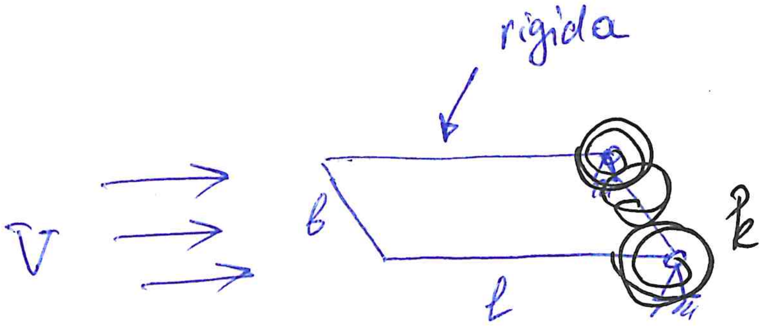
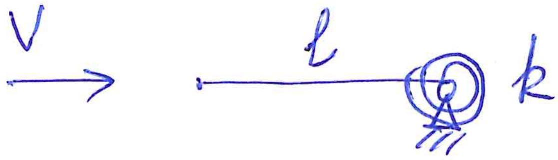
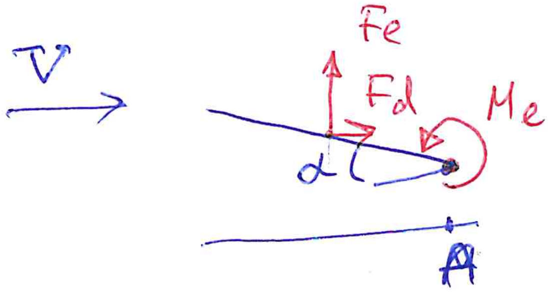
Esempio 1'

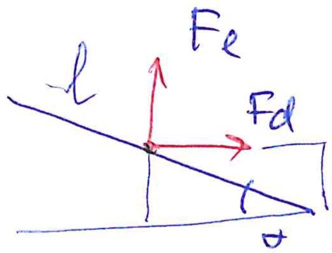
$\boxed{V_c - ?}$







$M_e = k\alpha$



$$M_A: \quad F_e \cdot \frac{l}{2}\cos\alpha + F_d \frac{l}{2}\sin\alpha - M_e = 0$$

$$F_e = \frac{1}{2}\rho_a V^2 l C_l \,, \quad F_d = \frac{1}{2}\rho_a V^2 l C_d$$

$$C_l = C_e'\alpha \,, \quad C_d = C_d'\alpha$$

$$\Rightarrow \underline{F_e = \frac{1}{2}\rho V^2 l C_e'\alpha} \,, \quad \underline{F_d = \frac{1}{2}\rho_a V^2 l C_d'\alpha}$$

$$F_l \frac{l}{2} \cos\alpha + F_d \sin\alpha \frac{l}{2} - M_e = 0$$

$$\alpha = 0 \quad \Rightarrow \quad \cos\alpha = 1, \quad \sin\alpha = \alpha, \ldots$$

$$\frac{1}{2}\rho_a V^2 l C_l' \alpha \frac{l}{2} + \cancel{\frac{1}{2}\rho_a V^2 l C_a' \alpha^2 \frac{l}{2}} - k\alpha = 0$$

