Introduction to thermonuclear fusion

Nuclear reactions

Mass number *A*=number of protons + number of neutrons
numero atomico *Z*=number of protons

H^{A}_{Z} → **A-Z=number of neutrons**

Composition of some elements of interest:
- Idrogeno, H^1_1, 1 proton and e 1 electron;
- Elio, H^4_2, 2 protons , 2 neutrons e 2 electrons;
- Litio, Li^6_3, 3 protons , 3 neutrons 3 electrons;

In a primary chemical element (mass number A), whose nucleus contains N neutrons and Z protons, the total mass of the isolated individual particles exceeds the actual nuclear mass (m_A)

$$Z \cdot m_p + N \cdot m_n > m_A$$

The difference in mass that is transformed into Binding Energy (E_B) by Einstein's famous relation

$$E_B = \Delta m \cdot c^2 = (Z \cdot m_p + N \cdot m_n - m_A) \cdot c^2$$

E_B is the amount of energy would have to be provided to the nucleus to break it apart into its separate components.

Mass-Energy equivalence

In a **nuclear reaction**, a reduction in the mass Δm releases an energy proportional to the reduction in mass Δm and to the speed of light c^2:

$$E = \Delta m \cdot c^2$$

being c^2 = 91016 [m^2/s^2] a high number, it follows that small variations in mass can lead to the generation of a large amount of energy, mainly in as kinetic energy of the reaction products.

Deuterium H_1^2

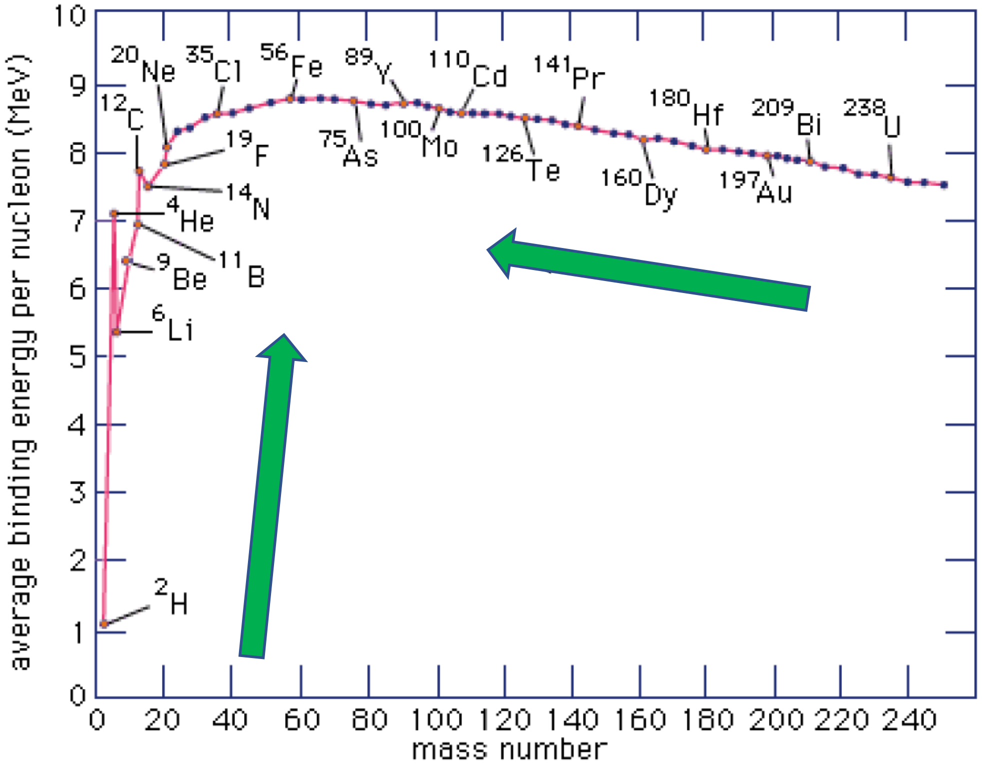
$$m_p = 938,272029 \quad m_n = 939,565360$$

Mass of the deuterium nucleus: m_D= 1875,612762

$$m_D < m_p + m_n$$

Experimentally it is found that if the deuterium nucleus is bombed with *2.2 MeV* the nucleus splits.

Mass–Energy equivalence



$$a + b \rightarrow c + d$$

$$\text{mass difference: } \Delta m = (m_c + m_d) - (m_a + m_b)$$

$$\Delta E = \Delta m\, c^2$$

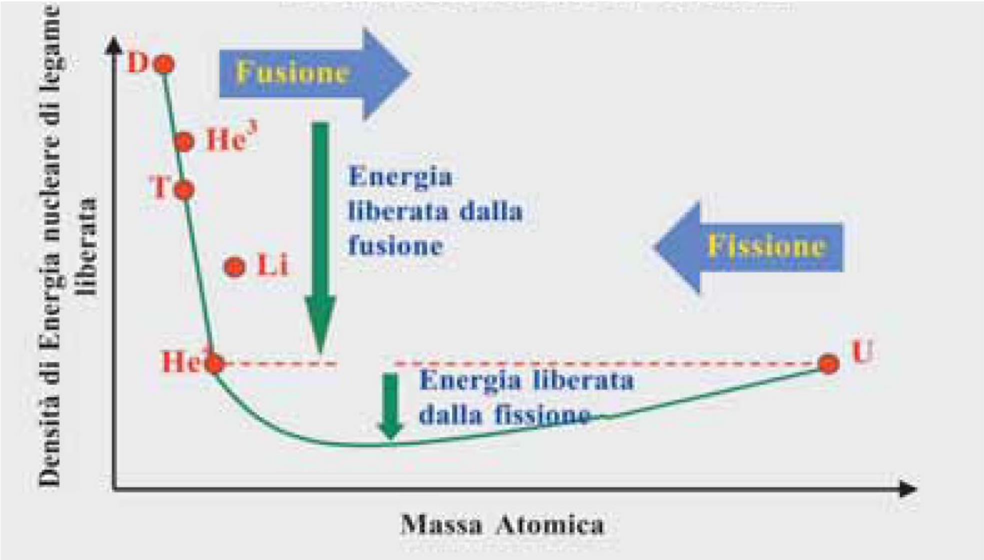
A convenient quantity to evaluate mass–energy equivalence is E_B/A which measures ***the average Binding Energy per nucleon***. If we calculate the Binding Energy per nucleon for all chemical elements, we get the ***nuclear mass defect diagram.***

E_B/A is low for both light and heavier atoms, so it's easier for them to start a nuclear reaction. The higher E_B/A is the more stable the nucleus because the nucleoli are more bounded.

The processes that generates elements with more stable nucleus release energy in support of the binding energy. In nature, evolving towards a more stable configuration means evolving *towards a condition of lower total energy*. But **decreasing the total energy means *decreasing the overall mass* because energy and mass are the same thing**.

Mass–Energy equivalence

- A <60: as the mass number increases, A increases and the nucleus is more bound → it is better ***to fuse the small nuclei*** to form a larger one with larger E_B/A
- A> 60: as the mass of the nucleus decreases, A decreases and the nucleus is more bound → it is better **to divide the large *nuclei*** into two smaller ones with larger E_B/A



Are stable reactions:

- ❖ the fusion between two light nuclei
- ❖ the fission in which a very heavy nucleus breaks down into two lighter nuclei.

In both cases the total mass of the reaction fragments is lower than the mass of the starting nuclei.

