Università degli Studi di Cagliari
Facoltà di Ingegneria e Architettura
Dipartimento di Ingegneria Civile, Ambientale e Architettura

Magistrale Ingegneria Civile 2 anno **a.a. 2022/23**

Structural Stability and Limit Analysis of Structures

(Instabilità delle strutture e calcolo a rottura)

> Lezione 19

Instability of an elastic arc
(Instabilità dell'arco)

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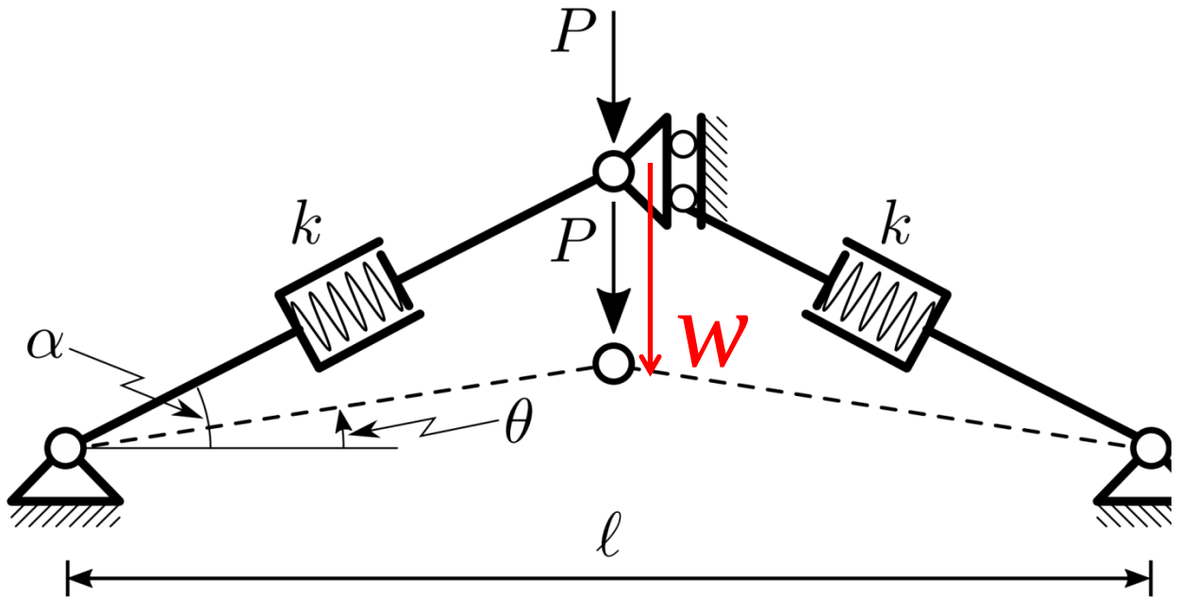
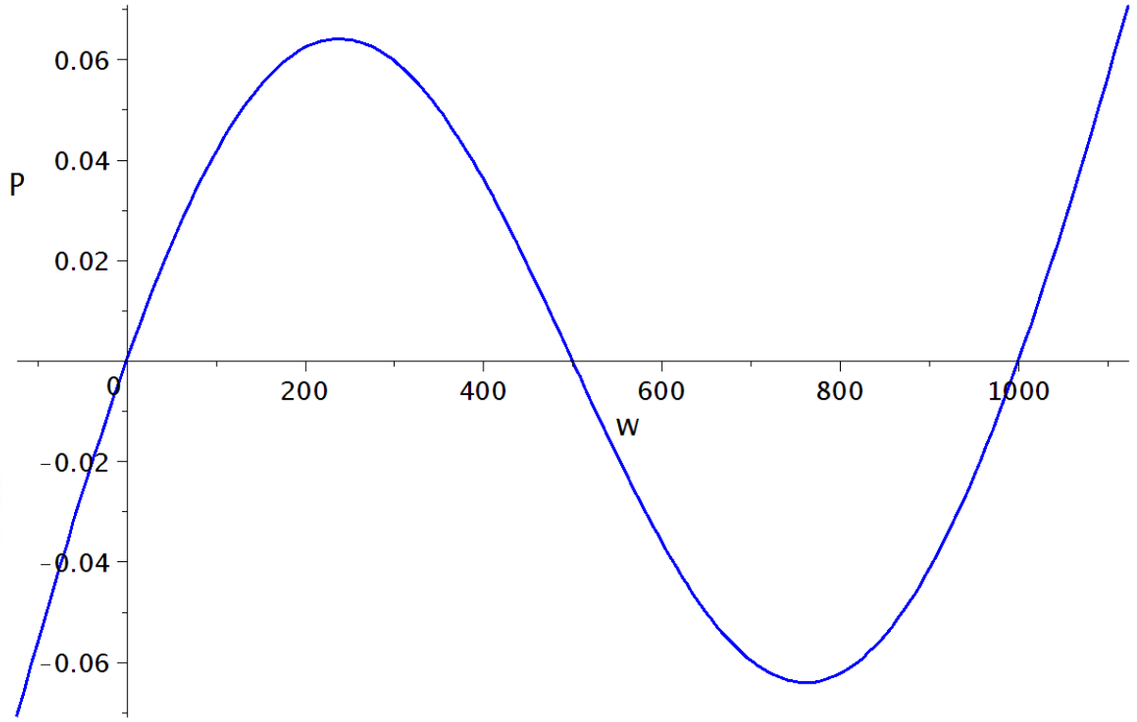
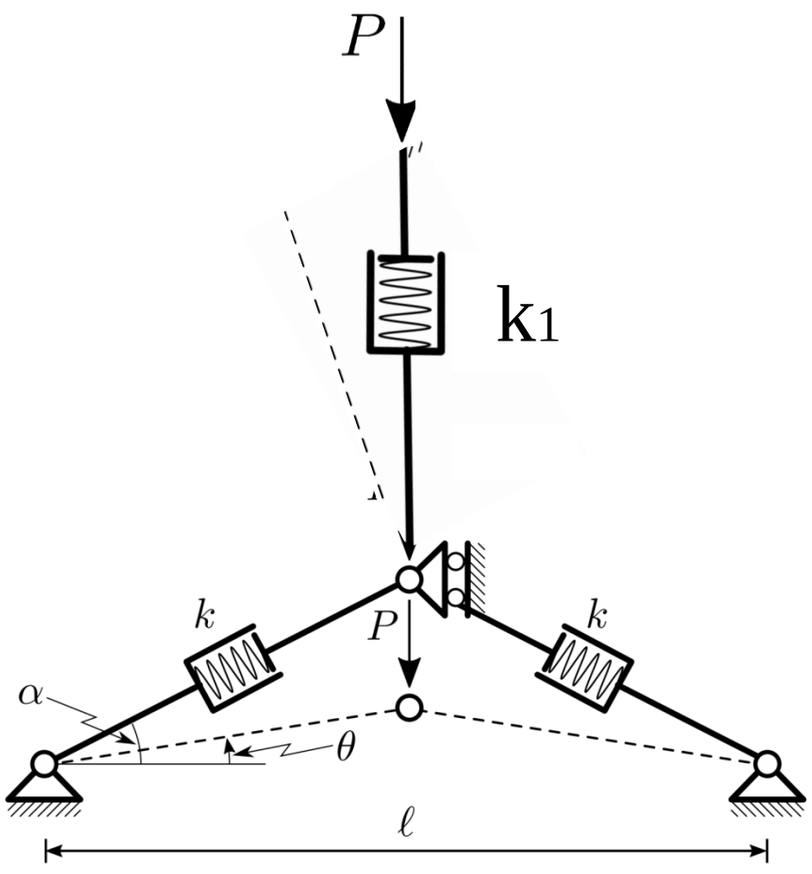
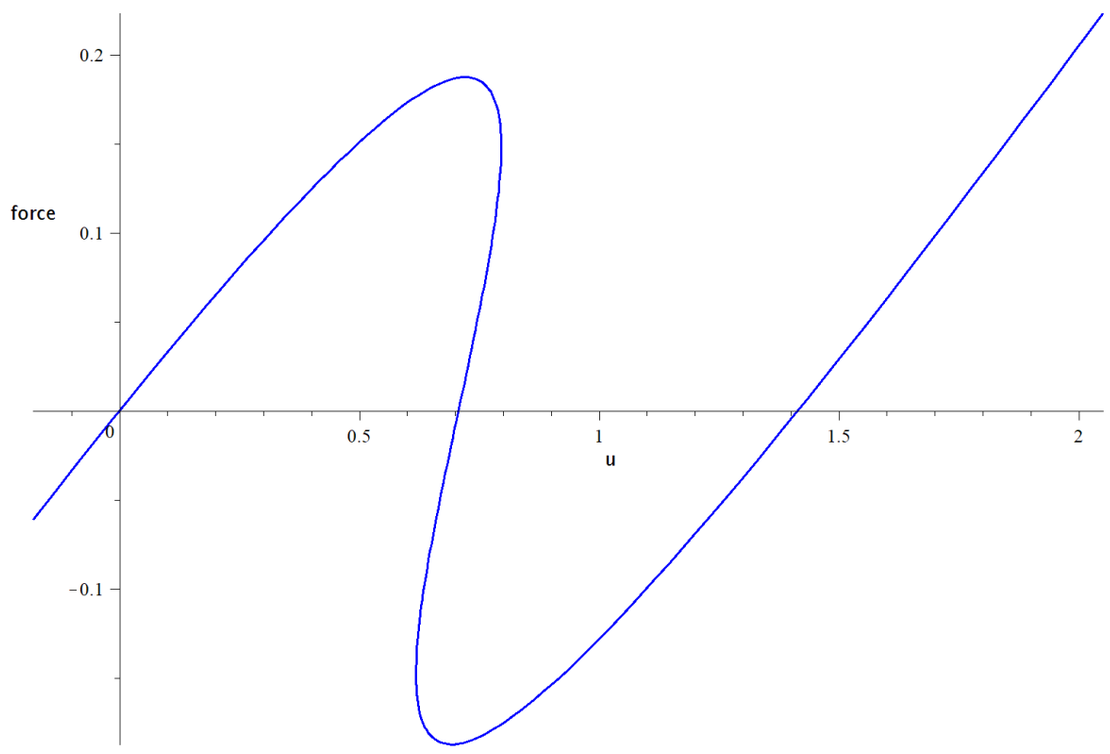
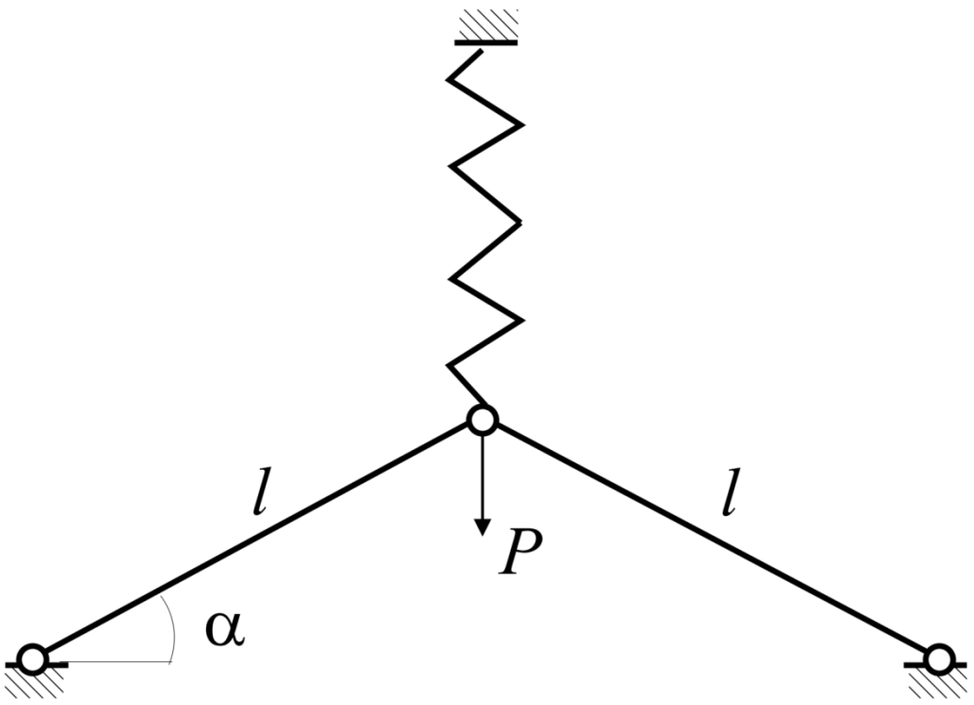
Snap-through and snap-back of von Mises truss

