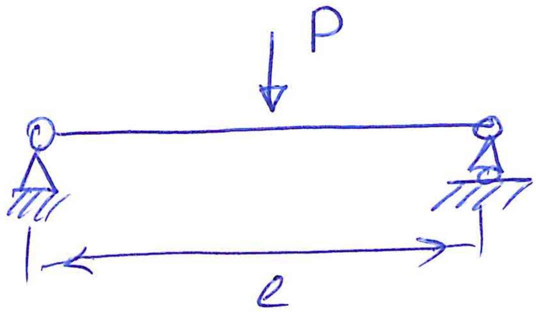
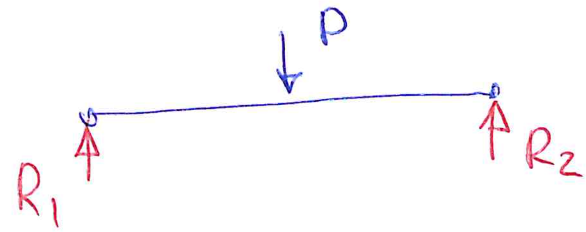
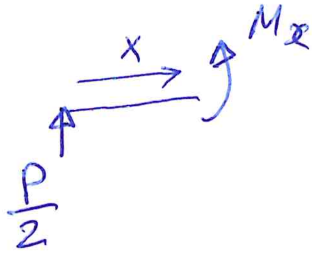
Esempio 1



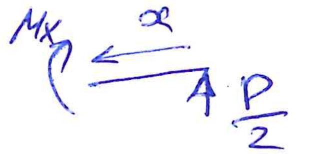
$EI, \; M_0$



$$R_1 = R_2 = \frac{P}{2}$$

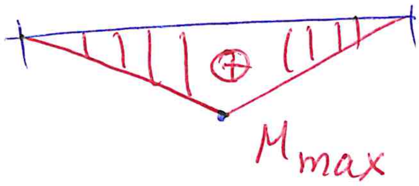


$$M_x = \frac{P}{2} x$$



$$M(x) = \frac{P}{2} x$$

$$M_x = \frac{P}{2} x$$

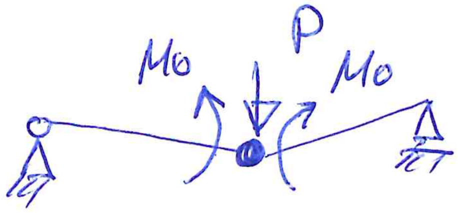


$$M_{max} = M\left(\frac{l}{2}\right) = \frac{P}{2}\frac{l}{2} = \frac{Pl}{4}$$

stato elastico: $\quad M_{max} < M_0$

$$\boxed{P < \frac{4M_0}{l}}$$

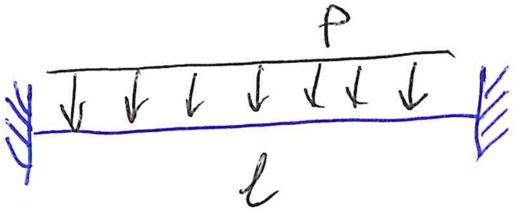
$$P = P_c = \frac{4 M_0}{l}$$



la cerniera plastica

$$P \geq P_c = \frac{4 M_0}{l} \rightarrow \boxed{\text{il collasso plastico}}$$

Il collasso plastico



$EI, \ M_0$

$M(x) - ?$

$|M| \le M_0$

l'equilibrio :

$$EI\,v^{(4)} = p \qquad \underline{v(x) - ?} \qquad \downarrow v$$

$$EI\,v^{(3)} = px + C_1$$

$$EI\,v'' = p\frac{x^2}{2} + C_1 x + C_2$$

$$EI\,v' = p\frac{x^3}{6} + \frac{C_1 x^2}{2} + C_2 x + C_3$$

$$EI\,v = p\frac{x^4}{24} + \frac{C_1 x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$

