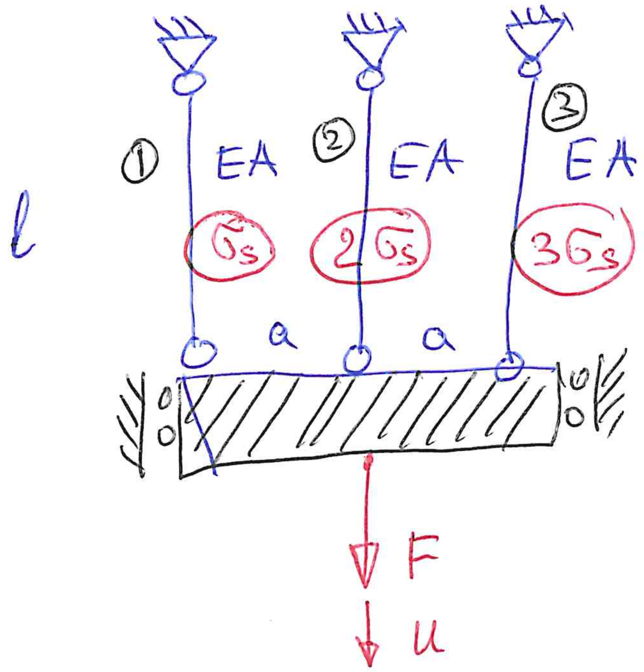
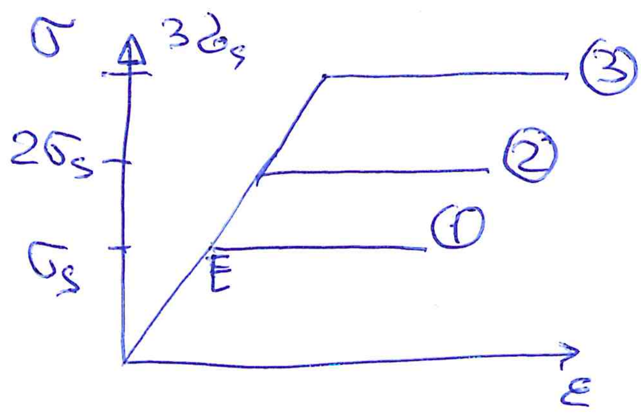
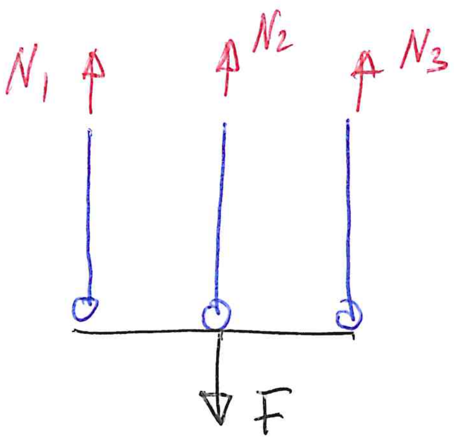
Esempio, Il collasso plastico



F–u ?





equilibrio:

$$N_1 + N_2 + N_3 = F$$

Dominio elastico: $\sigma = E\varepsilon$, $\varepsilon = \dfrac{\Delta l}{l}$

$$N = \sigma A$$

$$N_1 = \sigma_1 A, \quad N_2 = \sigma_2 A, \quad N_3 = \sigma_3 A$$

$$\Delta l_1 = \Delta l_2 = \Delta l_3$$

1. Stato 1 : risposta elastica.
Limite elastico.

$$\sigma_1 < \sigma_s, \quad \sigma_2 < 2\sigma_s, \quad \sigma_3 < 3\sigma_s$$

$$\Delta_c = \Delta l_{1c} = \frac{\sigma_s l}{E}, \quad \varepsilon_c = \frac{\sigma_s}{E}$$

