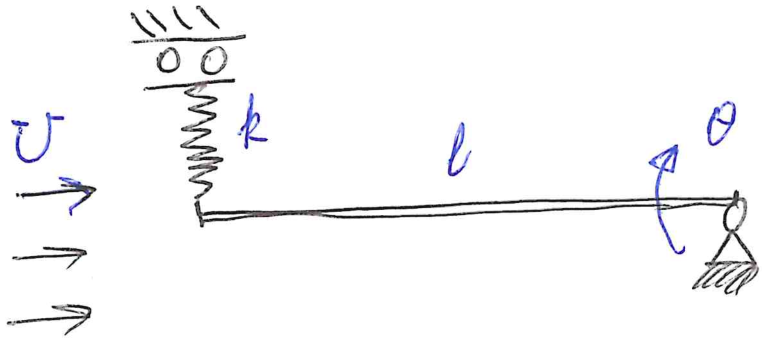
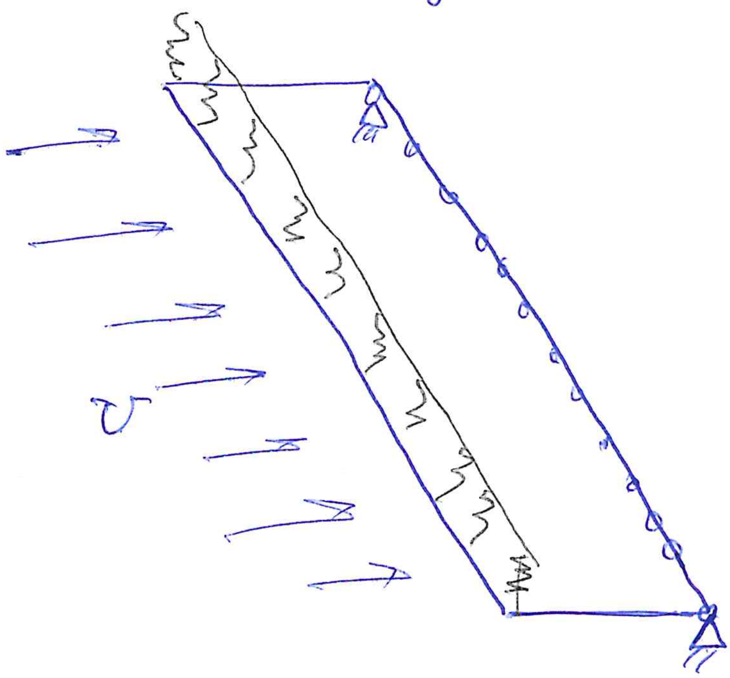
Example 1

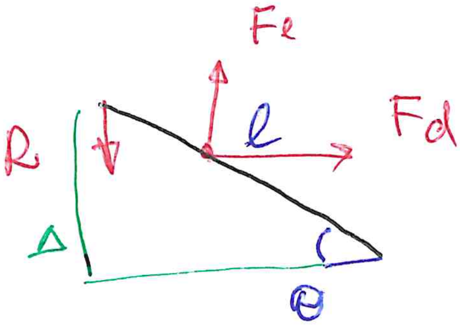


$\boxed{U_c - ?}$

rigid plate



Free body diagram



$$\underline{R = k\Delta = k\,l\sin\theta}$$

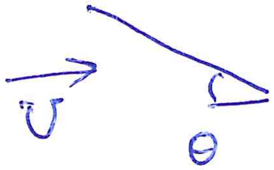
ρ_a - density of air

$$F_d = \underbrace{\tfrac{1}{2}\rho_a U^2}_{\text{kinetic pressure}}\ \underbrace{l}_{\text{characteristic length}}\ C_d$$

C_d – aerodyn. coeff.

$$\underline{F_e = \tfrac{1}{2}\rho_a U^2 l\, C_e}$$

$\theta \approx 0$



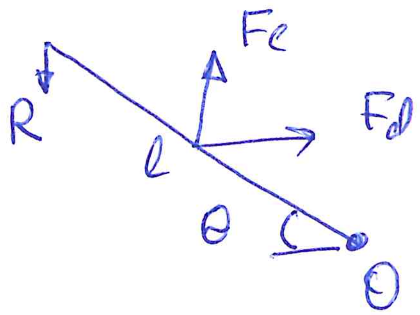
We have symmetry

$$C_d = C_d(\theta) \approx \cancel{C_d^0} + C_d'\,\theta$$
$$C_e = C_e(\theta) \approx \cancel{C_e^0} + C_e'\,\theta$$

θ is the attack angle

Balance of moments

M_O :



$$R\,l\cos\theta - F_e \frac{l}{2}\cos\theta - {}$$
$$- F_d \frac{l}{2}\sin\theta = 0$$

linearization : $\cos\theta \approx 1$, $\sin\theta \approx \theta$

$R = kl\theta$, $F_e = \frac{1}{2}\rho_a U^2 l\, C_e'\theta$

$F_d = \frac{1}{2}\rho_a U^2 l\, C_d'\theta$

Equation :

$$kl^2\theta - F_e\frac{l}{2} - \cancel{F_d\frac{l}{2}\theta} = 0$$

$$kl^2\theta - \frac{1}{2}\rho_a U^2 \frac{l^2}{2} C_e'\theta = 0$$

$$k\ell^2\theta - \frac{1}{4}\rho a v^2 e^2 c_\ell' \theta = 0$$

Solutions :

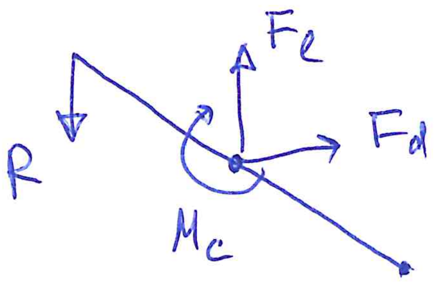
1. $\theta = 0$ trivial

2. $\theta \neq 0$ nontrivial

$$k\ell^2 - \frac{1}{4}\rho a v^2 e^2 c_\ell' = 0$$

$$v^2 = \frac{4k}{\rho a c_\ell'}$$

$$\boxed{v_c = 2\sqrt{\frac{k}{\rho a c_\ell'}}}$$



$$F_d = \frac{1}{2}\rho_a U^2 l \, C_d^0$$

$$M_c = \frac{1}{2}\rho_a U^2 l^2 \, C_m' \theta$$

Balance :

$$k l^2 \theta - \frac{1}{2}\rho_a U^2 \frac{l^2}{2} C_l' \theta - \frac{1}{2}\rho_a U^2 \frac{l^2}{2} C_d^0 \theta$$

$$- \frac{1}{2}\rho_a U^2 l^2 C_m' \theta = 0$$

$$k\theta - \frac{1}{4}\rho_a U^2 \left[C_l' + C_d^0 + 2 C_m' \right] \theta = 0$$

$$U^2 = \frac{4k}{\rho_a \left[C_l' + C_d^0 + 2 C_m' \right]}$$

$$U_c = 2\sqrt{\frac{k}{\rho_a \left[C_l' + C_d^0 + 2 C_m' \right]}}$$