le condizione al contorno :

$$v(0) = 0, \quad v(l) = 0$$

$$v'(0) = 0, \quad v'(l) = 0$$

$$v(0) = 0 \Rightarrow \boxed{C_4 = 0}$$

$$v'(0) = 0 \Rightarrow \boxed{C_3 = 0}$$

$$EIv = p\frac{x^4}{24} + c_1\frac{x^3}{6} + c_2\frac{x^2}{2}$$

$$EIv' = p\frac{x^3}{6} + c_1\frac{x^2}{2} + c_2 x$$

$$\left.\begin{array}{l} v(l)=0 \Rightarrow p\frac{l^4}{24} + c_1\frac{l^3}{6} + c_2\frac{l^2}{2} = 0 \\ v'(l)=0 \Rightarrow p\frac{l^3}{6} + c_1\frac{l^2}{2} + c_2 l = 0 \end{array}\right\}$$

$$\begin{cases} \frac{pl^2}{12} + c_1\frac{l}{3} + c_2 = 0 \rightarrow c_2 = -c_1\frac{l}{3} - p\frac{l^2}{12} \\ \frac{pl^2}{6} + c_1\frac{l}{2} + c_2 = 0 \end{cases}$$

$$\downarrow$$

$$\frac{pl^2}{6} + c_1\frac{l}{2} - c_1\frac{l}{3} - p\frac{l^2}{12} = 0$$

$$c_1\left(\frac{1}{2} - \frac{1}{3}\right) + pl\left(\frac{1}{6} - \frac{1}{12}\right) = 0$$

$$c_1\frac{1}{6} = -\frac{1}{12}pl \Rightarrow \boxed{c_1 = -\frac{1}{2}pl}$$

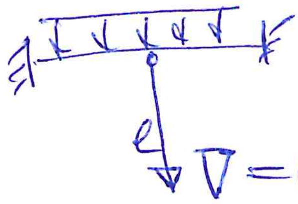
$$c_2 = -\frac{l}{3}\left(-\frac{1}{2}pl\right) - p\frac{l^2}{12} = \frac{pl^2}{6} - \frac{pl^2}{12}$$

$$\boxed{c_2 = \frac{1}{12}pl^2}$$

$$EI\,v(x) = \frac{p x^4}{24} - \frac{1}{2} p l \cdot \frac{x^3}{6} + \frac{1}{12} p l^2 \frac{x^2}{2}$$

$$EI\,v(x) = \frac{p x^4}{24} - \frac{p l}{12} x^3 + \frac{p l^2}{24} x^2$$

$$V = v(l/2)$$



$V = v\left(\frac{l}{2}\right)$

$$EIV = p \frac{l^4}{2^4 \cdot 24} - p l \frac{l^3}{12 \cdot 8} + p l^2 \frac{l^2}{24 \cdot 4}$$

$$EIV = p l^4 \left[\frac{1}{16 \cdot 24} - \frac{1}{12 \cdot 8} + \frac{1}{24 \cdot 4} \right] =$$

$$= \frac{p l^4}{8} \left[\frac{1}{24 \cdot 2} - \cancel{\frac{1}{12}} + \cancel{\frac{1}{3 \cdot 4}} \right] =$$

$$= \frac{p l^4}{8 \cdot 24 \cdot 2} = \frac{p l^4}{384}$$

$$\boxed{V \equiv v(l/2) = \frac{p l^4}{384\, EI}}$$

per la curva $p - V$.

