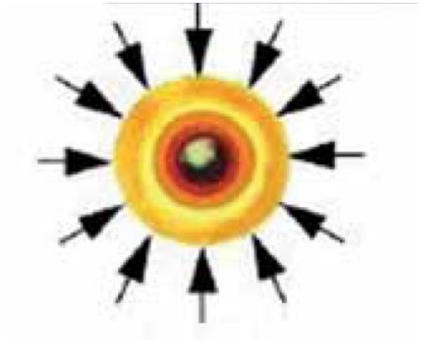
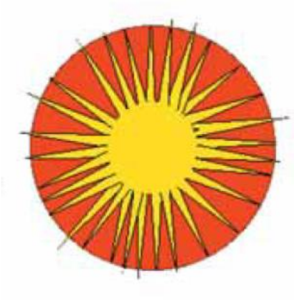
The plasma confinement

Inertial confinement

For the Inertial confinement, 0.1 mg of gaseous D-T mixture are contained in a small hollow spheres of plastic material are used, with a diameter of about 2 mm.

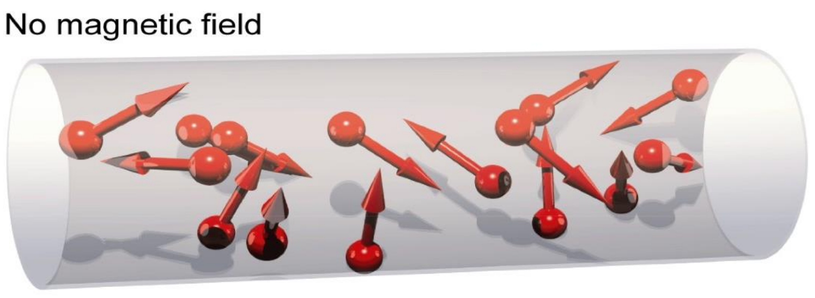


Several high-powered beams strike the sphere from multiple directions. The D-T, pushed towards the geometric center of the sphere, reaches very high densities n.
The time needed to cross the radius of the sphere (τ_E) is enough for the product $nT\tau_E$ to satisfy the condition of Lawson's criterion. When the compression develops a temperature that satisfies the Lawson condition.



In an inertial fusion reactor it would work similar to an internal combustion engine, indeed small spheres of D-T would have to be injected and burned in the vacuum chamber one after the other at a very high frequency.

Magnetic confinement



Disordered motion

The electromagnetic field is capable of transforming the disordered movement of charged particles into an ordered motion. At microscopic level, the single-particle interactions in a plasma controlled **by an electromagnetic field** can be:

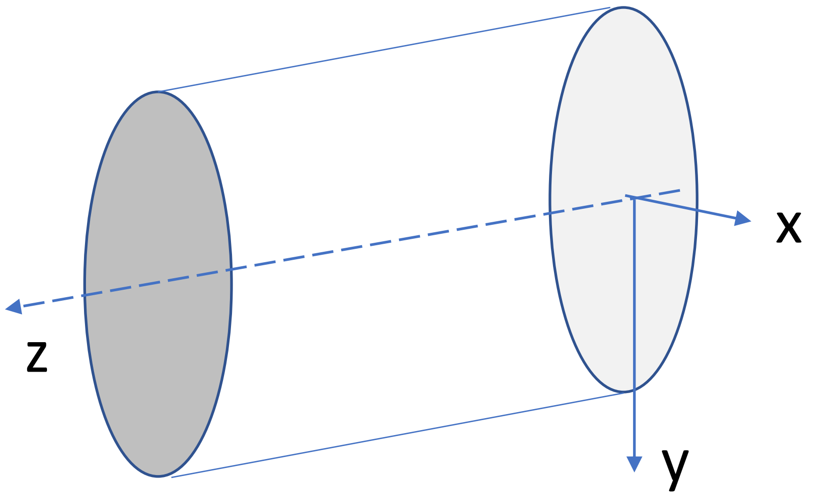
- ✓ **Electromagnetic interaction:** dominant long-range interaction governed by the Lorentz force
- ✓ **Coulomb collisions**: infrequent interactions in the neighborhood inter-particle Coulomb potential, responsible for transport
- ✓ **Nuclear fusion collisions**: even less frequent interactions in the very short-range nuclear potential

To determine the trajectories of charged particles in a prescribed electromagnetic field, assuming a **Collision-less behaviour,** where motion is dominated only by the long-range electromagnetic interanion, the simplest way to understand how an electromagnetic confinement works is to use **Newton's law of motion.** The electromagnetic field is assumed varying slowly in time and space.

$$\begin{cases} m\dfrac{d\bar{v}}{dt} = q(\bar{E} + \bar{v} \times \bar{B}) \\ \dfrac{d\bar{r}}{dt} = \bar{v} \end{cases}$$

How the magnetic confinement can be done?

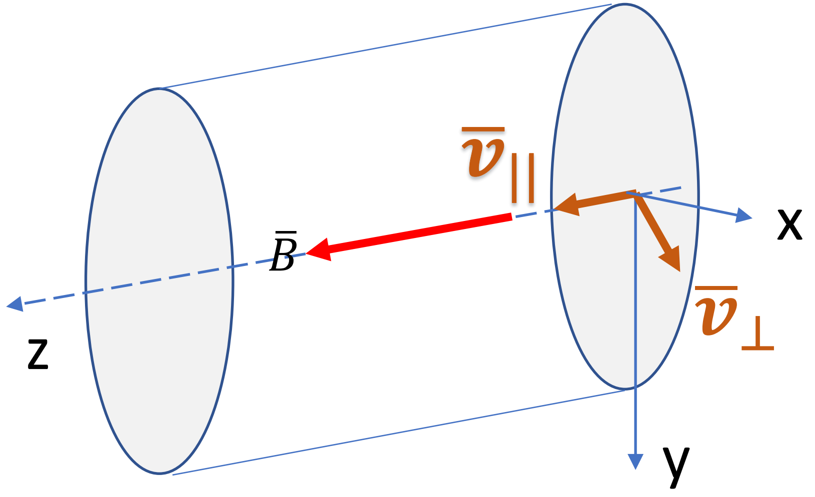
Assuming the plasma contained in a cylindrical geometry, the magnetic confinement can be achieved by applying magnetic and electric fields as in the following combinations:



1. Constant magnetic field
2. Constant magnetic and electric field
3. Magnetic field with perpendicular gradients
4. Magnetic field with parallel gradients

Magnetic confinement in constant B

1. Motion in constant B and E=0



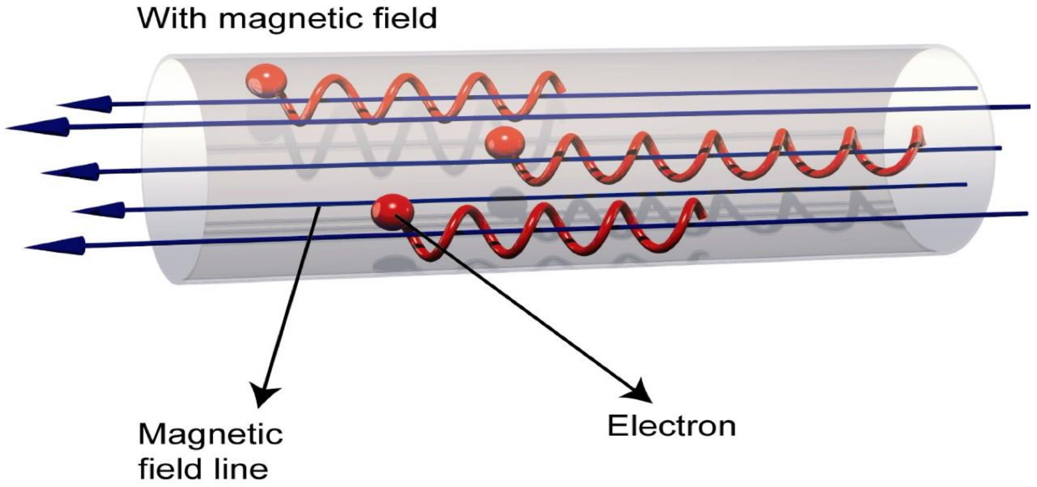
$$\begin{cases} \bar{E} = 0 \\ \bar{B} = B_z \vec{a}_z \end{cases}$$

$$m\frac{d\bar{v}}{dt} = q(\bar{v} \times \bar{B})$$

$$\bar{v} = \bar{v}_{||} + \bar{v}_{\perp} \rightarrow \begin{cases} \bar{v}_{||} = v_z \vec{a}_z \\ \bar{v}_{\perp} = (v_x \vec{a}_x + v_y \vec{a}_y) \end{cases}$$

1. $m\frac{d\bar{v}_{||}}{dt} = 0$ equation for parallel motion

2. $m\frac{d\bar{v}_{\perp}}{dt} = q(\bar{v}_{\perp} \times \bar{B})$ equation for perpendicular motion