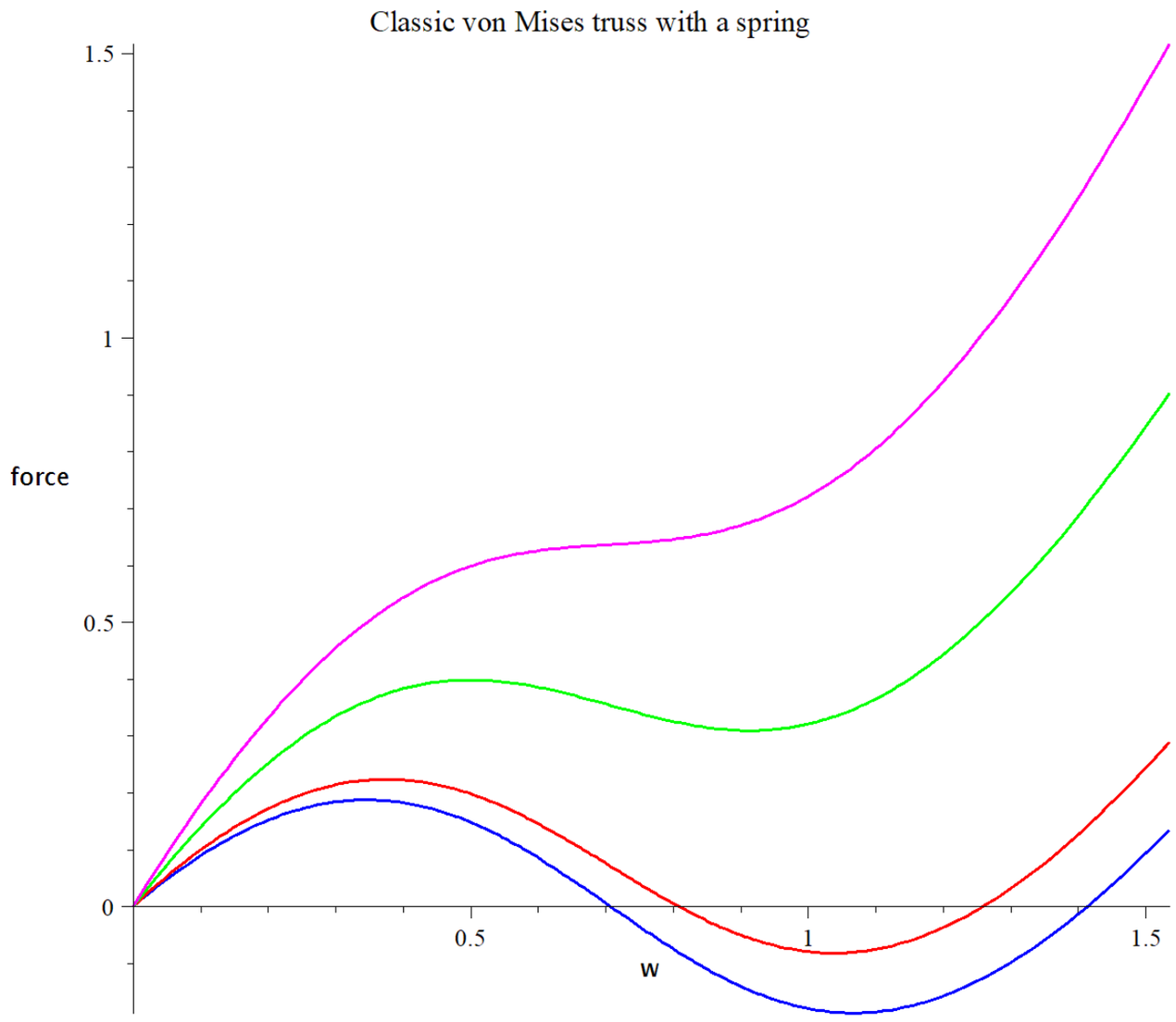




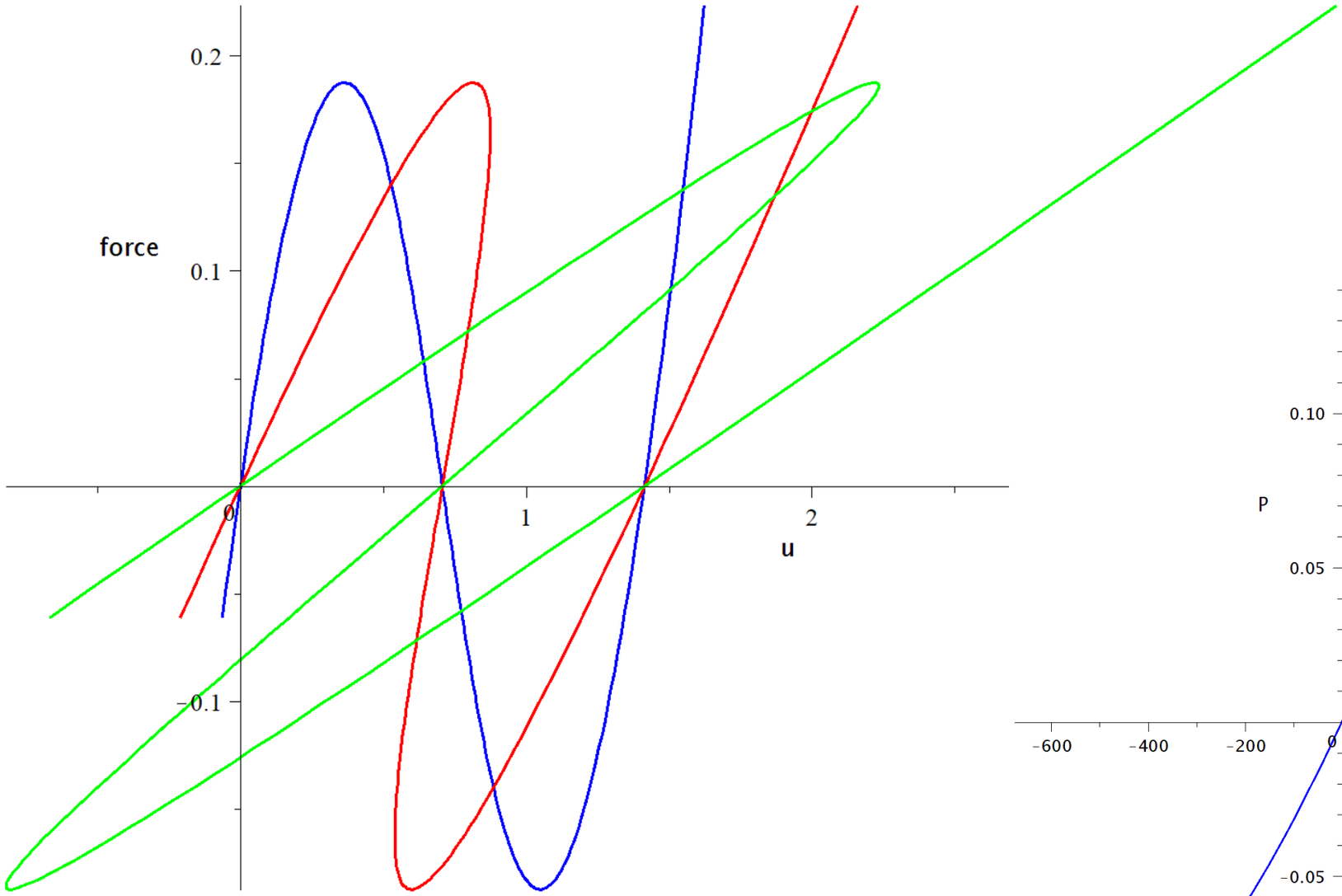


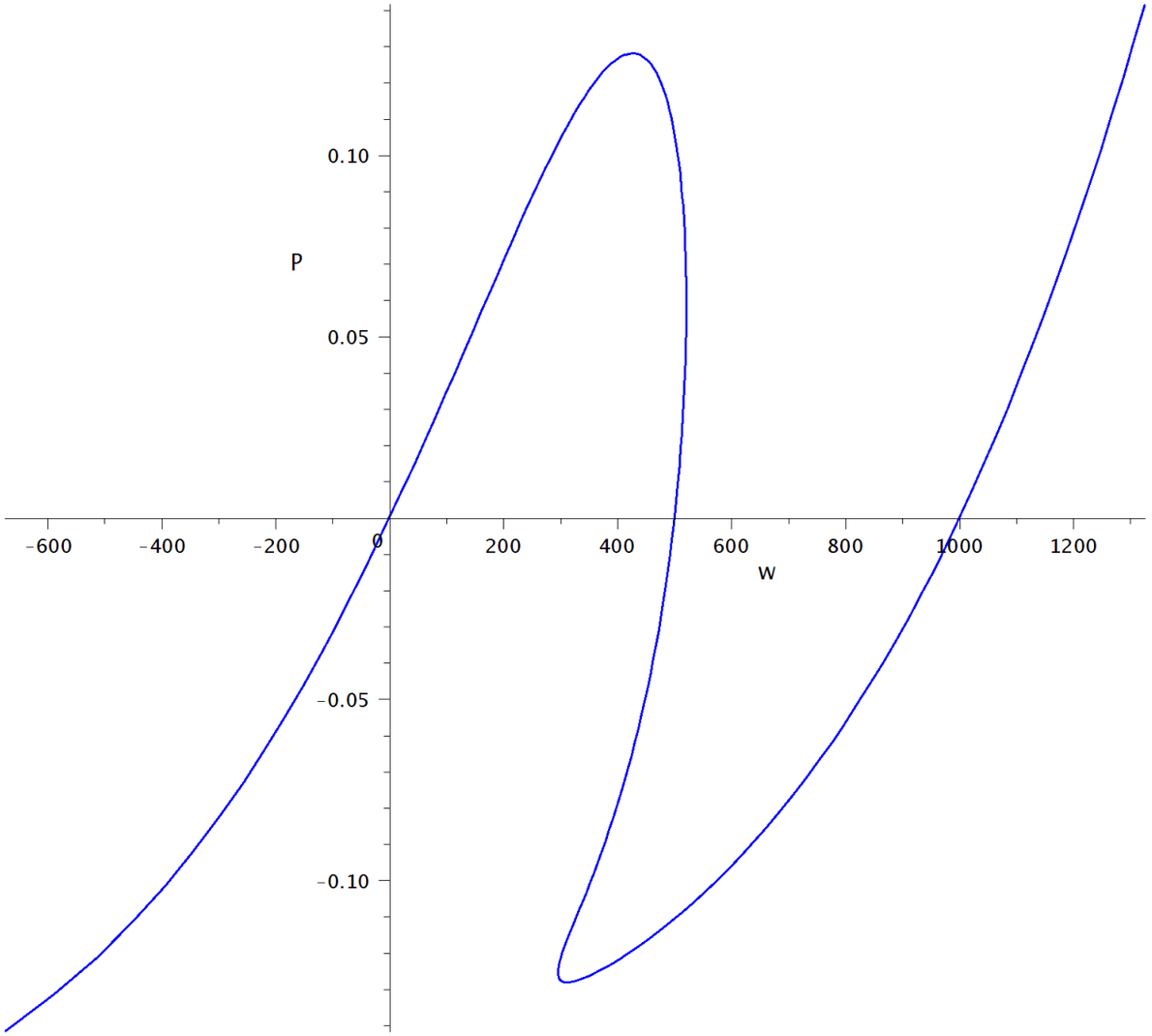



l
l
P
α


Classic von Mises truss with a spring
force
w
1.5
1
0.5
0
0.5