$M(x) - ?$ $\boxed{M(x) = -EI\,v''(x)}$

$$EI\,v(x) = \frac{px^4}{24} - \frac{pl}{12}x^3 + \frac{pl^2}{24}x^2$$

$$EI\,v'(x) = \frac{px^3}{6} - \frac{pl}{4}x^2 + \frac{pl^2}{12}x$$

$$\underline{EI\,v''(x)} = \frac{px^2}{2} - \frac{1}{2}plx + \frac{pl^2}{12}.$$

$M(0)$, $M(l)$, $M(l/2)$

$$M(0) = -EI\,v''(0) = -\frac{pl^2}{12}$$

$$M(l) = -EI\,v''(l) = -\frac{pl^2}{2} + \frac{1}{2}pl^2 - \frac{pl^2}{12}$$

$$M(l) = -\frac{pl^2}{12}$$

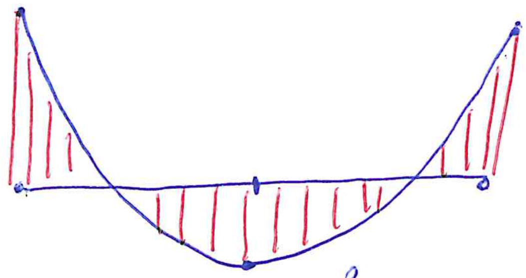
$$M\left(\frac{l}{2}\right) = -EI\,v''\left(\frac{l}{2}\right) =$$

$$= -\frac{pl^2}{2\cdot 4} + \frac{1}{2}pl\cdot\frac{l}{2} - \frac{pl^2}{12} =$$

$$= -\frac{pl^2}{8} + \frac{pl^2}{4} - \frac{pl^2}{12} =$$

$$= pl^2\Big[\underbrace{-\frac{1}{8} + \frac{1}{4} - \frac{1}{12}}_{1/24}\Big) = \frac{pl^2}{24}$$

Diagramma dei momenti $\qquad M(x)$



$$M_{max} = \frac{pl^2}{12}$$

$$|M(x)| \leq M_{max} \leq M_0 \quad \text{(il momento limite)}$$

$P_e = ?$ $\qquad$ $\underline{0 < p < p_e}$ lo stato elastico

$$M_0 = \frac{p_e l^2}{12} \Rightarrow \boxed{p_e = \frac{12 M_0}{l^2}}$$

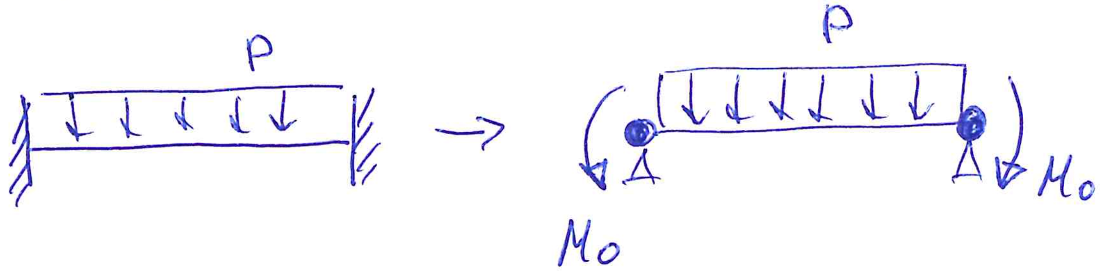
$$V_e = \frac{p_e l^4}{384 EI} = 12\frac{M_0}{l^2}\frac{l^4}{384 EI} = \frac{1}{32}\frac{M_0 l^2}{EI}$$

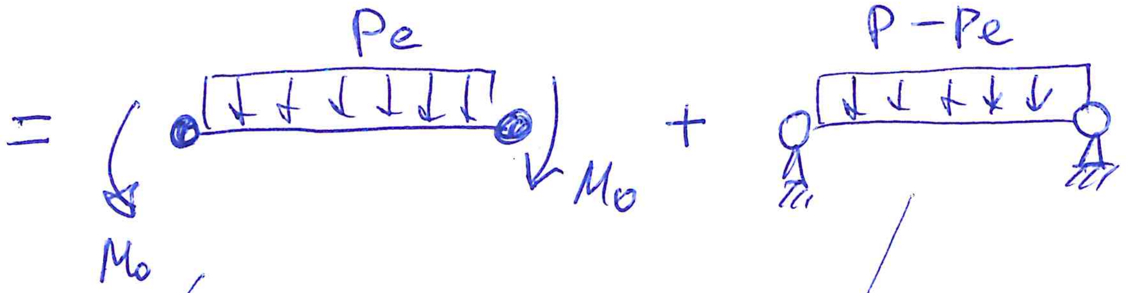
$$\boxed{p_e = 12\frac{M_0}{l^2}, \quad V_e = \frac{1}{32}\frac{M_0 l^2}{EI}}$$

