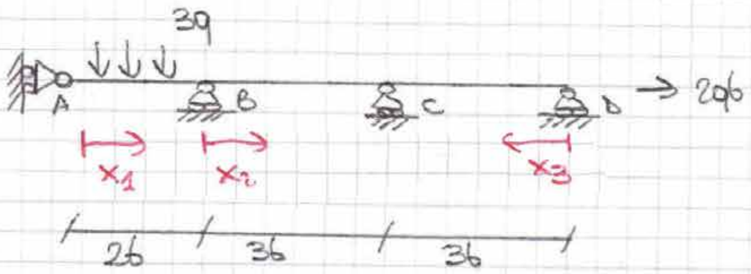
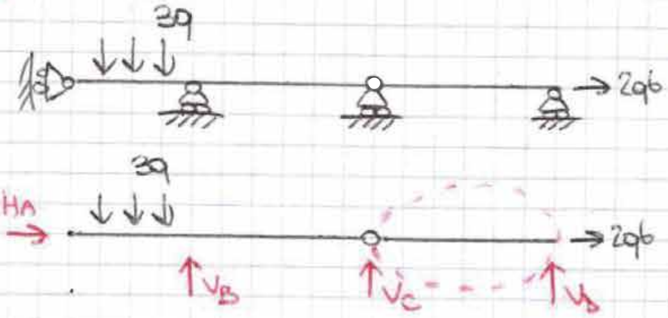
TRACCIA 2 - ESERCIZIO 1



P_0



$$\begin{cases} H_A + 2qb = 0 \Rightarrow H_A = -2b \\ 3q(2b) - V_B - V_C - \cancel{V_D} = 0 \\ 3q(2b)(b) - V_B 2b - V_C 5b - \cancel{V_D 8b} = 0 \end{cases}$$

eq. AUX

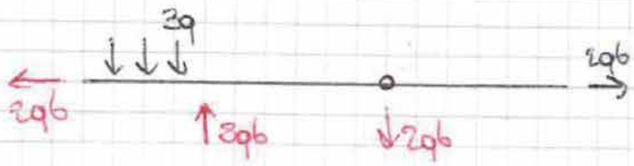
$$\begin{cases} V_D 3b = 0 \Rightarrow V_D = 0 \end{cases}$$

$$\begin{cases} H_A = -2b \\ V_B = -V_C + 6qb \quad [2] \\ 6qb^2 - 2b(-V_C + 6qb) - 5bV_C = 0 \quad [3] \end{cases}$$

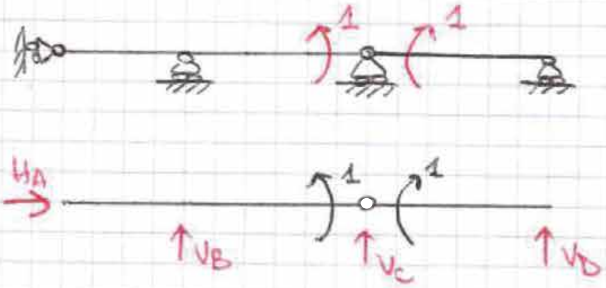
[3] $6qb^2 + 2V_C b - 12qb^2 - 5bV_C = 0$

$-3V_C b - 6qb^2 = 0 \Rightarrow V_C = -2qb$

[2] $V_B = +2qb + 6qb = 8qb$



P_1



$$\begin{cases} H_A = 0 \\ V_B + V_C + V_D = 0 \quad [2] \\ V_B(2b) + V_C(5b) + V_D(8b) = 0 \quad [3] \end{cases}$$

eq. AUX

$$\begin{cases} V_D(3b) - 1 = 0 \Rightarrow V_D = \frac{1}{3b} \end{cases}$$

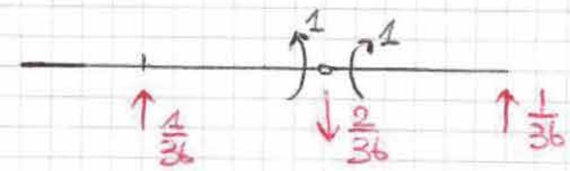
[2] $V_B + V_C + \frac{1}{3b} = 0 \quad V_B = -\frac{1}{3b} - V_C$

[3] $2b\left(-\frac{1}{3b} - V_C\right) + 5bV_C + \frac{8}{3} = 0$

$-\frac{2}{3} - 2bV_C + 5bV_C + \frac{8}{3} = 0$

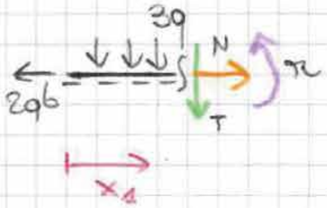
$3V_C b + 2 = 0 \Rightarrow V_C = -\frac{2}{3b}$