A nuclear reaction (either fission or fusion) can be written as

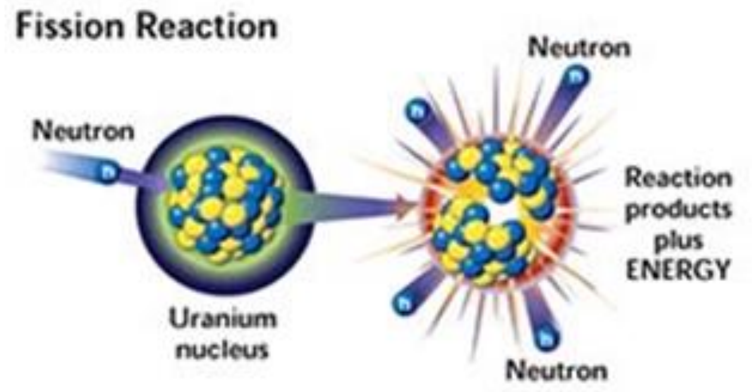
$$A_1 + A_2 + A_3 + \dots + A_i = B_1 + B_2 + B_3 + \dots + B_j + energy$$

Where:

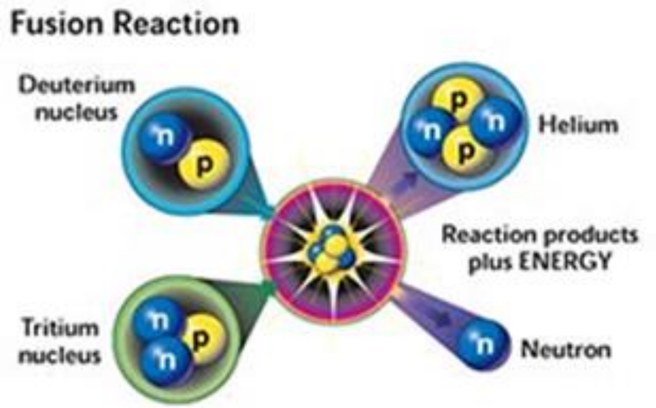
$$energy = (m_{A1} + m_{A2} + m_{A3} + \dots + m_{Ai} - m_{B1} - m_{B2} - m_{B3} - \dots - m_{Bi}) \cdot C^2$$

Nuclear reactions

Nuclear reactions concern the transformation of the nucleus of a specific chemical elements into another with a different atomic number, it is necessary to vary the mass number A, thus the number of neutrons and protons.



FISSION is obtained bombing the nucleus of a relatively rare isotope of uranium U^{235}. A neutron is used as bullet, since it doesn't interact with the charged particles (protons and electrons)



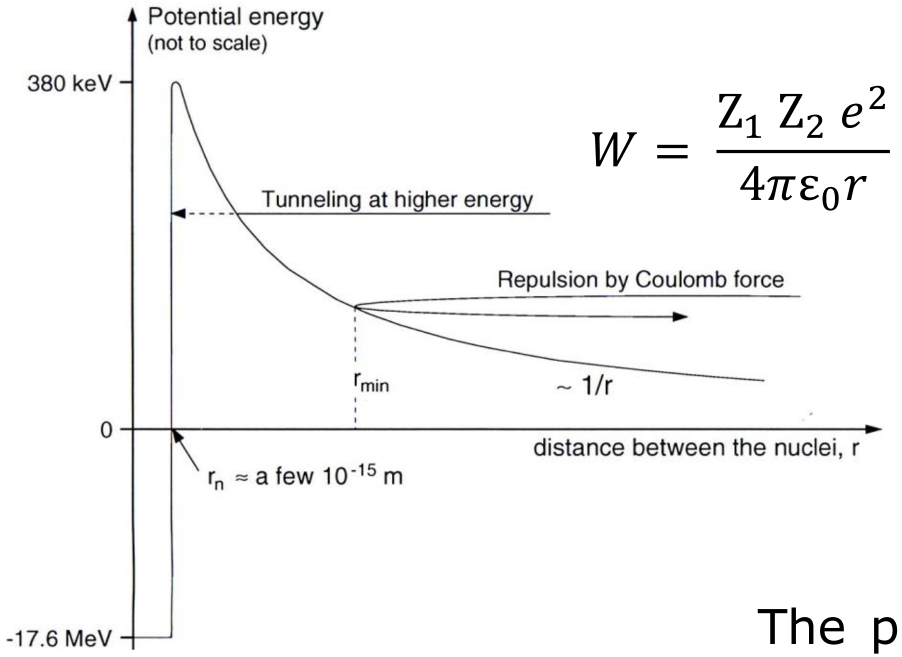
FUSION is obtained by combining nuclei of light elements. To achieve a fusion reaction is necessary to provide the nuclei enough energy to overcome the Potential energy barrier (Coulomb's law)

It is proved experimentally that for distances smaller than 5.10^{-12} m the electrostatic force is overcome by the attraction nuclear forces (*forces which allow more protons to remain linked in the same nucleus*). Thus, the nuclei must be brought to a distance where the binding energy overcomes the potential barrier.

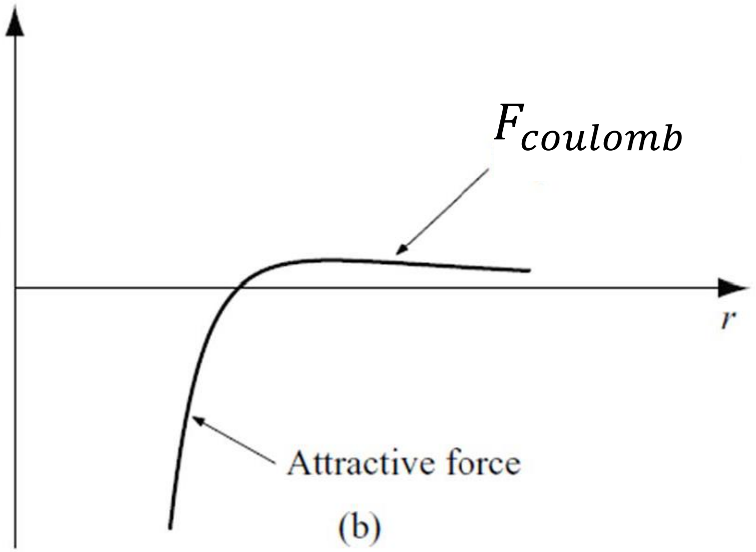
$$F_{coulomb} = \varepsilon \frac{q_1 \cdot q_1}{d^2}$$

Fusion reactions on the earth

To **trigger a fusion reaction**, two light atom nuclei must be very **close each other** so that the **short-range** nuclear attractive forces **exceed** the **long-range** repulsion forces.



$$W = \frac{Z_1\, Z_2\, e^2}{4\pi\varepsilon_0 r}$$



$$F = \frac{Z_1\, Z_2\, e^2}{4\pi\varepsilon_0 r^2}$$

The potential energy ***W*** which must be supplied to the nuclei to overcome the **Coulomb barrier. The coulomb repulsion barrier is most easily penetrated at high temperatures.**

where Z_1, Z_2 are the nucleus atomic numbers, *e* is the electon (elementary) charge, and r is the distance between the nucleus centres. For D-T reaction, at r=5 10^{-15} m the nuclear attractive forces become dominant.

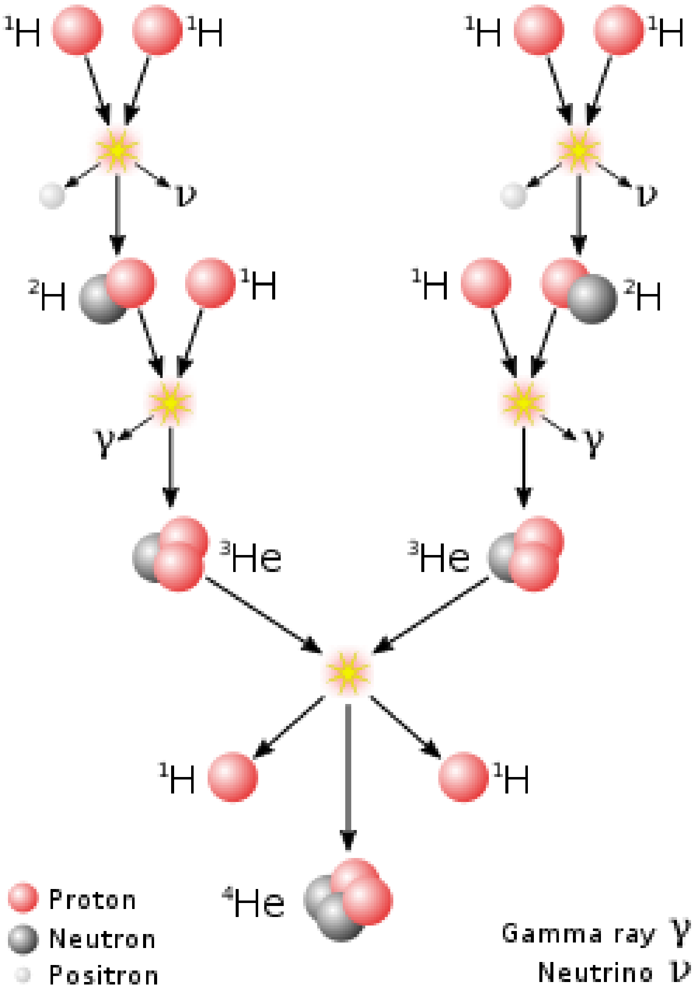
For D-T ➔ r=5 10^{-15}m, the W required is 288keV

The fusion in the Sun

The energy from the Sun, both heat and light energy, originates from a nuclear fusion process that occurs inside the core.

