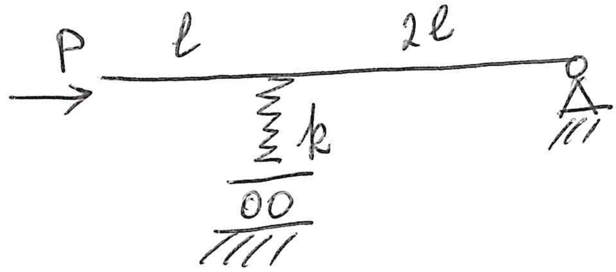
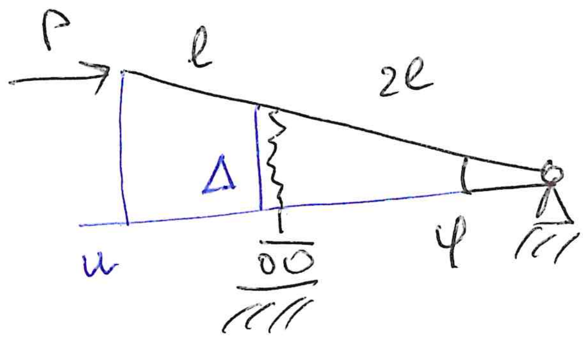
Problem 1



1. P_c - ?
2. Postbuckling
   P - φ - curve
   + stable - unstable

Total potential energy

1. $\Pi = U - V$
2. $\dfrac{d\Pi}{d\varphi} = 0 \rightarrow$ equilibrium condition
3. $\dfrac{d^2\Pi}{d\varphi^2} \gtrless 0$? - stable / unstable



$$U = \frac{1}{2} k \Delta^2$$

$$\Delta = 2l \sin\varphi$$

$$V = -Pu \quad , \quad u = 3l - 3l\cos\varphi$$

$$\Pi = \frac{1}{2} k \Delta^2 - P(3l - 3l\cos\varphi)$$

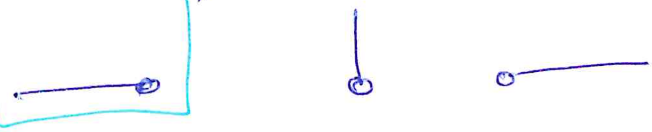
$$\Pi = \frac{1}{2} k \, 4l^2 \sin^2\varphi - 3lP(1 - \cos\varphi)$$

$$\frac{d\Pi}{d\varphi} = \frac{d}{d\varphi}\left[2kl^2\sin^2\varphi - 3lP + 3lP\cos\varphi\right]$$

$$= 4kl^2\sin\varphi\cos\varphi - 3lP\sin\varphi = 0$$

$$\sin\varphi\left(4kl^2\cos\varphi - 3lP\right) = 0$$

1. $\sin\varphi = 0 \Rightarrow \boxed{\varphi = 0}, \quad \varphi = \pi, \ 2\pi$

