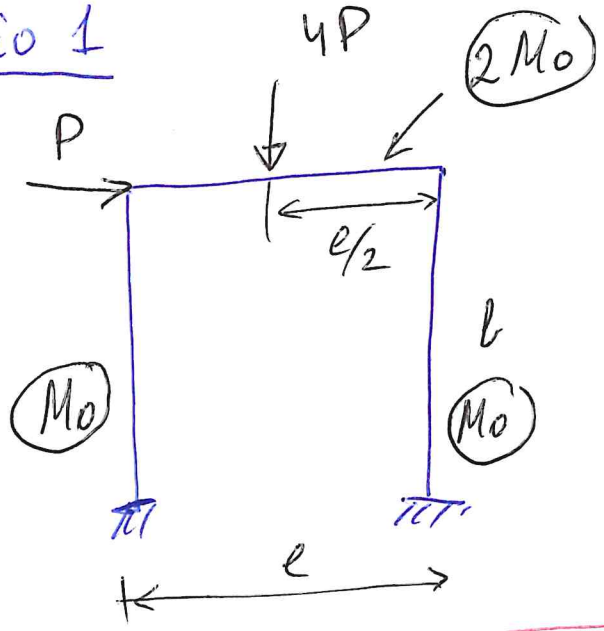
Esempio 1



$P \to M P_0$

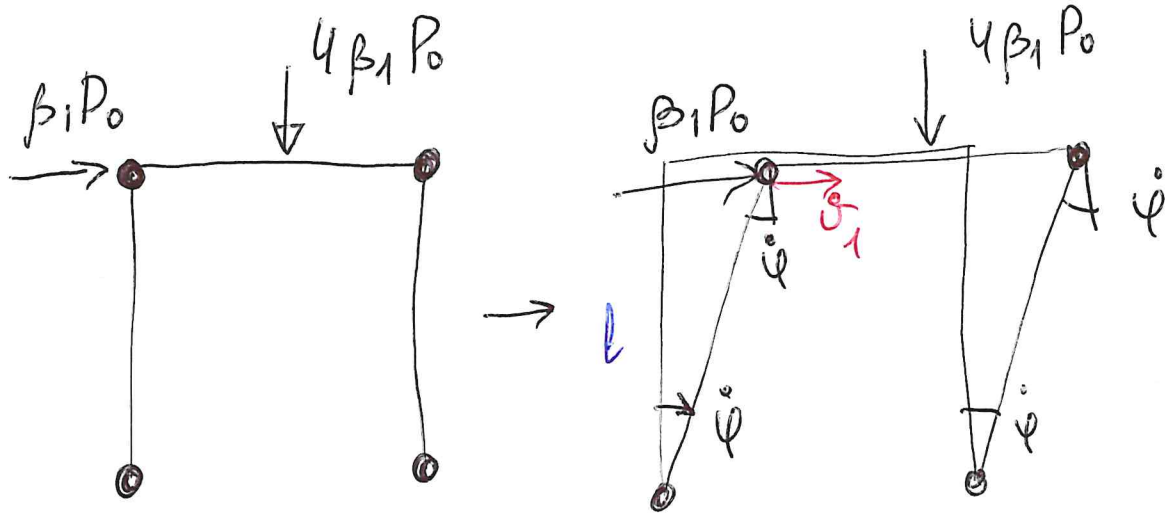
$$P_0 = \frac{M_0}{l}$$

(M) ?

S ?

$$S \int p_0 \ddot{v}\, dx = \sum M_{0k} \ddot{\psi}_k \qquad (*)$$

1.