Stato elastico ↑

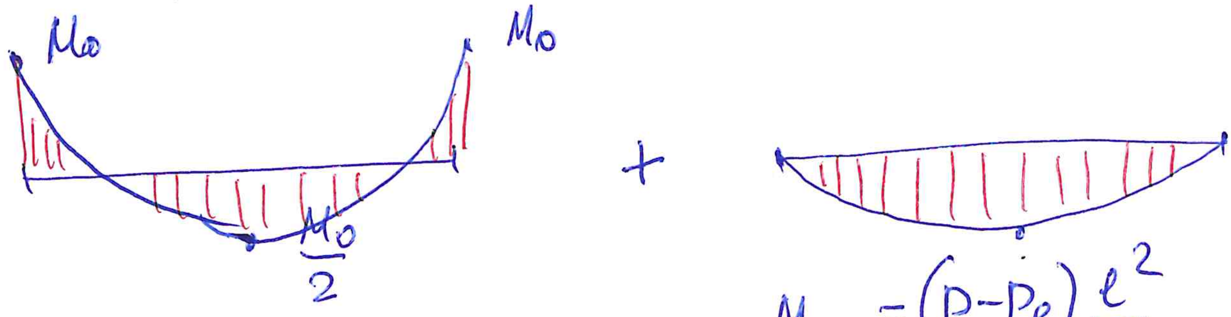
$P > P_e$

Stato elasto-plastico





$M(x)$ - ?

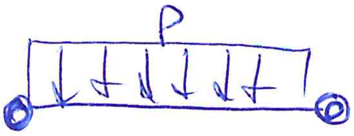


$M_{max} = (P - P_e)\dfrac{\ell^2}{8}$

$P = P_e$

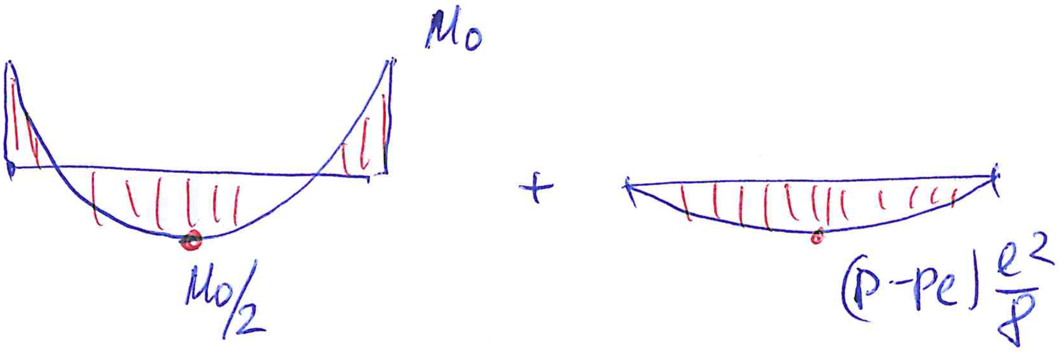
$R = \dfrac{q\ell}{2}$ $\quad M(x) = Rx - q\dfrac{x^2}{2} = x\dfrac{q\ell}{2} - q\dfrac{x^2}{2}$

$M(x) = q\dfrac{x}{2}(\ell - x), \quad M_{max} = M\left(\dfrac{\ell}{2}\right) = \dfrac{p\ell^2}{8}$



$$p > p_e$$

$M(x)$:

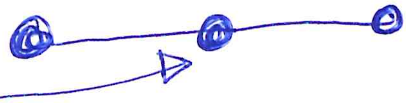


$$\Delta p = p - p_e$$

$$\boxed{\frac{M_0}{2} + \Delta p \frac{\ell^2}{8} = M_0}$$

$$\Delta p \frac{\ell^2}{8} = \frac{M_0}{2} \Rightarrow \boxed{\Delta p = \frac{4 M_0}{\ell^2}}$$

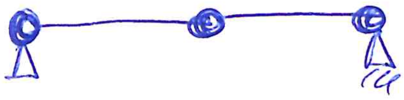
per $p = p_e + \Delta p$



$$p_e = 12 \frac{M_0}{\ell^2}$$

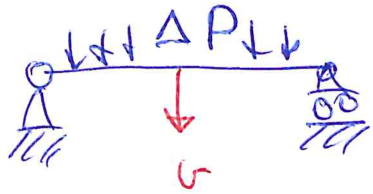
$$p_c = 12 \frac{M_0}{\ell^2} + 4 \frac{M_0}{\ell^2}, \quad \boxed{p_c = 16 \frac{M_0}{\ell^2}}$$

Il collasso plastico



$p > p_c \rightarrow$ collasso

Per la curva $p - v$, $v - ?$



$$M(x) = \Delta p \frac{x}{2}(\ell - x)$$

$$EI v'' = -M(x) = \Delta p \frac{x}{2}(x - \ell)$$

$$v(0) = 0 \quad , \quad v(\ell) = 0$$

$$EI v' = -\Delta p \frac{\ell}{2}\frac{x^2}{2} + \Delta p \frac{x^3}{6} + C_1$$

$$EI v = -\Delta p \frac{\ell}{2}\frac{x^3}{6} + \Delta p \frac{x^4}{24} + C_1 x + C_2$$

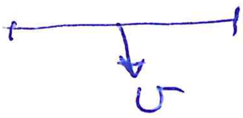
$$v(0) = 0 \Rightarrow \boxed{C_2 = 0}$$

$$v(\ell) = 0 \quad -\Delta p \frac{\ell^4}{12} + \Delta p \frac{\ell^4}{24} + C_1 \ell = 0$$

$$\boxed{C_1 = \frac{\Delta p \ell^3}{24}}$$

$$EIv(x) = -\Delta p\frac{\ell x^3}{12} + \Delta p\frac{x^4}{24} + \frac{\Delta p \ell^3}{24}x$$

$v\left(\frac{\ell}{2}\right)$ - ?



$$EIv\left(\frac{\ell}{2}\right) = \Delta p\left[-\frac{\ell}{12}\,\frac{\ell^3}{8} + \frac{\ell^4}{24\cdot 16} + \frac{\ell^3\cdot\ell}{24\cdot 2}\right]$$

$$= \Delta p\,\ell^4\left[-\frac{1}{96} + \frac{1}{48} + \frac{1}{384}\right] =$$

$$= \Delta p\,\ell^4\left[\frac{1}{96} + \frac{1}{384}\right] = \Delta p\,\ell^4\left[\frac{4+1}{384}\right]$$

$$\boxed{v\left(\frac{\ell}{2}\right) = \frac{\Delta p\,\ell^4}{EI}\,\frac{5}{384}.}$$

$$V_c = V_e + v\left(\frac{\ell}{2}\right).$$

$$\boxed{\begin{array}{l} V_e = \frac{1}{32}\frac{M_0\ell^2}{EI} \\ \Delta P_c = \frac{4M_0}{\ell^2} \end{array}}$$

$$v\left(\frac{\ell}{2}\right) = \frac{4M_0}{\ell^2}\,\frac{\ell^4}{EI}\,\frac{5}{384}$$