$$N_1 = \sigma_1 A$$

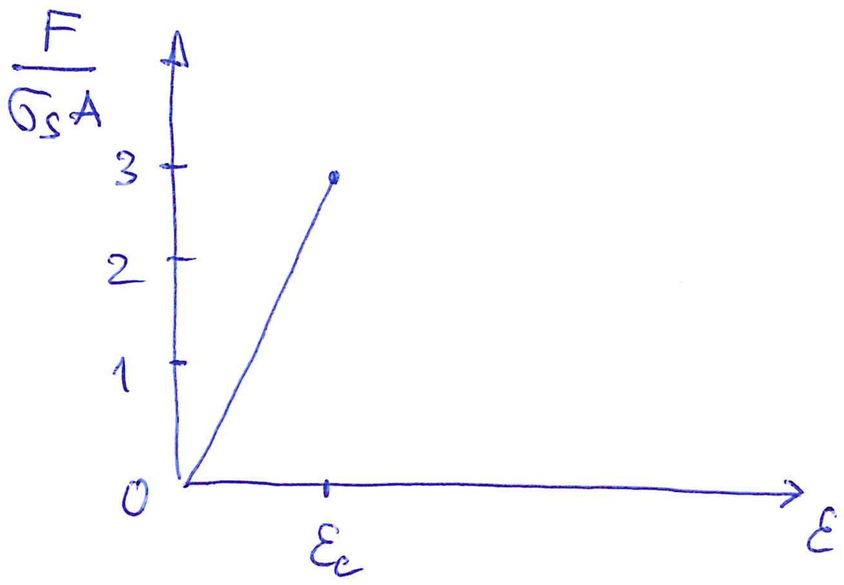
$$N_{1c} = \sigma_s A$$

$$\Delta l_1 = \Delta l_2 = \Delta l_3 = \Delta_c \equiv \Delta l_{1c}$$

$$\Rightarrow N_2 = N_3 = N_1 = N_{1c}$$

$$F = N_1 + N_2 + N_3$$

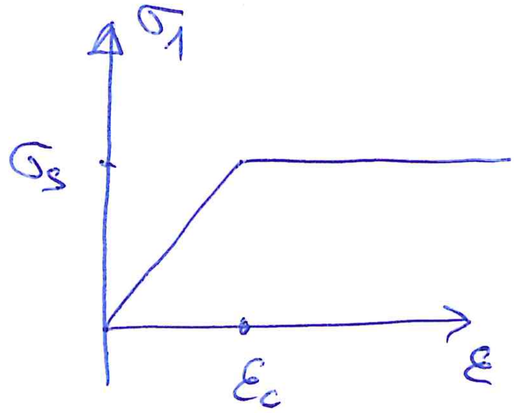
$$F_{1c} = \sigma_s A + \sigma_s A + \sigma_s A = 3\sigma_s A$$



Stato 2: risposta elasto-plastica fino allo snervamento del pendolo 2.

$$F > F_{1c} = 3\sigma_s A$$

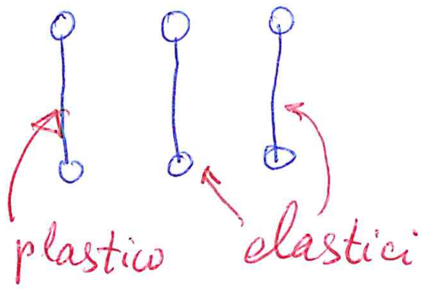
$$N_1 = \sigma_1 A = N_{1c} \equiv \sigma_s A$$



$$F = N_1 + N_2 + N_3 =$$

$$= \underline{\sigma_s A} + \sigma_2 A + \sigma_3 A$$

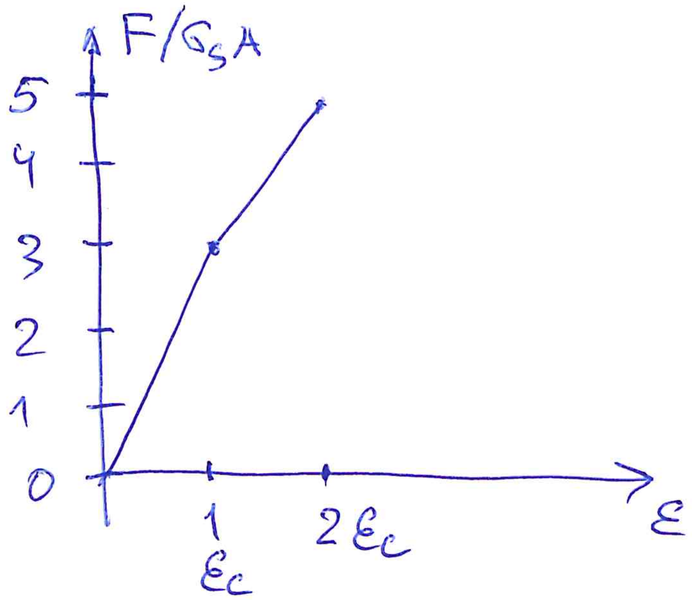
$$\underline{\sigma_2 < 2\sigma_s} \quad , \quad \sigma_3 < 3\sigma_s$$



$$\Delta l_{2c} = \frac{\sigma_{2c}}{E} l = \frac{2\sigma_s}{E} l$$

$$\Delta l_3 = \Delta l_{2c}$$

$$F_{2c} = \underset{N_{1c}}{\sigma_s A} + \underset{N_{2c}}{2\sigma_s A} + \underset{N_3}{2\sigma_s A} = 5\sigma_s A$$



$$\varepsilon_{2c} = \frac{\Delta l_{2c}}{l} =$$

$$= \frac{2\sigma_s}{E} =$$

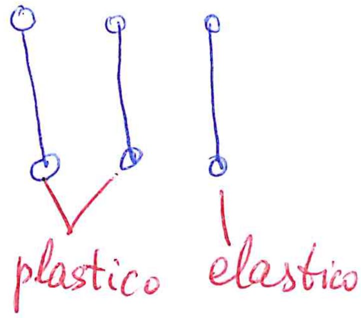
$$= 2\varepsilon_c$$

Stato 3: risposta elasto-plastica fino allo snervamento del pendolo 3.

$F > F_{2c}$ Colasso della struttura.

