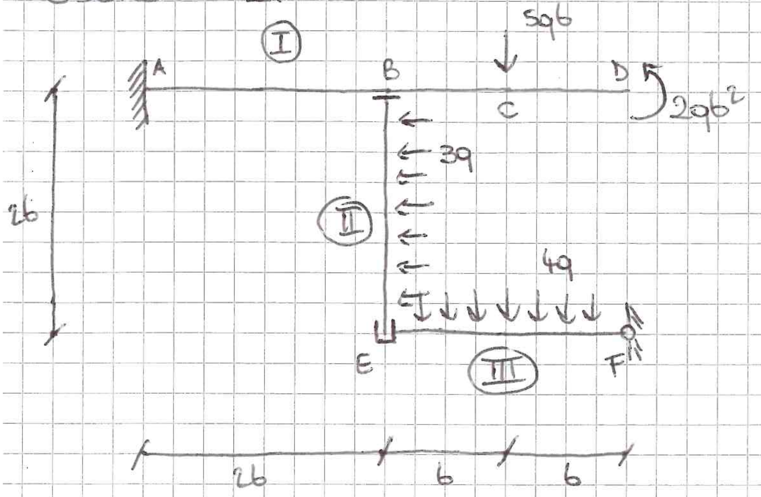
PROVA SCRITTA 24.01.2023

PARTE I - TESTO 2

Esercizio 1.

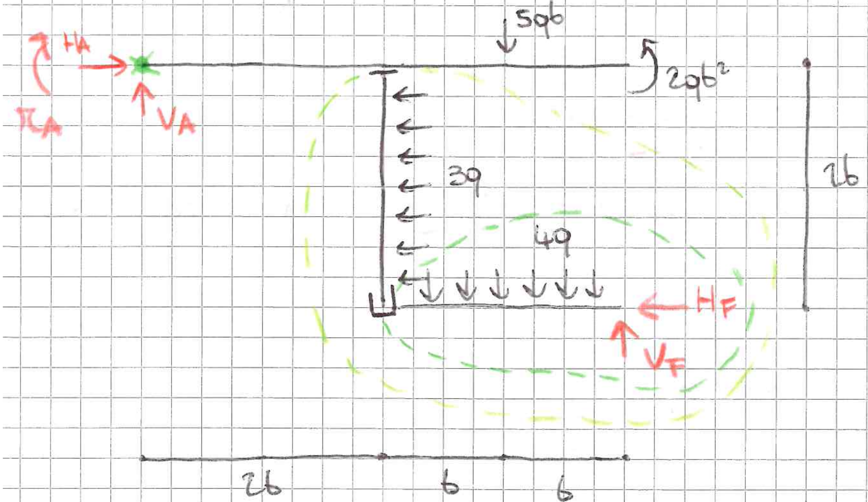


$GDL = 3(I) + 3(II) + 3(III) = 9$

$GDV = 3(A) + 2(B) + 2(E) + 2(F) = 9$

STRUTTURA ISOSTATICA

diagramma corpo libero

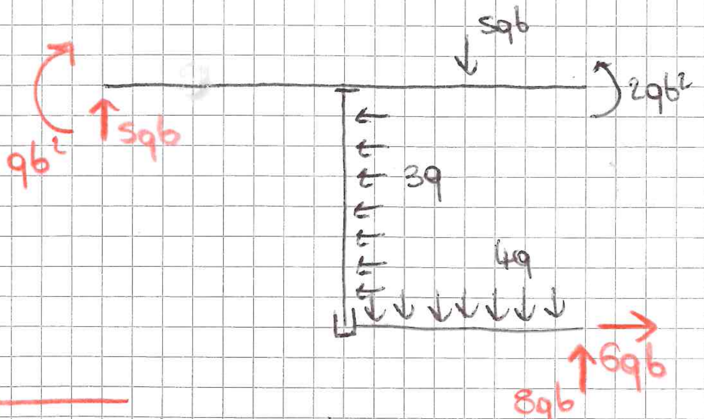


$$\begin{cases} R_x = 0 \\ R_y = 0 \\ M_{z(A)} = 0 \end{cases}$$

$$\begin{cases} H_A - H_F - 3q(2b) = 0 & [1] \\ V_A + V_F - 4q(2b) - 5qb = 0 & [2] \\ M_A + 5qb(3b) - 2qb^2 + 3q(2b)(b) + \\ + 4q(2b)(3b) + H_F 2b - V_F(4b) = 0 & [3] \end{cases}$$

eq. aux

$$\begin{cases} R_x^{(II+III)} = 0 \quad H_F + 3q(2b) = 0 & [4] \\ R_y^{(III)} = 0 \quad V_F - 4q(2b) = 0 & [5] \end{cases}$$