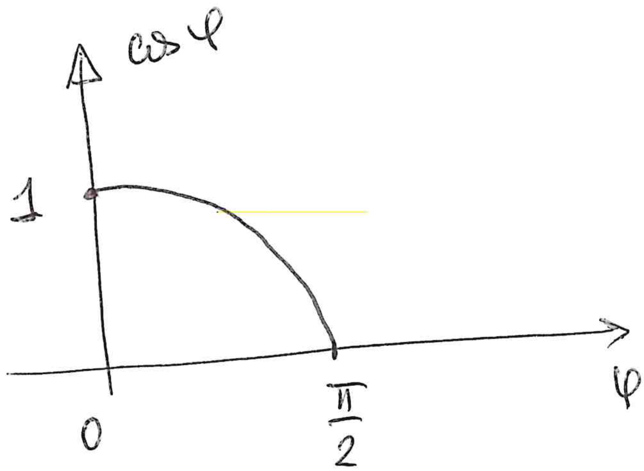


trivial solution

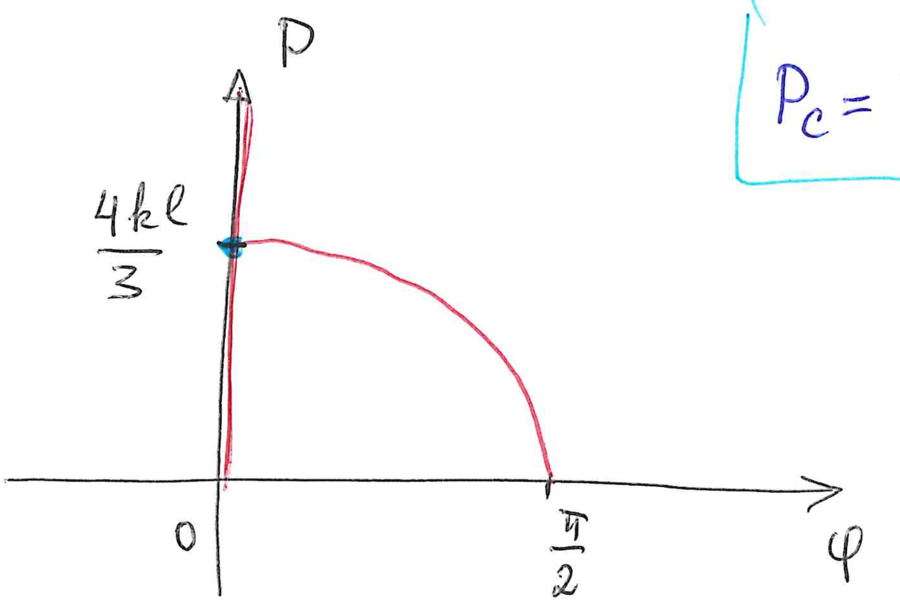
2. $4kl^2\cos\varphi - 3lP = 0$

$$\boxed{P = \frac{4kl}{3}\cos\varphi}$$

nontrivial solution



Both solutions are



$$\boxed{P_c = \frac{4kl}{3}}$$

$$\frac{d^2\Pi}{d\varphi^2} = \frac{d}{d\varphi}\left[4kl^2 \sin\varphi \cos\varphi - 3lP\sin\varphi\right]$$

$$= \frac{d}{d\varphi}\left[2kl^2 \sin 2\varphi - 3lP\sin\varphi\right]$$

$$= 4kl^2 \cos 2\varphi - 3lP\cos\varphi$$

1. Trivial solution: $\left.\frac{d^2\Pi}{d\varphi^2}\right|_{\varphi=0} = 4kl^2 - 3lP$

$$\Rightarrow \frac{d^2\Pi}{d\varphi^2} \begin{cases} >0, & P < 4kl/3 \equiv P_c \\ <0, & P > P_c \end{cases}$$

2. Nontrivial solution

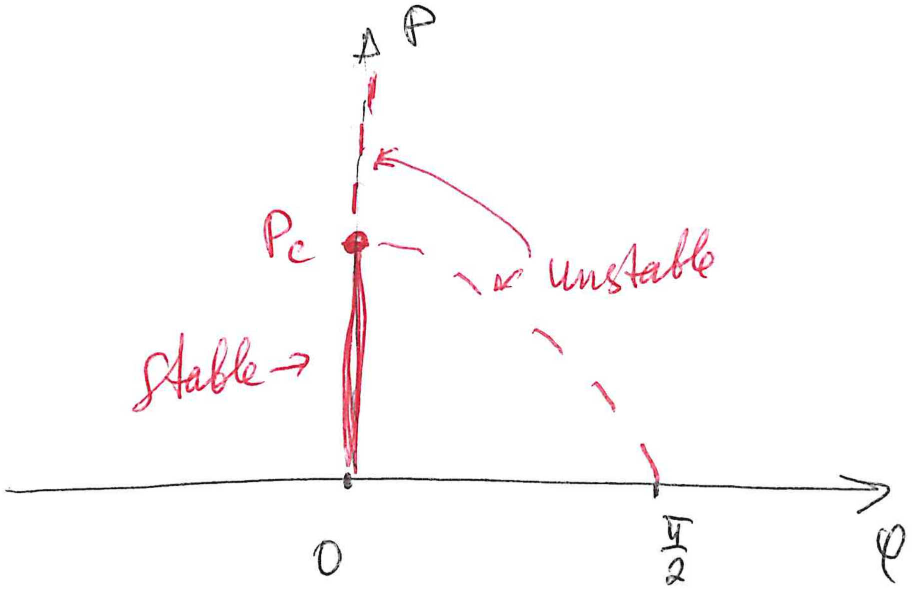
$$\frac{d^2\Pi}{d\varphi^2} = 4kl^2\cos 2\varphi - 3Pl\cos\varphi$$