The *proton-proton* fusion process:

1. Two protons within the Sun fuse forming *deuterium* + 1 positron and 1 neutrino
2. A proton collides with deuterium forming *helium-3* + γ ray
3. Two helium-3 nuclei collide, creating a helium-4 nucleus



4 H atoms: 4.03130 AMU(*)
1 He atom: 4.00268 AMU

$E = (2.8\ 10^{-6}\ \text{kg}) \times (3\ 10^{8}\ \text{m/sec})^{2} = 2.6\times10^{11}$ joules

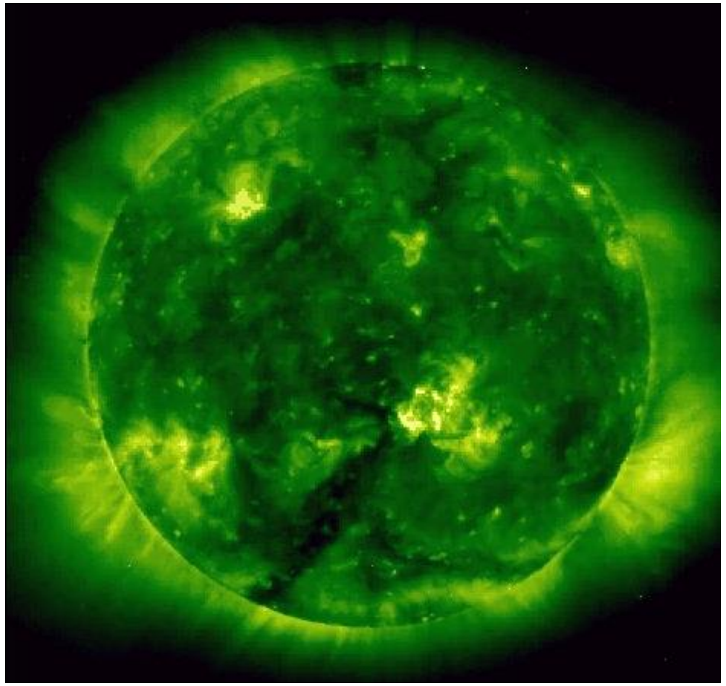
The difference between the mass of 4 H atoms and 1 He atom is 0.02862 AMU which is only 0.71% of the original mass

The sun converts 600 million tons of Hydrogen (H) → into 596 million tons of Helium (He) per second, producing energy with a power of 3.7 10^{17} GW

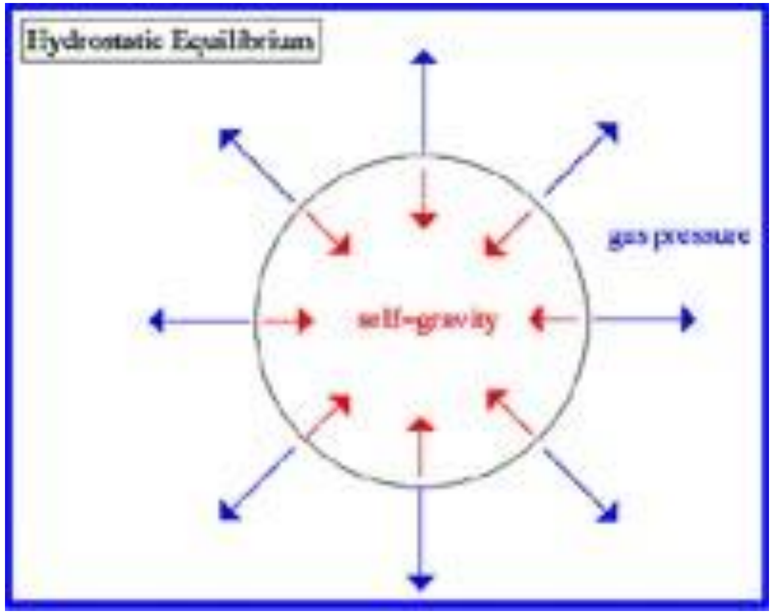
(*)1 Atomic Mass Unit (AMU)=1.67 10^{-27}kg

The fusion in the sun

The nuclear fusion reaction powers the Sun and the other stars



How to replicate the conditions in the core sun:
1) Create a plasma. In lab plasma is obtained superheating a gas.
2) Confine the plasma: apply pressure to keep the nuclei colliding each others.
3) Create a continuous chain of fusion reactions



The power generator in the Sun is in its centre, buried deeply within it. It is called "the core", with a radius close to 1/4 of that of the star.
In the core, pressures and temperatures are high enough to force fusion.

The SUN rely on gravity as confinement force

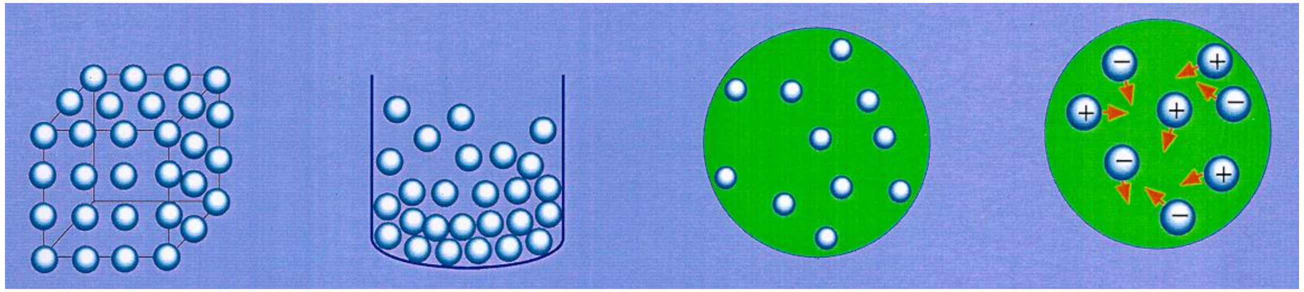
$$F = G\frac{M_1 M_2}{d^2}$$

G = 6.672·10^{-11} Nm^2 kg^{-2}= ***Cavendish constant***

The plasma

The energy needed to overcome the potential energy barrier and to achieve the fusion between two nuclei can be provided by bringing them to **very high pressure** (**very high temperature**, about 10^7 kelvin, and/or very high density).

The different states of matter depend on the balance between:
- the energy associated with the Coulomb barrier, which binds the particles together
- the kinetic energy of the particles (thermal energy that increases with temperature).



The plasma is the *fourth state of matter. It contains non negligible portion of charged particles (ions and/or electrons) even if the overall charge* is roughly zero. It is rather intuitive to think about it as an homogeneous distribution of charged particles, placed in such a way that the resulting electric field is zero.
An ionized substance becomes highly electrically conductive, thus the electric and magnetic fields are used to dominate its behaviour. Depending on temperature and density, a certain amount of neutral particles may also be present.

The plasma creation

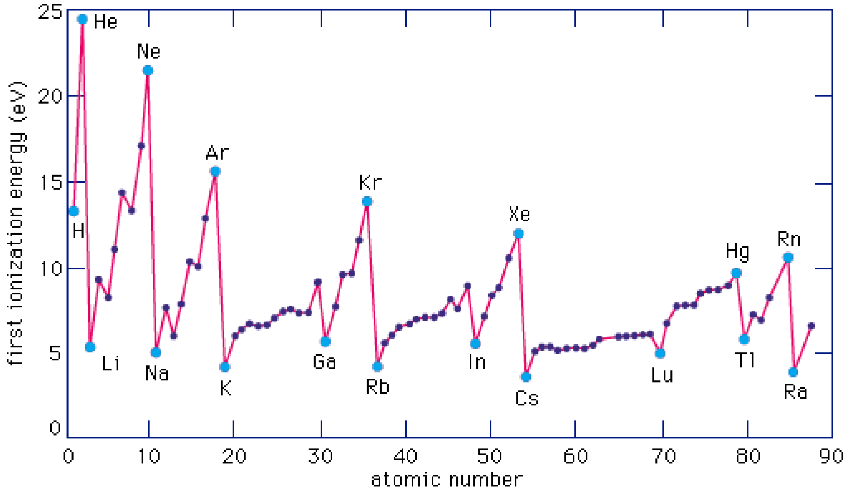
Crookes tube consists of a partially vacuum glass bulb, with two metal electrodes, the cathode and the anode at the end. The electrons are generated by the ionization of the residual air by a high DC voltage applied between the electrodes.
When high voltage (few kilovolts) is applied to the tube, the electric field accelerates the small number of ions and free electrons always present in the gas. The electrons collide with the neutral atoms creating more positive ions and free electrons, by a chain reaction which originate a discharge.