[2] $V_B = -\frac{1}{3b} + \frac{2}{3b} = \frac{1}{3b}$



Azioni interne

A → B $\quad 0 \le x_1 \le 2b$

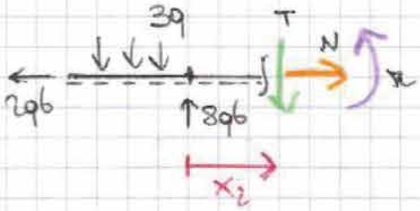


$N_{x_1} = 2qb$

$T + 3q x_1 = 0 \Rightarrow T = -3q x_1$

$\mathcal{M} + 3q x_1 \left(\frac{x_1}{2}\right) = 0 \Rightarrow \mathcal{M} = -\frac{3}{2} q x_1^2$

B → C $\quad 0 \le x_2 \le 3b$



$N_{x_2} = 2qb$

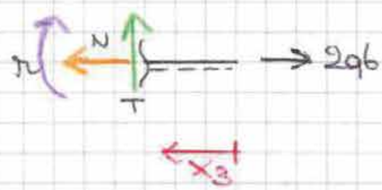
$T_{x_2} + 3q(2b) - 8qb = 0 \Rightarrow T = 2qb$

$\mathcal{M} - 8qb x_2 + 3q(2b)(b + x_2) = 0$

$\mathcal{M} - 8qb x_2 + 6qb^2 + 6qb x_2 = 0$

$\mathcal{M} = 2qb x_2 - 6qb^2$

D → C $\quad 0 \le x_3 \le 3b$

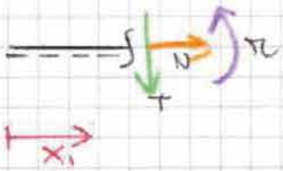


$N_{(x_3)} = 2qb$

$T_{(x_3)} = 0$

$\mathcal{M}_{(x_3)} = 0$

A → B $\quad 0 \le x_1 \le 2b$

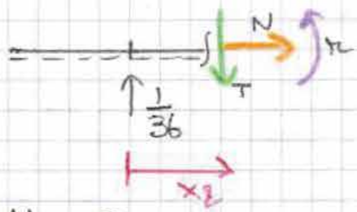


$N_{x_1} = 0$

$T_{x_1} = 0$

$\mathcal{M}_{x_1} = 0$

B → C $\quad 0 \le x_2 \le 3b$

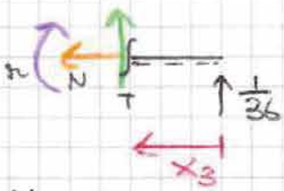


$N_{x_2} = 0$

$T_{x_2} = \frac{1}{3b}$

$\mathcal{M}_{x_2} - \frac{1}{3b} x_2 = 0 \Rightarrow \mathcal{M}_{x_2} = \frac{1}{3b} x_2$