$$= 4kl^2\cos 2\varphi - 3l\,\frac{4kl}{3}\cos^2\varphi$$

$$= 4kl^2\left[\cos 2\varphi - \cos^2\varphi\right]$$

$$= 4kl^2\left[\cos^2\varphi - \sin^2\varphi - \cos^2\varphi\right]$$

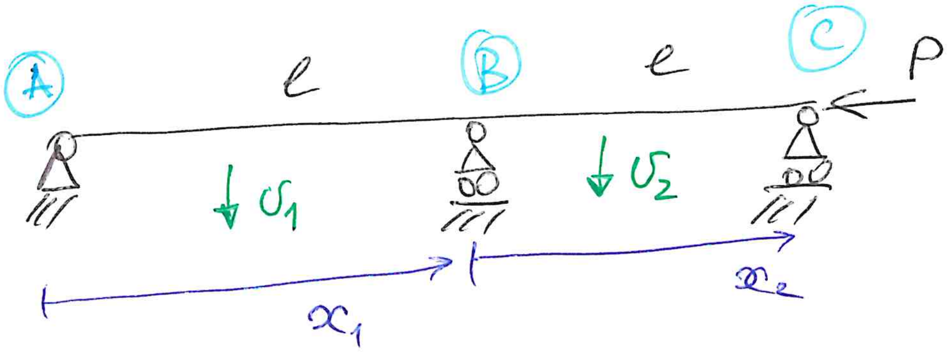
$$= -4kl^2\sin^2\varphi < 0$$



Problem 2

P_c - ?

EI



BCs:

A: $v_1(0)=0$, $v_1''(0)=0$

C: $v_2(l)=0$, $v_2''(l)=0$

B: $v_1(l)=0$, $v_2(0)=0$

$v_1'(l) = v_2'(0)$

$v_1''(l) = v_2''(0)$

$$\underline{EI\,v'''' + P v'' = 0}$$

$$v(x) = C_1 \cos\beta x + C_2 \sin\beta x + C_3 x + C_4$$

$$\beta = \sqrt{\frac{P}{EI}}$$

$$v_1(x) = C_1 \cos\beta x_1 + C_2 \sin\beta x_1 + C_3 x_1 + C_4$$

$$v_2(x) = C_5 \cos\beta x_2 + C_6 \sin\beta x_2 + C_7 x_2 + C_8$$

$$v_1(0) = C_1 + C_4 = 0 \quad \Rightarrow \quad C_4 = 0$$

$$v_1''(0) = -\beta^2 C_1 = 0 \quad \Rightarrow \quad C_1 = 0$$

$$v_1(x) = C_2 \sin \beta x_1 + C_3 x_1$$

$$v_1(l) = C_2 \sin \beta l + C_3 l = 0$$

$$v_2(x) = C_5 \cos \beta x_2 + C_6 \sin \beta x_2 + C_7 x_2 + C_8$$

$$v_2(0) = C_5 + C_8 = 0$$

$$v_2(l) = C_5 \cos \beta l + C_6 \sin \beta l + C_7 l + C_8 = 0$$

$$v_2''(l) = -C_5 \beta^2 \cos \beta l - C_6 \beta^2 \sin \beta l = 0$$

$$v_1'(l) = + C_2 \beta \cos \beta l + C_3 =$$

$$= -C_5 \beta \cdot 0 + C_6 \beta \cdot 1 + C_7 = v_2'(0)$$

$$C_2 \beta \cos \beta l + C_3 = C_6 \beta + C_7$$

$$v_1''(\ell) = -C_2\beta^2 \sin\beta\ell = -C_5\beta^2 = v_2''(0)$$

$$\begin{cases} C_2 \sin\beta\ell + C_3\ell = 0 & \longrightarrow C_3 = -C_2 \dfrac{\sin\beta\ell}{\ell} \\ C_5 + C_8 = 0 & \longrightarrow C_8 = -C_5 \\ C_5\cos\beta\ell + C_6 \sin\beta\ell + C_7\ell + C_8 = 0 \\ +C_5\cos\beta\ell + C_6 \sin\beta\ell = 0 \\ C_2\beta\cos\beta\ell + C_3 = C_6\beta + C_7 \\ C_2 \sin\beta\ell = C_5 \end{cases}$$

$$\begin{cases} C_5\cos\beta\ell + C_6\sin\beta\ell + C_7\ell - C_5 = 0 \\ C_5\cos\beta\ell + C_6\sin\beta\ell = 0 \\ C_2\left[\beta\cos\beta\ell - \dfrac{\sin\beta\ell}{\ell}\right] = C_6\beta + C_7 \\ C_2\sin\beta\ell - C_5 = 0 \end{cases}$$

$$
\begin{array}{cccc}
c_2 & c_5 & c_6 & c_7
\end{array}
$$

$$
\begin{vmatrix}
0 & \cos\beta l - 1 & \sin\beta l & l \\
0 & \cos\beta l & \sin\beta l & 0 \\
\beta\cos\beta l - \dfrac{\sin\beta l}{l} & 0 & -\beta & -1 \\
\sin\beta l & -1 & 0 & 0
\end{vmatrix} = 0
$$

$$
= \left(\beta\cos\beta l - \frac{\sin\beta l}{l}\right)
\begin{vmatrix}
\cos\beta l - 1 & \sin\beta l & l \\
\cos\beta l & \sin\beta l & 0 \\
-1 & 0 & 0
\end{vmatrix}
$$

$$
- \sin\beta l
\begin{vmatrix}
\cos\beta l - 1 & \sin\beta l & l \\
\cos\beta l & \sin\beta l & 0 \\
0 & -\beta & -1
\end{vmatrix} =
$$

$$= \left(\beta\cos\beta l - \frac{\sin\beta l}{l}\right)\left(+\, l\sin\beta l\right)$$

$$- \sin\beta l \left[-l\beta\cos\beta l - 1(\cos\beta l - 1)\sin\beta l - \cos\beta l\sin\beta l\right]$$

$$= l\sin\beta l\left(\beta\cos\beta l - \frac{\sin\beta l}{l}\right)$$

$$- \sin\beta l\left[-l\beta\cos\beta l - \cancel{\cos\beta l\sin\beta l} + \sin\beta l + \cancel{\cos\beta l\sin\beta l}\right]$$