$N_1 = \sigma_s A$

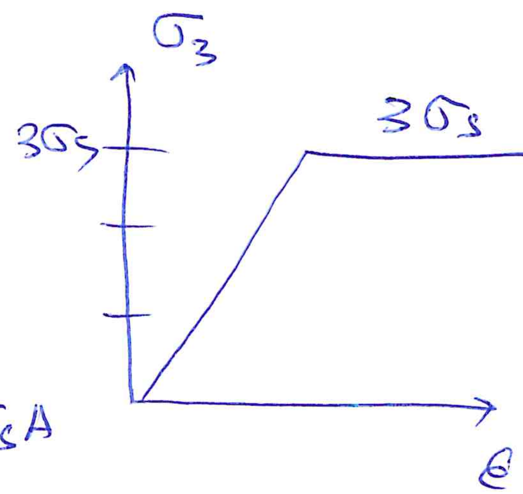
$N_2 = 2\sigma_s A$



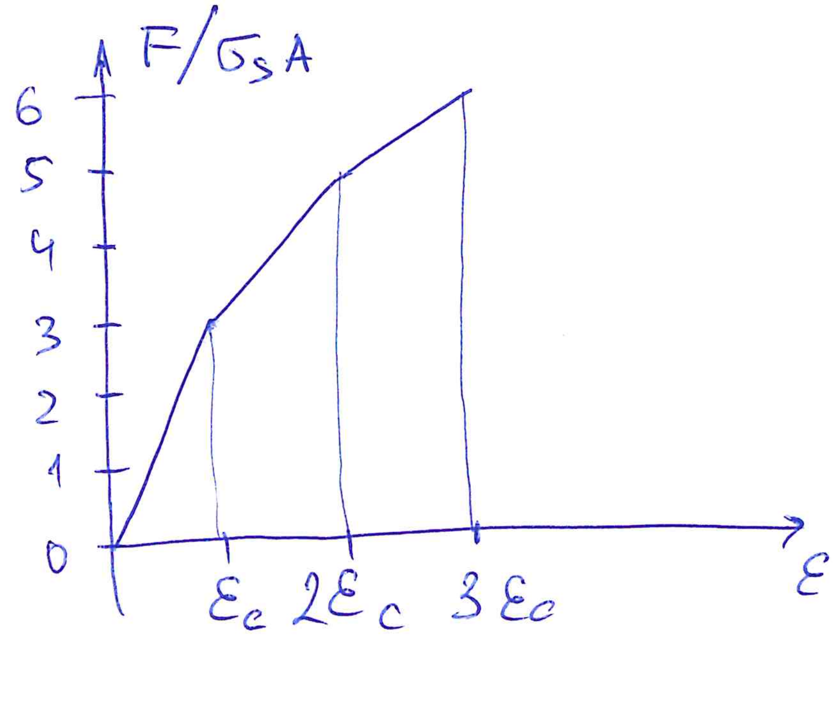
$F = N_1 + N_2 + N_3 = 3\sigma_s A + \sigma_3 A$