A **fusion plasma** is an excellent ***good conductor*** of electricity even if the number density of electrons is typically many orders of magnitude smaller than in good conductors (e.g. copper). Nevertheless, the electrical conductivity of a fusion plasma is about 40 times larger than that of copper. This is because at the fusion temperature, the coulomb collisions between electrons and ions rarely occur, this means very little resistance to the current flow.

The high conductivity of the plasma implies that:
- ❑ It is shielded from DC electric fields (as any good conductor)
- ❑ It can be penetrate by DC magnetic fields

The plasmas

Plasmas vary according to temperature and density.



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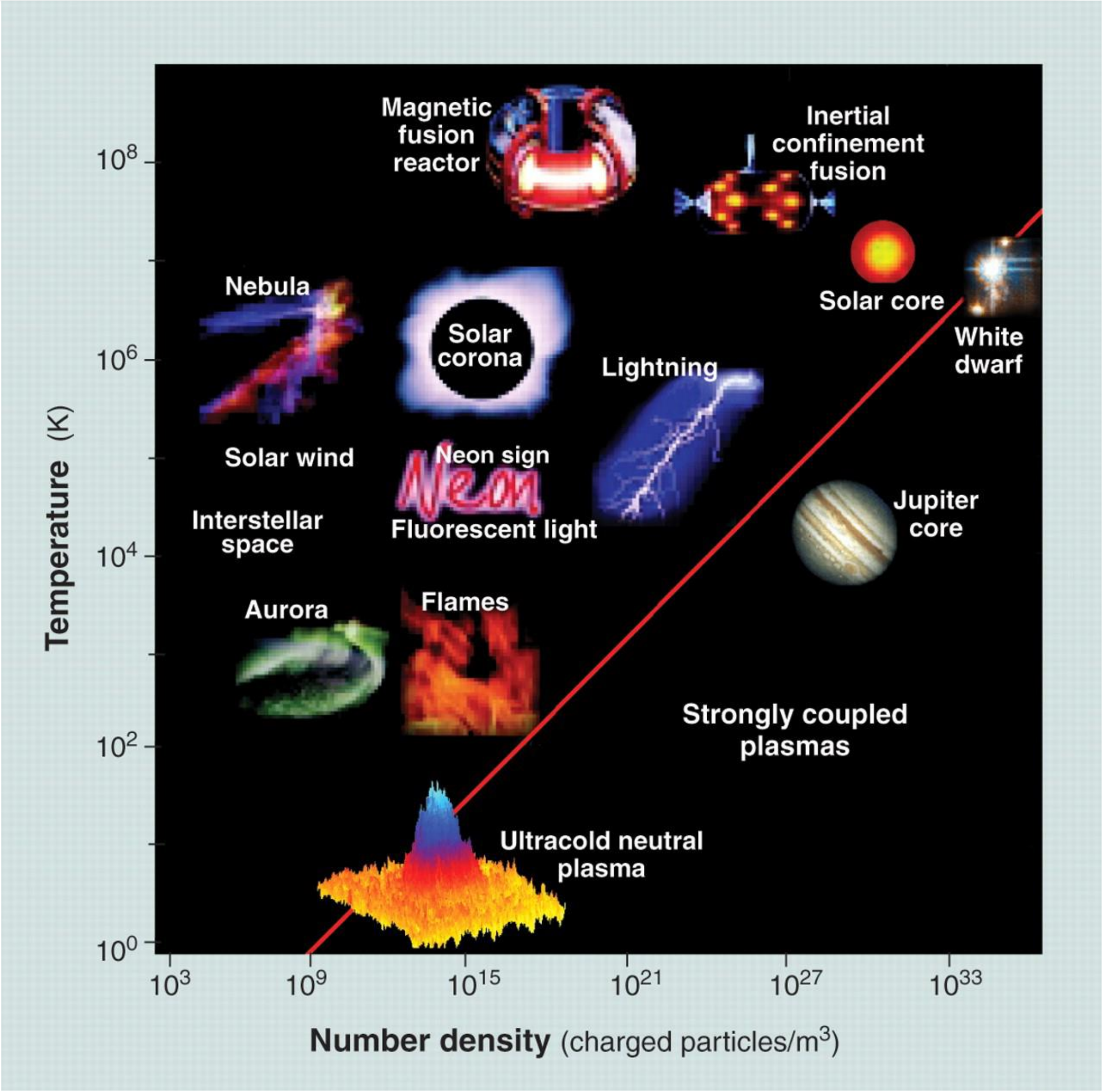
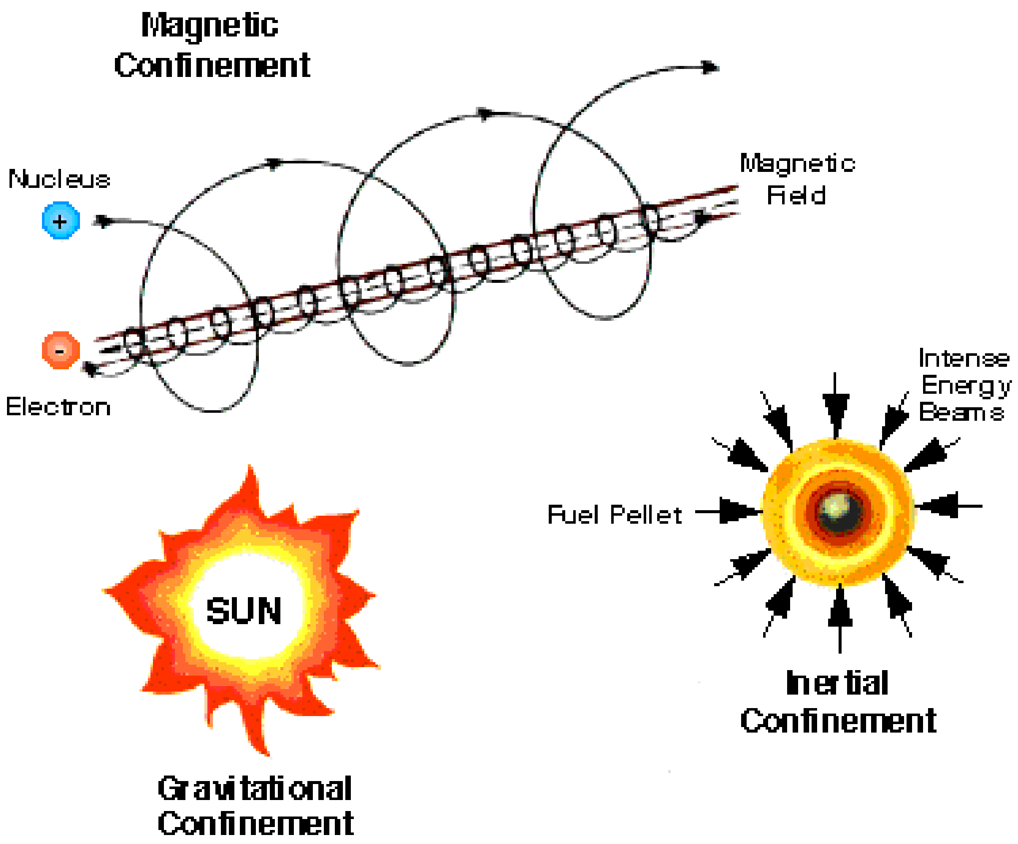
Plasmas are very common in the universe –The visible Universe is 99.999% plasma:

- The Sun and all the stars are about 100% plasma.
- Plasma makes up nearly 100% of the interplanetary, interstellar and intergalactic medium
- The Earth’s ionosphere is plasma



How to make the fusion

Since fusion needs a hot plasma held tightly together for long enough for the fusion reaction to occur, and since the plasma is so hot it would melt any container you could put it in, three methods for confining the plasma are possible: Gravity, Magnetic fields, or Inertial.