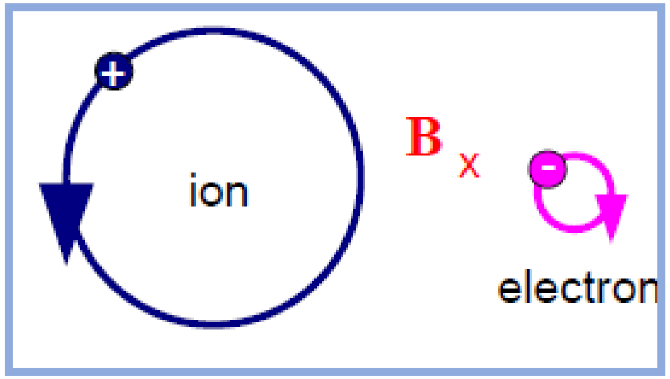
By integrating the two equation

1. $\bar{v}_{||}(t) = const$ → constant uniform motion **NO parallel confinement**

2. $v_x(t) = v_{\perp}\sin(\omega t - \Phi)$
   $v_y(t) = -v_{\perp}\sin(\omega t - \Phi)$ **radial confinement**

The combined perpendicular and parallel motion of a charged particle corresponds to a helical trajectory

Magnetic confinement in constant B



The balance between magnetic and centrifugal forces determines a helical trajectory of radius r in a direction parallel to the field line

cyclotron frequency $f_c = \dfrac{q \cdot B}{2\pi\, m} \rightarrow \quad \omega_c = \dfrac{q \cdot B}{m}$

rotation radius (Larmor radius) $r_L = \dfrac{v_\perp}{2\pi f} = \dfrac{v_\perp}{Bq} m$

Since the gyro radius is, in general, quite small in comparison to the plasma radius, there is **good confinement in the plane perpendicular** to the magnetic field.
The rotation radius are different for ions and electrons because of their mass. For typical values **B=5T** and **Ti=Te=15keV:**

– **Electrons**

- $\omega_{ce} = \dfrac{|e|\,B}{m_e} = 8.8\ 10^{11}\ {}^{r}/_{s} \Longrightarrow \quad f_{ce} = 140\ \text{GHz}$
- $r_{Le} = \dfrac{m_e v_e}{|e|B} = \dfrac{\sqrt{2m_e}}{|e|}\ \dfrac{\sqrt{T_e}}{B} = 83\ \mu\text{m}$

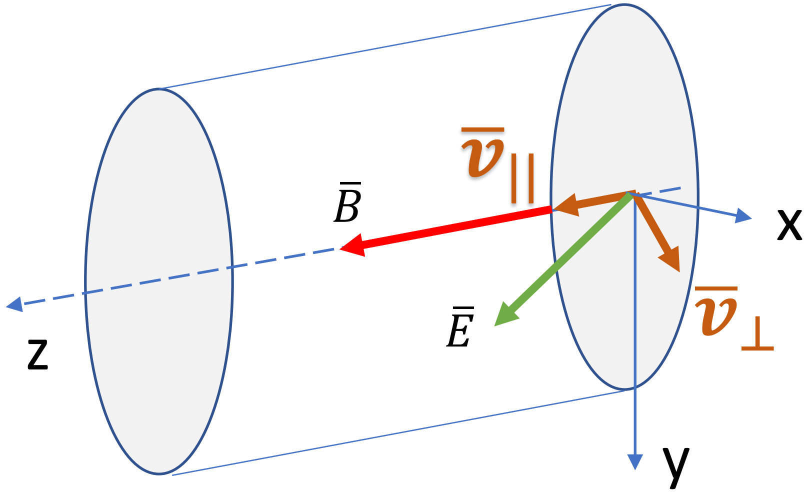
· **Ions**

- $\omega_{ci} = \dfrac{|e|\,B}{m_i} = 2.4\ 10^{8}\ {}^{r}/_{s} \Longrightarrow \quad f_{ci} = 40\ \text{MHz}$
- $r_{Li} = \dfrac{m_i v_i}{|e|B} = \dfrac{\sqrt{2m_i}}{|e|}\ \dfrac{\sqrt{T_i}}{B} = 5\text{mm}$

Magnetic confinement in constant B and E

2. Motion in constant $\bar{B}$ and $\bar{E}$

$$\begin{cases} \bar{E} = E_z\vec{a}_z + E_x\vec{a}_x = \bar{E}_{||} + \bar{E}_{\perp} \\ \bar{B} = B_z\vec{a}_z \end{cases}$$



$$\bar{v} = \bar{v}_{||} + \bar{v}_{\perp} = \begin{cases} \bar{v}_{||} = v_z\vec{a}_z \\ \bar{v}_{\perp} = \bar{v} \times \vec{a}_z = v_x\vec{a}_x + v_y\vec{a}_y \end{cases}$$

$$m\frac{d\bar{v}}{dt} = q(\bar{E} + \bar{v} \times \bar{B}) \rightarrow \begin{cases} m\frac{d\bar{v}_{||}}{dt} = q\bar{E}_{||} \\ m\frac{d\bar{v}_{\perp}}{dt} = q\bar{E}_{\perp} + q\bar{v}_{\perp} \times \bar{B} \end{cases}$$

1 for parallel motion

2 for perpendicular motion

$$\bar{v}_{\perp} = \bar{v}_e + \bar{v}'_{\perp} \rightarrow \begin{cases} \bar{v}_e \; as \; \bar{v}_e \times \bar{B} = -\bar{E}_{\perp} \\ \bar{v}'_{\perp} = (v'_x\vec{a}_x + v'_y\vec{a}_y) \end{cases}$$

Since $\bar{B}$ is along $\vec{a}_z$ direction , $\bar{v}_e$ is thus a component along $\vec{a}_y$

$$\bar{v}_e = \frac{\bar{E} \times \bar{B}}{B^2}$$

Magnetic confinement in constant B and E

The equation for perpendicular motion can be written

$$m\left(\frac{d\bar{v}_e}{dt}+\frac{d\bar{v}'_\perp}{dt}\right) = q\bar{E}_\perp + q(\bar{v}_e \times \bar{B} + \bar{v}'_\perp \times \bar{B}) = q\bar{E}_\perp - q\bar{E}_\perp + q\bar{v}'_\perp \times \bar{B}$$

$$\frac{d\bar{v}_e}{dt} = \frac{d}{dt}\left(\frac{\bar{E}\times\bar{B}}{B^2}\right)=0 \quad \Rightarrow \quad m\frac{d\bar{v}'_\perp}{dt} = q\bar{v}'_\perp \times \bar{B}$$

By integrating the two equation:

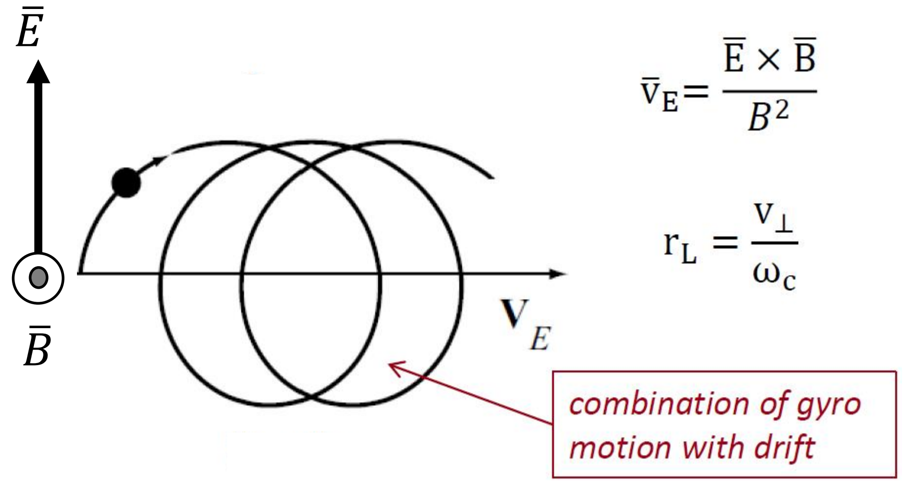
1 $$v_{||}(t) = v_{||}(0) + \frac{q}{m}E_{||}t$$