$$\Delta l_{3c} = \frac{\sigma_{3c}\, l}{E} = \frac{3\sigma_s\, l}{E}$$

$N_{3c} = \sigma_{3c} \cdot A = 3\sigma_s A$



$F_{c3} = 3\sigma_s A + 3\sigma_s A = 6\sigma_s A$

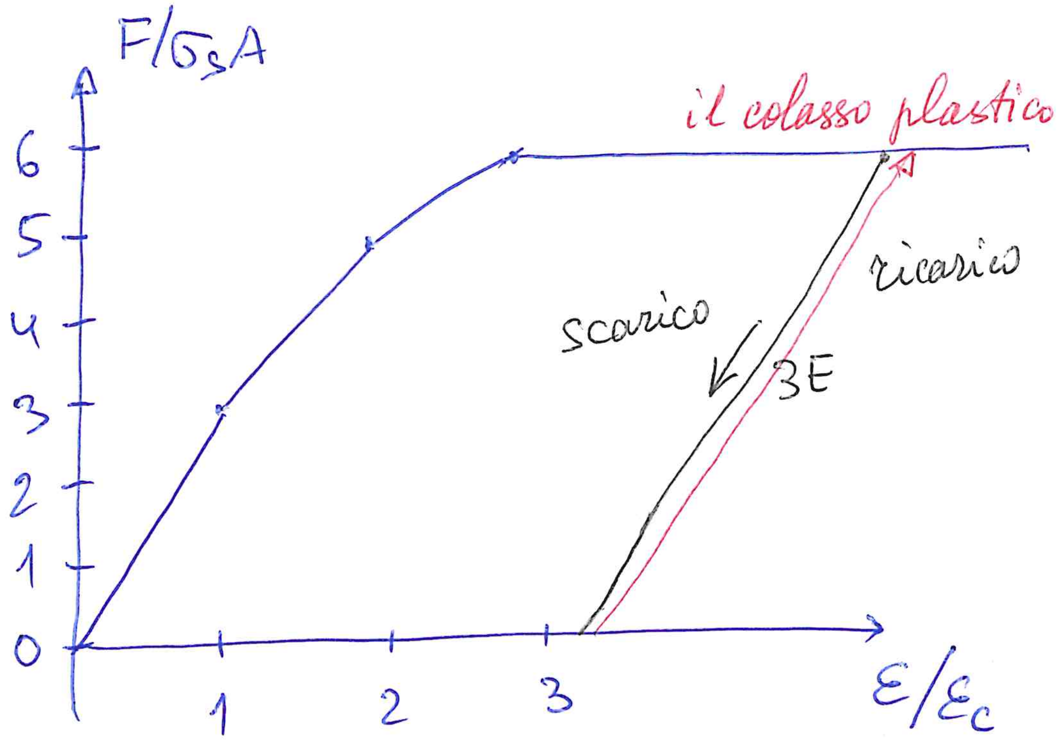


Stato 4: colasso - scarico elastico

$$F > F_{e3} = 6\sigma_s A$$

$$N_1 = \sigma_s A, \quad N_2 = 2\sigma_s A, \quad N_3 = 3\sigma_s A$$

$$F = N_1 + N_2 + N_3 = 6\sigma_s A$$

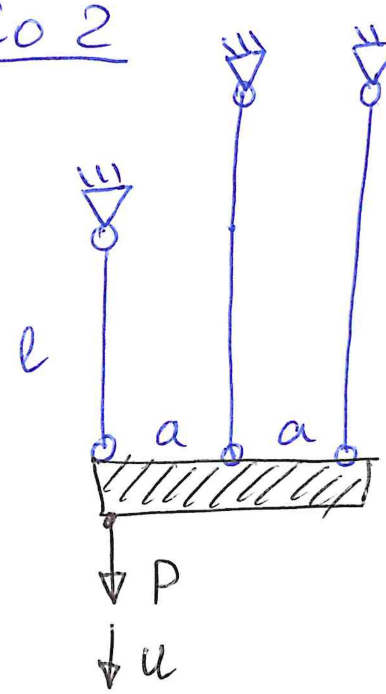


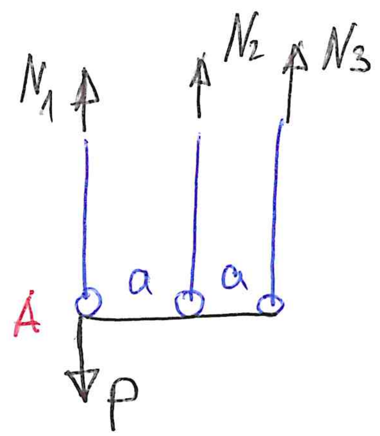
Esempio 2

EA

σ_s

$P-u$?





l'equilibrio:

$$P = N_1 + N_2 + N_3$$

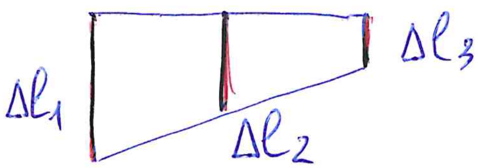
$$N_2 a + N_3 2a = 0$$

$$\Rightarrow \quad N_2 + 2N_3 = 0$$

$$\Rightarrow \quad \begin{cases} N_1 + N_2 + N_3 = P \\ N_2 + 2N_3 = 0 \end{cases}$$

$\Delta\ell_1, \ \Delta\ell_2, \ \Delta\ell_3$

$$\Rightarrow \quad \Delta\ell_2 = \frac{1}{2}(\Delta\ell_1 + \Delta\ell_3)$$



$$\Delta\ell_1 - 2\Delta\ell_2 + \Delta\ell_3 = 0$$

$$\begin{cases} N_1 + N_2 + N_3 = P \\ N_2 + 2N_3 = 0 \\ \Delta l_1 - 2\Delta l_2 + \Delta l_3 = 0 \end{cases}$$

$$\Delta l_{1,2,3} \rightarrow N_{1,2,3} \quad ?$$

$$\sigma = E\varepsilon, \quad \varepsilon = \frac{\Delta l}{l} \quad \Rightarrow \quad \Delta l = \frac{\sigma l}{E}$$

$$N = \sigma A \quad \Rightarrow \quad \boxed{\Delta l = \frac{Nl}{EA}}$$

$$\Delta l_1 = \frac{N_1 l}{EA},$$

$$\Delta l_2 = \frac{N_2 \cdot \frac{3}{2} l}{EA} = \frac{3}{2}\frac{N_2 l}{EA}$$

$$\Delta l_3 = \frac{3}{2}\frac{N_3 l}{EA}.$$

$$\Delta l_1 - 2\Delta l_2 + \Delta l_3 = 0$$

$$\frac{N_1 \cancel{l}}{\cancel{E}A} - 2 \cdot \frac{3}{2}\frac{N_2 \cancel{l}}{\cancel{E}A} + \frac{3}{2}\frac{N_3 \cancel{l}}{\cancel{E}A} = 0$$

$$\underline{N_1 - 3N_2 + \frac{3}{2}N_3 = 0}$$

$$\begin{cases} N_1 + N_2 + N_3 = P \\ N_2 + 2N_3 = 0 \\ N_1 - 3N_2 + \frac{3}{2}N_3 = 0 \end{cases}$$