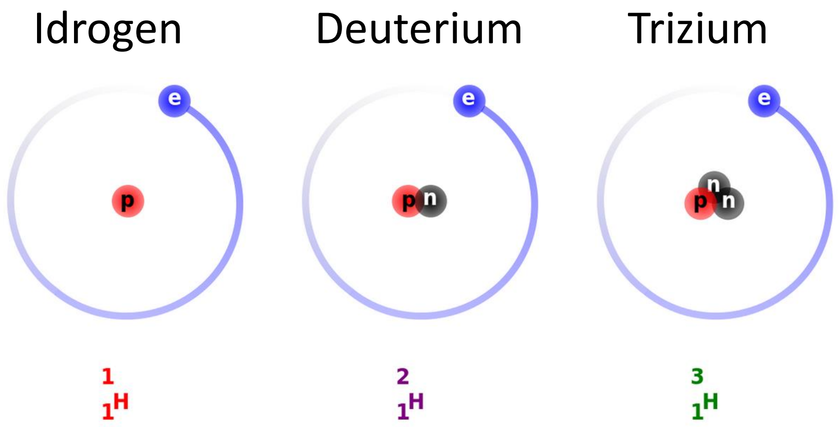
Gravity: the Sun and all other Stars use gravity to keep the hot plasma from flying apar.

Magnetic confinement: extremely powerful magnetic fields are used to dominate the plasma behaviour. The magnetic force acts on the plasma *to balance the expansive pressure force exerted on the plasma by the thermal energy*.

Inertial confinement: powerful lasers are used to implode a small pellet of fuel and make the fuel undergo fusion.

Isotopes of Hydrogen

Isotopes are atoms which belong to the same chemical element, having the *same atomic number Z (number of protons) and differ A (mass numbers)*, due to different numbers of neutrons in their nuclei.



Hidrogen isotopes :

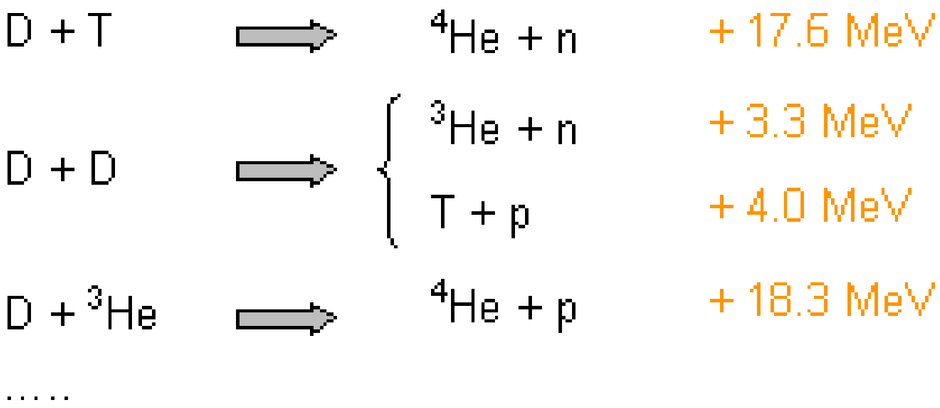
- ***Hydrogen (H)***, the nucleon is composed by ***1 proton***
- ***Deuterium (D)***, the nucleon composed by ***1 proton and 1 neutron.***
- ***Tritium (T),*** the nucleon is composed by ***1 proton and 2 neutrons.***

D constitutes 0.01% of the hydrogen existing in nature

T is not present in nature because it is an unstable element and spontaneously decays (in 12.32 years). T can be obtained by the reaction: **$Li^4 + n = He^4 + T + n^* - 2.5$ MeV**

Fusion reactions on the earth

Many reactions involving the combination of light elements (e.g. isotopes of hydrogen, helium, and lithium) are accompanied by a liberation of energy:



$$\mathrm{D} + \mathrm{T} \Rightarrow {}^{4}\mathrm{He} + \mathrm{n} \qquad +17.6\ \mathrm{MeV}$$

$$\mathrm{D} + \mathrm{D} \Rightarrow \begin{cases} {}^{3}\mathrm{He} + \mathrm{n} & +3.3\ \mathrm{MeV} \\ \mathrm{T} + \mathrm{p} & +4.0\ \mathrm{MeV} \end{cases}$$

$$\mathrm{D} + {}^{3}\mathrm{He} \Rightarrow {}^{4}\mathrm{He} + \mathrm{p} \qquad +18.3\ \mathrm{MeV}$$

.....

The 17.6 MeV released in the *D+T* reaction corresponds to an energy release many times greater than a typical chemical reaction (the combustion of octane releases only 57 eV, which is more than 300000 time less).

The **D-T** is the **most favourable** reaction and the easiest to be initiate, but involves tritium which is *not available in nature, radiative and produce large number of neutrons.*
The **D-D** is the **most desirable** reaction. It is virtually unlimited supply of inexpensive fuel (easily extracted from the ocean), but it is **the most difficult** to be initiate.
The **D–^{3}He** is the **less desirable**. The poor ^{3}He availability (there are no natural supplies of this isotope on earth) combined with the difficulty of initiating are the reasons that current fusion research is not focused on this reaction.

In spite to the problems associated with tritium in the D-T reaction, it is the central focus of worldwide fusion research

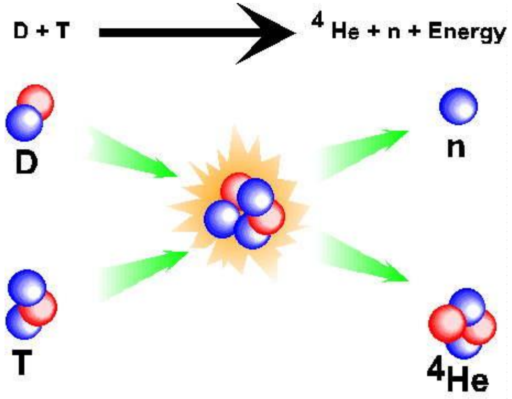
Fusion reactions on the earth

The large amounts of energy released in fusion reactions appear in the form of **kinetic energy of the end products**.

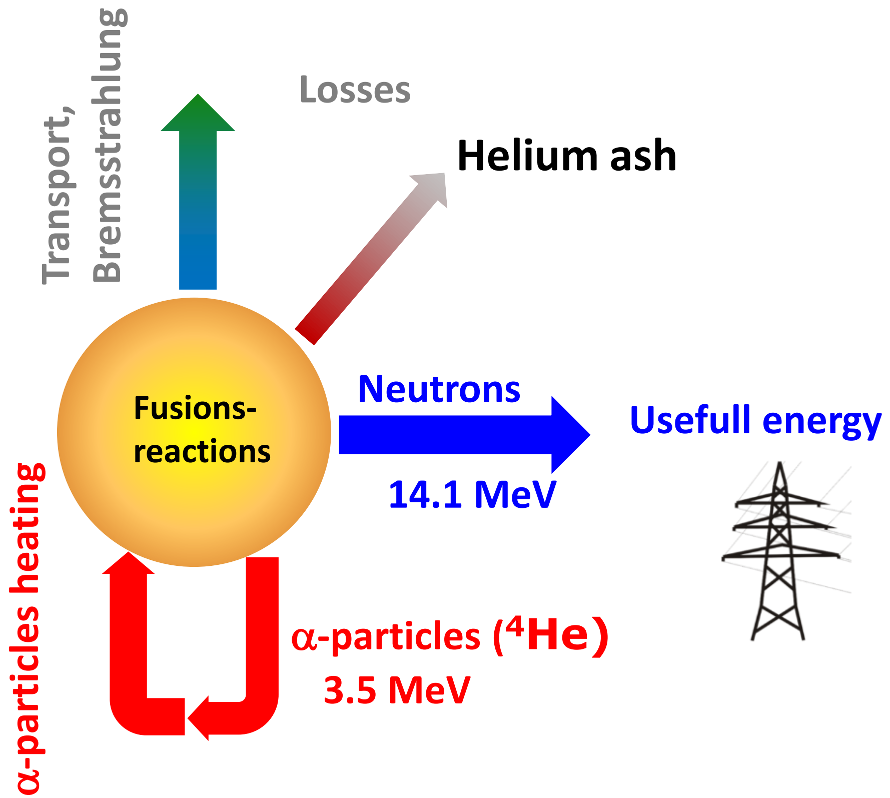
By making use of the well-satisfied assumption that the kinetic energy E inversely depends by the mass ($m_\alpha = 4\ m_n$), for the D–T reaction :

$$E_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 = \frac{m_n}{m_\alpha + m_n} E = 3.5 MeV \qquad E_n = \frac{1}{2} m_n v_n^2 = \frac{m_\alpha}{m_\alpha + m_n} E = 14.1 MeV$$

This is important to determine how the energy is shared between the two end products, in particular for reactions where one end product is electrically charged.