$$= l\sin\beta l\left(\beta\cos\beta l - \frac{\sin\beta l}{l}\right)$$

$$- \sin\beta l\left(-l\beta\cos\beta l + \sin\beta l\right) =$$

$$= 2\sin\beta l\left(\beta l\cos\beta l - \sin\beta l\right) = 0$$

$$2\sin\beta l\,(\beta l\cos\beta l - \sin\beta l) = 0$$

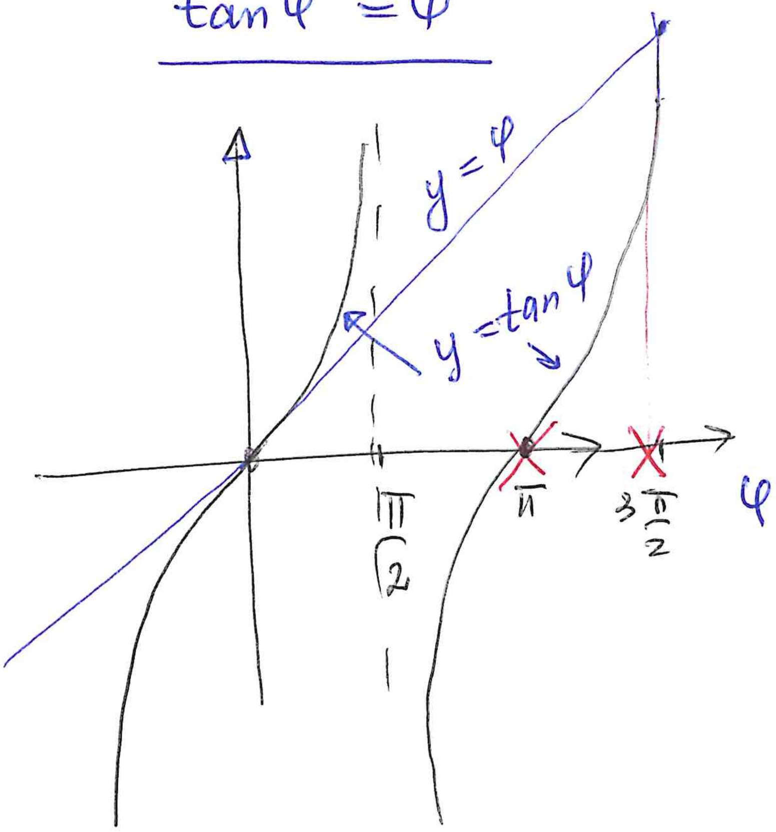
$\Rightarrow\ \sin\beta l = 0 \quad | \quad \underline{\sin\varphi = 0}$

$\beta l\cos\beta l - \sin\beta l = 0 \quad | \quad \varphi\cos\varphi - \sin\varphi = 0$

$$\underline{\tan\varphi = \varphi}$$

$\underline{\varphi = \pi}$

nearest root



$\beta l = \pi$

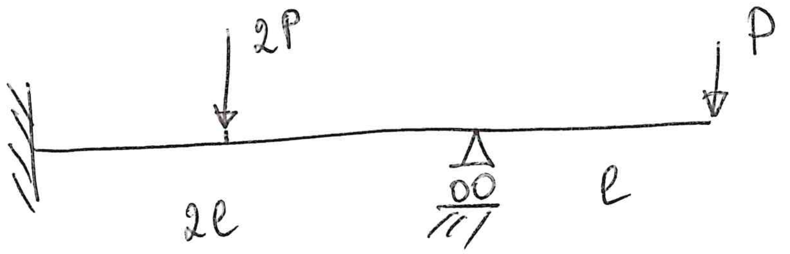
$$\sqrt{\frac{P}{EI}} = \frac{\pi}{l}$$

$$P_e = \frac{\pi^2 EI}{l^2}$$

Problem 3.

$$\boxed{s \int p_0 \dot{v}\, dx = \sum_k M_{0k} \dot{\varphi}_k}$$

$\boxed{s \leq \beta} \rightarrow \underline{\min \beta}$!



M_0, l

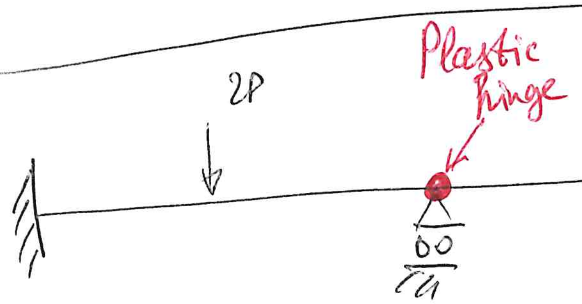
$P_0 = \frac{M_0}{l}$

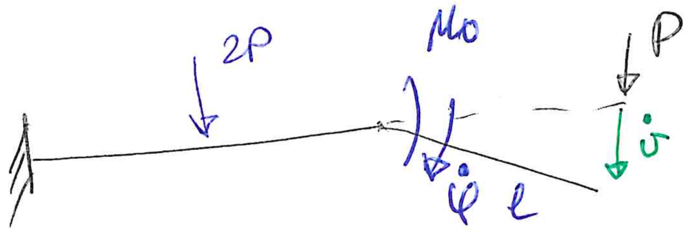
reference force

βP_0

?