[4] $H_F = -6qb$

[5] $V_F = 8qb$

[1] $H_A + \cancel{6qb} - \cancel{6qb} = 0 \Rightarrow H_A = 0$

[2] $V_A + \cancel{8qb} - \cancel{8qb} - 5qb = 0 \Rightarrow V_A = 5qb$

[5] $M_A + 5qb(3b) - 2qb^2 + 6qb^2 + 24qb^2 +$

$+ 2b(-6qb) - 4b(8qb) = 0$

$M_A + 15qb^2 - 2qb^2 + 6qb^2 + 24qb^2 - 12qb^2 +$

$- 32qb^2 = 0$

$M_A + 45qb^2 - 46qb^2 = 0$

$M_A = +qb^2$

REAZIONI VINCOLARI

$H_A = 0$

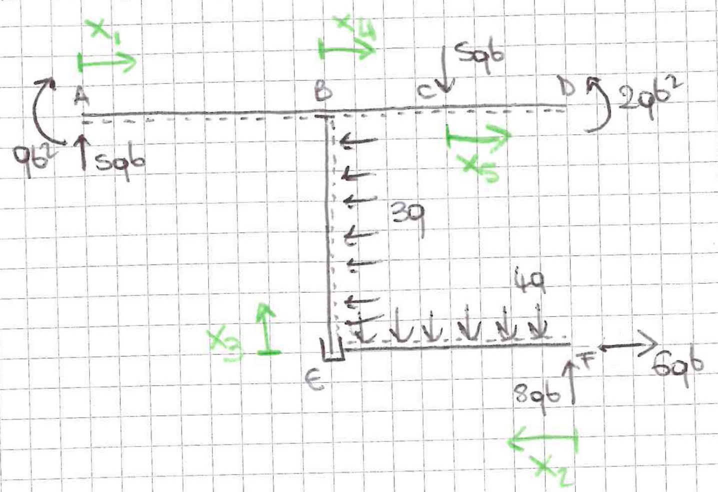
$H_F = -6qb$

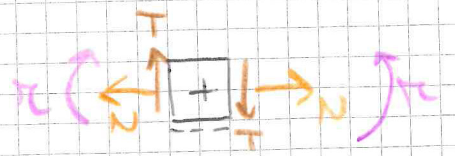
$V_F = 8qb$

$V_A = 5qb$

$M_A = qb^2$

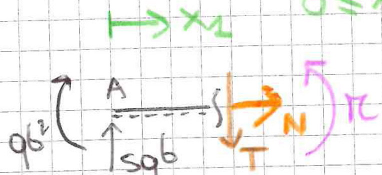
CALCOLO DELLE EQUAZIONI DELLE AZIONI INTERNE





A → B

$0 \le x_1 \le 2b$



$R_{\parallel} = 0$ $\quad N(x_1) = 0$

$R_{\perp} = 0$ $\quad T(x_1) - 5qb = 0 \Rightarrow T(x_1) = 5qb$

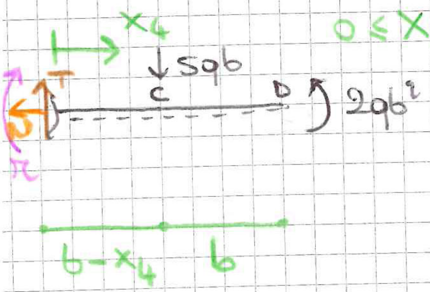
$\mathcal{M}_{z(x_1)} = 0$ $\quad \mathcal{M}_z(x_1) - qb^2 - 5qb(x_1) = 0$

$\mathcal{M}_{z(x_1)} = qb^2 + 5qb x_1$

• $\mathcal{M}_{z(x_1=0)} = qb^2$ • $\mathcal{M}_{z(x_1=2b)} = qb^2 + 10qb^2 = 11qb^2$

B → C

$0 \le x_4 \le b$



$R_{\parallel} = 0$ $\quad N(x_4) = 0$

$R_{\perp} = 0$ $\quad T(x_4) - 5qb = 0 \Rightarrow T(x_4) = 5qb$

$\mathcal{M}_{z(x_4)} = 0$ $\quad \mathcal{M}_{z(x_4)} - 2qb^2 + 5qb(b - x_4) = 0$

$\mathcal{M}_z(x_4) - 2qb^2 + 5qb^2 - 5qb x_4 = 0$

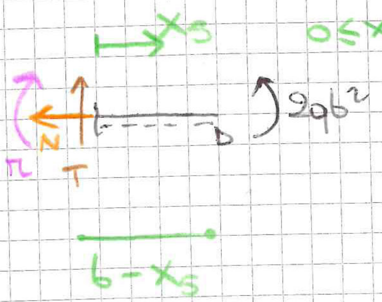
$\mathcal{M}_z(x_4) + 3qb^2 - 5qb x_4 = 0$