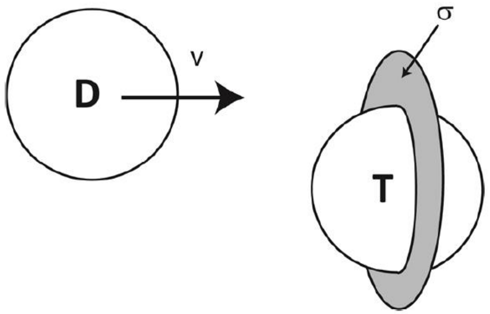
$$\begin{aligned}
&D+T\longrightarrow && \alpha(3.5MeV)+n(14.1MeV)\\
&D+D\longrightarrow && T(1.01MeV)+p(3.02MeV)\\
&D+D\longrightarrow && 3He(0.82MeV)+n(2.45MeV)\\
&D+{}^3He\longrightarrow && \alpha(3.6MeV)+p(14.7MeV)
\end{aligned}$$

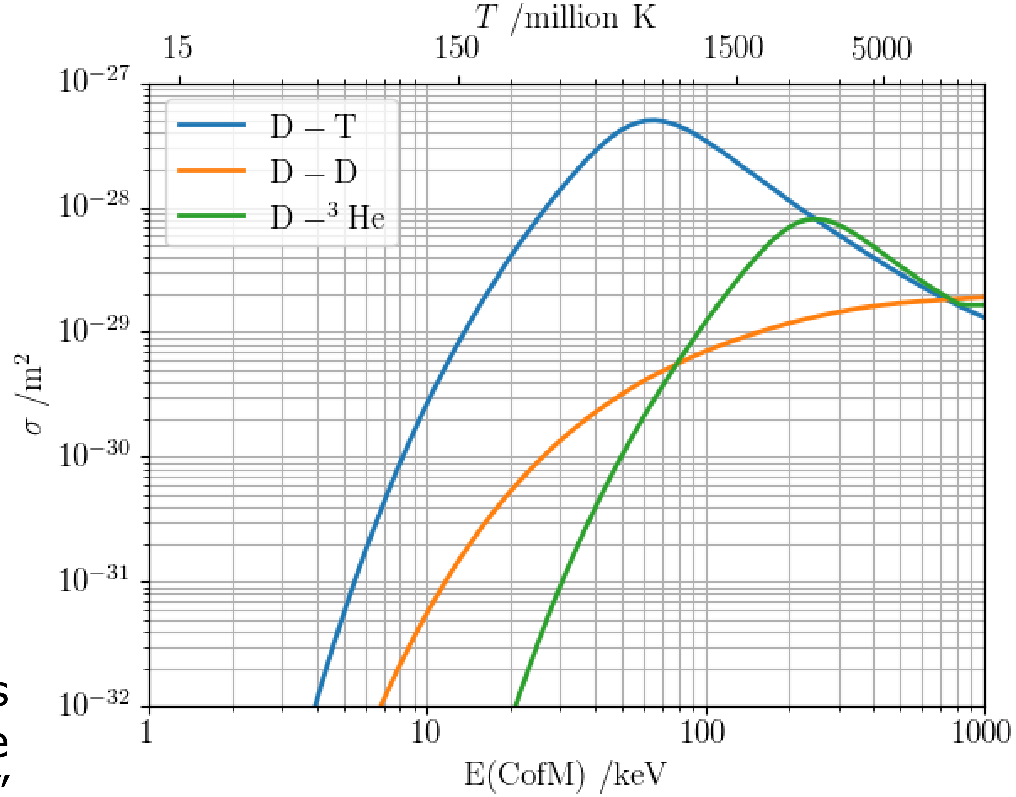


Fusion reactions on the eart

Let be two nuclei, D and T, T stationary and D moving toward T. assuming an hard-sphere" model , the **cross section *σ*** is the area, in the direction perpendicular to the D motion, of a symmetric force field, inside of which the incident particle has to pass through to undergo a nuclear fusion reaction. Otherwise, the force exerted on incident particle by the target particle is not sufficiently strong to activate a nuclear reaction.

σ gives is a measure of a fusion reaction probability as a function of the relative velocity of the two nuclei. For fusion reactions dominated by the **short-range nuclear force** the cross section is on the order of a **nuclear diameter**



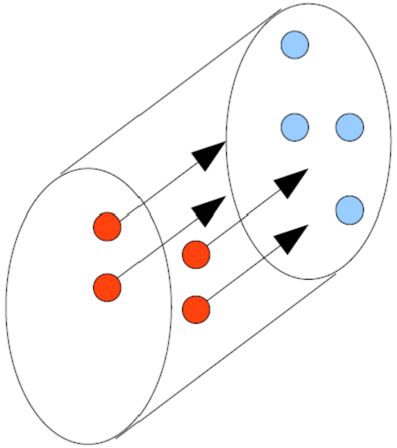


Actually, the correct cross section for fusion collisions involves also nuclear quantum mechanical effects, which introduce qualitatively modifications to the simple classical "hard-sphere" model.

The reason that the majority of research on experimental fusion reactors has focused on the D + T process is that it has the largest cross section, which peaks at the lowest temperature.

Fusion reactions on the earth

The **reaction rate** $\boldsymbol{R}$ is the number of particles having a collision per *unit of volume per unit of time* :



$$R = n_1 v_1 n_2 \sigma_2$$

Where n is the density, i.e. the number of particles per unit of volume, for each ion type

If the reagents have a distribution of velocities, e.g. a random distribution, then it is useful to perform the average of the product between the cross-section and velocity $\langle\sigma v\rangle$ distributions. This average is called the reactivity.

$$R = n_1 n_2 <\sigma v>$$

The fusion reaction rate (fusions per volume per time) is $\langle\sigma v\rangle$ times the product of the reactant number densities:

For a given ion density $n=n_1+n_2$, the maximum R is achieved for $\boldsymbol{n_1=n_2=n/2}$.

The thermonuclear power per unit of volume is $P = n_1 n_2 <\sigma v> E \ \ [W/m^3]$

Power balance in a D-T reaction

The thermonuclear fusion power in a D-T reaction per unit of volume is:

$$P_{fus} = n_D\, n_T <\sigma v> E \ \ [W/m^3]$$

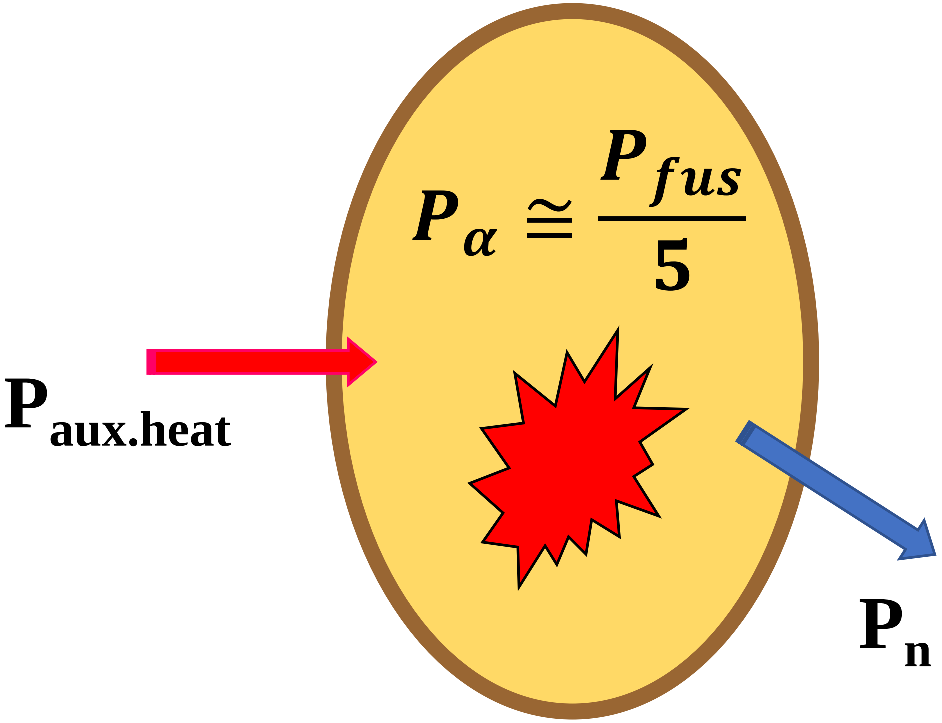
For the **optimum mixture** $n = n_D = n_T$ the total thermonuclear power is $P_{fus} = P_\alpha + P_n$