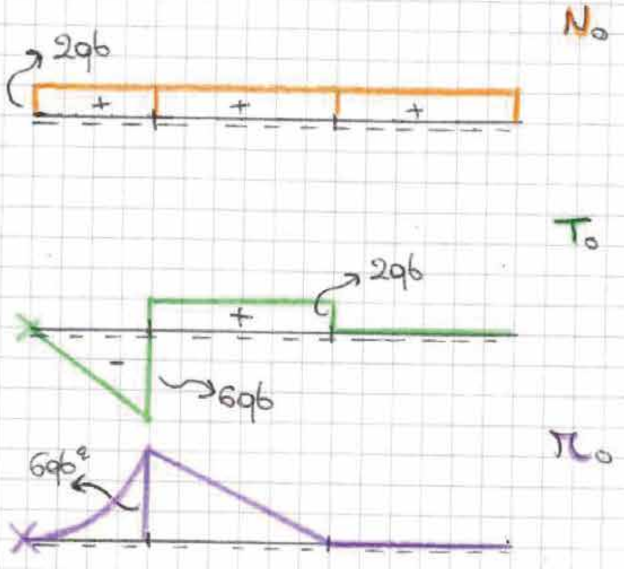
D → C $\quad 0 \le x_3 \le 3b$

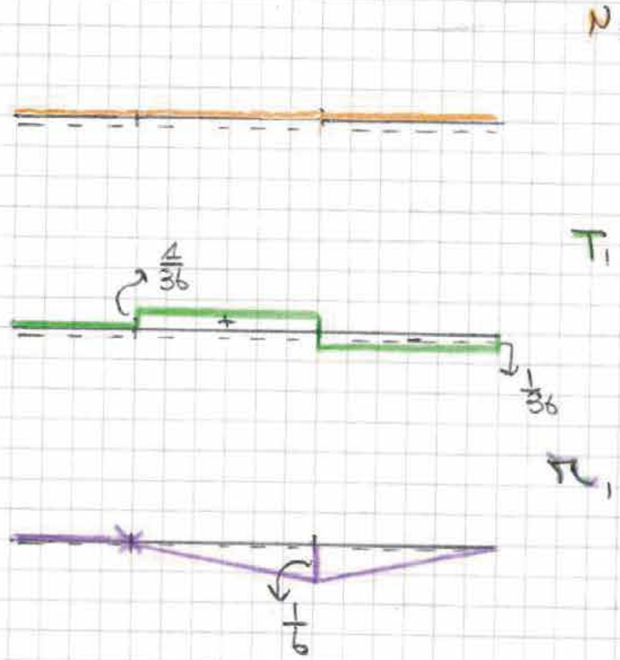


$N_{x_3} = 0$

$T_{x_3} = -\frac{1}{3b}$

$\mathcal{M}_{x_3} - \frac{1}{3b} x_3 = 0 \Rightarrow \mathcal{M}_{x_3} = \frac{1}{3b} x_3$





A→B	B→C	C→D	A→B	B→C	D→C
$M_0 = -\frac{3}{2}qx_1^2$	$M_0 = 2qbx_2 - 6qb^2$	$M_0 = 0$	$M_1 = 0$	$M_1 = \frac{1}{3b}x_2$	$M_1 = \frac{1}{3b}x_3$

PLV

$\mathcal{L}V_i = \mathcal{L}V_e$

$\mathcal{L}V_e = 1 \cdot \Delta\varphi_C = 1 \cdot 0 \Rightarrow \mathcal{L}V_e = 0$

$\mathcal{L}V_i = \int_s \cancel{N_1}\varepsilon_i + \int_s \cancel{T_1}\gamma_i + \int_s M_1 \chi_i$ $\qquad \chi_i = \frac{M_i}{EJ} \quad M_i = M_0 + XM_1$

$\mathcal{L}V_i = \int_s M_1 \left(\frac{M_0 + XM_1}{EJ}\right)$ $\qquad \chi_i = \frac{M_0 + XM_1}{EJ}$

TRATTO A→B $\Rightarrow M_1 = 0 \Rightarrow \mathcal{L}V_i^{(A\to B)} = 0$

$$\mathcal{L}V_i = \int_0^{3b} M_1 \left(\frac{M_0 + XM_1}{EJ}\right) dx_2 + \int_0^{3b} M_1 \left(\frac{\cancel{M_0} + XM_1}{EJ}\right) dx_3$$

$$= \int_0^{3b} \frac{1}{3b}x_2 \left[\frac{1}{EJ}\left(2qbx_2 - 6qb^2 + \frac{X}{3b}x_2\right)\right] dz_2 + \int_0^{3b} \frac{X}{EJ}\left(\frac{1}{3b}x_3\right)^2 dx_3$$

$$= \frac{1}{EJ}\int_0^{3b} \frac{1}{3b}x_2 \left(2qbx_2 - 6qb^2 + \frac{X}{3b}x_2\right) dz_2 + \frac{1}{EJ}\int_0^{3b} \frac{x_3^2}{9b^2} \cdot X \, dx_3$$

$$= \frac{1}{EJ}\int_0^{3b} \left(\frac{2}{3}qx_2^2 - 2qbx_2 + \frac{x_2^2}{9b^2} \cdot X\right) dz_2 + \frac{1}{EJ}\left[\frac{x_3^3}{27b^2} \cdot X\right]_0^{3b}$$

$$= \frac{1}{EJ}\left[\frac{2}{3}q\frac{x_2^3}{3} - 2qb\frac{x_2^2}{2} + \frac{x_2^3}{27b^2} \cdot X\right]_0^{3b} + \frac{1}{EJ}\left(\frac{27b^3}{27b^2} \cdot X\right)$$

$= \frac{1}{EJ}\left(\frac{2}{\cancel{3}}q(\cancel{27}^{3}b^3) - qb(9b^2) + \frac{1}{\cancel{27b^2}}(\cancel{27}b^3)\cdot X + Xb\right)$

$\delta_i = \frac{1}{EJ}\left(6qb^3 - 9qb^3 + 2Xb\right)$

$= \frac{1}{EJ}\left(-3qb^3 + 2Xb\right)$

$\delta V_i = 0 \Rightarrow \frac{1}{EJ}(-3qb^3 + 2Xb) = 0 \Rightarrow 2Xb = 3qb^3 \Rightarrow X = \frac{3}{2}qb^2$

REAZIONI VINCOLARI E AZIONI INTERNE DEL PROBLEMA INIZIALE

$H_A = H_{A0} + X H_{A1}$

$H_A = 2qb + \left(\frac{3}{2}qb^2\right)(0) = 2qb$

$V_B = V_{B0} + X V_{B1}$

$V_B = 8qb + \left(\frac{3}{2}qb^2\right)\left(\frac{1}{3b}\right) = 8qb + \frac{1}{2}qb = \frac{17}{2}qb$

$V_C = V_{C0} + X V_{C1}$

$V_C = -2qb + X\left(-2qb + \left(\frac{3}{2}qb^2\right)\left(-\frac{2}{3b}\right)\right) = -2qb - qb = -3qb$

