)\vec{a}_z$$

The z-pinch



Plasma current

Magnetic field



The following two cases will be discussed:

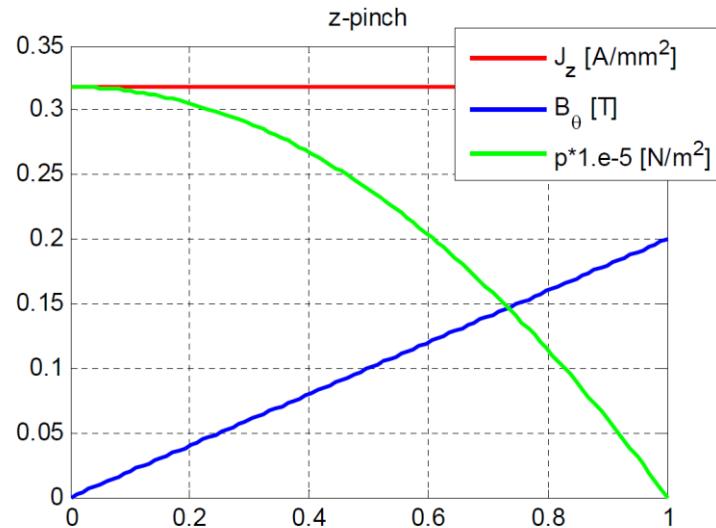
- 1) J_z with **uniform distribution** along r
- 2) J_z with **parabolic profile** along r

Static Plasma equilibrium: Z-pinch

1) J_z with **uniform distribution** along r

$$\begin{cases} J_z(r) = J_{z0} & r \leq a \\ J_z(r) = 0 & r > a \end{cases} \quad J_{z0} = \frac{I}{\pi a^2}$$

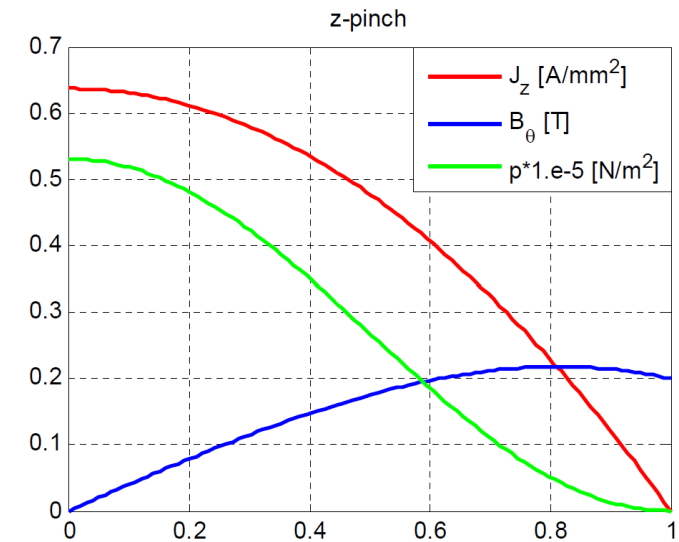
- I. From Amper'law we get $B_\theta(r)$
- II. Solving the momentum equation we get the pressure profile



2) J_z with **parabolic profile** along r

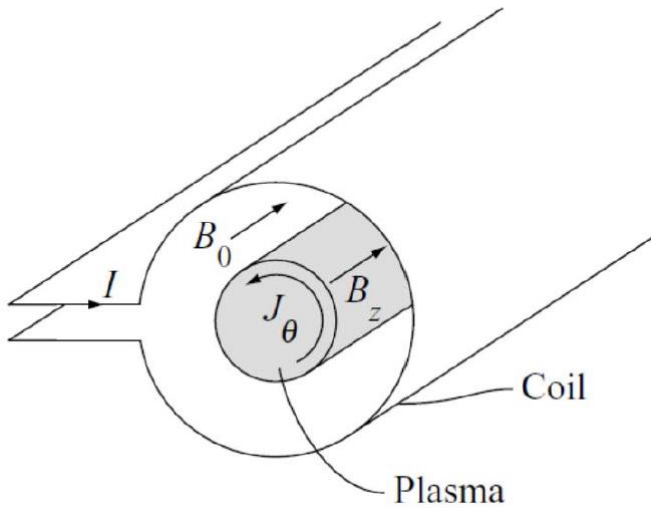
$$\begin{cases} J_z(r) = \frac{2I}{\pi a^2} \left[1 - \left(\frac{r}{a}\right)^2 \right] & r \leq a \\ J_z(r) = 0 & r > a \end{cases}$$

- I. From Amper'law we get $B_\theta(r)$
- II. Solving the momentum equation we get the pressure profile