$$N_2 = -2N_3$$

$$N_1 = 3N_2 - \frac{3}{2}N_3 = -6N_3 - \frac{3}{2}N_3 = -\frac{15}{2}N_3$$

$$-\frac{15}{2}N_3 - 2N_3 + N_3 = P$$

$$-\left(\frac{15}{2}+1\right)N_3 = P \Rightarrow \boxed{N_3 = -\frac{2}{17}P}$$

$$\boxed{N_2 = \frac{4}{17}P}$$

$$\boxed{N_1 = \frac{15}{17}P}$$

$$|N_1| \le \sigma_s A, \quad |N_2| \le \sigma_s A, \quad |N_3| \le \sigma_s A$$

$$N_1 = N_{1e} = \sigma_s A$$

$\Delta l_1 = u$

$$\Delta l_1 = \frac{N_1 l}{EA} = \frac{15}{17}\frac{Pl}{EA},$$

$$\Delta l_2 = \frac{3}{2}\frac{N_2 l}{EA} = \frac{6Pl}{17EA}, \quad \Delta l_3 = \frac{3}{2}\frac{N_3 l}{EA} = -\frac{3}{17}\frac{Pl}{EA}$$

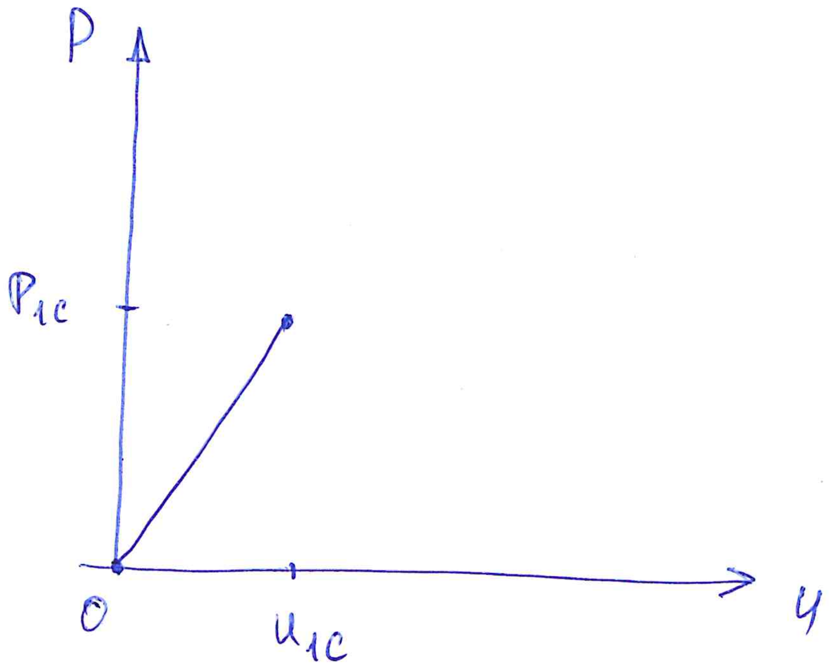
Stato 1 : limite elastica.

$$N_1 = N_{1c} = \sigma_s A \qquad N_1 = \frac{15}{17} P$$

$$\Delta l_1 = \frac{15}{17} \frac{Pl}{EA} \qquad \boxed{\Delta l_1 = \frac{N_1 l}{EA}}$$

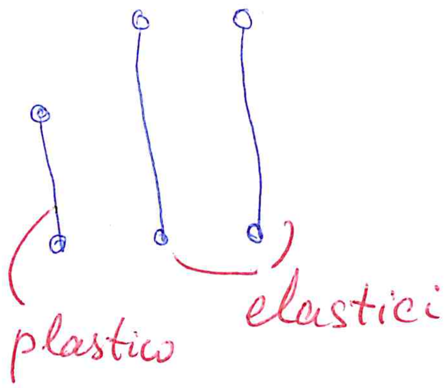
$$P_{1c} = \frac{17}{15} N_{1c} = \frac{17}{15} \sigma_s A$$

$$u_{1c} = \Delta l_1 = \frac{15}{17} \frac{P_{1c}\, l}{EA} = \frac{\sigma_s A l}{EA} = \frac{\sigma_s l}{E}$$



Stato 2: risposta elasto-plastica

$P > P_{1c}$



$N_1 = N_{1c} = \sigma_s A.$

$N_1 + N_2 + N_3 = P$

$N_2 + 2N_3 = 0$

$N_2 = -2N_3$

$N_1 - 2N_3 + N_3 = P$

$N_1 - N_3 = P$

$\boxed{N_3 = N_1 - P}$, $\boxed{N_2 = -2N_1 + P \cdot 2}$

$$\begin{cases} N_3 = N_{1c} - P \\ N_2 = 2P - 2N_{1c} \\ N_1 = N_{1c} = \sigma_s A \end{cases}$$

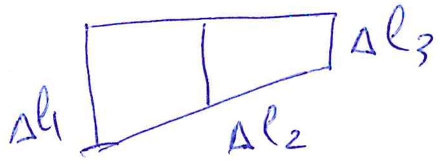
$\Delta l_2 = \frac{N_2}{EA} \frac{3}{2} l$, $\Delta l_3 = \frac{N_3}{EA} \frac{3}{2} l$