2 $$v_x(t) = \bar{v}'_\perp \sin(\omega t - \Phi)$$
$$v_y(t) = -\bar{v}'_\perp \sin(\omega t - \Phi)$$

$$\bar{v}_e = \frac{\bar{E}\times\bar{B}}{B^2}$$

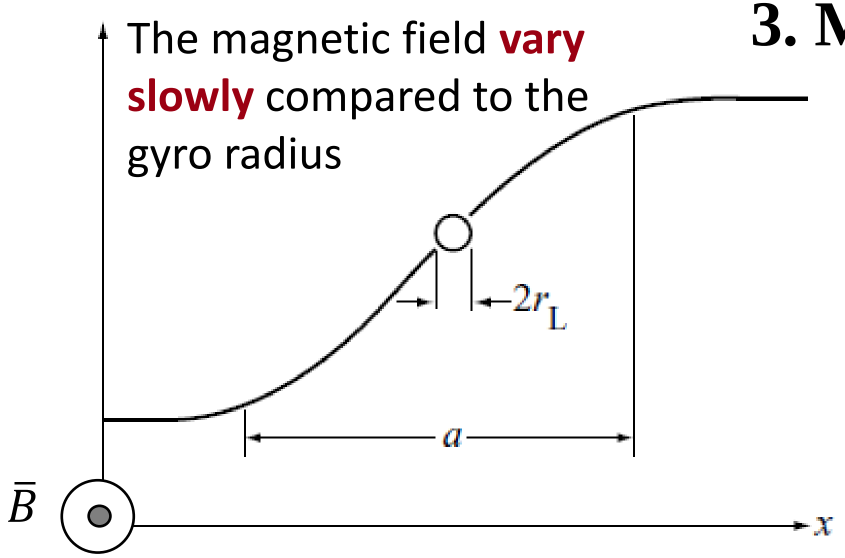
constant **accelerated motion** in the direction of B. The presence of Coulomb collisions, neglected in the **collisionless behaviour**, produces a small **frictional drag** on the parallel motion which **limits the maximum velocity.**

a constant drift velocity $\bar{v}_e$ is added on the gyro motion, perpendicular to both $\bar{E}$ and $\bar{B}$. The drift is independent from mass and charge (ions and electrons have suffer from the same drift), it depends from B.



Magnetic confinement in variable B

The magnetic field **vary slowly** compared to the gyro radius



3. Motion in $\bar{B}$ with perpendicular gradients ($\nabla_\perp \bar{B}$) and $\bar{E}$ =0

$$\begin{cases} \bar{E} = 0 \\ \bar{B} = B_z(x)\vec{a}_z \end{cases}$$

$B_z(x)$ **vary slowly** compared to the gyro radius

The equations are the same of case **1**

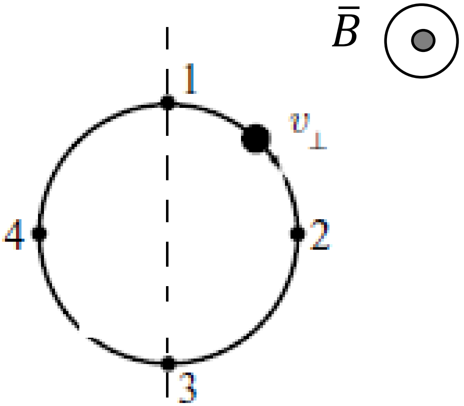
$$m\frac{d\bar{v}_\perp}{dt} = q\bar{v}_\perp \times \bar{B}(x)$$ equation for perpendicular motion

a) By integrating the equation ignoring $\nabla\bar{B}$

helical trajectory with $\bar{v}_\perp(t) = const$

$$r_L = \frac{v_\perp}{2\pi f} = \frac{m\, v_\perp}{q\, B}$$

The larger B, the smaller r_L
The smaller B, the larger r_L

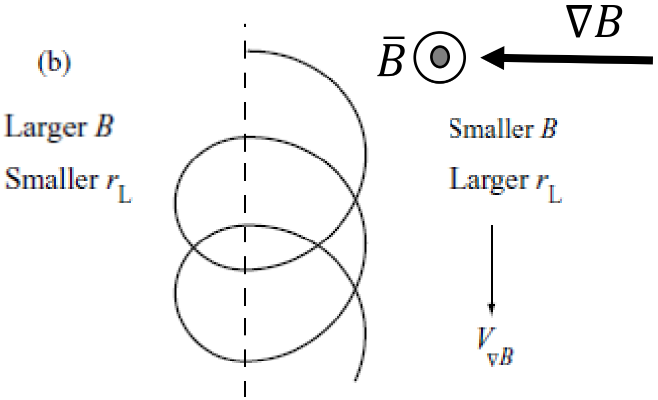


b) By integrating the equation considering ∇B

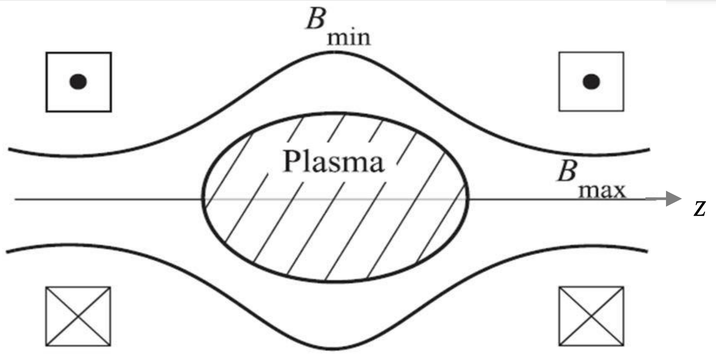
A constant acceleration (a=F/m) gives rise to an equivalent **drift in direction** $\bar{F} \times \bar{B}$ (i.e. $\nabla_\perp B \times \bar{B}$)

$$\bar{v}_{\nabla_\perp B} = \frac{\mu \nabla_\perp B \times \bar{B}}{qB^2}$$

$$r_L = \frac{v_\perp}{2\pi f} = \frac{v_\perp}{Bq}m$$



Magnetic confinement in parallel gradients



4. Motion in $\bar{B}$ with parallel gradients $(\nabla_{\|}\bar{B})$

magnetic confinement configuration known as the **mirror machine**

$$\begin{cases} \bar{E} = 0 \\ \bar{B} = B_z(z)\vec{a}_z \end{cases}$$

$$\bar{v} = \bar{v}_{\|} + \bar{v}_{\perp}$$

$$m\frac{d\bar{v}}{dt} = q(\bar{v} \times \bar{B})$$

$$\frac{1}{2}\bar{v} \cdot m\frac{d\bar{v}}{dt} = \frac{1}{2}\bar{v} \cdot q(\bar{v} \times \bar{B})$$

$$\frac{1}{2}\frac{d}{dt}mv^2 = 0 \rightarrow \frac{1}{2}mv^2 = \frac{1}{2}mv_{\perp}^2 + \frac{1}{2}mv_{\|}^2 = const = \mu B + \frac{1}{2}mv_{\|}^2$$

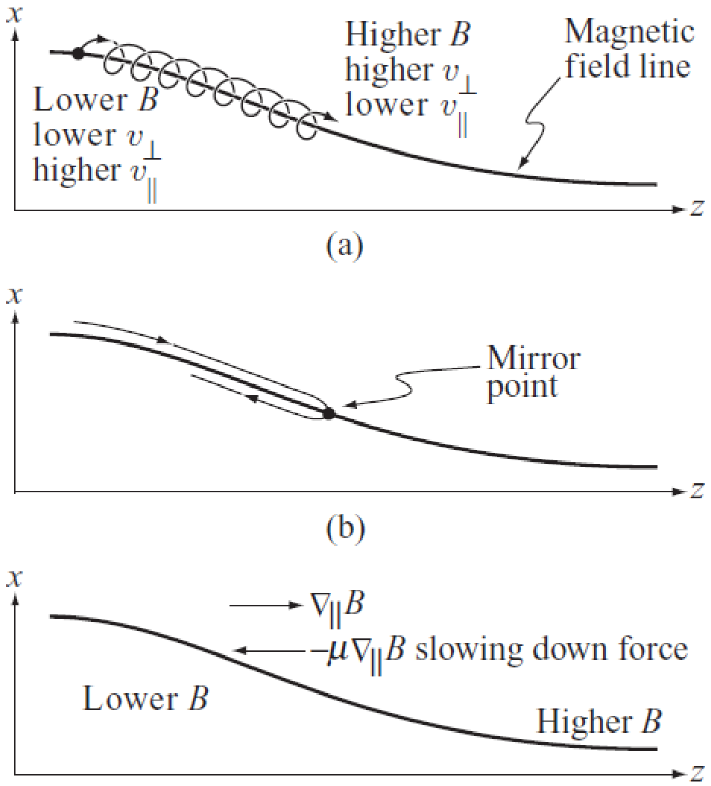
$\mathbf{F}_{\|} = -\mu\nabla_{\|}\mathbf{B}$ affects the parallel motion of the guiding centre

The larger **B**, the smaller $\boldsymbol{v}_{\|}$

The smaller **B**, the larger $\boldsymbol{v}_{\|}$

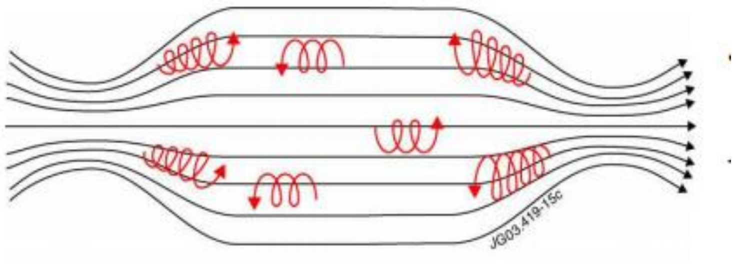
The key is to determine the relation between $\boldsymbol{v}_{\|}$ and $\boldsymbol{v}_{\perp}$ necessary to reflect a particle at a given point along the parallel field gradient.