1.





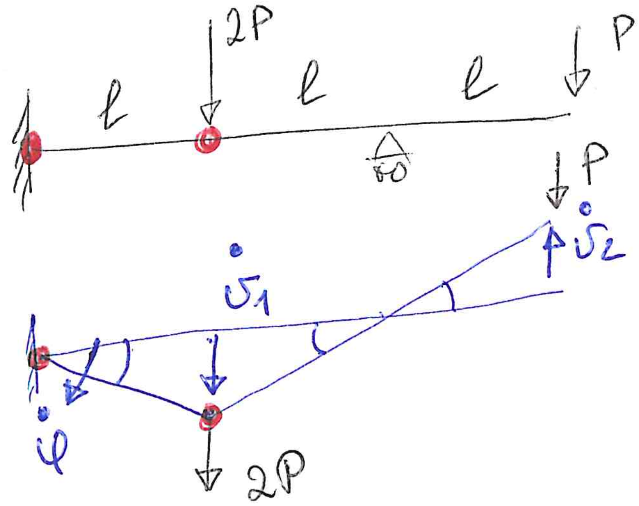
$P = \beta P_0$

$$\beta P_0 \dot{v} = M_0 \dot{\varphi} \qquad \dot{v} = l\dot{\varphi}$$

$$\beta P_0 l \dot{\varphi} = M_0 \dot{\varphi}$$

$$\beta \frac{M_0}{l} l \dot{\varphi} = M_0 \dot{\varphi} \implies \boxed{\beta = 1}$$

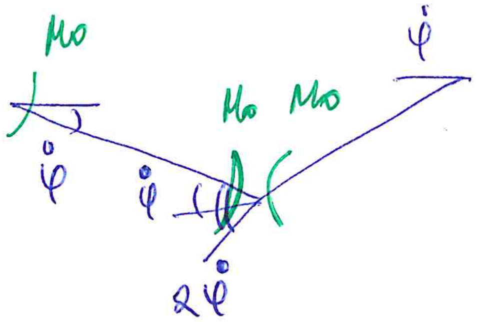
2,



$\dot{v}_1 = l\dot{\varphi}$, $\dot{v}_2 = l\dot{\varphi}$

$$\beta 2P_0 \dot{v}_1 + \beta P_0(-l\dot{\varphi}) = \beta 2P_0 l\dot{\varphi} - \beta P_0 l\dot{\varphi}$$

$$= \beta P_0 l\dot{\varphi}$$



$$\underline{M_0\dot{\varphi} + M_0 2\dot{\varphi} + M_0 2\dot{\varphi}}$$

$\beta P_0 l\dot{\varphi} = 5M_0\dot{\varphi}$, $P_0 = \frac{M_0}{l}$

$$\beta \frac{\cancel{M_0}}{\cancel{l}} \cancel{l\dot{\varphi}} = 5\cancel{M_0}\cancel{\dot{\varphi}}$$

$$\boxed{\beta = 5}$$

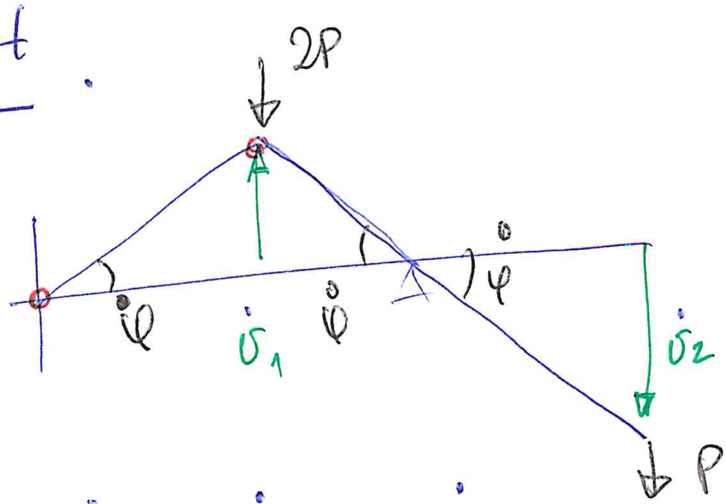
2 = 3

$\beta_1 = 1$, $\beta_2 = 5$

$\downarrow$

$\beta = 1$

Comment .