$|N_2| < \sigma_s A$, $|N_3| < \sigma_s A$

$$\underline{u = \Delta l_1}$$

$$\cancel{\underline{\Delta l_1 \sim N_1}}$$ non c'è

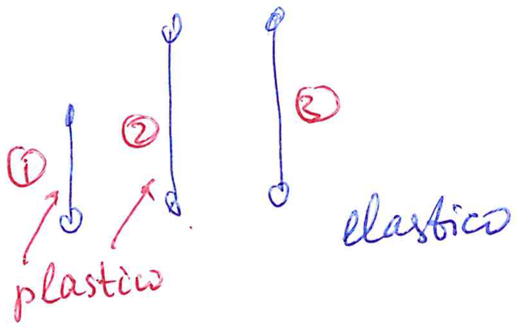
$$\underline{\Delta l_1 = 2\Delta l_2 - \Delta l_3}$$



$$\Delta l_2 = \frac{1}{2}(\Delta l_1 + \Delta l_3)$$

$$\Delta l_1 + \Delta l_3 - 2\Delta l_2 = 0$$

$$\underline{N_2 = \sigma_s A}$$



$$\Delta l_1 = 2\Delta l_2 - \Delta l_3$$

$$\Delta l_2 = \frac{3}{2}\frac{N_2 l}{EA} = \frac{3}{2}\frac{\sigma_s A l}{EA} = \frac{3}{2}\frac{\sigma_s l}{E}$$

$$\Delta l_3 = \frac{3}{2}\frac{N_3 l}{EA} \quad ; \quad N_3 = N_{1e} - P \Rightarrow \Delta l_3 = -\frac{3}{4}\frac{\sigma_s l}{E}$$

$$\cancel{\Delta l_3 = -\frac{3}{4}}$$

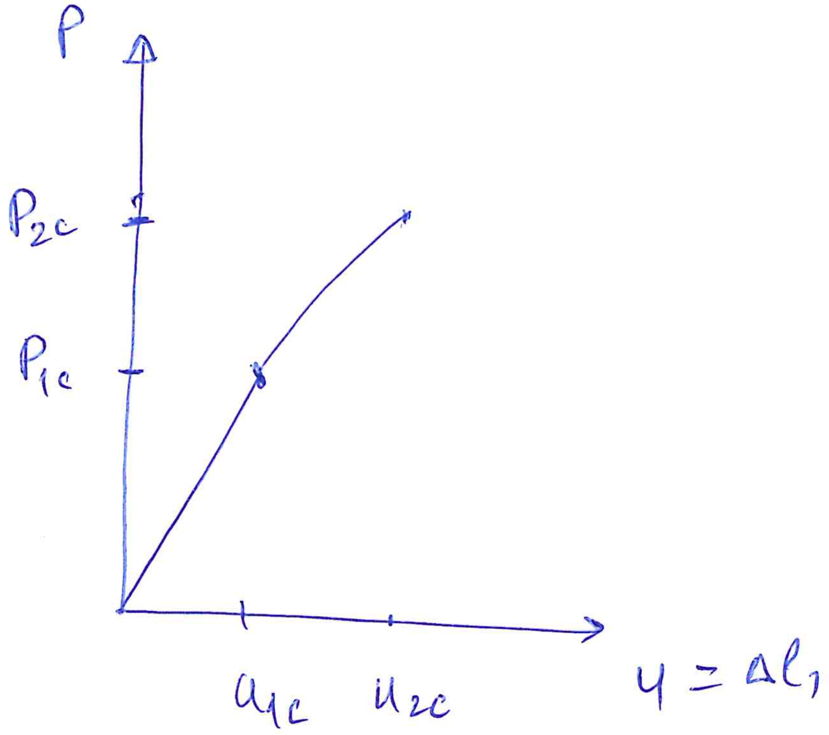
$$\boxed{\Delta l_1 = 3\frac{\sigma_s l}{E} + \frac{3}{4}\frac{\sigma_s l}{E} = \frac{15}{4}\frac{\sigma_s l}{E}}$$

$P_{2c} = ?$

$$N_1 + N_2 + N_3 = P$$

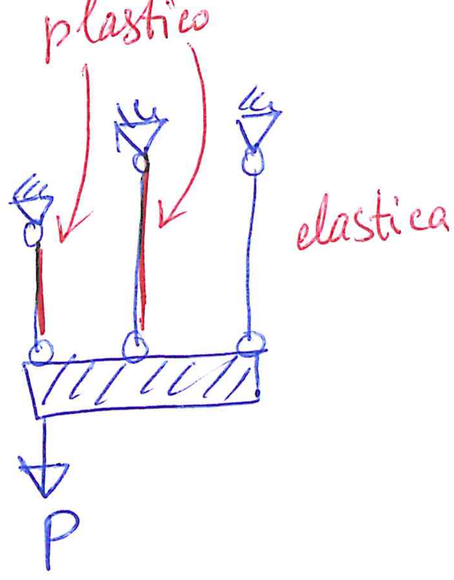
$$\sigma_s A + \sigma_s A + \left(\frac{1}{2}\sigma_s A\right) = P$$