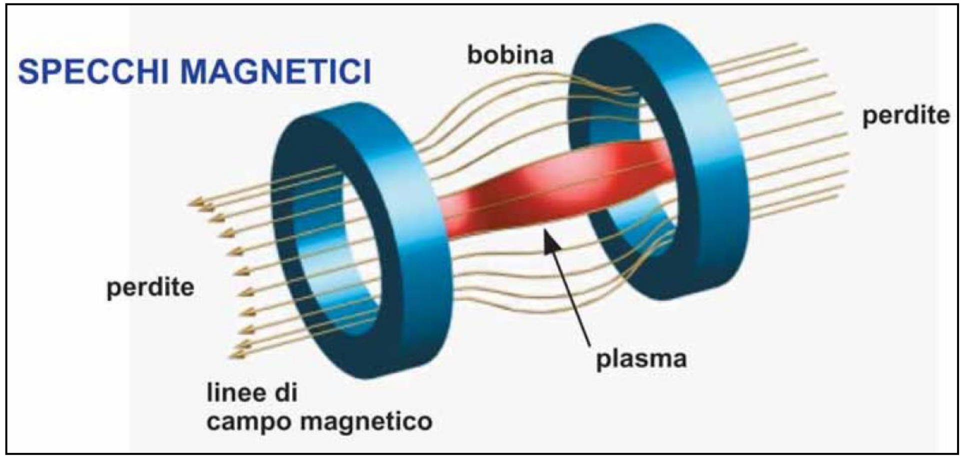
The direction of the parallel motion can be completely reversed: a particle **moving to the right** along a given field line at a certain instant of time can be **moving to the left** a short time later.



Magnetic confinement

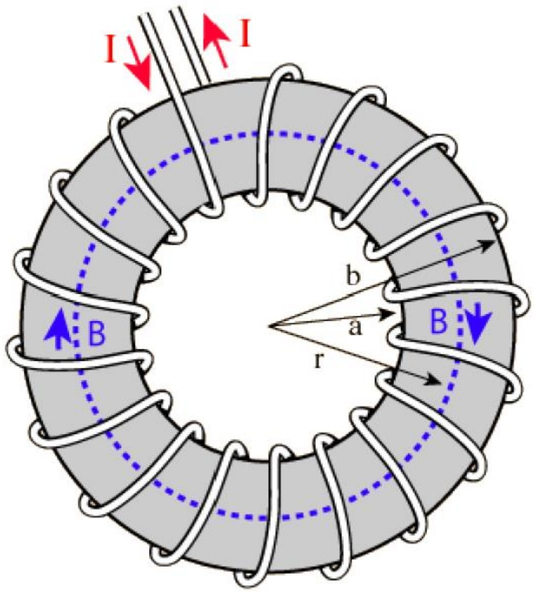
Inside a cylindrical chamber it is difficult to preserve the plasma state. The ions and electrons are not radially spread, but they hit the two ends of the container, thus losing their energy with a consequent cooling and decay of the plasma ionization.





To overcome this problem, the so called a magnetic mirror can be realized, in which a reflection of the particles is obtained by intensifying the field at the ends of the container. But the results obtained in are not acceptable for energy applications, the plasma neutralizes very quickly.

Magnetic confinement



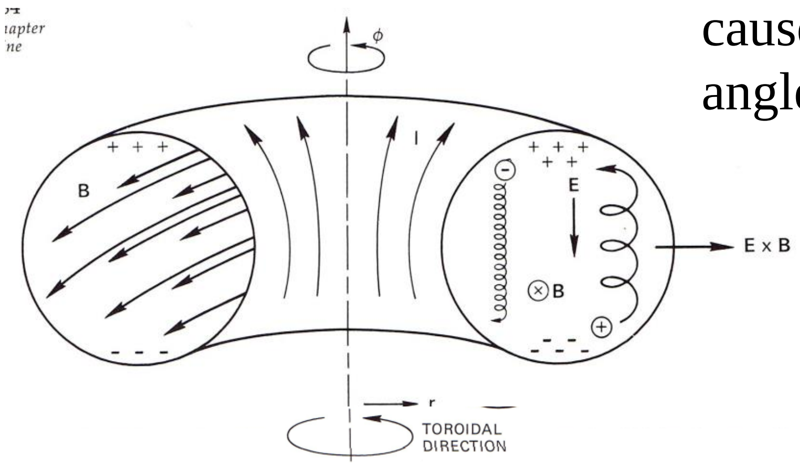
To overcome the cooling and decay effect happening at the finale interfaces of the cylindrical container, the plasma can be closed in a toroidal container. In a tours the magnetic field (B) lines are closed circles, crated by means of **solenoids arranged concentrically to the chamber,** at different toroidal angles, and equally spaced**.** *This should allow azimuthal symmetry of each line*.

Evaluating the toroidal magnetic field in a torus is a good example of Ampere's law:

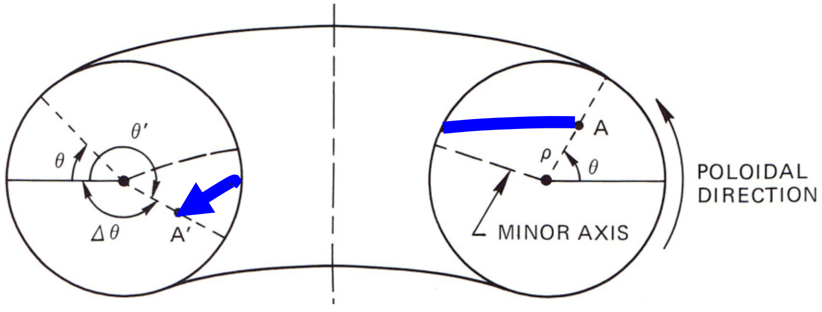
$$2\pi\, BR = \mu_0 IN$$

Thus, B∝ 1/R. This causes the motion of a charged particle in $\nabla_{\perp}$B $\neq 0$. The drift resulting from $\nabla_{\perp}$B causes a vertical charge separation which in turn causes an outward drift $(\bar{E} \times \bar{B})$. This determines a changes its azimuthal angle θ around the minor axis, resulting in a field line winding (blue line)

Major axis



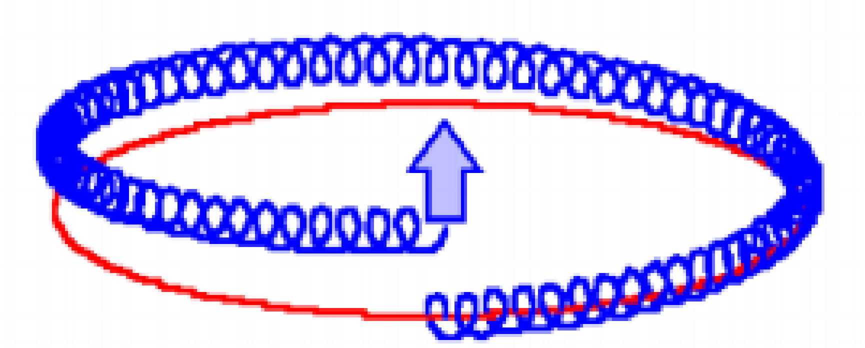
Drifts destabilize a toroidal plasma in a pure toroidal field.
E⊥B causes charge flow (not current, different from solid conductor where E = η j)



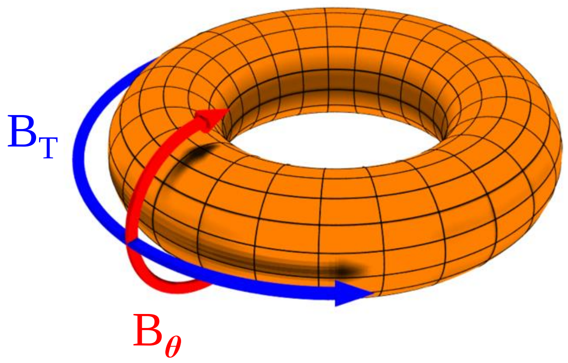
Magnetic confinement

In a magnetic confined plasma by a toroidal-shaped structure a particle drift motion is observed. mail due to:

1. the non-uniformity of the toroidal magnetic field which decays as 1/R
2. the centrifugal force on the non-linear moving particles because of the variable pressure.