$\mathcal{M}_{z(x_4)} = 5qb x_4 - 3qb^2$

• $\mathcal{M}_{z(x_4=0)} = -3qb^2$ • $\mathcal{M}_{z(x_4=b)} = 2qb^2$

C → D

$0 \le x_5 \le b$



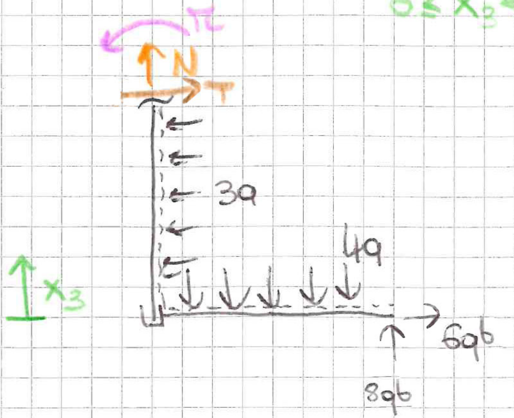
$R_{\parallel} = 0$ $\quad N(x_5) = 0$

$R_{\perp} = 0$ $\quad T(x_5) = 0$

$\mathcal{M}_{z(x_5)} = 0$ $\quad \mathcal{M}_{z(x_5)} - 2qb^2 = 0 \Rightarrow \mathcal{M}_{z(x_5)} = 2qb^2$

$E \rightarrow B$

$0 \le x_3 \le 2b$



$R_{\parallel} = 0 \qquad N(x_3) - 4q(2b) + 8qb = 0$

$R_{\perp} = 0 \qquad T(x_3) + 6qb - 3q(2b) = 0$

$M_z(x_3) = 0 \qquad M_z(x_3) - 3q(x_3)\left(\frac{x_3}{2}\right) - 4q(2b)(b) + 8qb(2b) + 6qb(x_3) = 0$

$N(x_3) = 0$

$T(x_3) = 0$

$M_z(x_3) - \frac{3}{2} q x_3^2 - 8qb^2 + 16qb^2 + 6qb x_3 = 0$

$M_z(x_3) - \frac{3}{2} q x_3^2 + 6qb x_3 + 8qb^2 = 0$

$M_z(x_3) = \frac{3}{2} q x_3^2 - 6qb x_3 - 8qb^2$

- $M_z(x_3=0) = -8qb^2$
- $M_z(x_3=2b) = -14qb^2$

$M'_z(x_3) = 3q x_3 - 6qb$

$M'_z(x_3) = 0 \qquad 3q x_3 - 6qb = 0 \qquad x_3 = 2b$

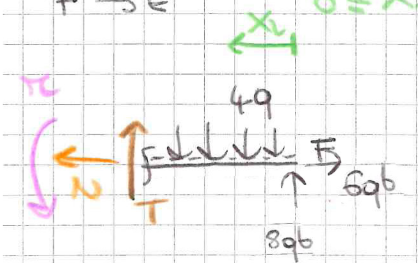
$M''_z(x_3) = 3q \Rightarrow$ CONCAVITÀ VERSO L'ALTO