$$\alpha^2$$

$$\frac{1}{2}\rho_a V^2 \frac{l^2}{2} C_l' \alpha - k\alpha = 0$$

$$\left[\frac{\rho_a V^2 l^2 C_l'}{4} - k\right]\alpha = 0$$

$\Rightarrow$ 1. $\alpha = 0$ banale

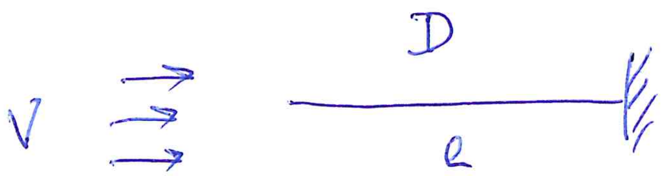
2. $\alpha \neq 0$ non-banale

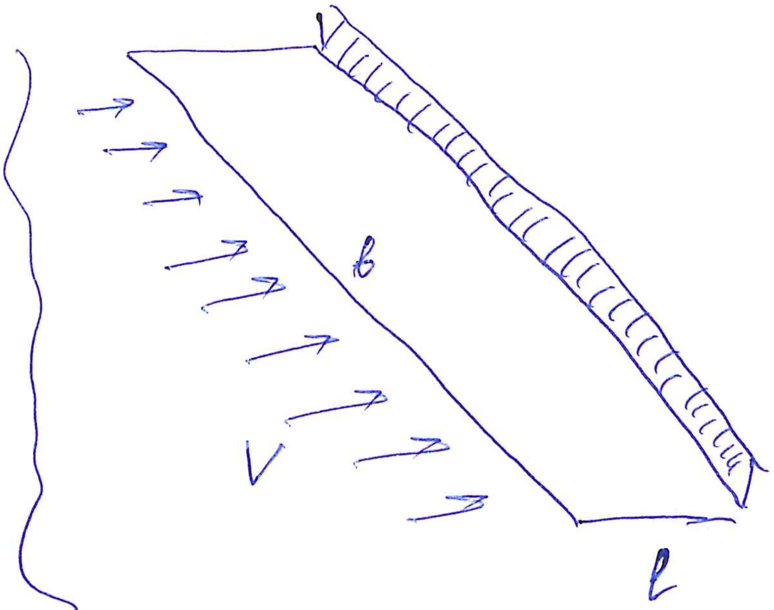
$$\frac{\rho_a V^2 l^2 C_l'}{4} - k = 0$$

$$V^2 = \frac{4k}{\rho_a l^2 C_l'}$$

$$V = V_c \equiv 2\sqrt{\frac{k}{\rho_a l^2 C_l'}}$$

Esempio 2



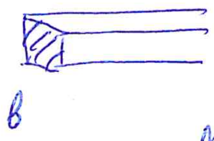


$$D = \frac{E h^3}{12(1-\nu^2)}$$

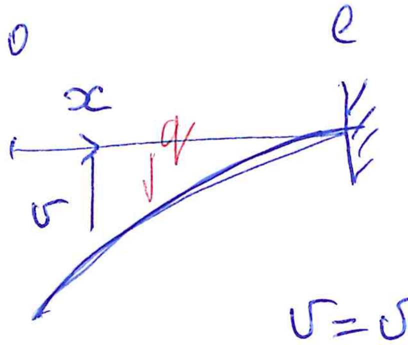
$EI \neq D$

$EI = D, \quad \nu = 0, \quad b = 1$

EI

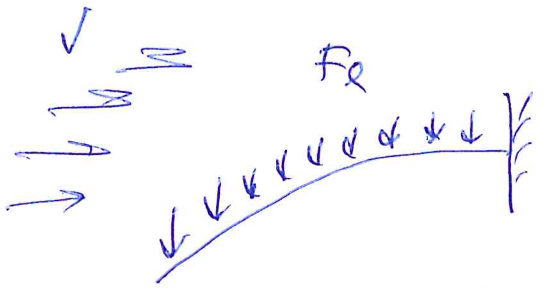


$$I = \frac{\cancel{b} h^3}{12}$$



$v = v(x)$

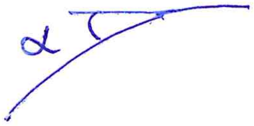
$$\boxed{D\, v^{(4)} = q}$$



$q = -C v'$

$C = \frac{1}{2} \rho_a V^2 \ell\, C_\ell'(\alpha)$

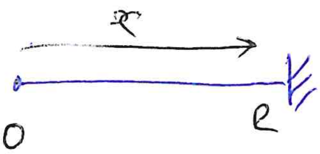
$F_\ell \sim v'$



$\alpha \simeq v'$

$$\boxed{D\, v^{(4)} = -C v'}$$

$D \sim EJ$



$$v^{(4)} + s^3 v' = 0$$

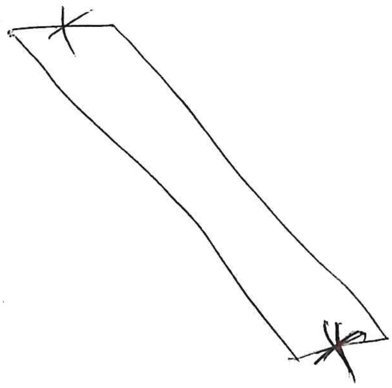
$$s = \sqrt[3]{\frac{C}{D}}$$

$$\left[\begin{array}{ll} v''(0)=0 & v(\ell)=0 \\ v'''(0)=0 & v'(\ell)=0 \end{array}\right.$$