$V_D = V_{D0} + X V_{D1}$

$V_D = 0 + \left(\frac{3}{2}qb^2\right)\left(\frac{1}{3b}\right) = \frac{1}{2}qb$

$A \rightarrow B$

$N = N_0 + XN_1$

$2qb + X(0) = 2qb$

$T = T_0 + XT_1$

$T = -\frac{3}{2}qx_1 + X(0) = -3qx_1$

$M = M_0 + XM_1$

$M = -\frac{3}{2}qx_1^2 + X(0) = -\frac{3}{2}qx_1^2$

$B \rightarrow C$

$N = N_0 + XN_1$

$N = 2qb$

$T = T_0 + XT_1$

$T = 2qb + \left(\frac{3}{2}qb^2\right)\left(\frac{1}{3b}\right)$

$= 2qb + \frac{1}{2}qb = \frac{5}{2}qb$

$M = M_0 + XM_1$

$M = 2qbx_2 - 6qb^2 + \left(\frac{3}{2}qb^2\right)\left(\frac{1}{3b}x_2\right)$

$M = 2qbx_2 - 6qb^2 + \frac{1}{2}qbx_2$

$M = \frac{5}{2}qbx_2 - 6qb^2$

$C \rightarrow D$

$N = N_0 + XN_1$

$N = 2qb$

$T = T_0 + XT_1$

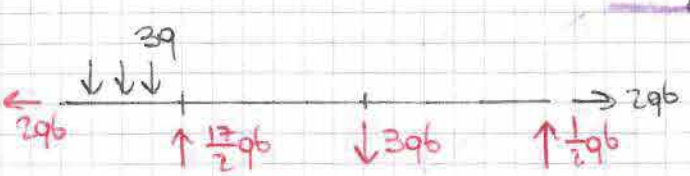
$T = 0 + \left(\frac{3}{2}qb^2\right)\left(-\frac{1}{3b}\right)$

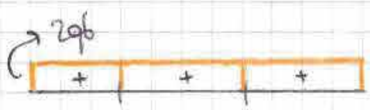
$T = -\frac{1}{2}qb$

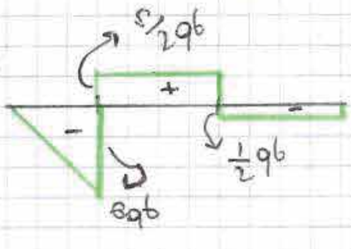
$M = M_0 + XM_1$

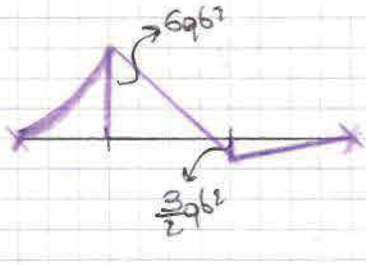
$M = 0 + \left(\frac{3}{2}qb^2\right)\left(\frac{1}{3b}x_3\right)$

$M = \frac{1}{2}qbx_3$

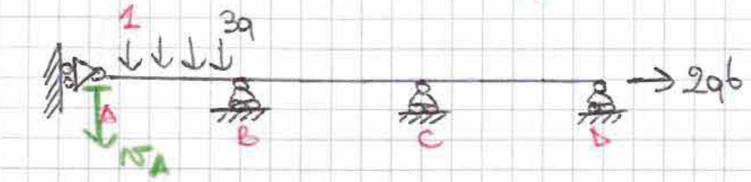




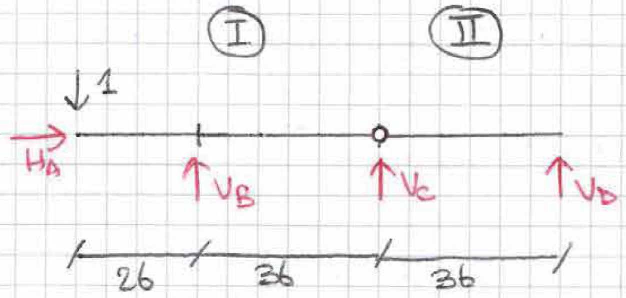




CALCOLO ABBASSAMENTO IN A: v_A ?