LA PARABOLA RAGGIUNGE IL PUNTO DI MINIMO

PER $x_3 = 2b \Rightarrow M_z(x_3=2b) = -14qb^2$

$F \rightarrow E$

$0 \le x_2 \le 2b$



$R_{\parallel} = 0 \qquad N(x_2) - 6qb = 0 \Rightarrow N(x_2) = 6qb$

$R_{\perp} = 0 \qquad T(x_2) + 8qb - 4q x_2 = 0$

$M_z(x_2) = 0 \qquad M_z(x_2) + 8qb x_2 - 4q(x_2)\left(\frac{x_2}{2}\right) = 0$

$T(x_2) = 4q x_2 - 8qb$

- $T(x_2=0) = -8qb$
- $T(x_2=2b) = 0$

$M_z(x_2) = 2q x_2^2 - 8qb x_2$

- $M_z(x_2=0) = 0$
- $M_z(x_2=2b) = -8qb^2$

$M'_z(x_2) = 4q x_2 - 8qb$

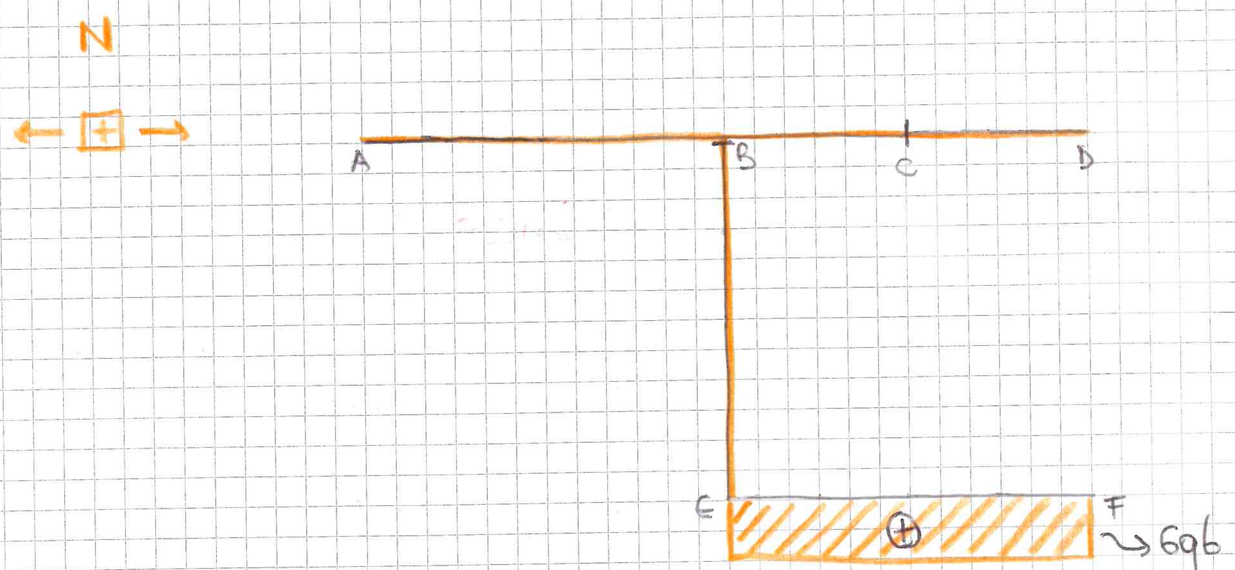
$M'_z(x_2) = 0 \Rightarrow 4q x_2 - 8qb = 0 \qquad x_2 = 2b$

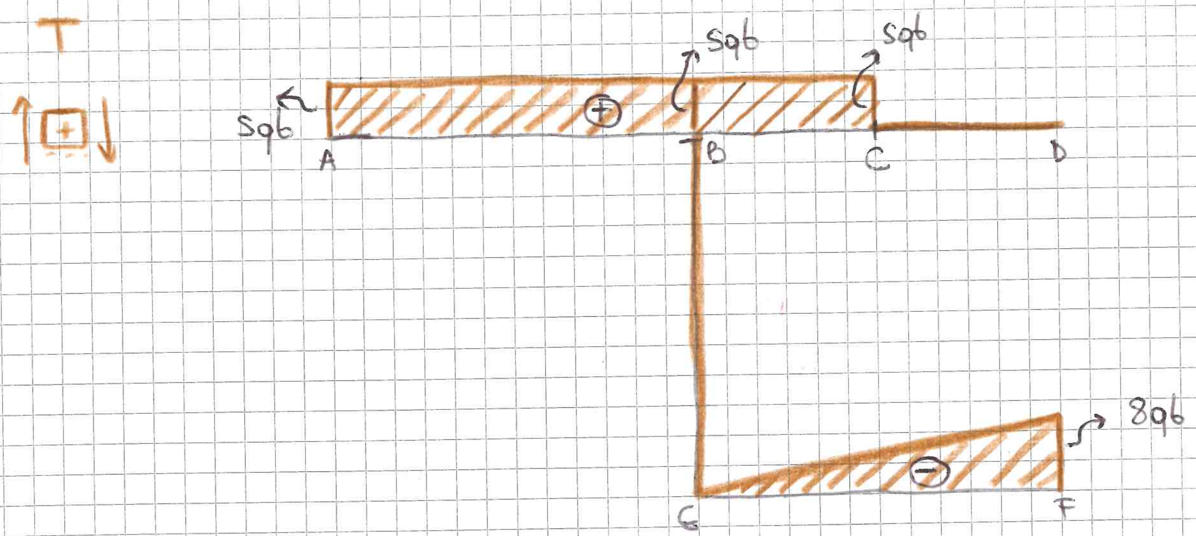
$M''_z(x_2) = 4q \Rightarrow$ CONCAVITÀ VERSO L'ALTO