le condizione al contorno

$$\underline{v = v_0 e^{\lambda x}}$$

$$\cancel{v_0} \lambda^4 \cancel{e^{\lambda x}} + s^3 \lambda \cancel{v_0} \cancel{e^{\lambda x}} = 0$$



$$\boxed{\lambda^4 + s^3 \lambda = 0}$$

$$\lambda(\lambda^3 + s^3) = 0 \quad \Rightarrow \quad \underline{\lambda = 0} \; !$$

$$\lambda^3 + s^3 = 0 \qquad \underline{\lambda = -s}$$

$$(-s)^3 + s^3 = -s^3 + s^3 = 0$$

$$\underline{\lambda^3 + s^3 = 0}$$

$$(\lambda^3 + s^3) = (\lambda + s)(\lambda^2 - \lambda s + s^2)$$

$$\lambda^2 - \lambda s + s^2 = 0$$

$$\begin{array}{l|l} \lambda^3 + s^3 & \lambda + s \\ \hline \lambda^3 + \lambda^2 s & \lambda^2 - \lambda s + s^2 \\ \hline -\lambda^2 s + s^3 & \\ -\lambda^2 s - \lambda s^2 & \\ \hline s^3 + \lambda s^2 & \\ \lambda s^2 + s^3 & \\ \hline \end{array}$$

$$\lambda^2 - \lambda s + s^2 = 0$$

$$\lambda_\pm = \frac{s \pm \sqrt{s^2 - 4s^2}}{2} = \frac{s \pm \sqrt{\cancel{s^2} - 3s^2}}{2}$$

$$\lambda_\pm = \frac{s \pm s\sqrt{-3}}{2} = \frac{s \pm s\, i\sqrt{3}}{2}$$

$$\Rightarrow \boxed{\begin{aligned} &\lambda_1 = 0, \quad \lambda_2 = -s \\ &\lambda_3 = \frac{s}{2} + is\frac{\sqrt{3}}{2} \\ &\lambda_4 = \frac{s}{2} - is\frac{\sqrt{3}}{2} \end{aligned}}$$

$$v(x) \sim e^{\lambda x} \qquad \lambda_1 = 0 \qquad v_1 = c_1$$

$$\lambda_2 = -s \qquad v_2 = c_2 e^{-sx}$$

$$\lambda_3 = \frac{s}{2} + is\frac{\sqrt{3}}{2} \qquad v_{3,4} = e^{s/2x}\left[c_3 \sin\left(\frac{\sqrt{3}}{2}xs\right) + \right.$$

$$\lambda_4 = \frac{s}{2} - is\frac{\sqrt{3}}{2} \qquad \left. + c_4 \cos\left(\frac{\sqrt{3}}{2}xs\right)\right]$$

$$\Rightarrow \boxed{\begin{aligned} v(x) &= c_1 + c_2 e^{-sx} + \\ &+ c_3 e^{s/2x} \sin\frac{\sqrt{3}}{2}sx + c_4 e^{\frac{s}{2}x} \cos\frac{\sqrt{3}}{2}sx \end{aligned}}$$

$$v'(x) = -sC_2 e^{-sx}$$

$$+ C_3 \left[\frac{s}{2} e^{\frac{s}{2}x} \sin\frac{\sqrt{3}}{2}sx + \frac{\sqrt{3}}{2}s\, e^{\frac{s}{2}x}\cos\frac{\sqrt{3}}{2}sx \right]$$

$$+ C_4 \left[\frac{s}{2} e^{\frac{s}{2}x} \cos\frac{\sqrt{3}}{2}sx - \frac{\sqrt{3}}{2}s\, e^{\frac{s}{2}x}\sin\frac{\sqrt{3}}{2}sx \right];$$

$$v'' = s^2 C_2 e^{-sx}$$

$$+ C_3 \Big[\frac{s^2}{4} e^{\frac{s}{2}x} \sin\frac{\sqrt{3}}{2}sx + \frac{s}{2} e^{\frac{s}{2}x} \frac{\sqrt{3}}{2}s \cos\frac{\sqrt{3}}{2}sx$$

$$+ \frac{\sqrt{3}}{2}s\,\frac{s}{2} e^{\frac{s}{2}x} \cos\frac{\sqrt{3}}{2}sx + \frac{3}{4}s^2 e^{\frac{s}{2}x}\left(-\sin\frac{\sqrt{3}}{2}sx\right)\Big]$$

$$+ C_4 \Big[\frac{s^2}{4} e^{\frac{s}{2}x} \cos\frac{\sqrt{3}}{2}sx - \frac{s^2}{4}\sqrt{3}\, e^{\frac{s}{2}x} \sin\frac{\sqrt{3}}{2}sx$$