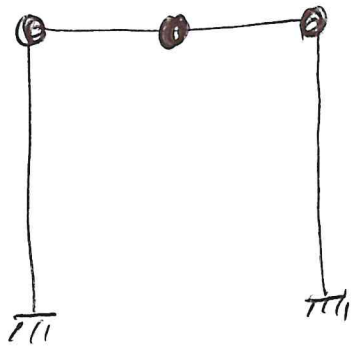


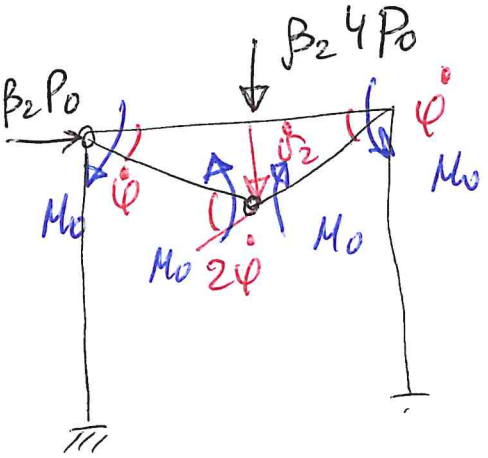
$(*)$ $\quad \beta_1 P_0 \dot{v}_1 = 4 M_0 \ddot{\varphi}$, $\quad \dot{v}_1 = l \dot{\varphi}$

$\beta_1 P_0 l \ddot{\varphi} = 4 M_0 \ddot{\varphi}$, $\quad P_0 = \frac{M_0}{l}$

$\beta_1 \frac{M_0}{l} l \ddot{\varphi} = 4 M_0 \ddot{\varphi} \Rightarrow$ $\boxed{\beta_1 = 4}$

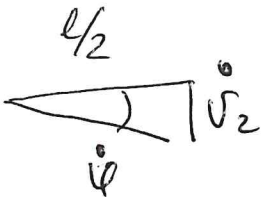
2.





(*) $4\beta_2 P_0 \dot{v}_2 = \underbrace{M_0\dot{\varphi} + M_0\dot{\varphi} + M_0\cdot 2\dot{\varphi} + M_0 2\dot{\varphi}}_{6M_0\dot{\varphi}}$

$\dot{v}_2 = \frac{\ell}{2}\dot{\varphi}$



$4\beta_2 P_0 \frac{\ell}{2}\dot{\varphi} = 6M_0\dot{\varphi}$, (*) $P_0 = \frac{M_0}{\ell}$

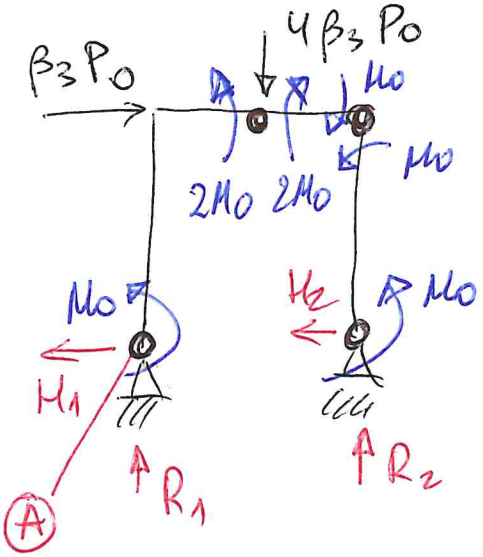
$4\beta_2 \frac{M_0}{\ell}\frac{\ell}{2}\dot{\varphi} = 6M_0\dot{\varphi}$

$2\beta_2 = 6 \Rightarrow \boxed{\beta_2 = 3}$

$\beta_1 = 4$

$\boxed{S = \min \beta}$

β_3: $\beta_3 = \frac{8}{3}$



$M(x) - ?$

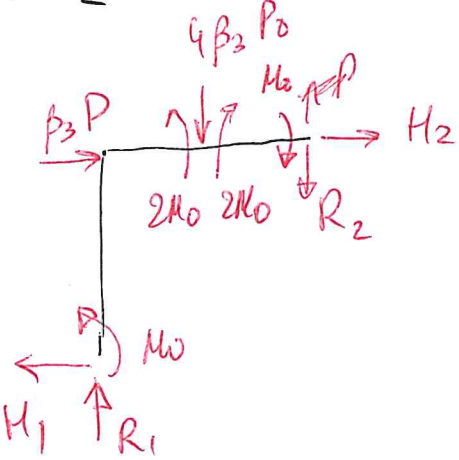
$\beta_3 P_0 = \frac{8}{3} \frac{M_0}{\ell}$