$\dot{v}_1 = l\dot{\varphi}$, $\dot{v}_2 = l\dot{\varphi}$

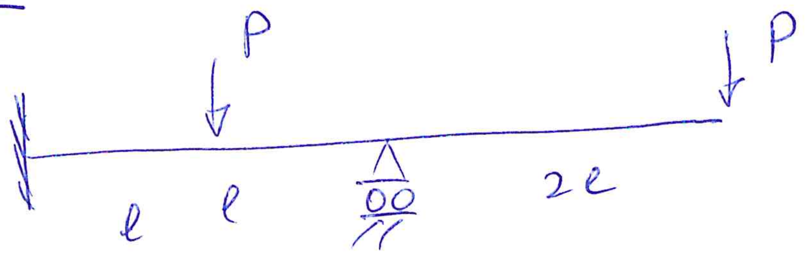
$$-2P\dot{v}_1 + P\dot{v}_2 = -Pl\dot{\varphi} = -\beta P_0 l\dot{\varphi}$$

?

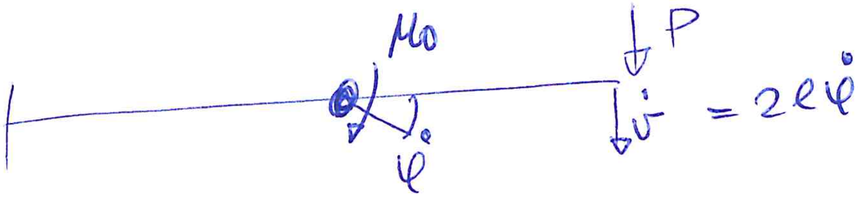
$$\cancel{-\beta P_0 l\dot{\varphi} = 5M_0\dot{\varphi}}$$

Problem 3'

$\underline{M_0}$



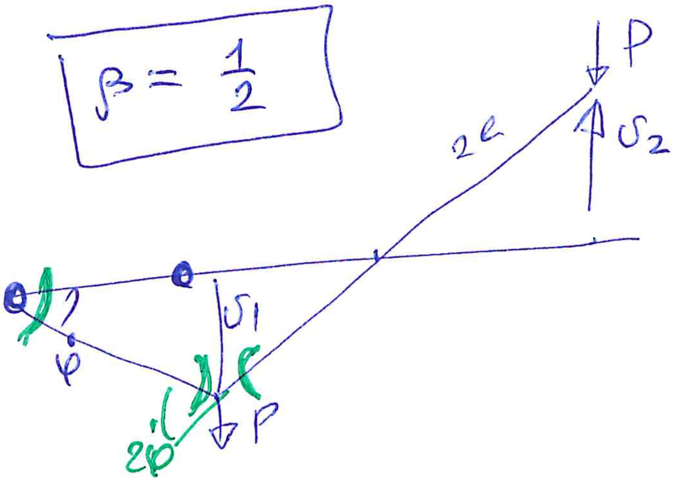
1.



$$\beta P_0 \cdot 2l\dot{\varphi} = M_0 \dot{\varphi}$$

$$\boxed{\beta = \frac{1}{2}}$$

2.



$$\beta P_0 \dot{v}_1 - \beta P_0 \dot{v}_2 = \beta P_0 l\dot{\varphi} - \beta P_0 2l\dot{\varphi} =$$

$$= -\beta P_0 l\dot{\varphi}$$

$$\beta P_0 l\dot{\varphi} = 5 M_0 \dot{\varphi}$$

$$\boxed{\beta = 5} \quad \rightarrow \quad \beta = \frac{1}{2}$$