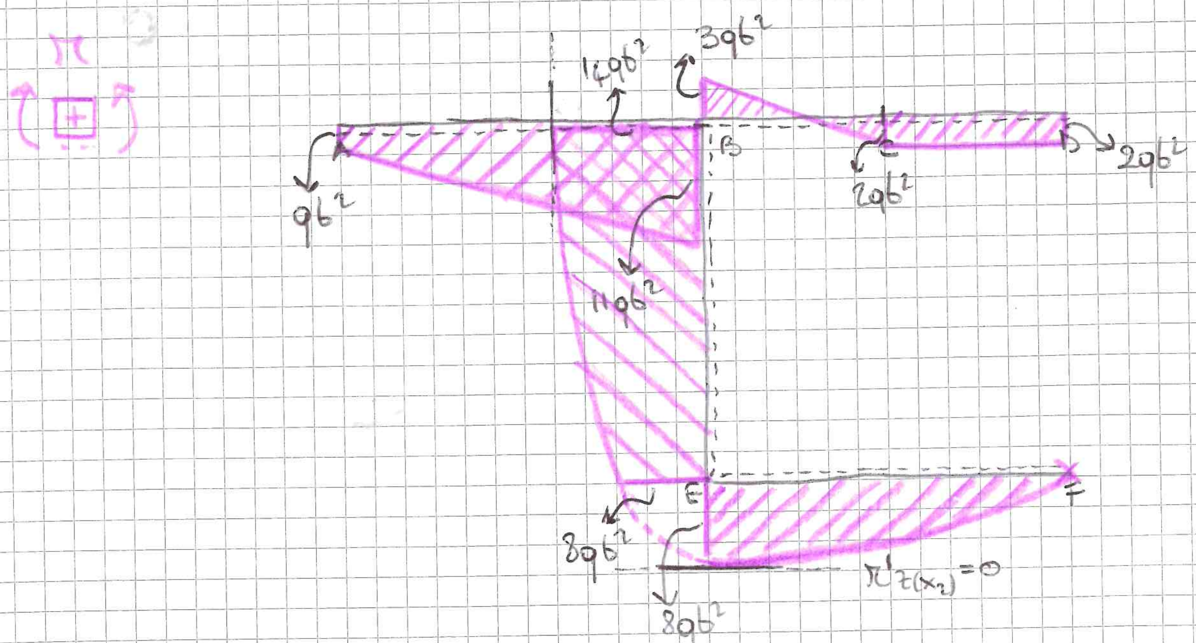
LA PARABOLA RAGGIUNGE IL PUNTO DI MINIMO