1
1.5

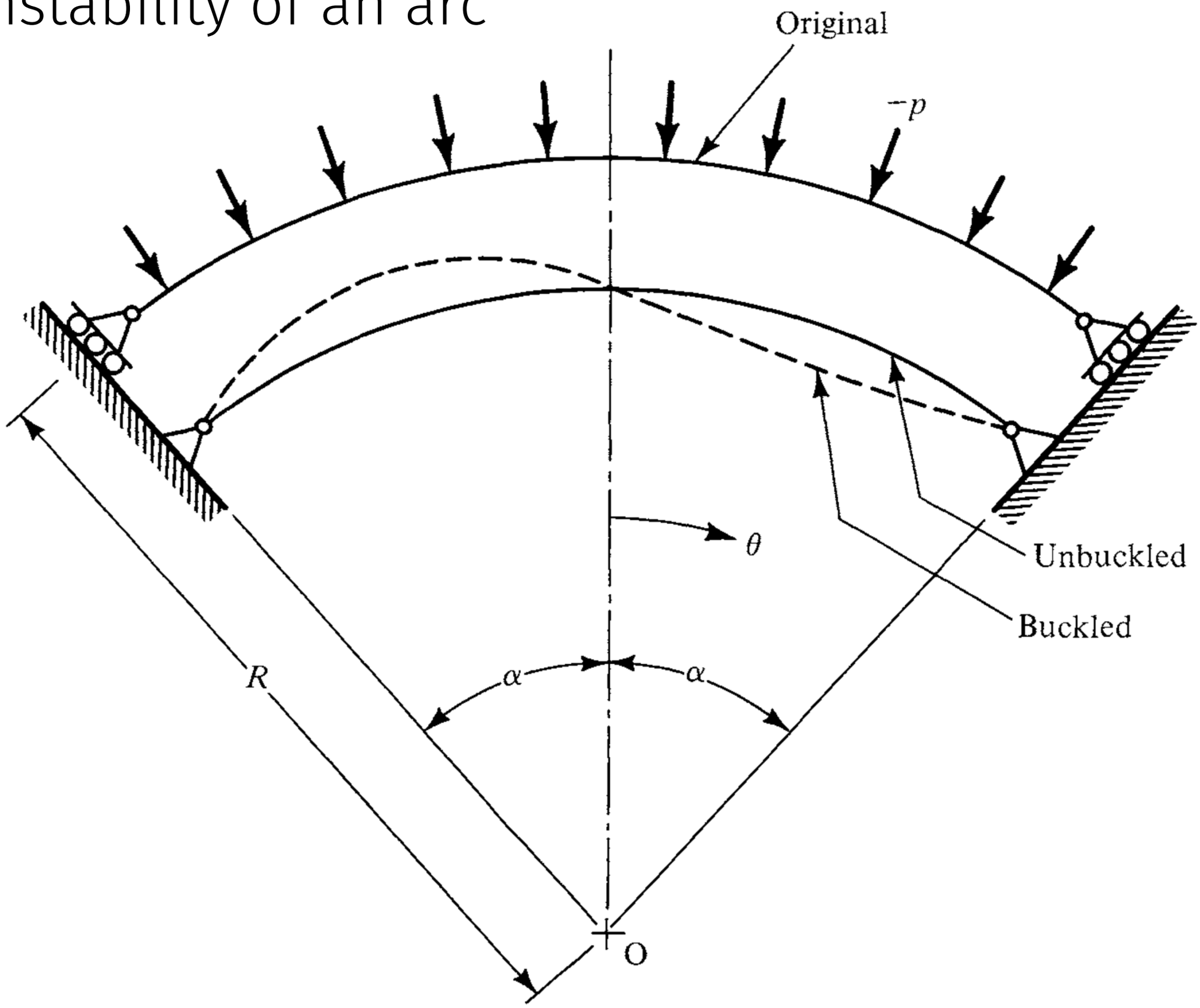
Snap-back of the truss



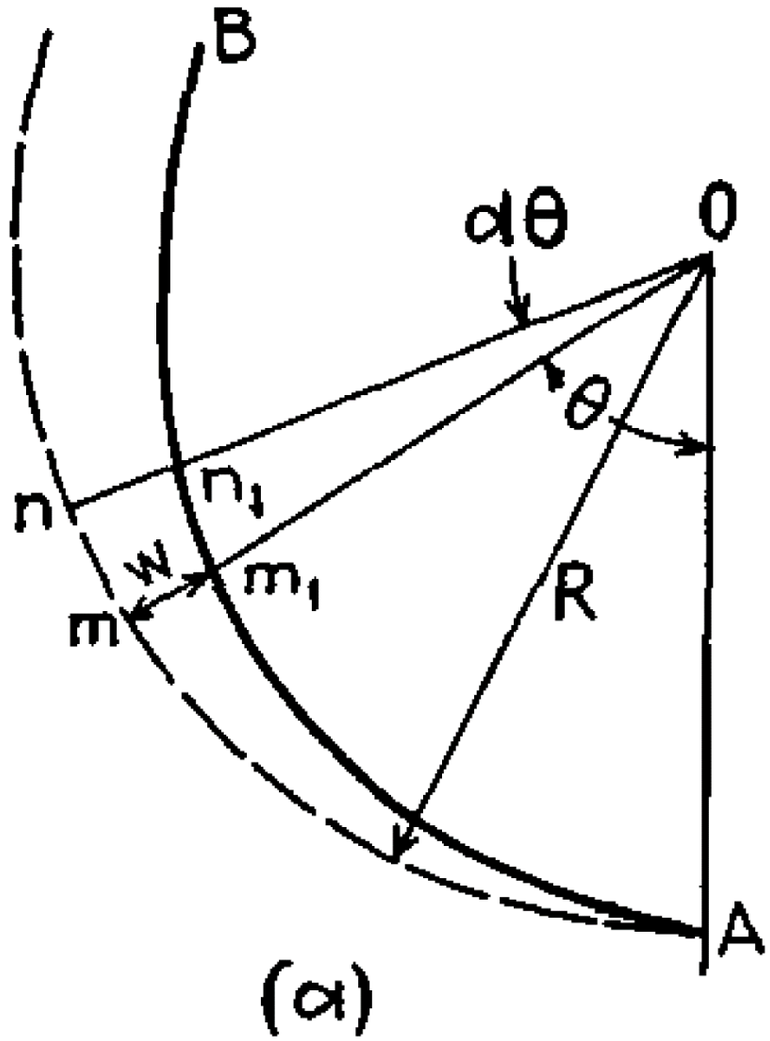


Instability of an elastic arc:
Linearized approach

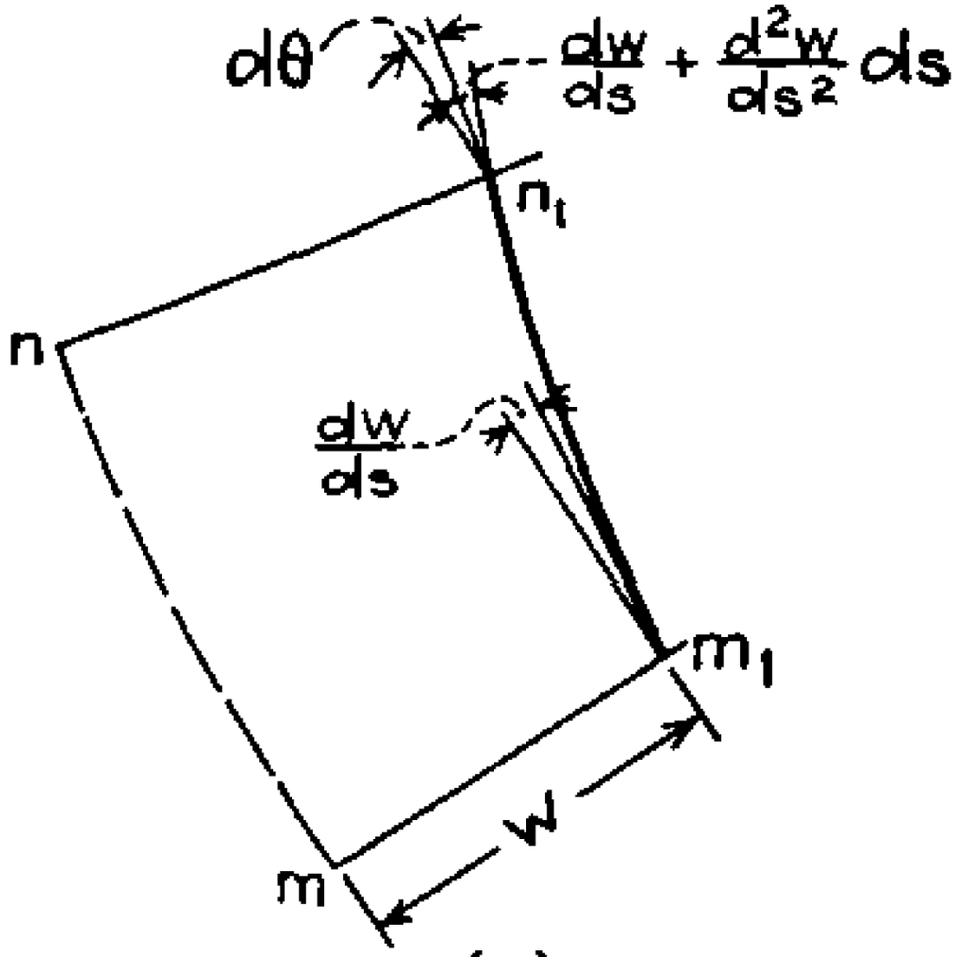
Instability of an arc



Equilibrium equation



(a)



(b)

$$ds = R\,d\theta \qquad \frac{d\theta}{ds} = \frac{1}{R} \qquad \frac{1}{\rho} = \frac{d\theta + \Delta\, d\theta}{ds + \Delta\, ds}$$

