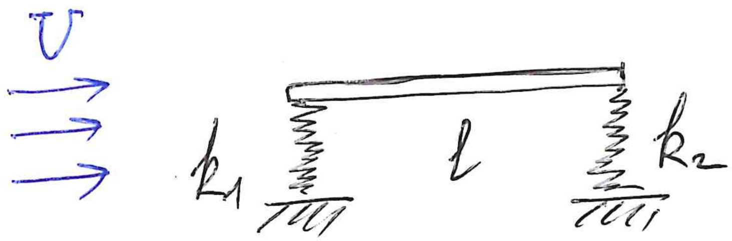
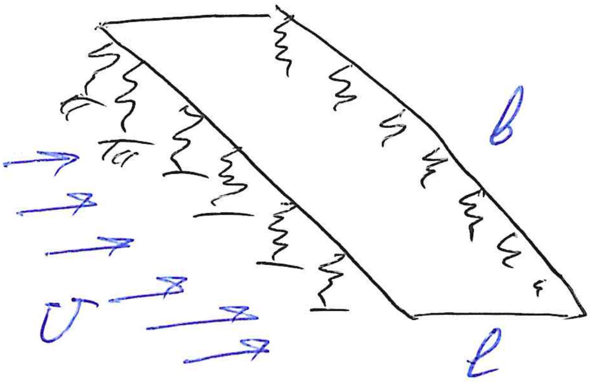
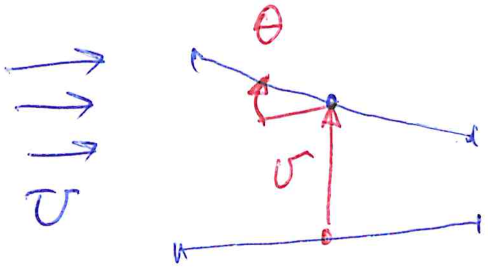
Il flutter





U_c

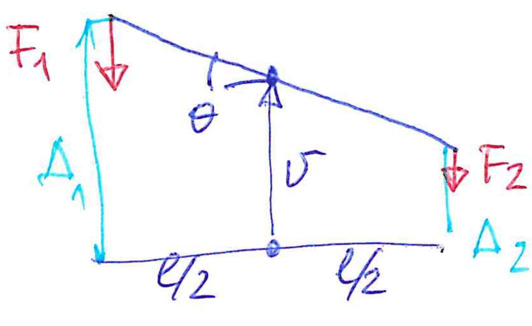


$$\begin{cases} v = v(t) \\ \theta = \theta(t) \end{cases}$$

1. Stazionaria: v, θ = cost

$v = 0$ e $\theta = 0$ — banale

$v \neq 0$, $\boxed{\theta \neq 0}$ — non-banale ?



$$F_1 = k_1 \Delta_1$$

$$F_2 = k_2 \Delta_2$$

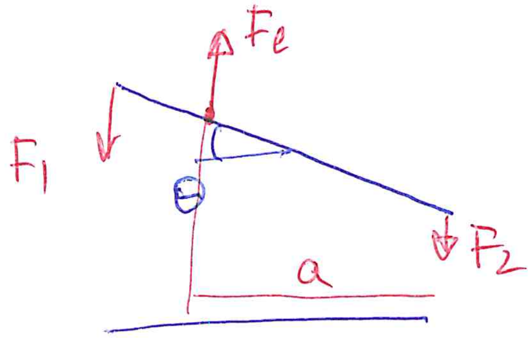
$\boxed{\theta \ll 1}$ — $\sin\theta \sim \theta$, $\tan\theta \sim \theta$