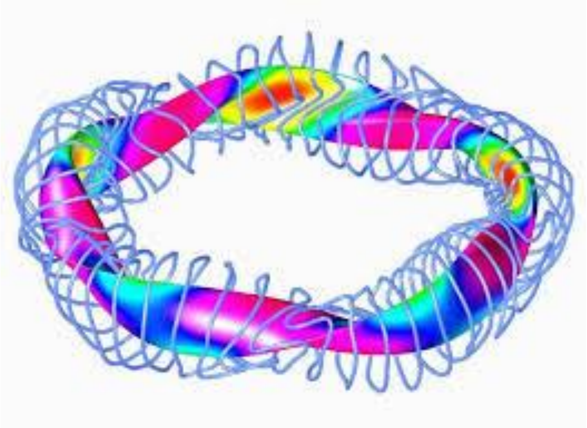


Even if the ends of the container in a toroidal camber are eliminated the magnetic toroidal field alone does not allow the particle confinement.



The solution to the drift motion of particles is to **twist the toroidal field lines into a helix**. This can be done in 2 ways:

- ❑ with the superposition of a poloidal magnetic field B_θ (tokamak configuration)
- ❑ Generating an elicoidal fiels by elicoidal external coils.

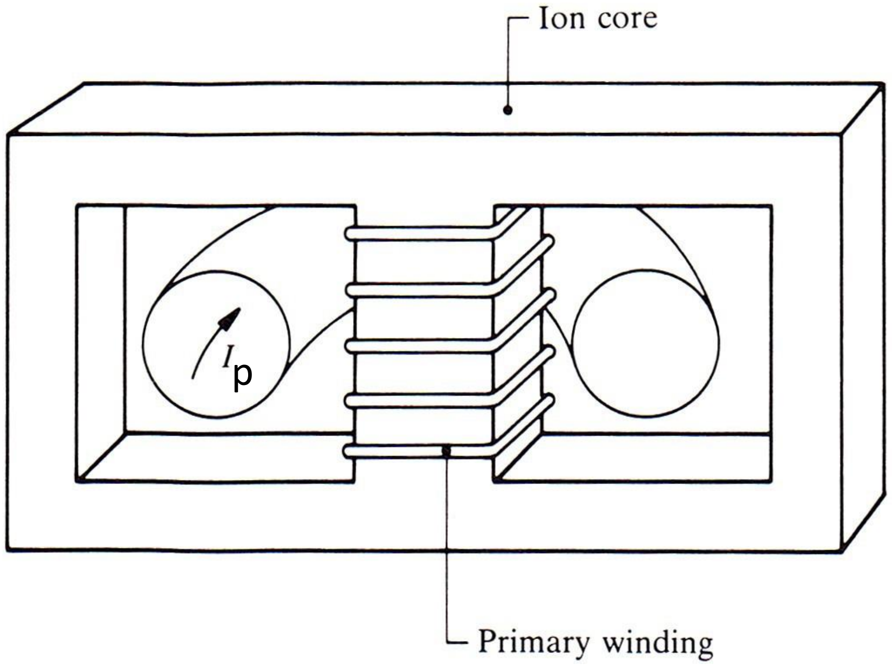


The variation of B_{tor} across the plasma has an important effect also on the transport.

Tokamak configuration

The poloidal magnetic field is generated from toroidal plasma current (I_p) driven by a toroidal electric field induced by a magnetic flux change in the *primary* (OH = ohmic) coils.
An iron transform core can be used to reduce the stray magnetic field.

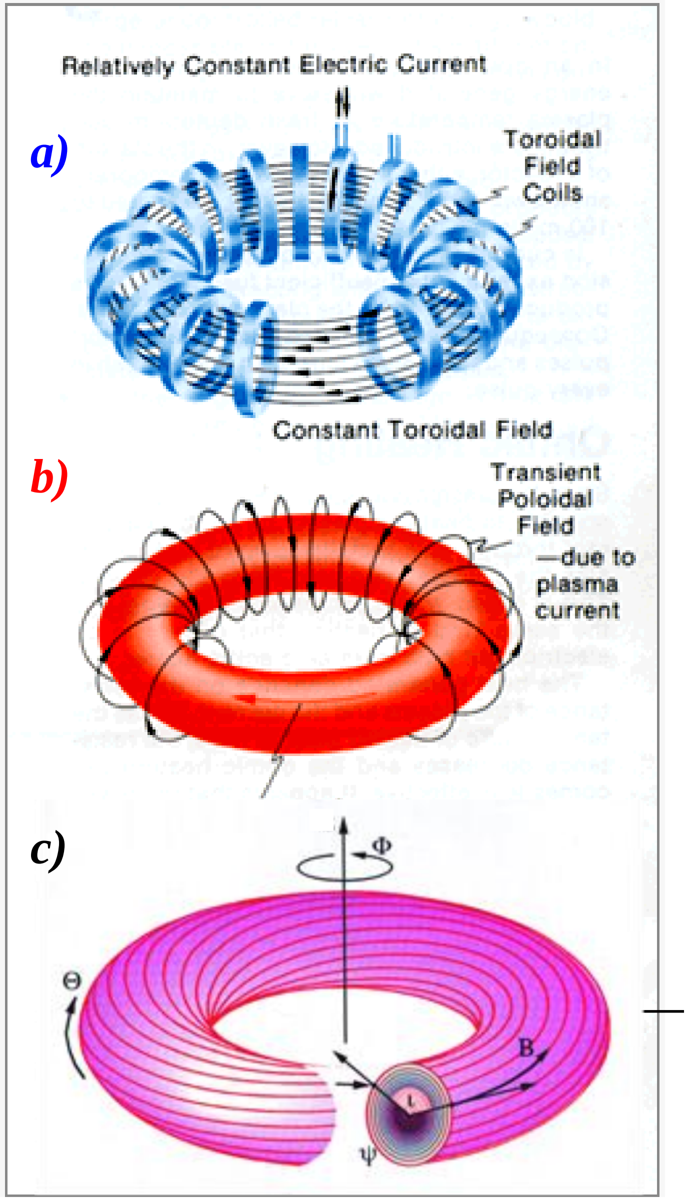
- The current in the primary coils is brought to I_{OH}
- Fuel gas is fed into the chamber
- The I_{OH} current is reversed, builds up the toroidal plasma current (Ip), and must continue to decrease to maintain the plasma current against resistive losses

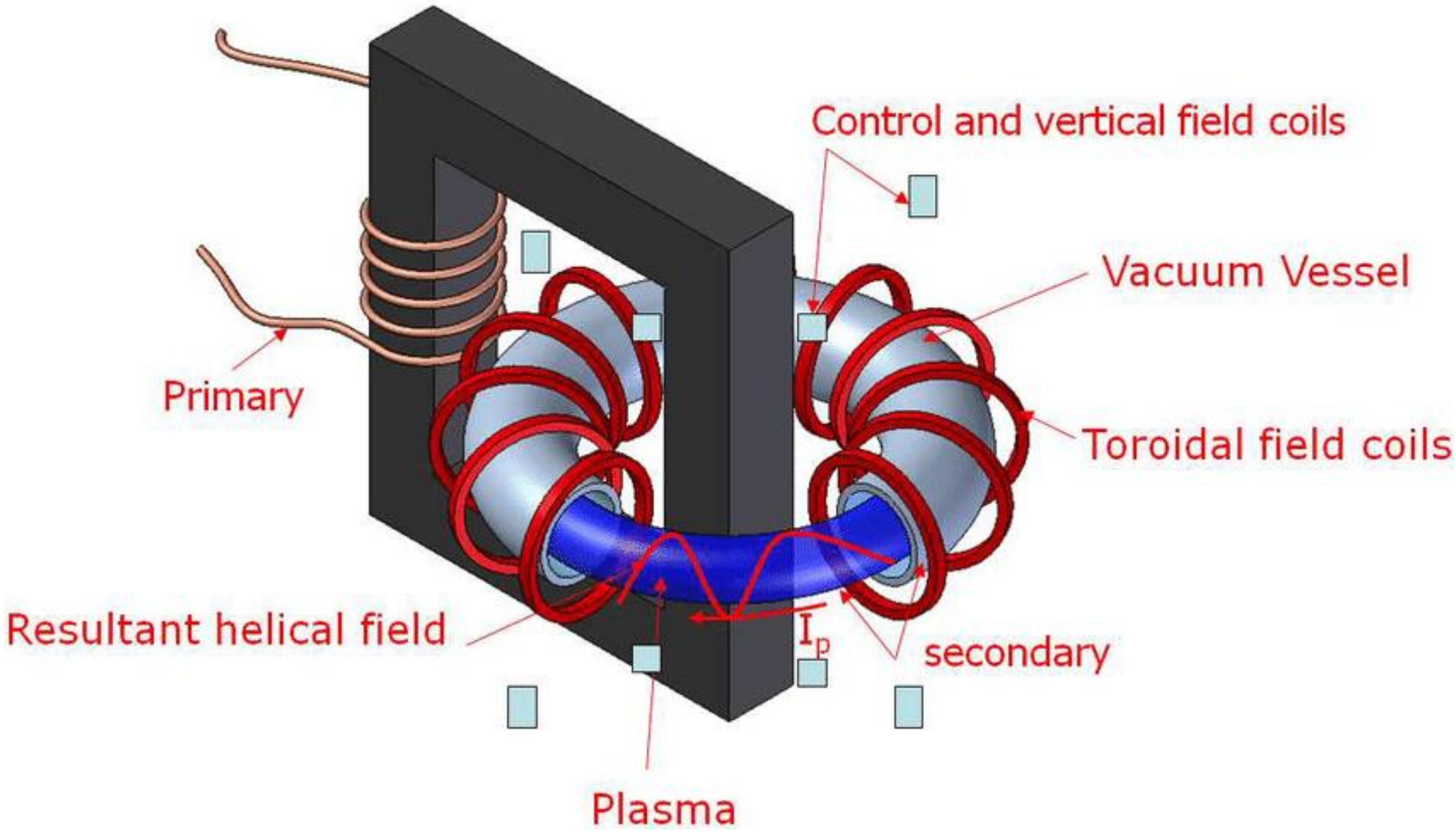


$$\int_S \frac{dB_{OH}}{dt} dS = L_p \frac{dI_p}{dt} + R_p I_p$$

Primary   Plasma (secondary)