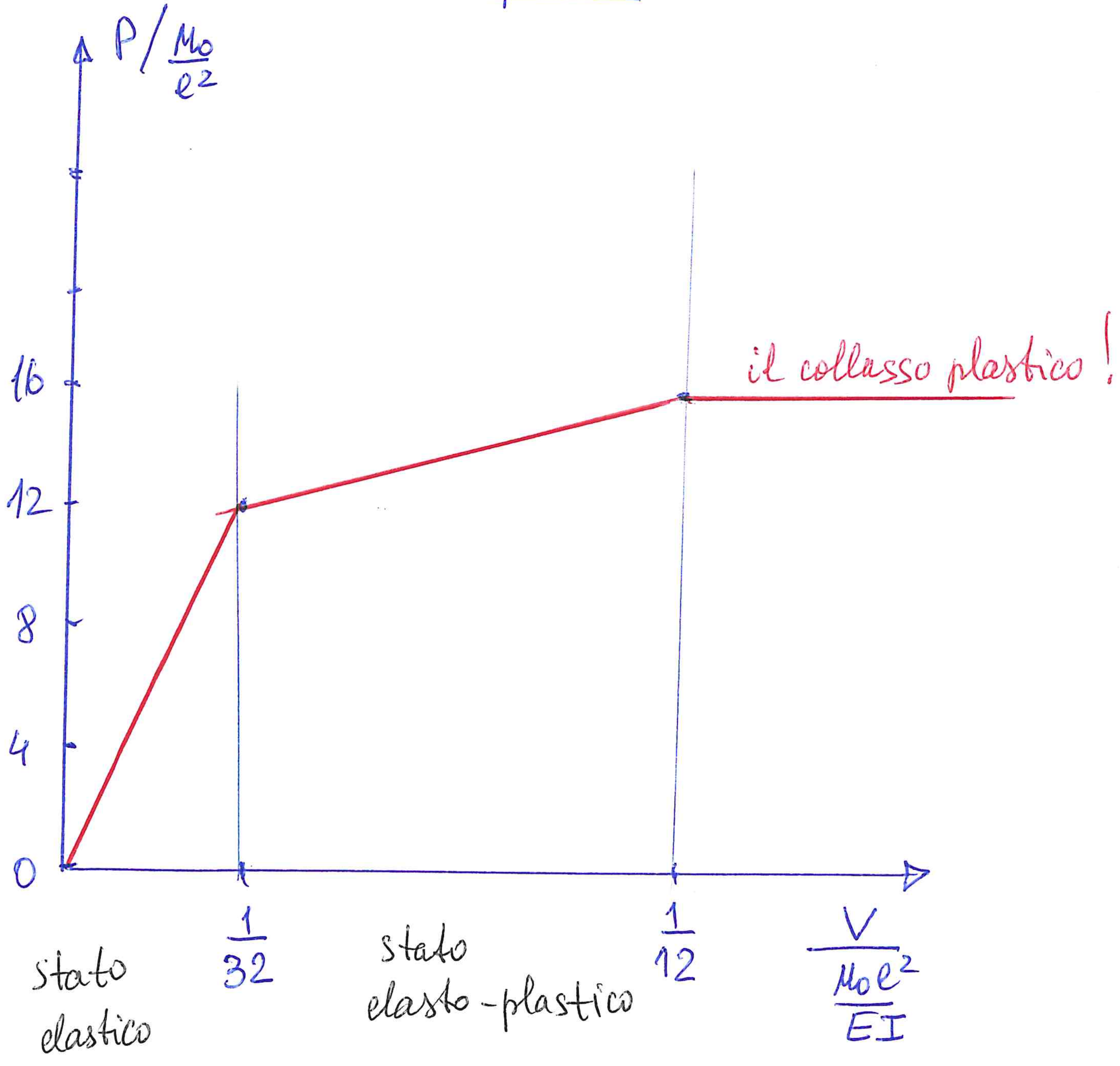
$$v(\ell/2) = \frac{5}{96}\,\frac{M_0\ell^2}{EI}$$

$$V_c = \frac{1}{32}\,\frac{M_0\ell^2}{EI} + \frac{5}{96}\,\frac{M_0\ell^2}{EI} = \frac{1}{12}\,\frac{M_0\ell^2}{EI}$$