PER $x_2 = 2b \Rightarrow M_z(x_2=2b) = -8qb^2$

DIAGRAMMI DI AZIONE INTERNA

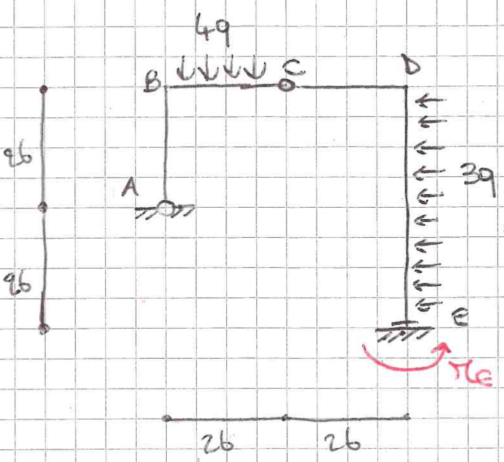


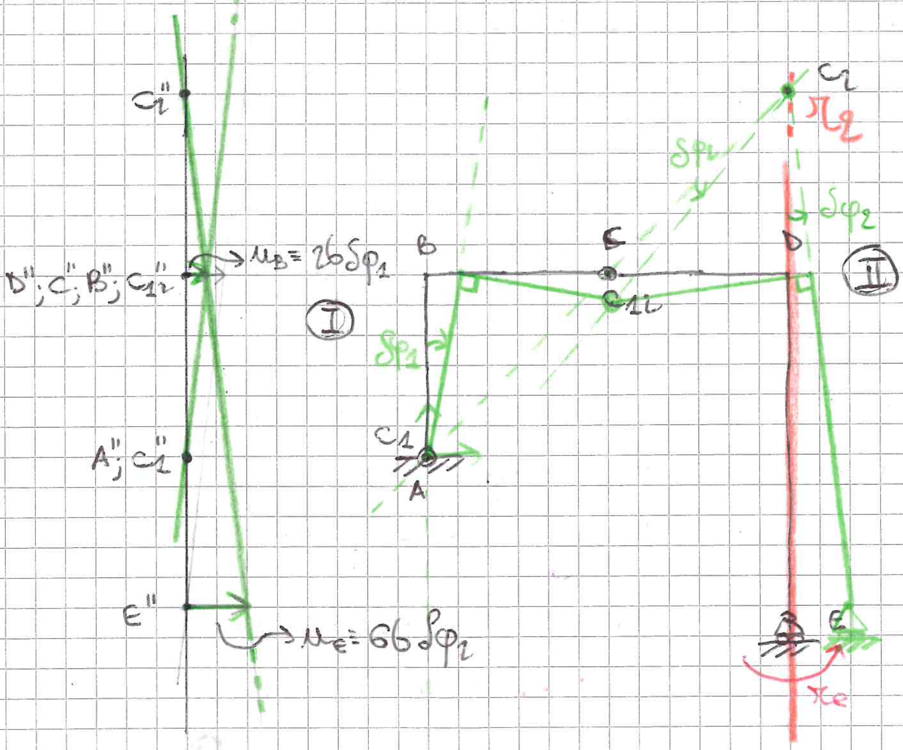




Esercizio 2.

• PARTE I





C.I.R. = C_I ; C_{II}

C.I.R.R = $C_{I,II}$

Condizioni cinematiche

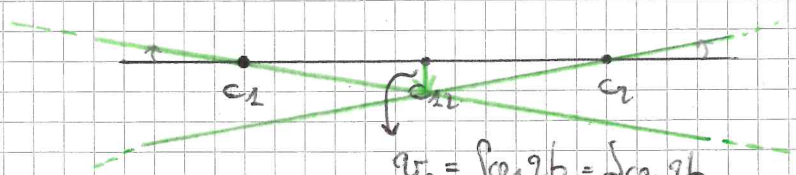
$$\begin{cases} C_1 \leftrightarrow C_{12} \leftrightarrow C_2 \\ C_2 \in r_2 \end{cases}$$

$C_1 = A \quad A(0;0)$

$C_{12} = C \quad C(2b;2b)$

$C_2 = (4b;4b)$

$\delta\varphi_1 = \delta\varphi_2$



$v_C = \delta\varphi_1 2b = \delta\varphi_2 2b$

Principio dei lavori virtuali

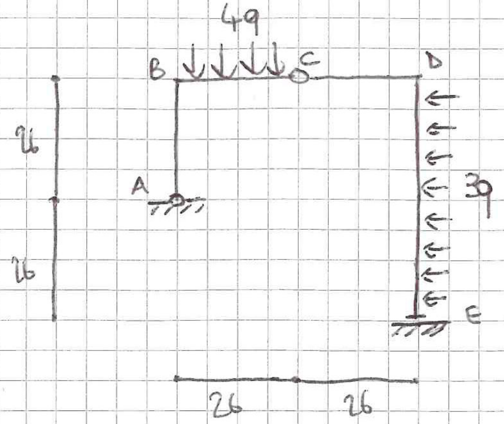
$M_E \delta\varphi_2 - 3q(4b)(4b\,\delta\varphi_2) + 4q(2b)(b\,\delta\varphi_1) = 0$