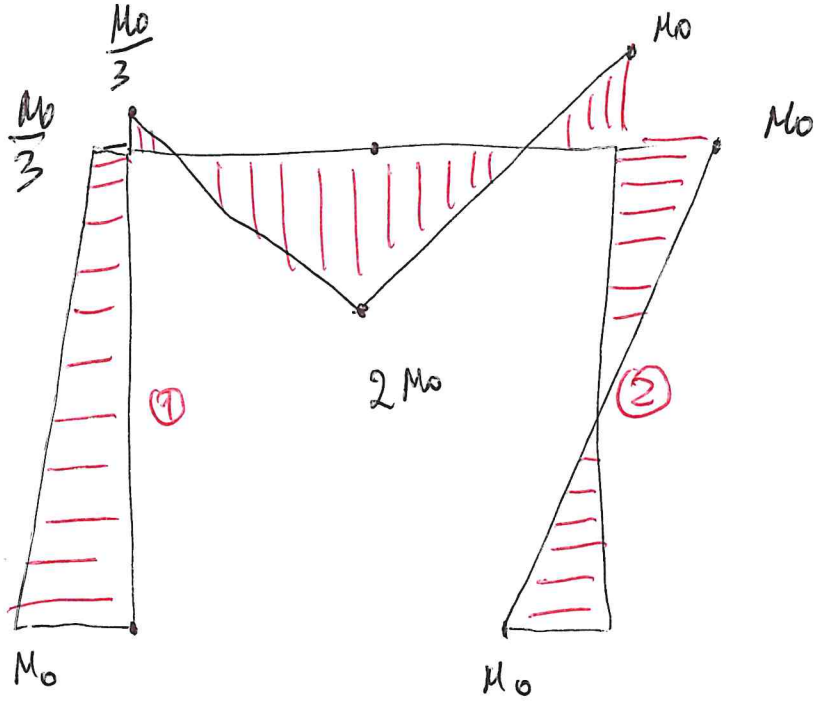
$4\beta_3 P_0 = 4 \cdot \frac{8}{3} \frac{M_0}{\ell} = \frac{32}{3} \frac{M_0}{\ell}$

$$\begin{cases} R_1 + R_2 = \frac{32}{3} \frac{M_0}{\ell} & \Sigma \uparrow \\ H_1 + H_2 = \frac{8}{3} \frac{M_0}{\ell} & \Sigma \rightarrow \end{cases}$$

M_A: $M_0 - \beta_3 P_0 \ell - 4\beta_3 P_0 \frac{\ell}{2} + R_2 \ell - H_2 \ell - M_0 = 0$

$$\begin{cases} H_1 = \frac{2}{3} \frac{M_0}{\ell} \\ H_2 = 2 \frac{M_0}{\ell} \\ R_1 = \frac{14}{3} \frac{M_0}{\ell} \\ R_2 = \frac{18}{3} \frac{M_0}{\ell} \end{cases}$$





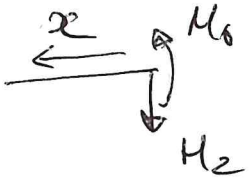
① : $M_x = M_0 - H_1 x = M_0 - \frac{2}{3}\frac{M_0}{e} x$