P_2

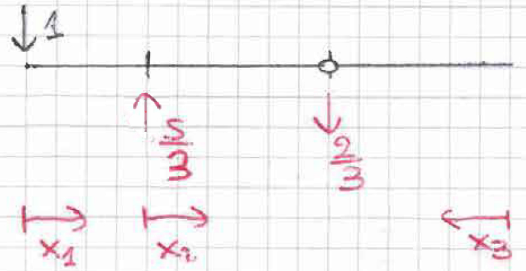


$$\begin{cases} H_A = 0 \\ V_B + V_C + \cancel{V_D} - 1 = 0 \Rightarrow V_B = 1 - V_C \quad [2] \\ V_B(2b) + V_C(5b) + \cancel{V_D(8b)} = 0 \quad [3] \end{cases}$$

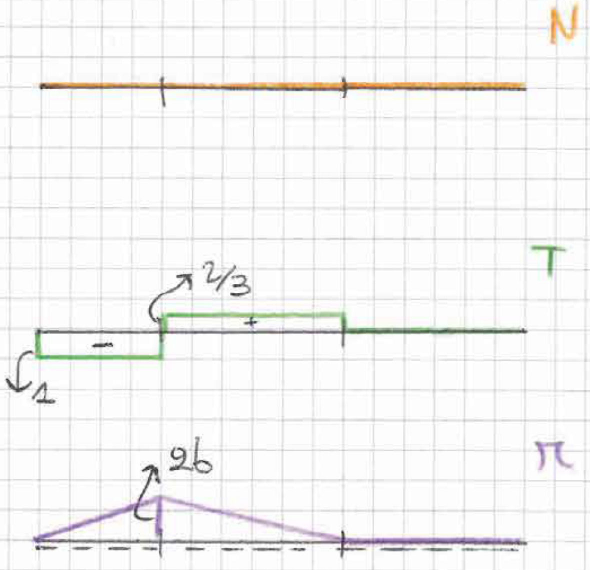
$$\begin{cases} V_D\, 3b = 0 \quad V_D = 0 \end{cases}$$

$[3] \quad 2b - 2bV_C + 5bV_C = 0 \quad V_C = -\frac{2}{3}$

$[2] \quad V_B = 1 + \frac{2}{3} = \frac{5}{3}$



A → B	B → C
$N = 0$	$N = 0$
$T_{x_1} = -1$	$T_{x_2} = \frac{2}{3}$
$\mathcal{M}_{x_1} = -x_1$	$\mathcal{M}_{x_2} = -2b - x_2 + \frac{5}{3}x_2 = -2b + \frac{2}{3}x_2$



D → C

$N_{x_3} = 0$

$T_{x_3} = 0$

$\mathcal{M}_{x_3} = 0$

$\mathcal{L}V_e = \mathcal{L}V_i$

$\mathcal{L}V_e = 1 \cdot v_A = v_A$

$$\mathcal{L}V_i = \int_S \underbrace{N_{2(x)}}_{\text{SI ANNULLA}} \varepsilon_x dx + \int_S \underbrace{T_{2(x)} \gamma_x dx}_{\text{TRASCURIAMO IL TAGLIO}} + \int_S \mathcal{M}_{2(x)} \chi_x dx$$

$$\mathcal{L}V_i = \int_S \mathcal{M}_{2(x)} X_x \, dx$$

$$\mathcal{L}V_i = \int_0^{2b} \mathcal{M}_{2x_1} X_{x_1} dx + \int_0^{3b} \mathcal{M}_{2(x_2)} + X_{x_2} dx$$

$$X_{(x_1)} = -\frac{3}{2} q x_1^2 \cdot \frac{1}{EI} \qquad X_{(x_2)} = \frac{1}{EI}\left(\frac{5}{2} qb\, x_2 - 6qb^2\right)$$

$$\mathcal{L}V_i = \int_0^{2b} -x_1\left(-\frac{3}{2}\frac{q x_1^2}{EI}\right) dx + \int_0^{3b} \frac{1}{EI}\left(-2b + \frac{2}{3}x_2\right)\left(\frac{5}{2}qb\,x_2 - 6qb^2\right) dx$$

$$\mathcal{L}V_i = \frac{1}{EI}\int_0^{2b} +\frac{3}{2} q x_1^3 \, dx + \frac{1}{EI}\int_0^{3b} \left(-5qb^2 x_2 + 12qb^3 + \frac{5}{3}qb\, x_2^2 - 4qb^2 x_2\right) dx$$

$$= \frac{1}{EI}\left[\frac{3}{2} q \frac{x_1^4}{4}\right]_0^{2b} + \frac{1}{EI}\left[-9qb^2 \frac{x_2^2}{2} + 12qb^3 x_2 + \frac{5}{3} qb \frac{x_2^3}{3}\right]_0^{3b}$$

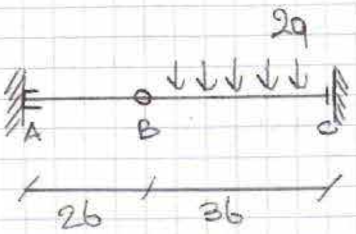
$$= \frac{1}{EI}\frac{3}{\cancel{8}} q\left(\overset{2}{\cancel{16}} b^4\right) + \frac{1}{EI}\left[-\frac{9}{2}qb^2 (9b^2) + 12qb^3(3b) + \frac{5}{\cancel{9}} qb\left(\overset{3}{\cancel{27}} b^3\right)\right] =$$