Tokamak configuration





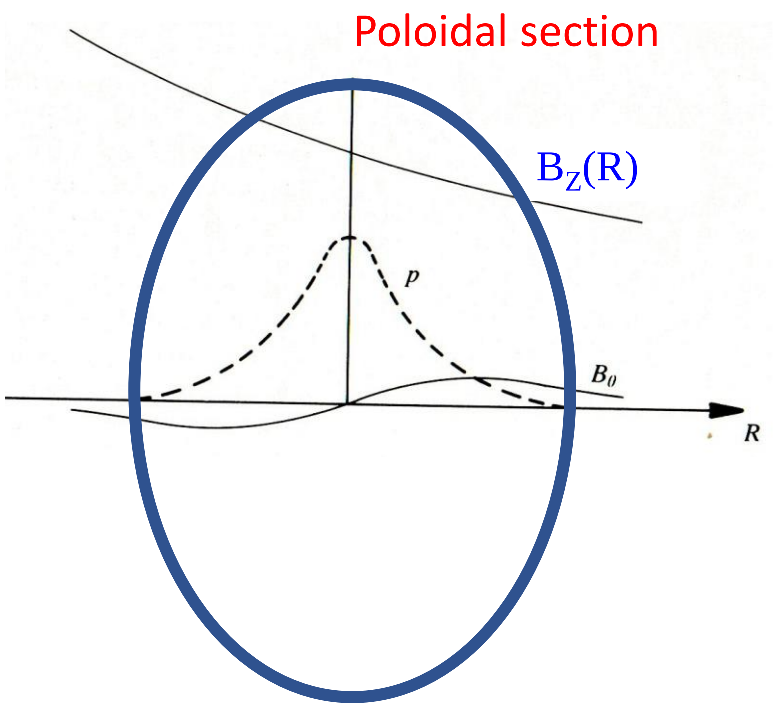
Tokamak configuration

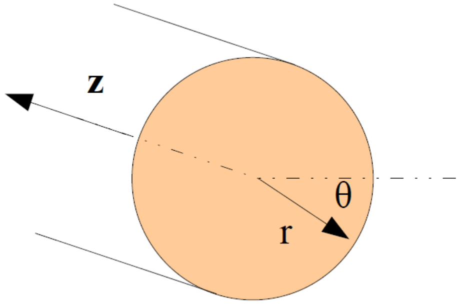
The poloidal magnetic field is given by the plasma current distribution through Ampere's law ($\nabla \times \bar{B} = \mu_0 \bar{J}$). In cylindrical coordinates:

$$\mu_0 \, j_z = \frac{1}{r}\frac{d}{dr}(r B_\theta)$$

$$j_z = \text{const}$$

$$B_\theta = \begin{bmatrix} \frac{\mu_0}{2} j_z r, & r \leq a \\ \frac{\mu_0}{2} j_z \frac{a^2}{r}, & r > a \end{bmatrix}$$

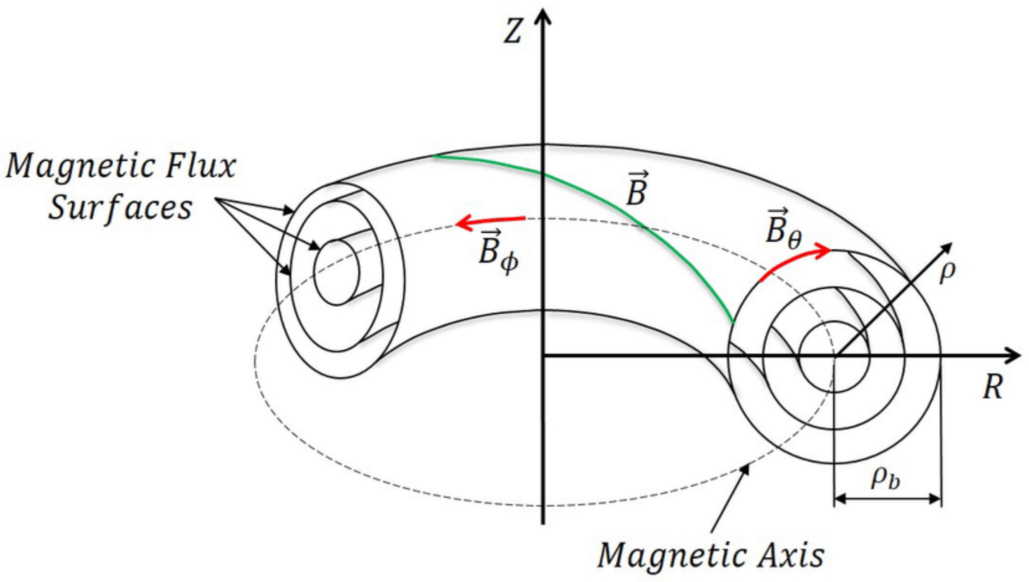




In cylindrical coordinates

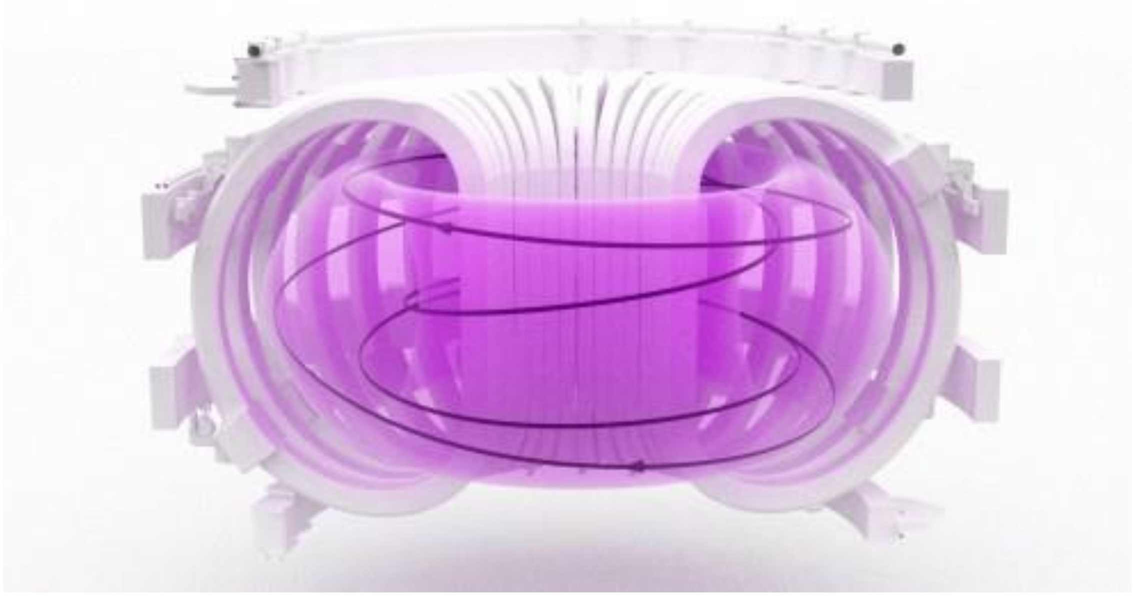
- ❑ z is the toroidal coordinate, B_z is the toroidal field
- ❑ ϑ is the poloidal coordinate and $\bar{B}_\vartheta$ is the poloidal field
- ❑ r is the radial coordinate, a is the radius of the plasma column and R is torus radius