$x = e \quad M_x = \frac{1}{3} M_0$

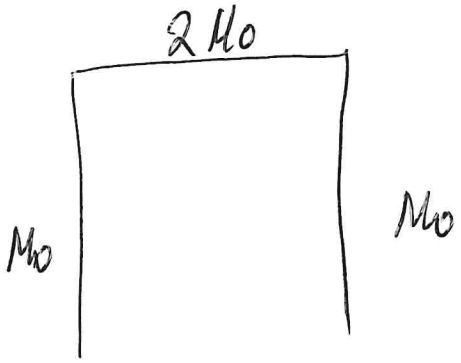
② : $M_x = M_0 - H_2 x =$

$= M_0 - 2\frac{M_0}{e} x$

$x = e \quad M_x = -M_0$



$$-M_0(x) \leq M(x) \leq M_0(x)$$



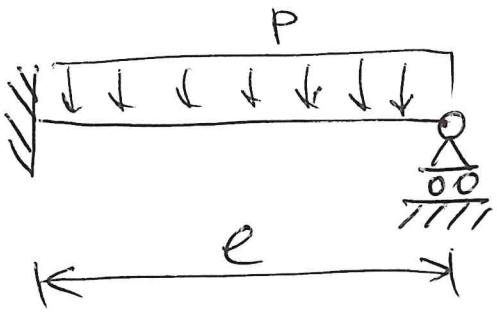
$\beta_3 = \psi$

$s = \min \beta \quad e \quad s = \max \psi$

$\beta_3 = \psi \Rightarrow \boxed{s = \beta_3 = 8/3}$

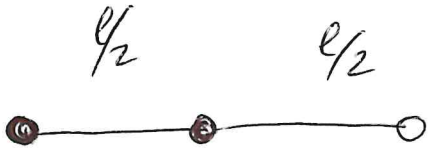
Esempio 2.

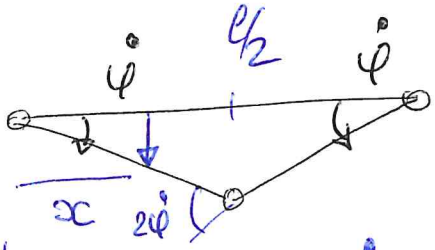
M_0



$p = \mu p_0$

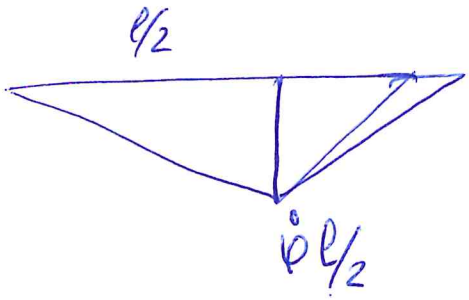
$p_0 = \frac{M_0}{\ell^2}$





$$\beta \int_0^{\ell} p_0 \dot{v}\, dx = M_0 \dot{\varphi} + M_0 2\dot{\varphi} \qquad \dot{v} = \begin{cases} x\dot{\varphi}, & x < \ell/2 \\ (\ell - x)\dot{\varphi}, & x > \ell/2 \end{cases}$$

$$2\beta p_0 \frac{\ell}{2} \dot{\varphi} \frac{\ell}{2} \frac{1}{2} = 3 M_0 \dot{\varphi}$$



$$\beta p_0 \frac{1}{4} \ell^2 \dot{\varphi} = 3 M_0 \dot{\varphi}$$

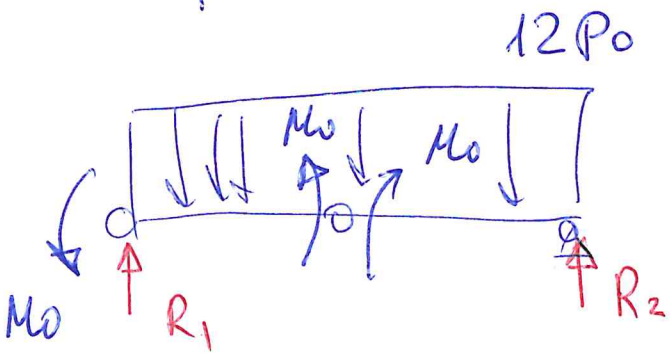
$$p_0 = \frac{M_0}{\ell^2}$$

$$\beta \frac{M_0}{4\ell^2} \ell^2 \dot{\varphi} = 3 M_0 \dot{\varphi}$$

$$\boxed{\beta = 12}$$

$$\boxed{\beta \geq 5}$$

$\beta = 12$



$M_x(x) - ?$

$$R_1 + R_2 = 12 p_0$$

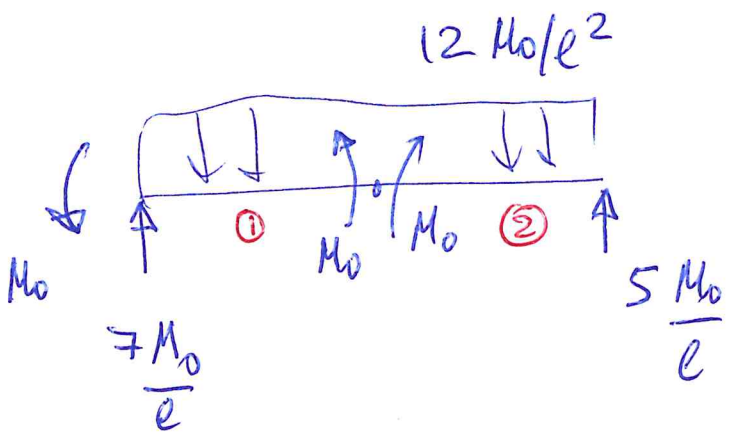
$$R_2 \ell + M_0 - 12 p_0 \frac{\ell^2}{2} = 0$$