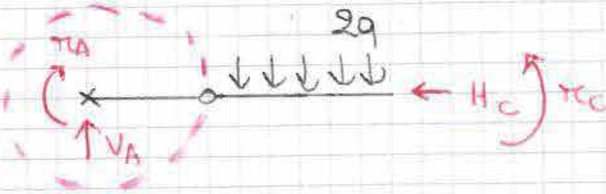
$$= \frac{6qb^4}{EI} + \frac{1}{EI}\left[-\frac{81}{2}qb^4 + 36qb^4 + 15qb^4\right]$$

$$\mathcal{L}V_i = \frac{1}{EI}\left(6qb^4 + 51qb^4 - \frac{81}{2}qb^4\right) = \frac{33}{2}\frac{qb^4}{EI}$$

$v_A = \dfrac{33}{2}\dfrac{qb^4}{EI}$ $\quad v_A > 0 \Rightarrow$ SPOSTAMENTO DIRETTO VERSO IL BASSO

TRACCIA 1 - ESERCIZIO 2



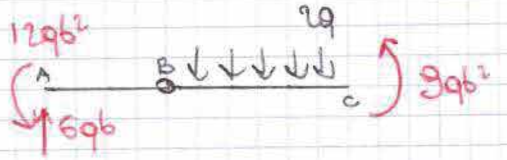


$$\begin{cases} R_x = 0 \\ R_y = 0 \\ \mathcal{M}_{z(A)} = 0 \end{cases} \quad \begin{cases} H_C = 0 \\ V_A - 2q(3b) = 0 \implies V_A = 6qb \\ \mathcal{M}_A - \mathcal{M}_C + \cancel{2}q(3b)\left(\frac{7}{\cancel{2}}b\right) = 0 \implies -12qb - \mathcal{M}_C + 21qb^2 = 0 \quad \mathcal{M}_C = 9qb^2 \end{cases}$$

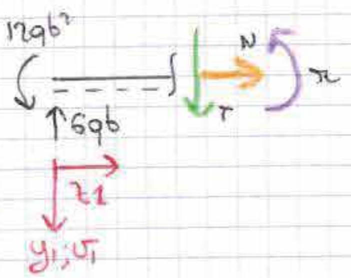
eq. aux

I

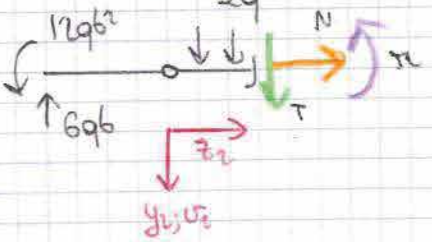
$$\left\{ \mathcal{M}_{z(B)} = 0 \right. \quad \left\{ \mathcal{M}_A + V_A(2b) = 0 \implies \mathcal{M}_A + 12qb^2 = 0 \quad \mathcal{M}_A = -12qb^2 \right.$$



A → B



B → C



$N_{z_1} = 0$

$T_{z_1} - 6qb = 0 \implies T_{z_1} = 6qb$

$\mathcal{M}_{z_1} + 12qb^2 - 6qb\,z_1 = 0 \implies \mathcal{M} = 6qb\,z_1 - 12qb^2$

$N_{z_2} = 0$

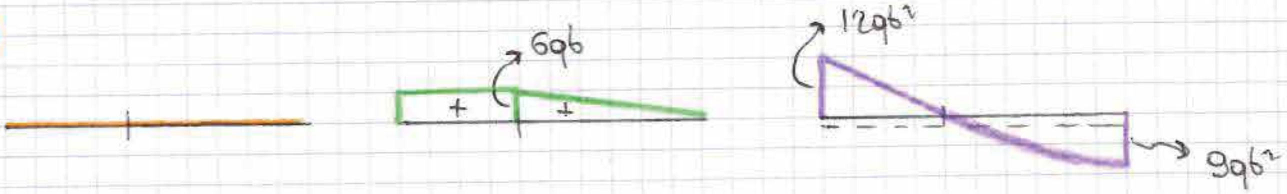
$T_{z_2} + 2q(z_2)\left(\frac{z_2}{\cancel{2}}\right) - 6qb = 0 \implies T = 6qb - 2qz_1$

$\mathcal{M}_{z_2} + 12qb^2 - 6qb(2b + z_2) + \cancel{2}q(z_2)\left(\frac{z_2}{\cancel{2}}\right) = 0$