Tokamak configuration



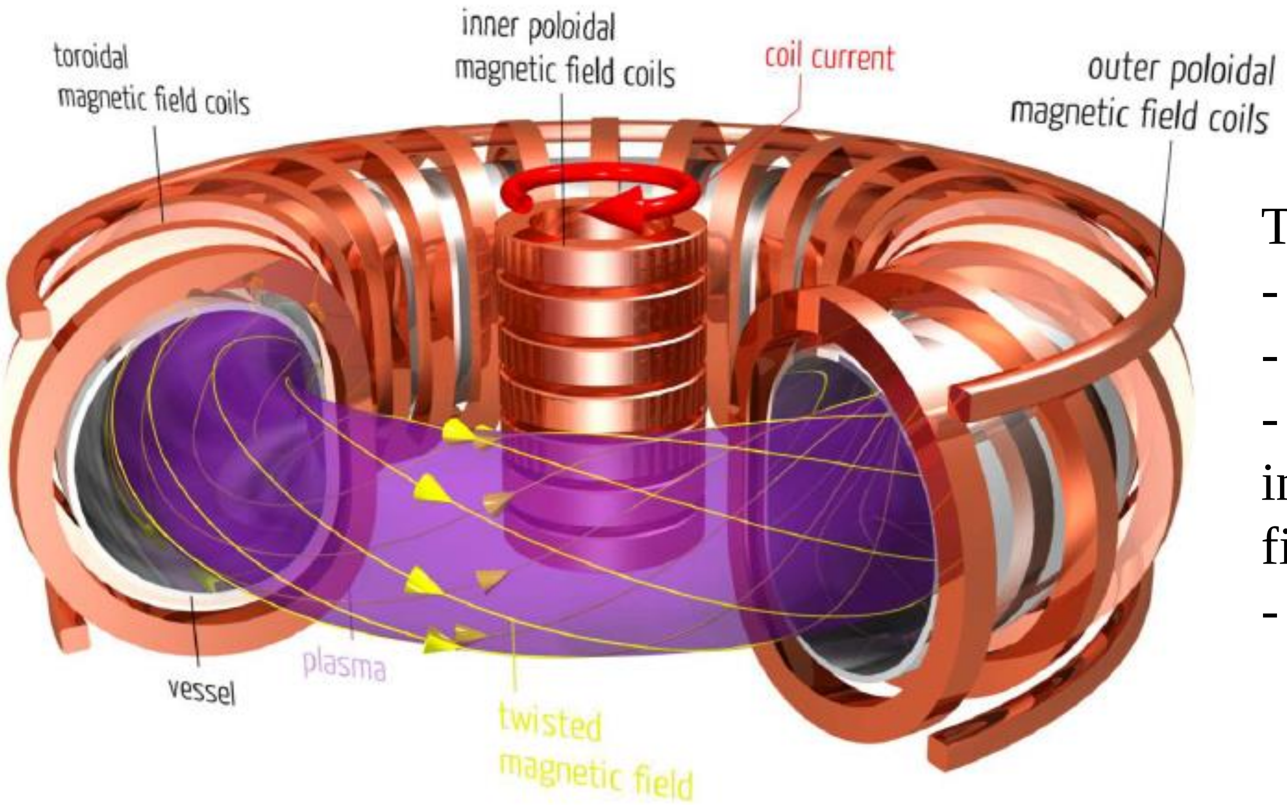
In Toroidal coordinates

- z is the axisymmetric axis
- Φ is the toroidal coordinate and $\bar{B}_\Phi$ is the toroidal field
- r is the radial coordinate, a is the radius of the plasma column and R is torus radius
- ϑ is the poloidal coordinate and $\bar{B}_\vartheta$ is the poloidal field



Tokamak configuration

Tokamak is today the machine for controlled thermonuclear fusion that has made it possible to reach the highest of $n\tau_E T$ and ion temperature values



The essential component of the machine are:
- the vacuum vessel
- the toroidal magnetic field coils (poloidal coils)
- the primary coils, used to induce a toroidal current in the plasma, which generates a poloidal magnetic filed
- The outer poloidal magnetic field coil

Large toroidal field is desirable for the *good performance* of a thermonuclear reactor, but its maximum value is limited by technological constrains.

Tokamak configuration

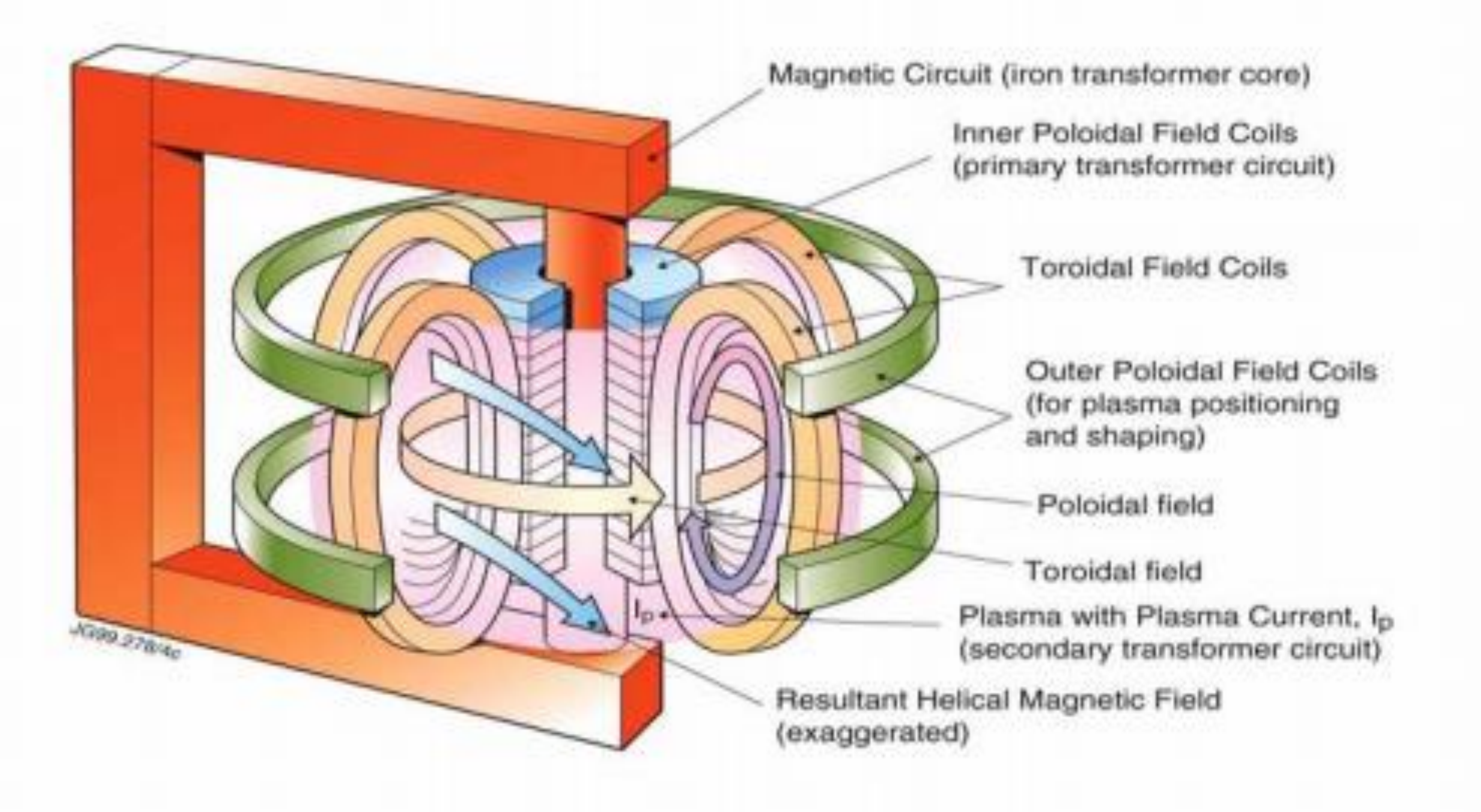
Poloidal coils → toroidal field

Inner coils → poloidal field

Tokamak configuration

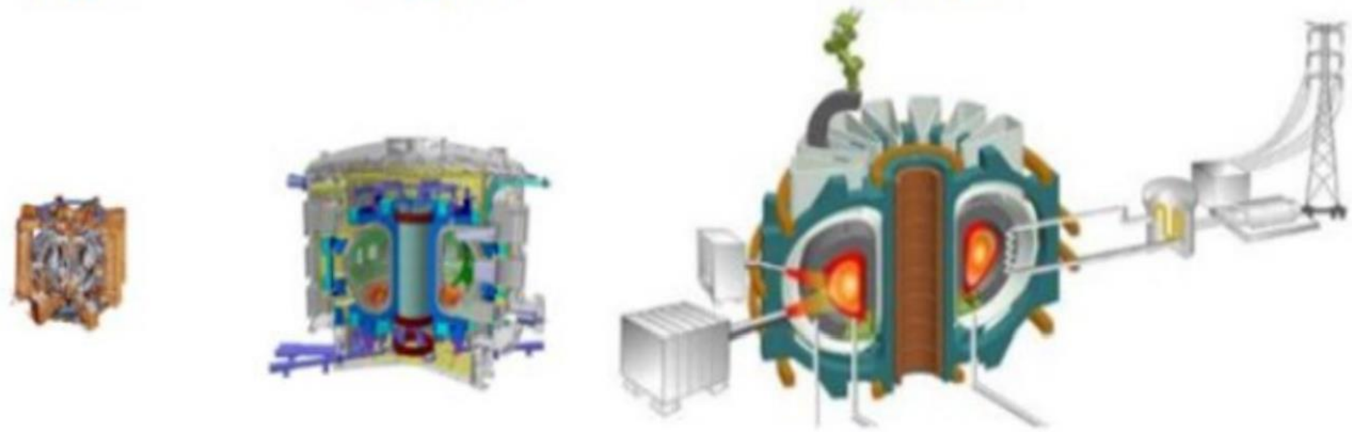
In addition to the coils of the transformer primary and the toroidal coils, there are also outer poloidal coils which have the main purpose of generating a magnetic field with a vertical component (parallel to the z axis of the torus and normal to the toroidal field) which allows to control:

- ❑ Equilibrium
- ❑ shape
- ❑ position



Tokamaks

JET	ITER	DEMO



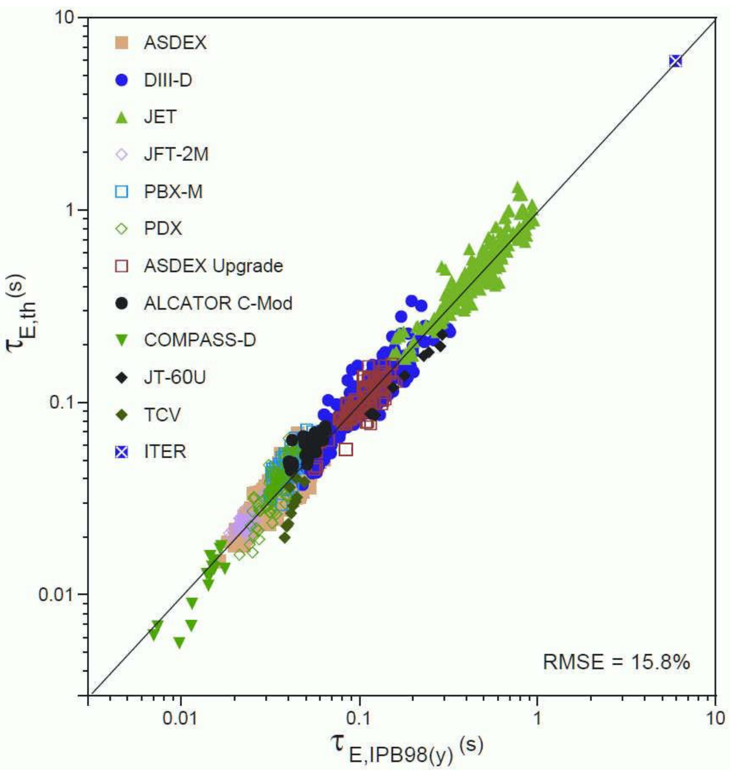
$D = 5.92$ m	$D = 12.4$ m	$D \approx 13\text{–}19$ m
$V = 80$ m^3	$V = 800$ m^3	$V \approx 1000\text{–}3500$ m^3
$Q \approx 0.6$	$Q \approx 10$	$Q \approx 25$
$P \approx 16$ MW$_{th}$	$P \approx 500$ MW$_{th}$	$P \approx 2000\text{–}4000$ MW$_{th}$

Increasing size

Increasing power

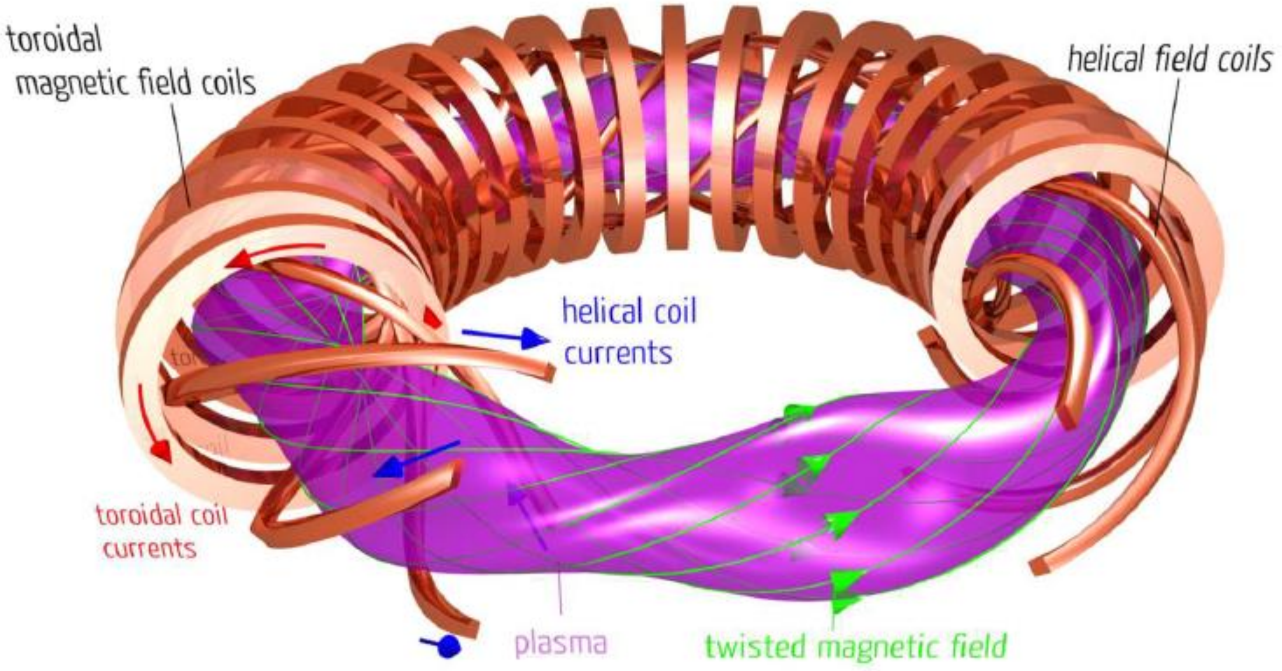
Plasma experiments

Demonstration of electricity generation



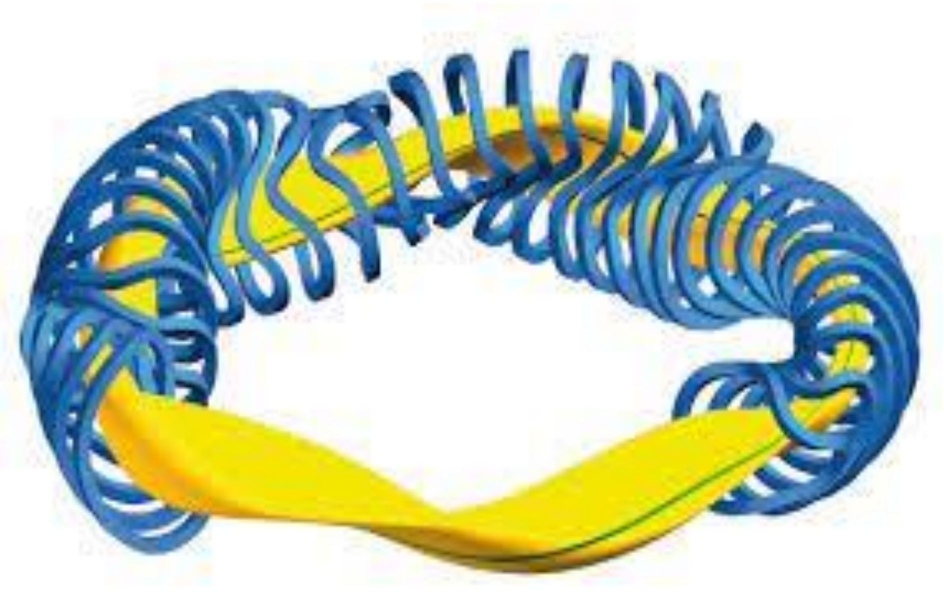
Stellarator configuration

In this machine the twisted magnetic field is entirely generated by external coils. Together with the ordinary toroidal solenoids helical coils are placed around the torus. These additional coils generate a helical magnetic field in the toroidal chamber that twists the toroidal magnetic field. No plasma current is needed in this machine configuration. Therefore the stellarator does not require a transformer for its operation, and it can operate continuously.



Stellarator configuration

The stellarator can be made with only the toroidal coils having the final shape of the plasma column. The geometry of the toroidal magnets are different according to the position occupied around the machine. In particular, certain number of modules having different coil shapes are repeated around the machine.



Stellarator configuration

New stellarators exhibit much better performance thanks to the numerical optimization of the magnetic configuration.

Advantages:
- ✓ less injected power to sustain the plasma
- ✓ quiescent steady state at high pressure
- ✓ The MHD instabilities due to toroidal plasma current are usually absent

Disadvantages:
- ✓ more complex coil configuration
- ✓ more complex divertor/structures
- ✓ poor energy and particle confinement