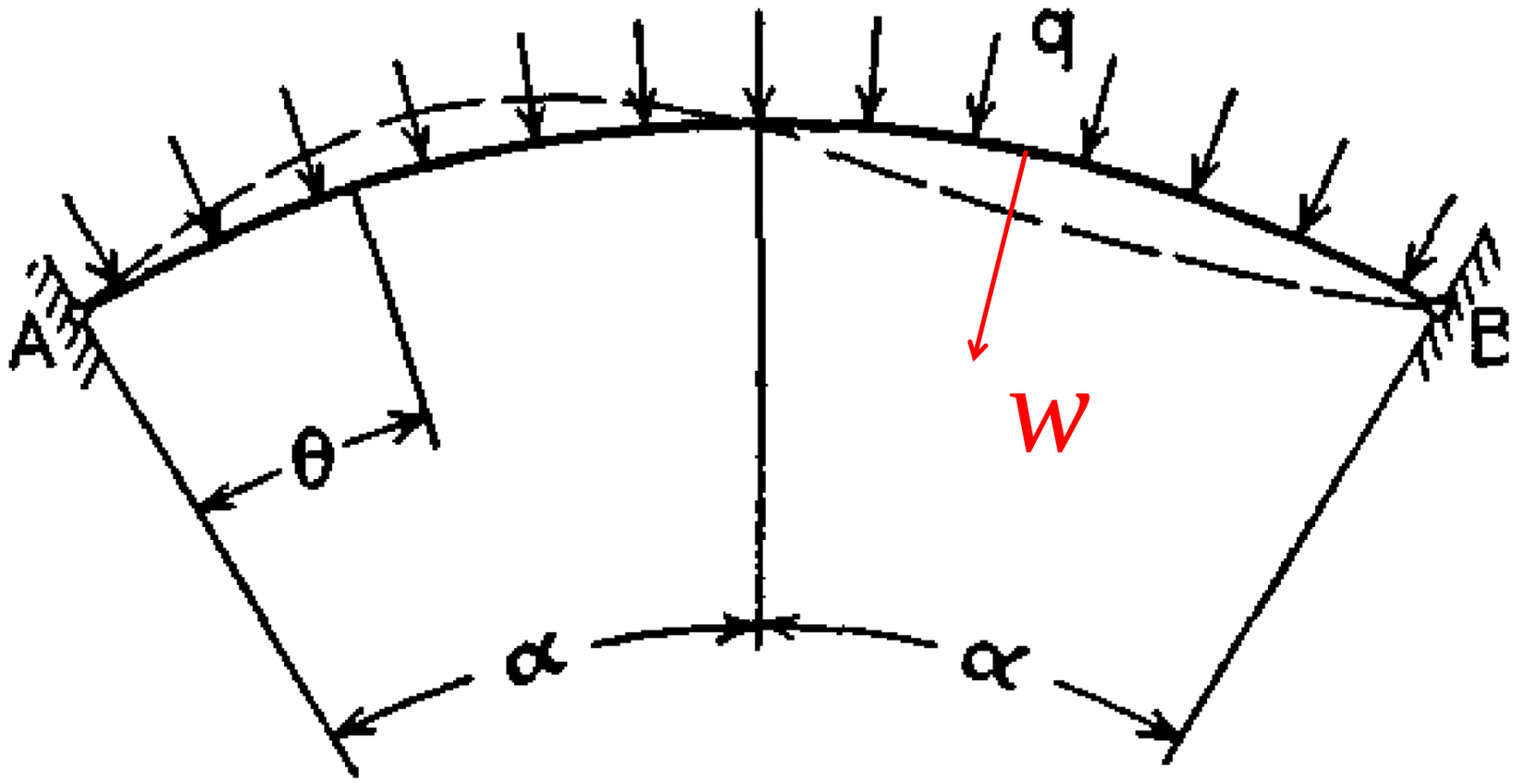
Equilibrium equation

$$EI\left(\frac{1}{\rho}-\frac{1}{R}\right)=-M$$

$$\frac{1}{\rho}=\frac{1}{R}\left(1+\frac{w}{R}\right)+\frac{d^2w}{ds^2}$$

$$\frac{d^2w}{ds^2}+\frac{w}{R^2}=-\frac{M}{EI}$$

$$\frac{d^2w}{d\theta^2}+w=-\frac{MR^2}{EI}$$



$$\frac{d^2w}{d\theta^2} + w = -\frac{R^2Sw}{EI} \qquad S = qR$$

$$\frac{d^2w}{d\theta^2} + w = -\frac{R^2Sw}{EI}$$

$$k^2 = 1 + \frac{qR^3}{EI}$$

$$\frac{d^2w}{d\theta^2} + k^2w = 0$$

Solution is given by

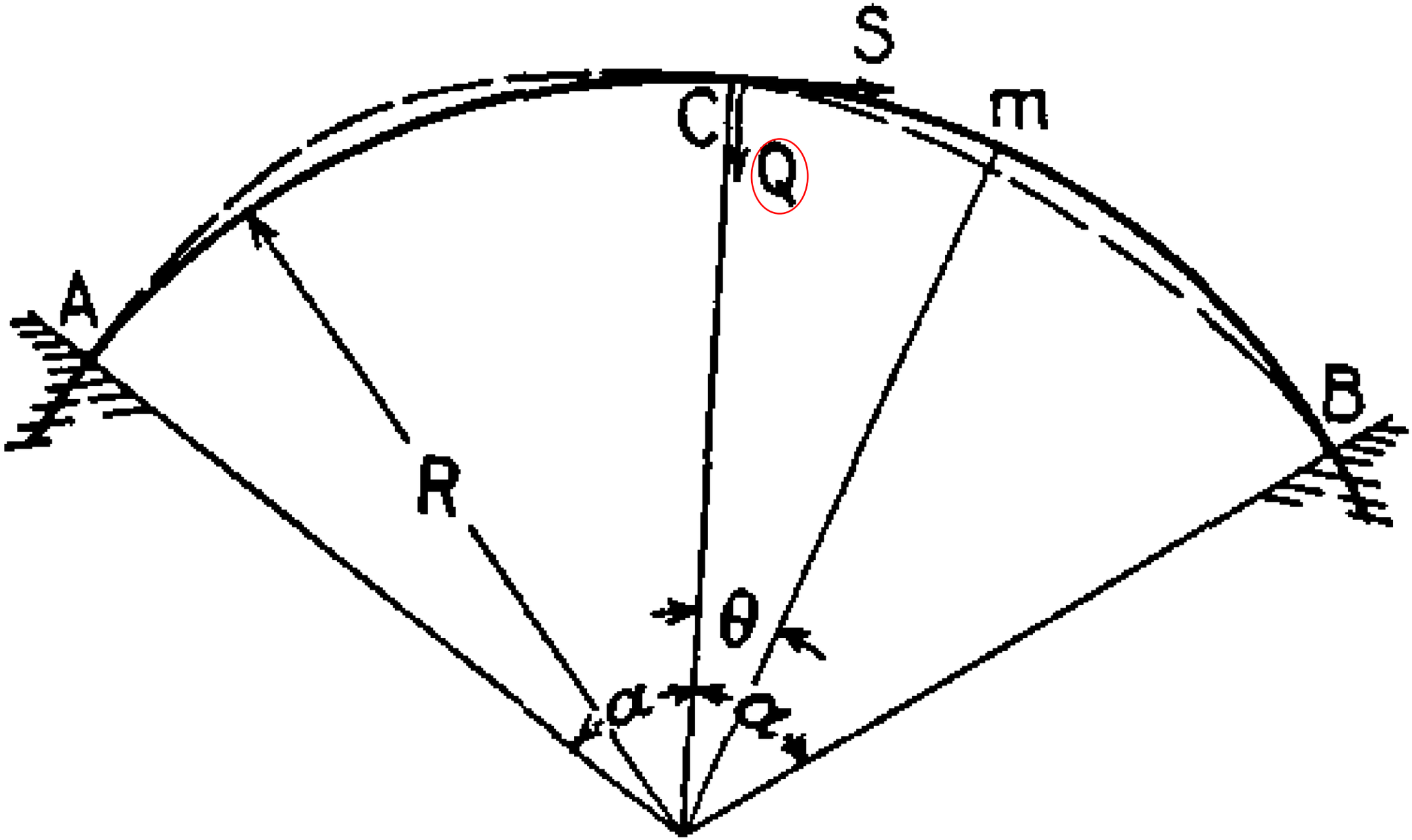
$$w = A \sin k\theta + B \cos k\theta$$