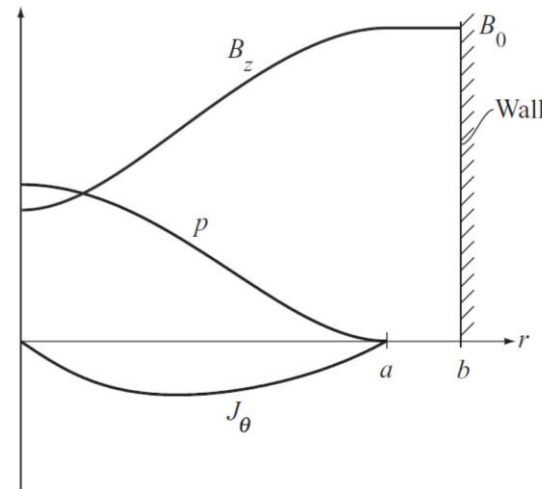
Static Plasma equilibrium: θ -pinch

The θ -pinch can be obtained driving a current in a external poloidal coil, this produces an applied axial magnetic field B_z inside the plasma, then a poloidal current is induced in the plasma

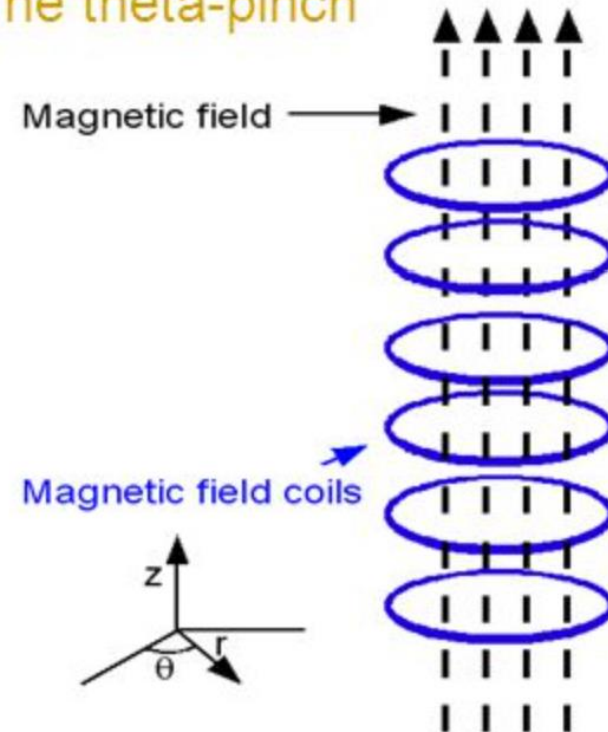


$$\vec{J} = J_\theta(r) \vec{a}_\theta$$

$$\vec{B} = B_z(r) \vec{a}_z$$



The theta-pinch



The θ -pinch has a magnetic field directed in the z direction and a large diamagnetic current directed in the θ direction.

Static Plasma equilibrium: -pinch

- ❖ A **pure Z-pinch** has good toroidal equilibrium and can therefore be easily twisted into a torus however, it has very poor MHD stability.
- ❖ A **pure ϑ -pinch** does not have a good toroidal equilibrium, but a linear ϑ -pinch have good stability properties.

An optimum solution is the **«screw pinch»** configuration that consists of an arbitrary combination of θ -pinch and Z-pinch fields. In this configuration **the magnetic lines twist around the plasma surface** giving the appearance of a screw thread.

$$\vec{B} = B_{\theta}(r)\vec{a}_{\theta} + B_z(r)\vec{a}_z$$

$$\vec{J} = J_{\theta}(r)\vec{a}_{\theta} + J_z(r)\vec{a}_z$$

of toroidal and poloidal magnetic fields that can stably confine plasmas in toroidal equilibrium

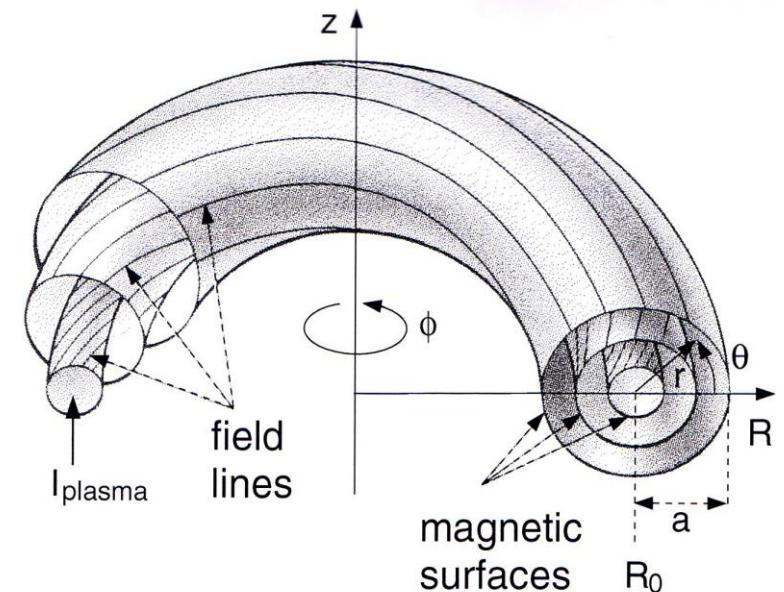
In general one is free to specify two arbitrary functions, for instance:

– $B_{\theta}(r)$

– $B_z(r)$

MHD then determines the third function: $p(r)$, as solution of the momentum equation

$$\frac{d}{dr} \left(p + \frac{B_{\theta}^2}{2\mu_0} + \frac{B_z^2}{2\mu_0} \right) = -\frac{B_{\theta}^2}{\mu_0 r}$$



Hydromagnetic equilibrium in a tokamak

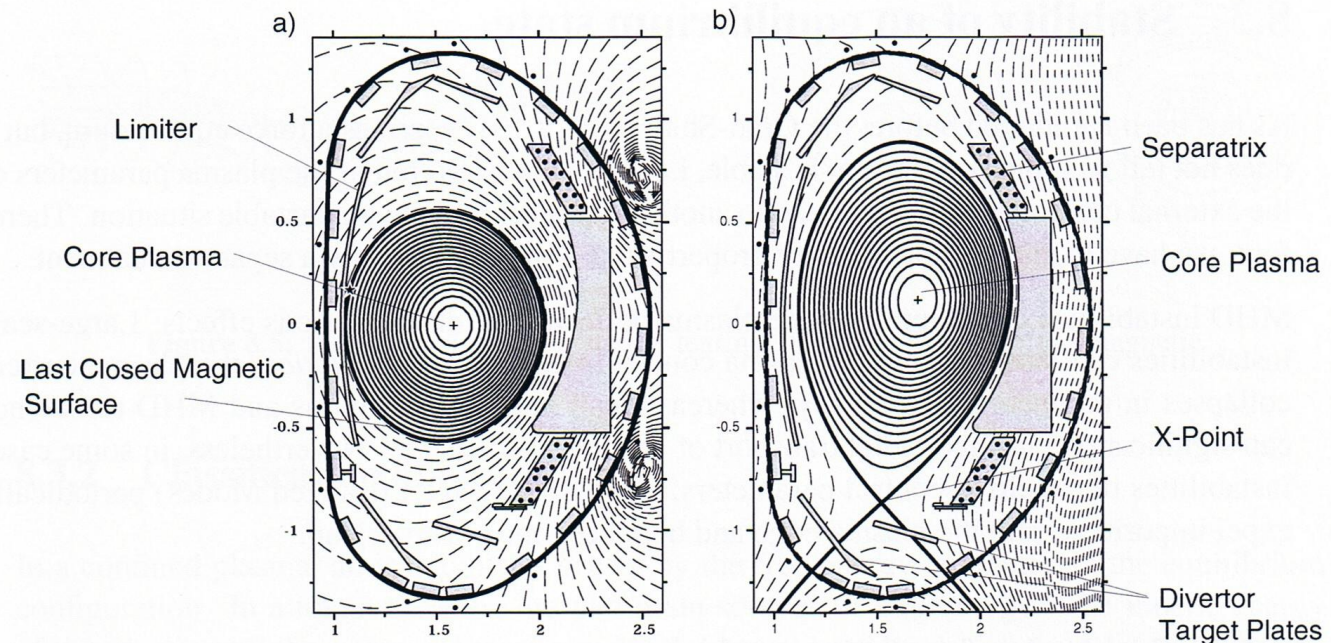
A time-dependent plasma configuration can be a sequence of equilibrium states. An equilibrium is a state in which all forces are balanced.

Calculating the plasma equilibrium means finding $\mathbf{B}(\mathbf{r})$, sum of the magnetic fields created by the external coils and by the plasma, in which plasma pressure and $\mathbf{j} \times \mathbf{B}$ forces balance:

$$(\bar{\mathbf{J}} \times \bar{\mathbf{B}}) - \nabla p = 0$$

The equilibrium equation for an axisymmetric system can be written as a differential equation (the *Grad-Shafranov equation*, GSE) for the poloidal flux function. The GSE can be solved analytically or for practical cases the *GSE* is solved numerically.

$$(\bar{\mathbf{J}} \times \bar{\mathbf{B}}) - \nabla p = 0 \rightarrow f(\psi) = 0$$



When $\psi(R,z)$ is known, several geometrical and other parameters can be defined.