$$\boxed{P_{2c} = \frac{3}{2}\sigma_s A}$$



$$P_{1c} = \frac{17}{15}\sigma_s A$$

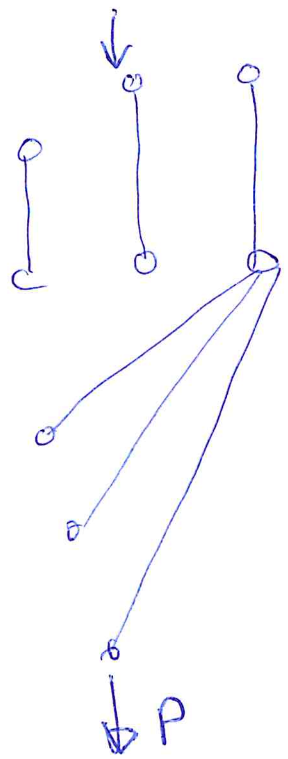
Stato 3. $\underline{\underline{P > P_{2c}}}$



$$N_1 = \sigma_s A, \quad N_2 = \sigma_s A$$

$$\begin{cases} N_1 + N_2 + N_3 = P \\ N_2 + 2N_3 = 0 \end{cases}$$

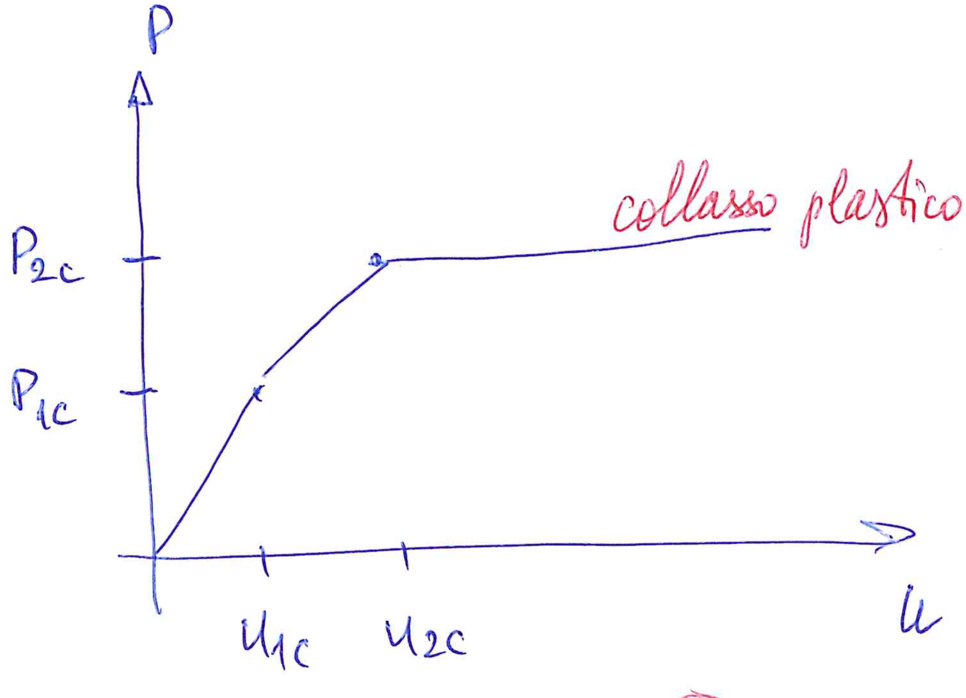
$$\begin{cases} \sigma_s A + \sigma_s A + N_3 = P \\ \sigma_s A + 2N_3 = 0 \end{cases}$$



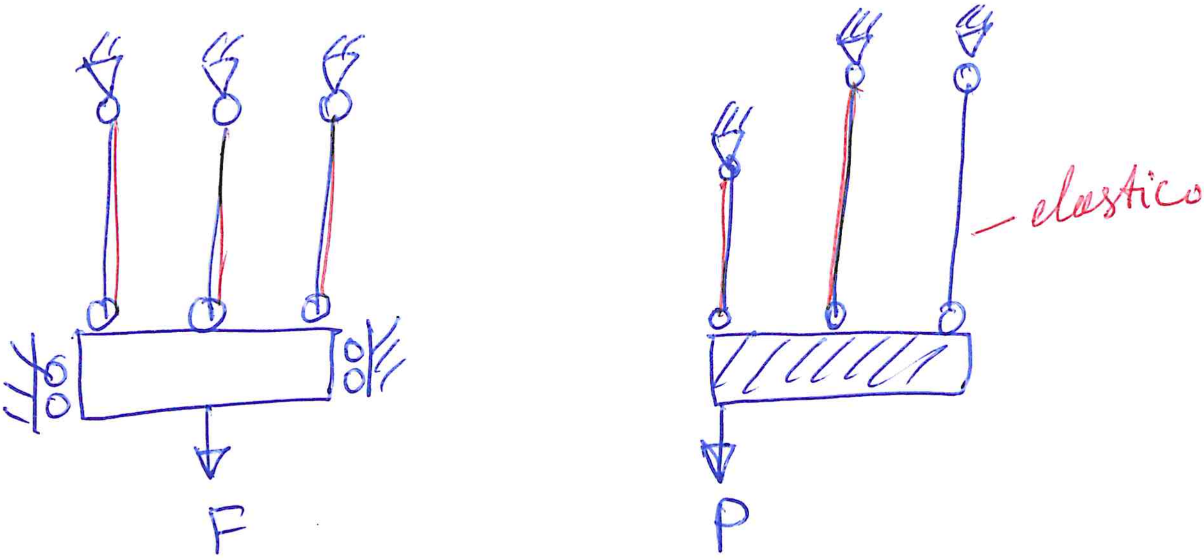
$\Rightarrow$ no soluzione !!!