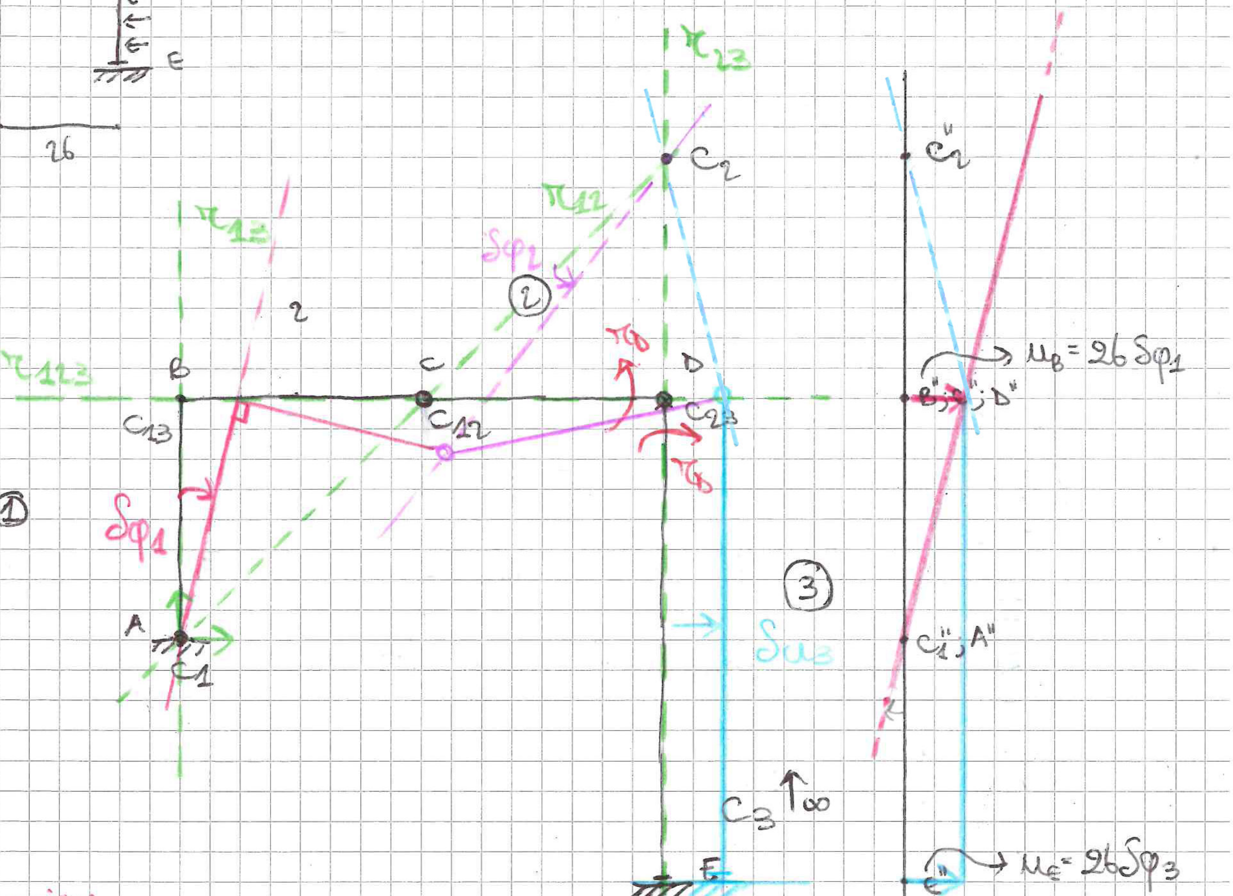
$M_E - 48qb^2 + 8qb^2 = 0$

$M_E = 40qb^2$

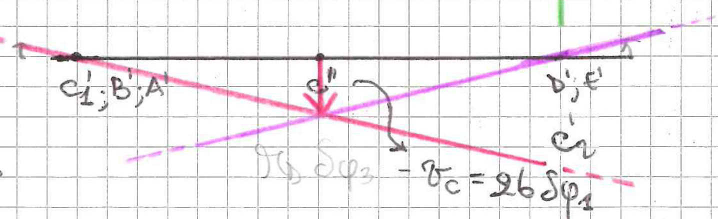
ESERCIZIO 2.

• PARTE II





CONDIZIONI CINEMATICHE



$$\begin{cases} C_1 \leftrightarrow C_{12} \leftrightarrow C_2 \\ C_2 \leftrightarrow C_{23} \leftrightarrow C_3 \end{cases}$$

$$\begin{cases} C_1 \leftrightarrow C_{13} \leftrightarrow C_3 \\ C_{12} \leftrightarrow C_{23} \leftrightarrow C_{31} \end{cases}$$

$u_D = 2b\,\delta\varphi_2 \; ; \; v_B = 0$

PRINCIPIO DEI LAVORI VIRTUALI

$r_D\,\cancel{\delta\varphi_2} - 3q(4b)(2b\,\cancel{\delta\varphi_3}) + 4q(2b)(b\,\cancel{\delta\varphi_1}) = 0$