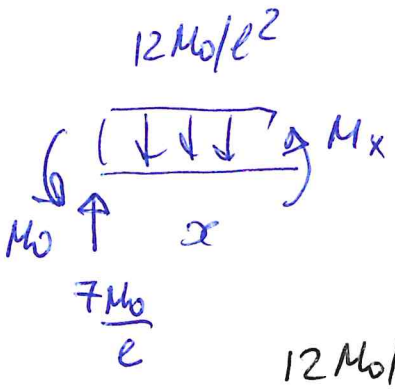
$$R_2 \ell + M_0 - 12 \frac{M_0}{\ell^2} \frac{\ell^2}{2} = 0$$

$$R_2 = \frac{5 M_0}{\ell}$$

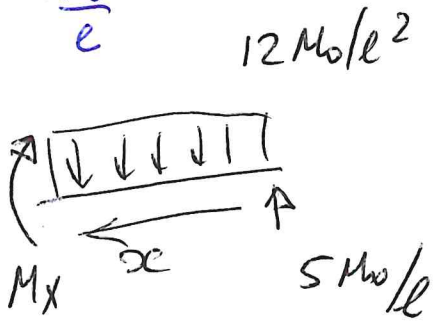
$$R_1 = 12 p_0 \ell - R_2 = 12 \frac{M_0}{\ell^2} \ell - 5 \frac{M_0}{\ell}$$

$$R_1 = 7 \frac{M_0}{\ell}$$





① $M_x = -M_o + \frac{7M_o}{\ell} x - 12 M_o/\ell^2 \frac{x^2}{2}$

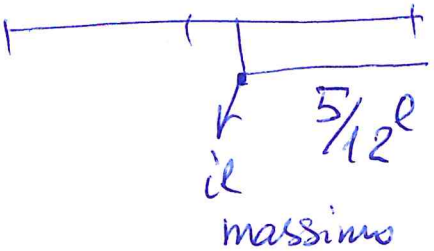


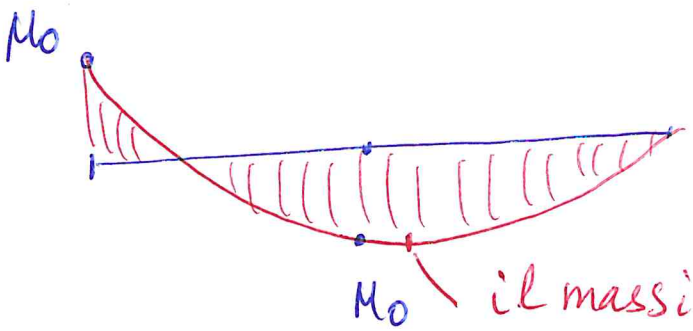
② $M_x = 12 \frac{M_o}{\ell^2} \frac{x^2}{2} - \frac{5M_o}{\ell} x$

$M_x = 6 \frac{M_o}{\ell^2} x^2 - \frac{5M_o}{\ell} x$

$M_x' = 0 \qquad M_x' = 12 \frac{M_o}{\ell^2} x - \frac{5M_o}{\ell} = 0$

$x = \frac{5}{12} \ell$





$M_x = 6 \frac{M_o}{\ell^2} \left(\frac{5}{12}\ell\right)^2 - \frac{5M_o}{\ell} \frac{5}{12} \ell = -\frac{25}{24} M_o$

$$-M_0 \leq M(x) \leq M_0$$

$$\max|M(x)| = \frac{25}{24} M_0 > M_0$$

$$\psi = \frac{24}{25}\beta = \frac{24}{25} \cdot 12 \approx 11.520$$

$$\psi \leq S \leq \beta$$

$$11.520 \leq S \leq 12.0$$

$S = 11.657$

essatto