non c'e l'equilibrio

il collasso plastico



Differenza:

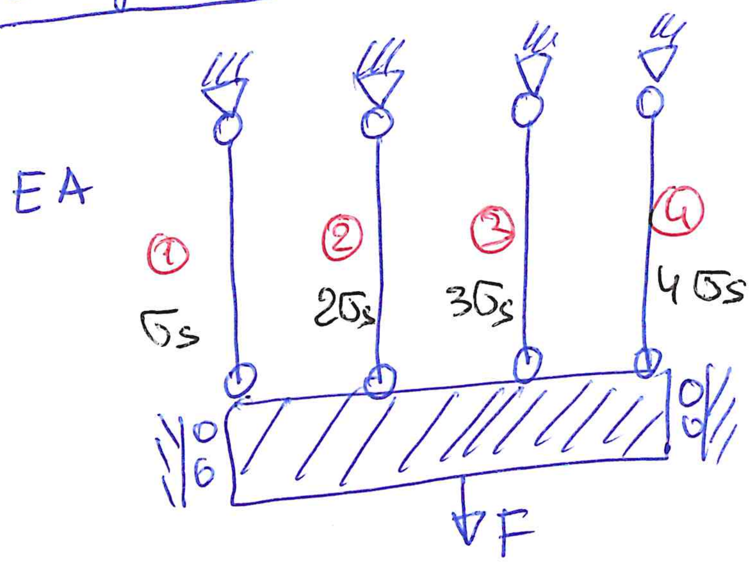


il collasso plastico

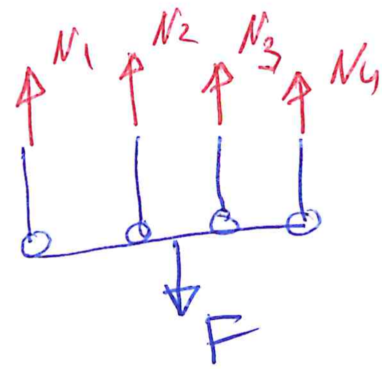
tutti plastici

due plastici

Esempio 3



$\Delta l_1, \Delta l_2\, \Delta l_3,$
Δl_4



$$N_1 + N_2 + N_3 + N_4 = F$$

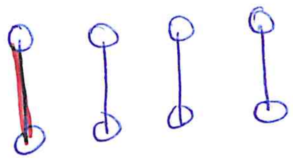
$$\Delta l_1 = \Delta l_2 = \Delta l_3 = \Delta l_3 = \Delta l$$

Stato 1: limite elastica

$$N_1 = \sigma_s A \quad , \quad F_{1c} = 4\sigma_s A, \quad \Delta l = \frac{\sigma_s l}{E}$$

Stato 2: $F > F_{1c}$ stato elasto-plastico

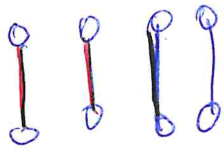
$$N_2 = 2\sigma_s A$$



$$F_{2c} = \sigma_s A + 2\sigma_s A + 2\sigma_s A + 2\sigma_s A$$
$$= 7\sigma_s A$$

Stato 3: elasto-plastico: $F > F_{2c} = 7\frac{\sigma_s A}{E}$

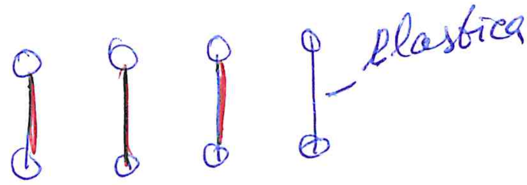
$$N_3 = 3\sigma_s A$$



$$F_{3c} = \underbrace{\sigma_s A}_{N_1} + \underbrace{2\sigma_s A}_{N_2} + \underbrace{3\sigma_s A}_{N_3} + \underbrace{3\sigma_s A}_{N_4}$$

$$F_{3C} = 9\sigma_s A$$

Stato 4 : $F > F_{3c}$



$$F_{4c} = \underset{N_1}{\sigma_s A} + \underset{N_2}{2\sigma_s A} + \underset{N_3}{3\sigma_s A} + \underset{N_4}{4\sigma_s A}, \qquad N_4 = 4\sigma_s A$$

$$F_{4c} = 10\,\sigma_s A$$

Stato 5: il collasso plastico

$$F > F_{4c}$$

$F_{1c} = 4\sigma_s A$

$F_{2c} = 7\sigma_s A$

$F_{3c} = 9\sigma_s A$

$F_{4c} = 10\sigma_s A$