$$- \frac{\sqrt{3}}{4}s^2 e^{s/2\,x} \sin\frac{\sqrt{3}}{2}sx - \frac{3}{4}s^2 e^{s/2\,x} \cos\frac{\sqrt{3}}{2}sx \Big];$$

$$v'''(x) = -s^3 C_2 e^{-sx}$$

$$+ C_3 \Big[\frac{s^3}{8} \cancel{e^{s/2\,x} \sin\frac{\sqrt{3}}{2} sx} + \frac{s^3}{8}\sqrt{3} \cos\frac{\sqrt{3}}{2} sx$$

$$+ \frac{s^3}{8}\sqrt{3}\, e^{\frac{s}{2}x} \cos\frac{\sqrt{3}}{2} sx - \frac{s^3}{8}\, 3\, \cancel{e^{\frac{s}{2}x} \sin\frac{\sqrt{3}}{2} sx}$$

$$+ \frac{\sqrt{3}\, s^3}{8} e^{\frac{s}{2}x} \cos\frac{\sqrt{3}}{2} sx - \frac{3}{8} s^3 e^{s/2\,x} \cancel{\sin\frac{\sqrt{3}}{2} sx}$$

$$- \frac{3}{8} s^3 \cancel{e^{s/2\,x}} \sin\frac{\sqrt{3}}{2} sx - \frac{3\sqrt{3}}{8} s^3 e^{s/2\,x} \cos\frac{\sqrt{3}}{2} sx \Big]$$

$$+ C_4 \Big[\quad \ldots$$

$$v''(0)=0$$

$$\cancel{s^2}C_2 + C_3\left[\frac{\sqrt{3}}{4}\cancel{s^2} + \frac{\sqrt{3}}{4}\cancel{s^2}\right] +$$

$$+ C_4\left[\frac{\cancel{s^2}}{4} - \frac{3}{4}\cancel{s^2}\right] = 0$$

$$v'''=0$$

$$-\cancel{s^3}C_2 + C_3\left[\frac{\sqrt{3}}{8}\cancel{s^3} + \frac{\sqrt{3}}{8}\cancel{s^3} + \frac{\sqrt{3}}{8}\cancel{s^3} - \frac{3\sqrt{3}}{8}\cancel{s^3}\right]$$

$$+ C_4\left[\qquad\qquad\qquad\right] = 0$$

$$v(l) = C_1 + C_2e^{-sl} + C_3e^{s/2\,l}\sin\frac{\sqrt{3}}{2}sl + C_4e^{s/2\,l}\cos\frac{\sqrt{3}}{2}sl$$

$$=0$$

$$v'(l) = -\cancel{s}C_2e^{-sl} + C_3\left[\frac{\cancel{s}}{2}e^{s/2\,l}\sin\frac{\sqrt{3}}{2}sl + \frac{\sqrt{3}}{2}\cancel{s}e^{s/2\,l}\times\right.$$

$$\left.\times\cos\frac{\sqrt{3}}{2}sl\right]$$

$$+ C_4\left[\frac{\cancel{s}}{2}e^{s/2\,l}\cos\frac{\sqrt{3}}{2}sl - \cancel{s}\frac{\sqrt{3}}{2}\cancel{s}e^{\frac{s}{2}l}\sin\frac{\sqrt{3}}{2}sl\right]$$

$$= 0$$

Per c_1, c_2, c_3, c_4

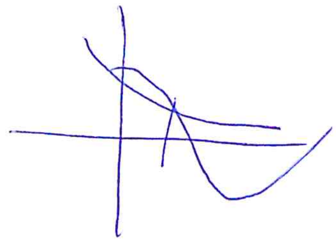
$$A \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = 0$$

$$\Rightarrow \quad \underline{\det A = 0}$$

$$A = \begin{bmatrix} 0 & 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 1 & 1 & 0 \\ 1 & e^{-sl} & e^{+\frac{sl}{2}} \cos \frac{\sqrt{3}}{2} sl & e^{\frac{sl}{2}} \sin \frac{\sqrt{3}}{2} sl \\ 0 & -e^{-sl} & \boxed{e^{\frac{sl}{2}} \left(\cos \frac{\sqrt{3}}{2} sl - \sqrt{3} \sin \frac{\sqrt{3}}{2} sl \right)} & \boxed{e^{\frac{sl}{2}} \left(\sqrt{3} \cos \frac{\sqrt{3}}{2} sl + \sin \frac{\sqrt{3}}{2} sl \right)} \end{bmatrix}$$