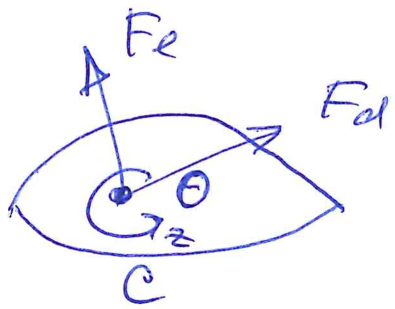
$$\Delta_1 = v + \frac{l}{2}\theta$$

$$\Delta_2 = v - \frac{l}{2}\theta$$

$$F_1 = k_1 A_1 = k_1 \left(v + \frac{l}{2}\theta \right)$$

$$F_2 = k_2 A_2 = k_2 \left(v - \frac{l}{2}\theta \right)$$





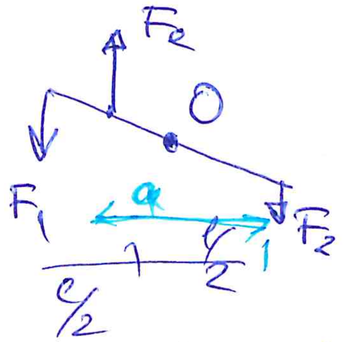
θ?: $F_e, C_z \rightarrow F_e$

$$F_e = \frac{1}{2}\rho v^2 c_e' l \theta$$

l'equilibrio:

$\Sigma F = 0 \quad F_e - F_1 - F_2 = 0$

$\Sigma M = 0 \quad F_1 \frac{l}{2} - F_2 \frac{l}{2} - F_e \left(a - \frac{l}{2} \right) = 0$



$\cos\theta \approx 1$

$$\begin{cases} \frac{1}{2}\rho v^2 c_e' l \theta - k_1 \left(v + \frac{l}{2}\theta \right) - k_2 \left(v - \frac{l}{2}\theta \right) = 0 \\ -\frac{1}{2}\rho v^2 c_e' l \theta \left(a - \frac{l}{2} \right) + k_1 \left(v + \frac{l}{2}\theta \right)\frac{l}{2} - k_2 \left(v - \frac{l}{2}\theta \right)\frac{l}{2} = 0 \end{cases}$$

$$-(k_1+k_2)v+\left[\frac{1}{2}\rho U^2 C_e' l - k_1\frac{l}{2}+k_2\frac{l}{2}\right]\theta=0$$

$$\frac{l}{2}(k_1-k_2)v+\left[-\frac{1}{2}\rho U^2 C_e' l\left(a-\frac{l}{2}\right)+k_1\frac{l^2}{4}+k_2\frac{l^2}{4}\right]\theta=0$$

$$A\cdot\begin{pmatrix} v \\ \theta \end{pmatrix}=0$$

$\Rightarrow$ 1. triviale

$v=0$

$\theta=0$

2. non-banale $\quad \theta\neq0,\ v\neq0$

$\det A=0$

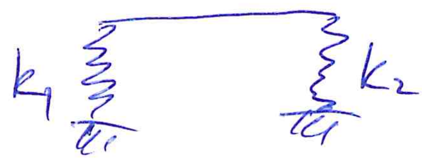
$$\begin{vmatrix} -(k_1+k_2) & \alpha-k_1\frac{l}{2}+k_2\frac{l}{2} \\ k_1-k_2 & -\alpha\left(a-\frac{l}{2}\right)+k_1\frac{l^2}{4}+k_2\frac{l^2}{4} \end{vmatrix}=0$$

dove $\alpha=\frac{1}{2}\rho U^2 C_e' l$

$$-(k_1+k_2)\left[-\alpha\left(a-\frac{l}{2}\right)+k_1\frac{l^2}{4}+k_2\frac{l^2}{4}\right]-$$
$$-(k_1-k_2)\left[\alpha-k_1\frac{l}{2}+k_2\frac{l}{2}\right]\frac{l}{2}=0$$

per $U=U_c$ - ?

a) $k_1 = k_2 = k$



$$\alpha\left(a - \frac{\ell}{2}\right) = (k_1 + k_2)\frac{\ell^2}{4}$$

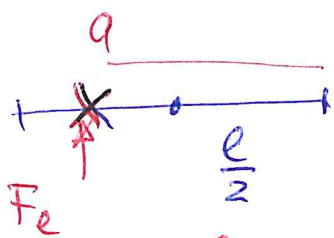
$$\alpha = \frac{2k\,\ell^2/4}{a - \ell/2} \quad ; \quad \alpha = \frac{1}{2}\rho v^2 C_e' \ell$$

$$v^2 = \frac{k\ell^2}{\rho C_e' \ell (a - \ell/2)} = \frac{k\ell}{\rho C_e' \left(a - \frac{\ell}{2}\right)}$$