$r_D - 24qb^2 + 8qb^2 = 0$

$r_D = 16qb^2$

$C_1 = A \qquad A(0;0)$

$C_{12} = C \qquad C(2b;2b)$

$C_2 \in r_{12} \qquad C_2 \in r_{23}$

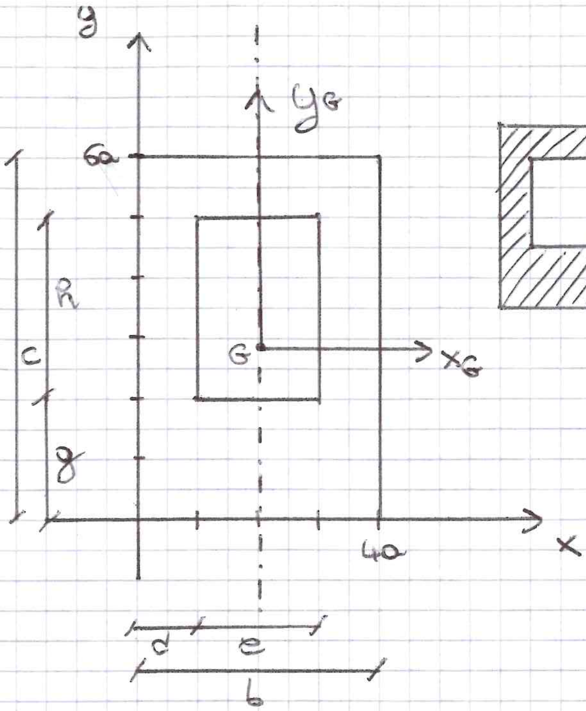
$C_3(\infty;\infty)$

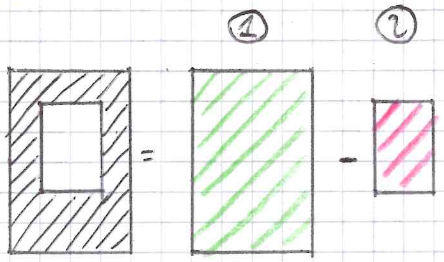
$C_{23} = D \qquad D(4b;2b)$

$C_2 = (4b;4b) \qquad C_{13}(0;2b)$

$C_{13} \in r_{13} \; ; \; C_{13} \in r_{123}$

Esercizio 3





$b = 4a$ $\qquad A = 18a^2$

$c = 6a$ $\qquad S_x = 51a^3$

$d = 1a$ $\qquad S_y = 36a^3$

$e = 2a$ $\qquad G\left(2a; \frac{17}{6}a\right)$

$f = 0$ $\qquad J_{x_G} = \frac{131}{2}a^4 \cong 65{,}5\,a^4$

$g = 2a$

$h = 3a$ $\qquad J_{y_G} = 30\,a^4$

$A_1 = 4a \cdot 6a = 24a^2$

$A_2 = 2a \cdot 3a = 6a^2$

$A = A_1 + A_2 = 18a^2$

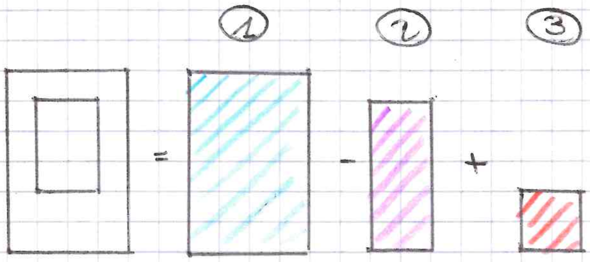
$G_1 = (2a; 3a)$

$G_2 = (2a; 7/2\,a)$

$$S_x = S_x^1 - S_x^2 = A_1 y_{G_1} - A_2 y_{G_2} = (24a^2)(3a) - (\cancel{6}^{3}a^2)\left(\frac{7}{\cancel{2}_1}a\right) = 72a^3 - 21a^3 = 51a^3$$

$$S_y = S_y^1 - S_y^2 = A_1 x_{G_1} - A_2 x_{G_2} = (24a^2)(2a) - (6a^2)(2a) = 48a^3 - 12a^3 = 36a^3$$

$$x_G = \frac{S_y}{A} = \frac{36a^3}{18a^2} = 2a \qquad y_G = \frac{S_x}{A} = \frac{51a^3}{18a^2} = \frac{17}{6}a \cong 2{,}83a$$



$$J_x = J_{x_1} - J_{x_2} + J_{x_3} = \frac{b_1 h_1^3}{3} - \frac{b_2 h_2^3}{3} + \frac{b_3 h_3^3}{3} =$$

$$= \frac{4a(6a)^3}{3} - \frac{2a(5a)^3}{3} + \frac{2a(2a)^3}{3} =$$

$$= \frac{864}{3}a^4 - \frac{250}{3}a^4 + \frac{16}{3}a^4 = \frac{\cancel{630}}{\cancel{3}}a^4 = 210a^4$$

$$J_{x_G} = J_x - A y_G^2 = 210a^4 - 18a^2\left(\frac{17}{6}a\right)^2 =$$

$$= 210a^4 - \cancel{18}a^2\left(\frac{289}{\cancel{36}_2}a^2\right) = \frac{131}{2}a^4$$

$$J_{y_G} = J_{y_{G_1}} - J_{y_{G_2}} = \frac{h_1 b_1^3}{12} - \frac{h_2 b_2^3}{12} =$$

$$= \frac{6a \cdot (4a)^3}{12} - \frac{3a(2a)^3}{12} = \frac{384}{12}a^4 - \frac{24a^4}{12} = \frac{360}{12}a^4 = 30a^4$$

$J_{x_G y_G} = 0 \Rightarrow y_G$ ASSE DI SIMMETRIA

$$tg\,2\theta = \frac{-2\cancel{J_{x_G y_G}}}{J_{x_G} - J_{y_G}} = 0 \qquad tg\,2\theta = 0 \Longrightarrow J_{x_G} > J_{y_G} \Longrightarrow \theta = 0$$

$$J_\xi = J_{max} = J_{x_G} = 65{,}5\,a^4$$

$$J_\eta = J_{min} = J_{y_G} = 30\,a^4$$