$\mathcal{M}_{z_2} + \cancel{12qb^2} - \cancel{12qb^2} - 6qb\,z_2 + z_2^2 q = 0$

$\mathcal{M}_{z_2} = 6qb\,z_2 - z_2^2 q$

N



Eq. LINEA ELASTICA

A → B $\quad 0 \le z_1 \le 2b$

$$v''_{1(z_1)} = -\frac{M}{EI} \Rightarrow v''_{1(z_1)} = -\frac{6qbz_1}{EI} + \frac{12qb^2}{EI}$$

$$v'_{1(z_1)} = -\frac{\cancel{6}^{3}qb}{EI}\frac{z_1^2}{\cancel{2}} + \frac{12qb^2 z_1}{EI} + A_1$$

$$= -\frac{3qb}{EI}z_1^2 + \frac{12qb^2}{EI}z_1 + A_1$$

$$v_{1(z_1)} = -\frac{\cancel{3}qb}{EI}\frac{z_1^3}{\cancel{3}} + \frac{\cancel{12}^{6}qb^2}{EI}\frac{z_1^2}{\cancel{2}} + A_1 z_1 + A_2$$

$$= -\frac{qb}{EI}z_1^3 + \frac{6qb^2}{EI}z_1^2 + A_1 z_1 + A_2$$

B → C $\quad 0 \le z_2 \le 3b$

$$v''_{2(z_2)} = -\frac{M}{EI} \Rightarrow v''_{2(z_2)} = -\frac{6qbz_2}{EI} + \frac{qz_2^2}{EI}$$

$$v'_{2(z_2)} = -\frac{\cancel{6}^{3}qb}{EI}\frac{z_2^2}{\cancel{2}} + q\frac{z_2^3}{3} + B_1$$

$$= -\frac{3qb}{EI}z_2^2 + \frac{q\,z_2^3}{3EI} + B_1$$

$$v_{2(z_2)} = \frac{\cancel{3}qb}{EI}\frac{z_2^3}{\cancel{3}} + \frac{q}{3}\frac{z^4}{4} + B_1 z_2 + B_2$$

$$= \frac{qb}{EI}z_2^3 + \frac{q}{12}z^4 + B_1 z_2 + B_2$$

COSTANTI A_1; A_2; B_1; B_2 ⇒ CONDIZIONI AL CONTORNO

A) MANICOTTO — IMPEDISCE $\begin{cases} \text{SPOSTAMENTO} \updownarrow \Rightarrow v_{z_1}(A) = 0 \\ \text{ROTAZIONE} \Rightarrow v'_{z_1}(A) = 0 \end{cases}$

B) CERNIERA — IMPONE $\begin{cases} \text{UGUALE ABBASSAMENTO IN } B_1 \text{ E } B_2 \Rightarrow v_{(z_1)}(B) = v_{(z_2)}(B) \end{cases}$

C) PATTINO — IMPEDISCE $\{ \varphi = 0 \Rightarrow v'_{z_2}(C) = 0$

IN A

$v'_{z_1}(A) = 0 \Rightarrow z_1 = 0$

$A_1 = 0$

$v_{z_1}(A) = 0 \Rightarrow z_1 = 0$

$A_2 = 0$

IN B

$v_{z_1}(B) = v_{z_2}(B) \quad z_1 = 2b;\ z_2 = 0$

$$-\frac{qb}{EI}(8b^3) + \frac{6qb^2}{EI}(4b^2) = B_2$$

$$B_2 = -\frac{8qb^4}{EI} + \frac{24qb^4}{EI} = \frac{16}{EI}qb^4$$

IN C

$v'_{z_2}(C) = 0 \quad z_2 = 3b$

$$-\frac{3qb}{EI}(9b^2) + \frac{q(\cancel{27}^{9}b^3)}{\cancel{3}EI} + B_1 = 0$$

$$-\frac{27}{EI}qb^3 + \frac{9}{EI}qb^3 + B_1 = 0$$

$$B_1 = \frac{18}{EI}qb^3$$

$$v_{1(z_1)} = \frac{1}{EI}\left(-qbz_1^3 + 6qb^2z_1^2\right)$$

$$v'_{1(z_1)} = \frac{1}{EI}\left(-3qbz_1^2 + 12qb^2z_1\right)$$

$$v_{2(z_2)} = \frac{1}{EI}\left(-qbz_2^3 + \frac{z^4}{12}q + 18qb^3z_2 + 16qb^4\right)$$

$$v'_{2(z_2)} = \frac{1}{EI}\left(-3qbz_2^2 + \frac{1}{3}qz_2^3 + 18qb^3\right)$$

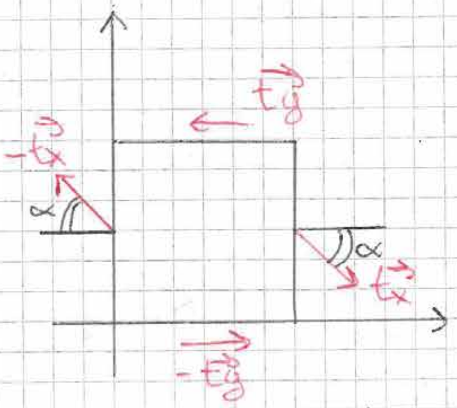
φ_B^1? $\quad v_C^2$?