$$\det A = \cos \frac{\sqrt{3}}{2} sl + \frac{1}{2} e^{-\frac{3sl}{2}} = 0$$

$$\boxed{sl \approx 1.85}$$



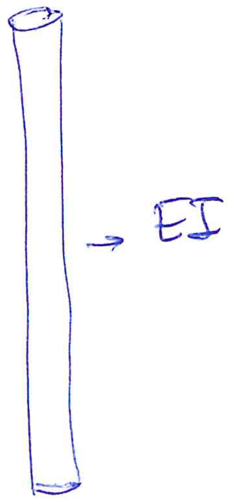
$$S = \sqrt[3]{\frac{C}{D}} \qquad C = \frac{1}{2}\rho_a V^2 \ell C_\ell'$$

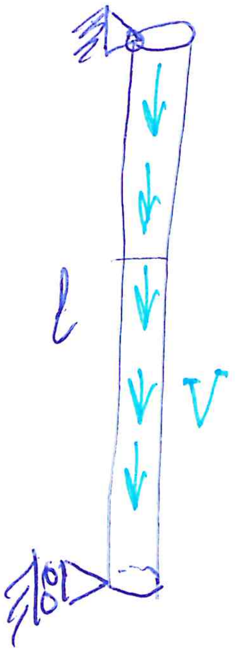
$$S\ell = 1{,}85$$

$$\frac{C}{D} = \left(\frac{1{,}85}{\ell}\right)^3 \qquad \frac{1}{2}\rho_a V^2 \ell C_\ell' = D\,\frac{1{,}85^3}{\ell^3}$$

$$\boxed{V_c = \sqrt{\frac{2 \cdot D \cdot 1{,}85^3}{\ell^3 \rho_a \ell C_\ell'}}}$$

Esempio 3



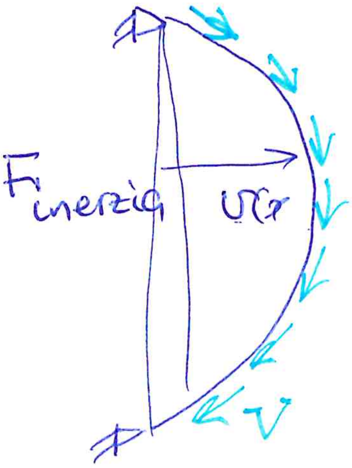


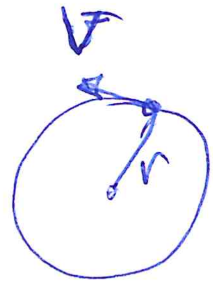
V_c

$$F_{inerzia} = -m\frac{V^2}{r}$$

m

$$\frac{1}{r} \sim v''$$





$$a \sim \frac{V^2}{r}$$

$$F_{inerzia} = -m V^2 v''$$

$$\boxed{EI\,v'''' = -m V^2 v''}$$

$$v^{(4)} + s^2 v'' = 0 \quad , \quad s^2 = \frac{mV^2}{EI}$$

$$v \sim e^{\lambda x}$$

$$\lambda^4 + s^2 \lambda^2 = 0$$

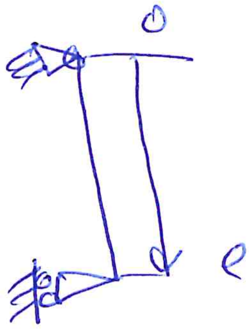
$$\Rightarrow \lambda^2 (\lambda^2 + s^2) = 0$$

$$\lambda = 0 \quad \rightarrow \quad c_1 + c_2 x$$

$$\lambda_\pm = \pm s \quad \rightarrow \quad c_3 \sin sx + c_4 \cos sx$$

$$v(x) = c_1 + c_2 x + c_3 \sin sx + c_4 \cos sx$$

$$v(0)=0, \quad v''(0)=0$$
$$v(l)=0, \quad v''(l)=0$$



l'equazione caratteristico :

$$\sin sl = 0 \quad \Rightarrow sl = \pi$$

$$s = \frac{\pi}{l} \quad \Rightarrow \quad \frac{mV^2}{EI} = \frac{\pi^2}{l^2}$$

$$\boxed{V_c = \frac{\pi}{l} \sqrt{\frac{EI}{m}}}$$

2