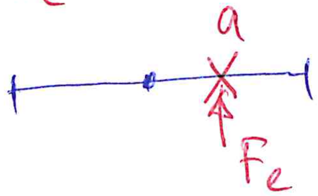
$$\boxed{v_c = \sqrt{\frac{k\ell}{\rho C_e' (a - \ell/2)}}}$$

$$C_e'\left(a - \frac{\ell}{2}\right) > 0$$

$C_e' > 0 \rightarrow a - \frac{\ell}{2} > 0$

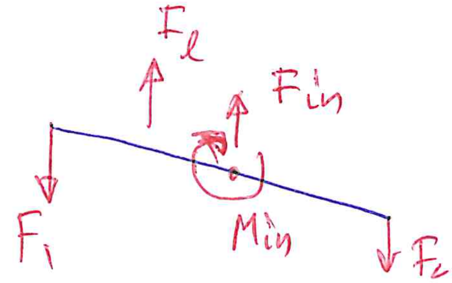


$C_e' < 0 \rightarrow a - \frac{\ell}{2} < 0$

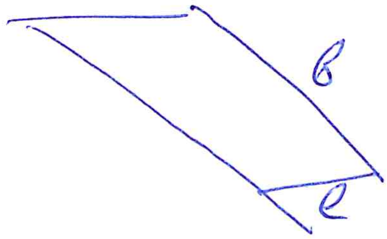


2. Quasi-Stacionaria

F_{in} , M_{in}



m per area



$$\begin{cases} m b l \ddot{v} = F_l - F_1 - F_2 \\ \dfrac{m b l^3}{12} \ddot{\theta} = F_l\left(a - \dfrac{l}{2}\right) - F_1 \dfrac{l}{2} + F_2 \dfrac{l}{2} \end{cases}$$

moto

$$\begin{cases} m b l \ddot{v} = \alpha \theta - k_1 \left(v + \dfrac{l}{2}\theta\right) - k_2 \left(v - \dfrac{l}{2}\theta\right) \\ \dfrac{m b l^3}{12} \ddot{\theta} = \alpha \theta \left(a - \dfrac{l}{2}\right) - k_1 \dfrac{l}{2}\left(v + \dfrac{l}{2}\theta\right) + \\ \qquad + k_2 \dfrac{l}{2}\left(v - \dfrac{l}{2}\theta\right) \end{cases}$$

$$v(t) = V e^{i\omega t}$$
$$\theta(t) = T e^{i\omega t}$$

$$\ddot{v} + a_{11} v + a_{12} \theta = 0$$

$$\ddot{\theta} + a_{21} v + a_{22} \theta = 0$$

dove

$$a_{11} = \frac{k_1 + k_2}{m b l}$$

$$a_{12} = -\frac{\alpha - k_1 \frac{l}{2} + k_2 \frac{l}{2}}{m b l}$$

$$a_{21} = \frac{(k_1 - k_2) l/2}{m b l^3/12}$$

$$a_{22} = -\frac{\alpha (a - l/2) - k_1 \frac{l^2}{4} - k_2 \frac{l^2}{4}}{m b l^3/12}$$

$$-\omega^2 V + a_{11} V + a_{12} T = 0$$

$$-\omega^2 T + a_{21} V + a_{22} T = 0$$

per V e T :

$$\begin{pmatrix} -\omega^2 + a_{11} & a_{12} \\ a_{21} & a_{22} - \omega^2 \end{pmatrix} \begin{pmatrix} V \\ T \end{pmatrix} = 0$$

$$(a_{11}-\omega^2)(a_{22}-\omega^2)-a_{12}a_{21}=0$$

$$\omega^4-(a_{11}+a_{22})\omega^2+a_{11}a_{22}-a_{12}a_{21}=0$$

$$\omega^2=\frac{a_{11}+a_{22}\pm\sqrt{(a_{11}+a_{22})^2-4(a_{11}a_{22}-a_{12}a_{21})}}{2}$$

$\omega^2>0$ reale ω

$\boxed{\omega=0}$

Quando $\omega \to \omega_0+i\eta$

$v=V e^{i\omega_0 t} e^{\pm\eta t}$

↓

instabilità

1. $(a_{11}+a_{22})^2-4(a_{11}a_{22}-a_{12}a_{21})>0$
   $\Rightarrow \omega^2$ è reale