$$v'_{(z_1)}(z_1 = 2b) \Rightarrow v'_{z_1} = -\frac{3qb(4b^2)}{EI} + \frac{12qb^2(2b)}{EI} \Rightarrow \varphi'_C = -\frac{12qb^3}{EI} + \frac{24qb^3}{EI} = +\frac{12qb^3}{EI}$$

$$v_{z_2}(z_2 = 3b) = \frac{1}{EI}\left(-qb(27b^3) + \frac{1}{\cancel{12}\,4}q\,(\cancel{81}^{27}b^4) + 18qb^3(3b) + 16qb^4\right)$$

$$v_C = \frac{1}{EI}\left(-qb^4\,27 + \frac{27}{4}qb^4 + 54qb^4 + 16qb^4\right) = \frac{qb^4}{EI}\left(\frac{-108 + 27 + 216 + 64}{4}\right) = \frac{199}{4}\frac{qb^4}{EI}$$

ESERCIZIO 3 - TRACCIA 2



$\alpha = -45°$

$|t_x| = 60\ MPa$

$\sigma_x ; \sigma_y$

$\sigma_x = |t_x| \cos\alpha = \cancel{60}^{30} \cdot \frac{\sqrt{2}}{\cancel{2}} = 30\sqrt{2}\ MPa$

$\sigma_x = 42,426\ MPa$

$\sigma_y = 0$

$\tau_{xy} = |t_x| \operatorname{sen}\alpha = \cancel{60}^{30}\sqrt{2}(-\sqrt{2}/\cancel{2})$

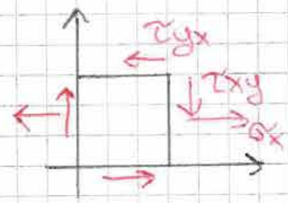
$= -30\sqrt{2}\ MPa = -42,426\ MPa$

$\tau_{xy} = \tau_{yx} = -30\sqrt{2}\ MPa$

$$\underline{\underline{\sigma}} = \begin{bmatrix} 30\sqrt{2} & -30\sqrt{2} & 0 \\ -30\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$P_x = (\sigma_x ; \tau_{xy})$ $[\tau_{xy} \circlearrowright]$

$P_y = (\sigma_y ; -\tau_{yx})$ $[\tau_{yx} \circlearrowleft]$



$P_x = (30\sqrt{2} ; 30\sqrt{2})$

$P_y = (0 ; -30\sqrt{2})$

$C = \left(\frac{\sigma_x + \sigma_y}{2} ; 0\right)$

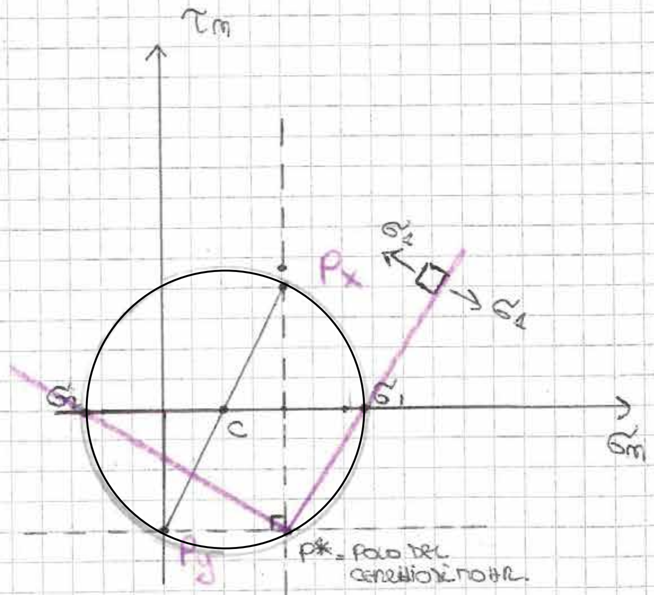
$C = \left(\frac{\cancel{30}^{15}\sqrt{2}}{\cancel{2}} ; 0\right) = (21,213 ; 0)$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{30\sqrt{2}}{2}\right)^2 + (30\sqrt{2})^2}$

$= \sqrt{(21,213)^2 + (42,426)^2} = 47,433$

$\sigma_1 = \sigma_c + R = 21,213 + 47,433 = 68,646\ MPa$

$\sigma_2 = \sigma_c - R = 21,213 - 47,433 = -26,22\ MPa$



$\tau_{max} = R \Rightarrow 47,433\ MPa$

$\operatorname{sen} 2\varphi = -\tau_{xy}$; $\cos 2\varphi = \sigma_x - \sigma_c$

$tg 2\varphi = \frac{\operatorname{sen} 2\varphi}{\cos 2\varphi} = \frac{-\tau_{xy}}{\sigma_x - \sigma_c} = \frac{-30\sqrt{2}}{30\sqrt{2} - 15\sqrt{2}} = \frac{-42,426}{21,213} = -2$

$2\varphi = \arctan g(-2) = -63,434° \Rightarrow \varphi = -31,717°$