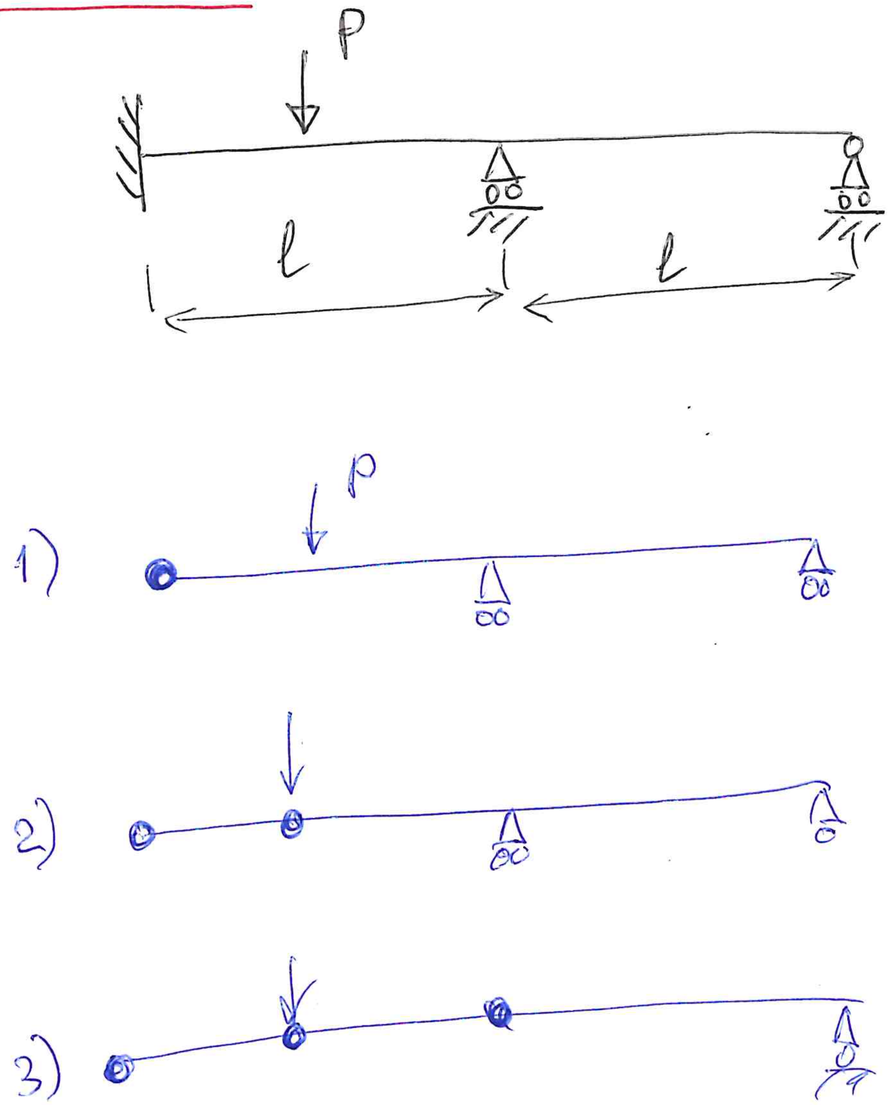
$$P_e = 12\,\frac{M_0}{\ell^2}\,, \quad V_e = \frac{1}{32}\,\frac{M_0 \ell^2}{EI}$$

$$P_c = 16\,\frac{M_0}{\ell^2}\,, \quad V_c = \frac{1}{12}\,\frac{M_0 \ell^2}{EI}$$

La curva p – V



Esempio



il collasso plastico