2. $a_{11}a_{22}-a_{12}a_{21}>0$ per $\omega^2>0$

v_c :

$$a_{11} a_{22} - a_{12} a_{21} = 0$$

$$(a_{11} + a_{22})^2 - 4 (a_{11} a_{22} - a_{12} a_{21}) = 0$$

$$a_{11} a_{22} - a_{12} a_{21} = 0$$

$$\frac{k_1 + k_2}{m b l} \left[- \frac{\alpha (a - \frac{l}{2}) - k_1 \frac{l^2}{4} - k_2 \frac{l^2}{4}}{m b l^3 / 12} \right]$$

$$+ \frac{(\alpha - k_1 \frac{l}{2} + k_2 l/2)}{m b l} \frac{(k_1 - k_2) \, l/2}{m b l^3 12} = 0$$

$$(k_1 + k_2) \left(-\alpha (a - \tfrac{l}{2}) + (k_1 + k_2) \frac{l^2}{4} \right)$$

$$+ \left(\alpha + (k_2 - k_1) \frac{l}{2} \right) (k_1 - k_2) \frac{l}{2} = 0$$

$$\alpha \left[-(k_1 + k_2)(a - \tfrac{l}{2}) + (k_2 - k_1) \frac{l}{2} \right]$$

$$+ (k_1 + k_2)^2 \frac{l^2}{4} - (k_2 - k_1)^2 \frac{l^2}{4} = 0$$

$\alpha = \dots$ $$v_c = \sqrt{\frac{2 \alpha}{\rho c_l' l}}$$

Quando $a_{11} a_{22} - a_{12} a_{21} = 0$

$$\omega^4 - (a_{11} + a_{22})\,\omega^2 = 0 \Rightarrow \omega^2 = 0$$

$$+ \quad \omega^2 = a_{11} + a_{22}$$

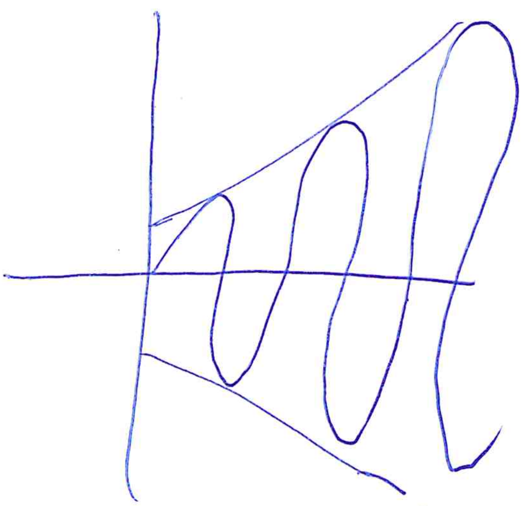
$$(a_{11} + a_{22})^2 - 4\,(a_{11} a_{22} - a_{12} a_{21}) = 0$$

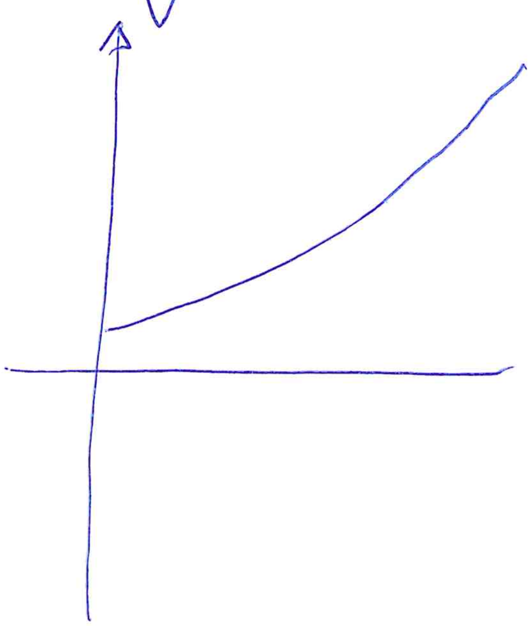
$$a_{11}^2 + 2\,a_{11} a_{22} + a_{22}^2 - 4\,a_{11} a_{22} + 4\,a_{12} a_{21} = 0$$

$$(a_{11} - a_{22})^2 + 4\,a_{12} a_{21} = 0$$

$V_c \Rightarrow \omega^2 = a_{11} + a_{22}$

ú $\omega \to \omega_0 + i\eta$ $\qquad$ $\omega \to i\eta$





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