$$P_\alpha = \frac{1}{4} n^2 <\sigma v> E_\alpha \qquad E_n \cong 4\, E_\alpha \longrightarrow P_{fus} = P_\alpha + 4P_\alpha = \frac{1}{4} n^2 <\sigma v> E_{Fus}$$

$$P_n = \frac{1}{4} n^2 <\sigma v> E_n$$

$$\boldsymbol{Q = \frac{P_{fus}}{P_{aux.heat}}}$$

$P_{aux.heat}$ is the power needed to trigger the fusion reactions



- **Breakeven (Q=1): $P_{fus} = P_{ext}$**
- **Ignition (Q→∞): $P_{fus} \neq 0$, $P_{aux.heat} = 0$**
- **fusion Reactor (Q~10)**

Power balance in a plasma per unit volume

$$\frac{dE_{th}}{dt} = P_{heath} - P_{loss} = P_{aux.heath} + P_{\alpha} - P_{loss}$$

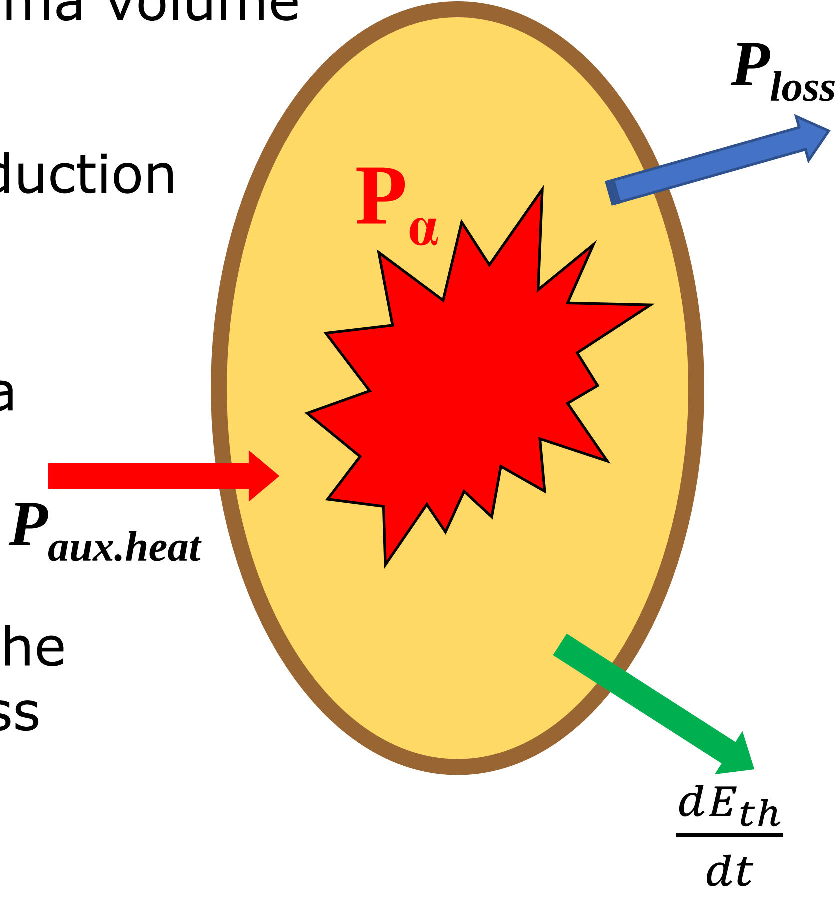
$\mathbf{E_{TH}} = 3\,\mathrm{nkT}v$, is the thermal energy, where v is the plasma volume

$\mathbf{P_{heat}}$ = total plasma heating power

$\mathbf{P_{loss}}$ = plasma looses by radiation (Bremsstrahlung), conduction and convection (transport)

$\mathbf{P_{aux.heat}}$ = heating power from auxiliary systems

$\mathbf{P_{\alpha}}$ = heating power from α-particle trapped in the plasma



$$P_{loss} = \frac{E_{th}}{\tau_E} = \frac{3\,nkTv}{\tau_E}$$

τ_E = confinement time. It is the cooling time constant of the plasma, introduced to describe the total plasma power loss

$$\tau_E = \frac{E_{th}}{\mathrm{P_{loss}}} = \frac{E_{th}}{\mathrm{P_{heat}} - \frac{dE_{th}}{dt}}$$

The way towards ignition

$$\frac{dE_{th}}{dt} > 0 \qquad \Rightarrow \qquad \mathrm{P}_{\mathrm{aux.heath}} + \mathrm{P}_{\alpha} > \mathrm{P}_{\mathrm{loss}}$$

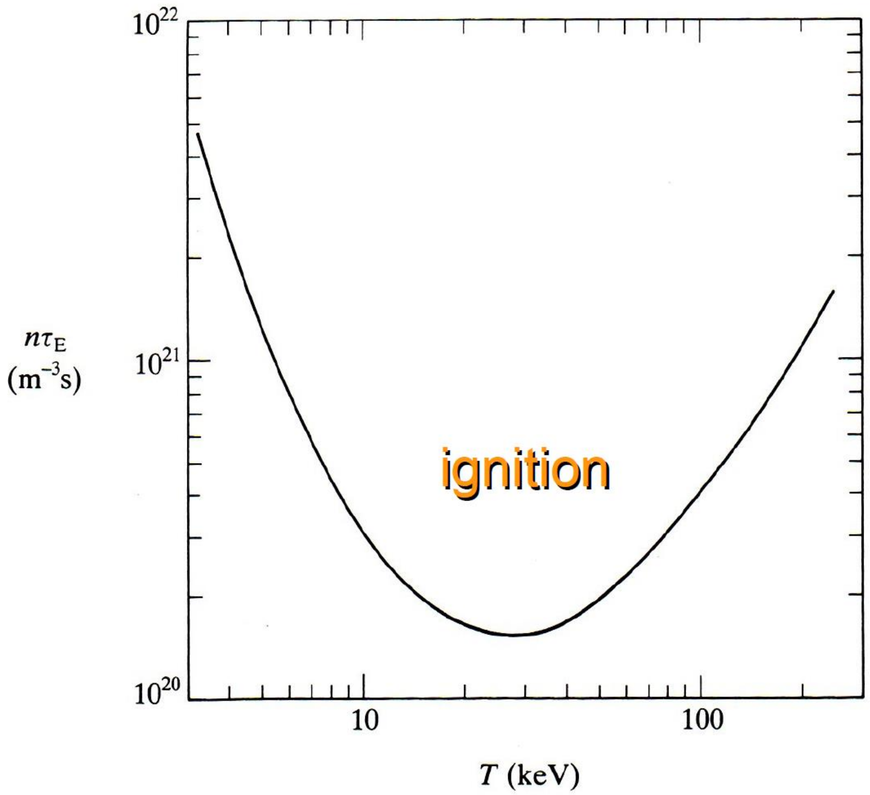
D-T reaction, optimum mixture $n = n_D = n_T$

$$\frac{P_{fus}}{Q} + P_\alpha \geq \frac{E_{th}}{\tau_E} = \frac{3\,n\,k\,T\,v}{\tau_E}$$

$$P_{fus}\left(\frac{1}{Q} + \frac{P_\alpha}{P_{fus}}\right) \geq \frac{3\,nkTv}{\tau_E}$$

$$\textbf{Ignition } \boldsymbol{Q} = \infty \rightarrow P_\alpha \geq \frac{3\,nkTv}{\tau_E} \qquad\qquad P_\alpha = \frac{1}{4} n^2 \langle\sigma v\rangle\, E_\alpha$$

$$\frac{1}{4} n^2 \langle\sigma v\rangle\, E_\alpha v > \frac{3\,nkTv}{\tau_E} \qquad \Rightarrow \qquad n \langle\sigma v\rangle\, E_\alpha > \frac{3\,T}{\tau_E}$$



$$nT\,\tau_E > \frac{3\,T^2}{\langle\sigma v\rangle\, E_\alpha}$$

where $\langle\sigma v\rangle = 1.1\ 10^{-24}\ T^2$
for $T = 20\ keV$ $\langle\sigma v\rangle = 4\ 10^{-22}\ m^3\ s$
(10 keV = 1.16 10^8K)

Lawson's Criteria calculates the requirements for more energy to be created than is put in, and came up with a dependence on three quantities: temperature (T), density (n) and confinement time (τ_E)

Ignition condition in terms of triple product: $n\ \tau_E\ T > 3{\times}10^{21}\ m^{-3}\ keVs$

The triple product

The Ignition is the ideal condition at which the plasma is self-sustaining, without any external input power (similar what happen in the sun and other stars): this condition implies P$_{aux.heath}$ = 0

$$P_{fus}\left(\frac{1}{Q}+\frac{P_\alpha}{P_{fus}}\right) \geq \frac{3\,nkTv}{\tau_E}$$

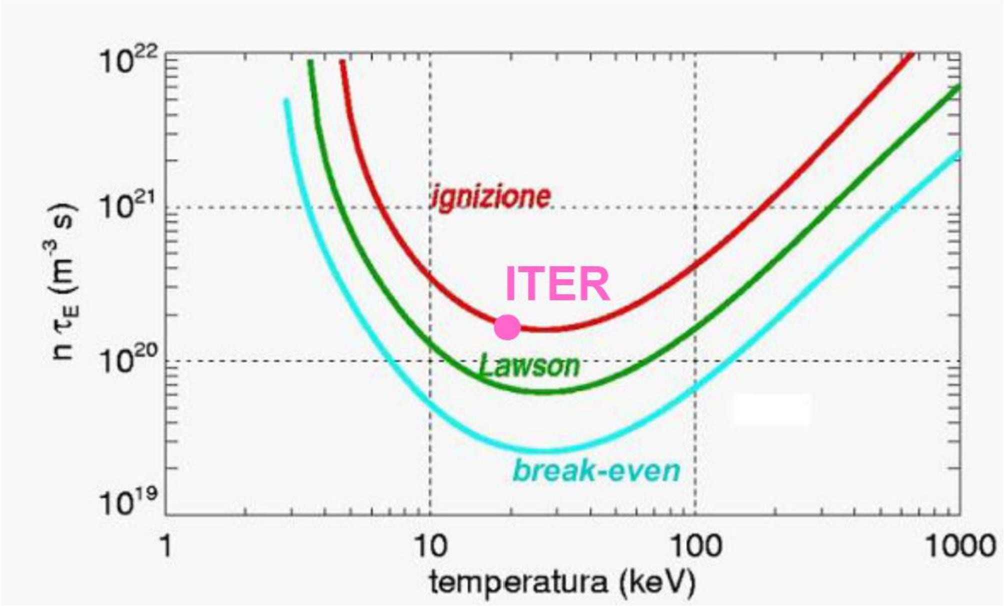
triple product

by using the fact that the plasma energy W and fusion power P_{fusion} depend on density and temperature T of the plasma, we obtain a relation expressing the constraints on the plasma parameters (density, temperature and energy confinement time) if we want to obtain a pulse with a given Q.

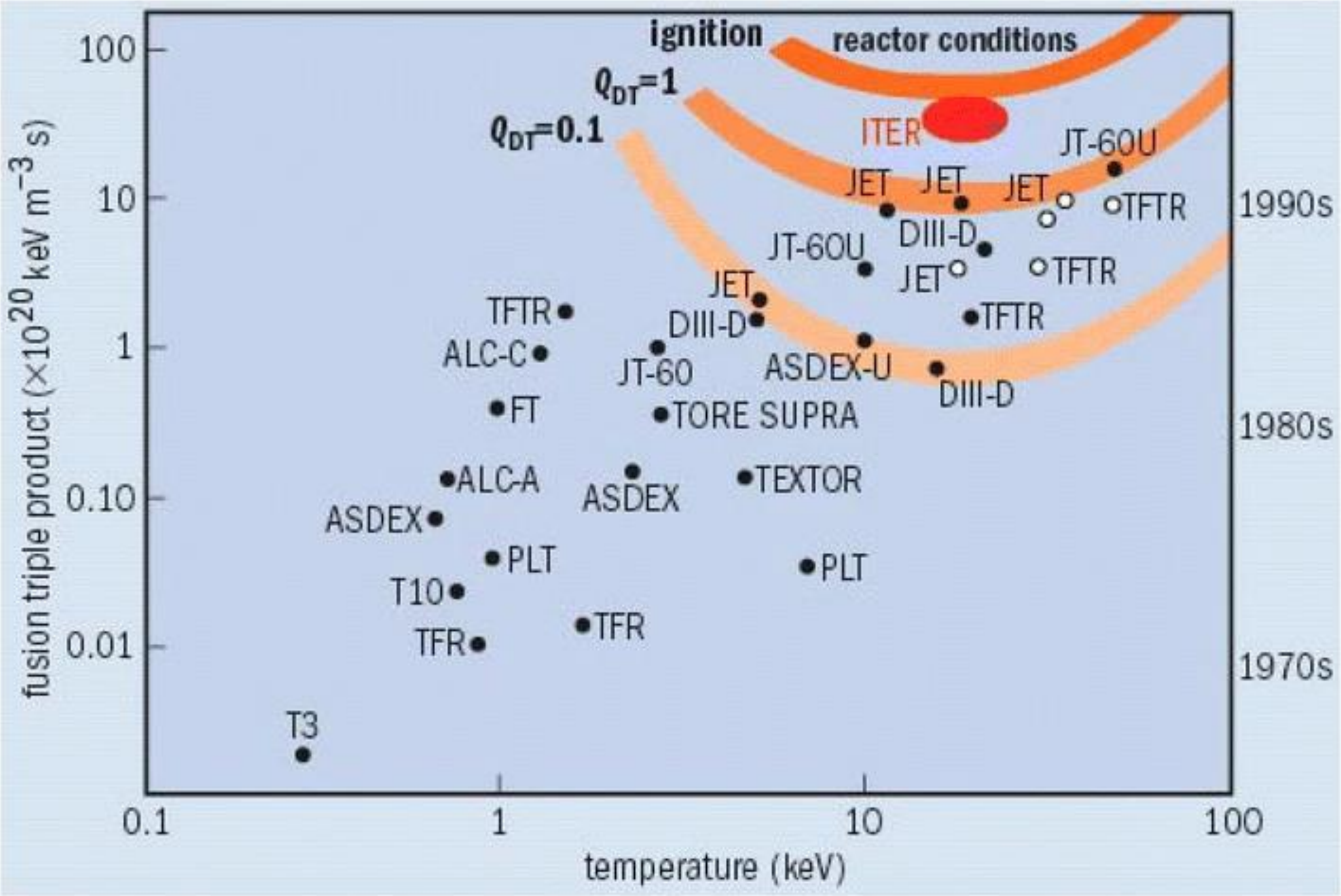
Per raggiungere i criteri necessari a un reattore, si può operare in due modi differenti, rimanendo a temperature *T* "relativamente basse":

☐ basse densità ***n*** (≈ 1020 m^{-3}) e tempi di confinamento τ_E alti (≈ 1 secondo) (**macchine a *confinamento magnetico***) (volume plasma ≈ 1000 m^3)

☐ alte densità ***n*** (≈ 1031 m^{-3}) e tempi di confinamento τ_E piccoli (≈ 10^{-10} secondi) (**macchine a *confinamento inerziale***) (volume plasma ≈ 10^{-12} m^3)



The triple product



(**10 keV = 1.16 10^8K**)