$$B = 0, \longrightarrow \sin 2\alpha k = 0$$

$$k = \frac{\pi}{\alpha}$$

$$q_{cr} = \frac{EI}{R^3}\left(\frac{\pi^2}{\alpha^2} - 1\right)$$

1908


S
C
Q
m
A
B
R
θ
α
α

$$M = Sw - QR \sin \theta$$

$$\frac{d^2w}{d\theta^2} + w = -\frac{R^2}{EI}(Sw - QR \sin \theta)$$

$$\frac{d^2w}{d\theta^2} + k^2w = \frac{QR^3 \sin \theta}{EI}$$

$$w = A \sin k\theta + B \cos k\theta + \frac{QR^3 \sin \theta}{(k^2 - 1)EI}$$

$$w = \frac{d^2w}{d\theta^2} = 0 \qquad \text{at } \theta = 0$$

$$w = \frac{dw}{d\theta} = 0 \qquad \text{at } \theta = \alpha$$

$$A \sin k\alpha + Q \frac{R^3 \sin \alpha}{(k^2 - 1)EI} = 0$$

$$Ak \cos k\alpha + Q \frac{R^3 \cos \alpha}{(k^2 - 1)EI} = 0$$

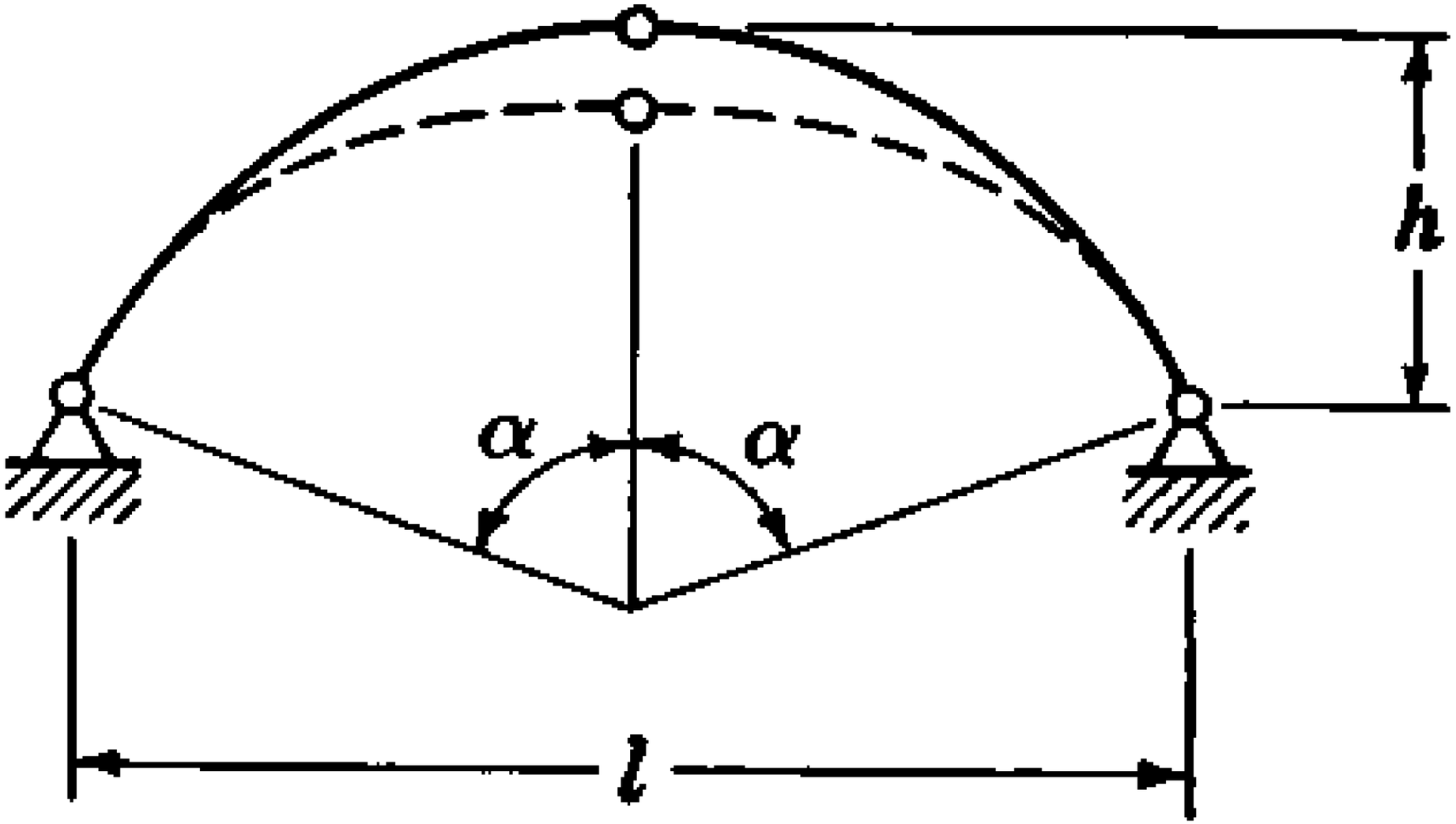
$$k \tan \alpha \cot k\alpha = 1$$

α	30°	60°	90°	120°	150°	180°
k	8.621	4.375	3	2.364	2.066	2

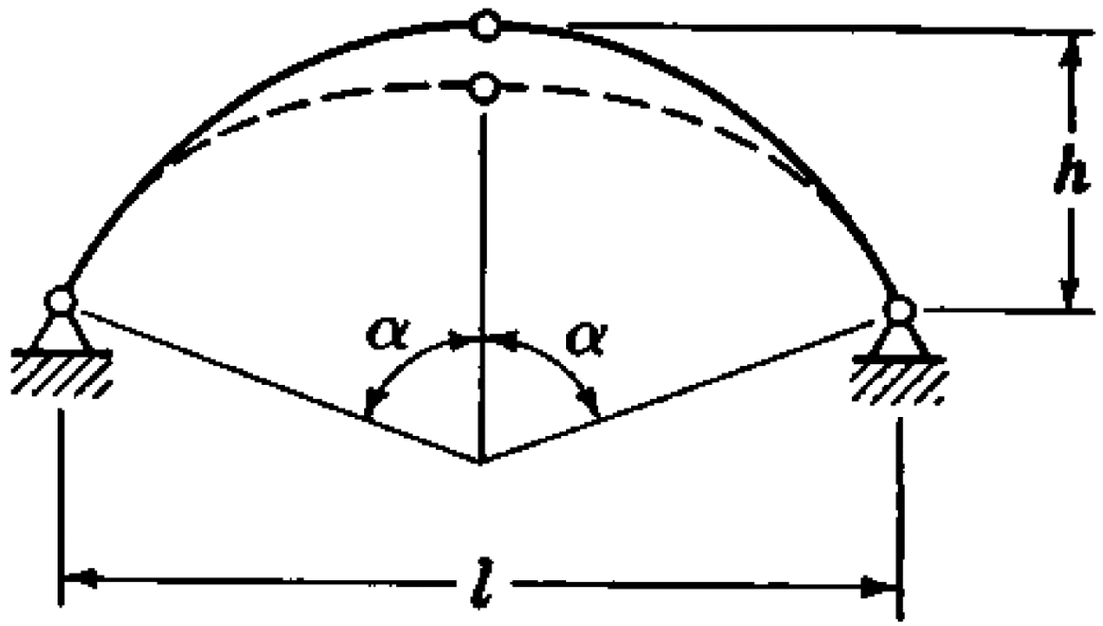
$$q_{cr} = \frac{EI}{R^3}(k^2 - 1)$$

$$q_{cr} = \gamma_1 \frac{EI}{R^3}$$

2α (deg)	No hinges	One hinge	Two hinges	Three hinges
30	294	162	143	108
60	73.3	40.2	35	27.6
90	32.4	17.4	15	12.0
120	18.1	10.2	8	6.75
150	11.5	6.56	4.76	4.32
180	8.0	4.61	3.00	3.00


h
α
α
l

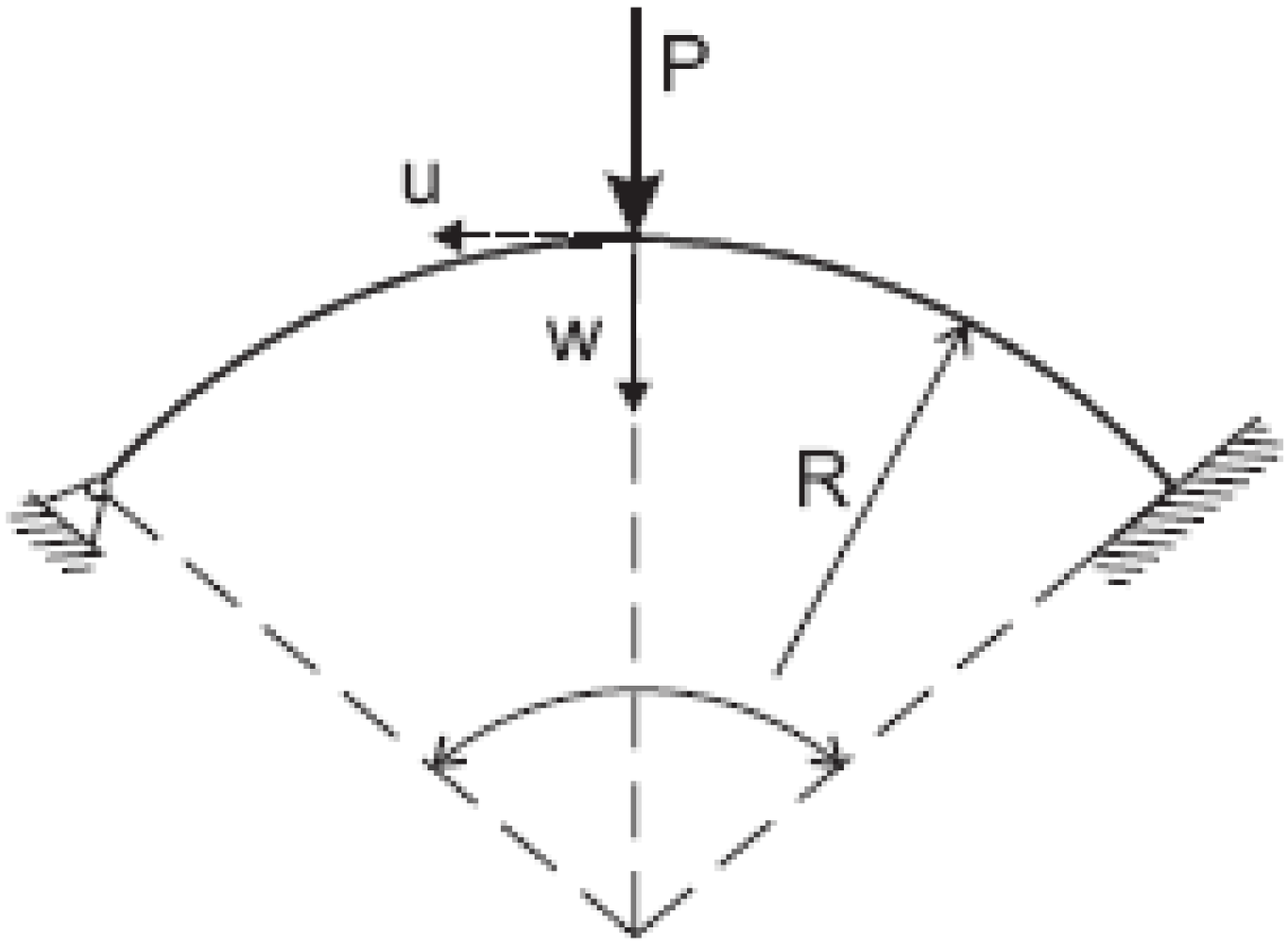
$$q_{cr} = \gamma_2 \frac{EI}{l^3}$$



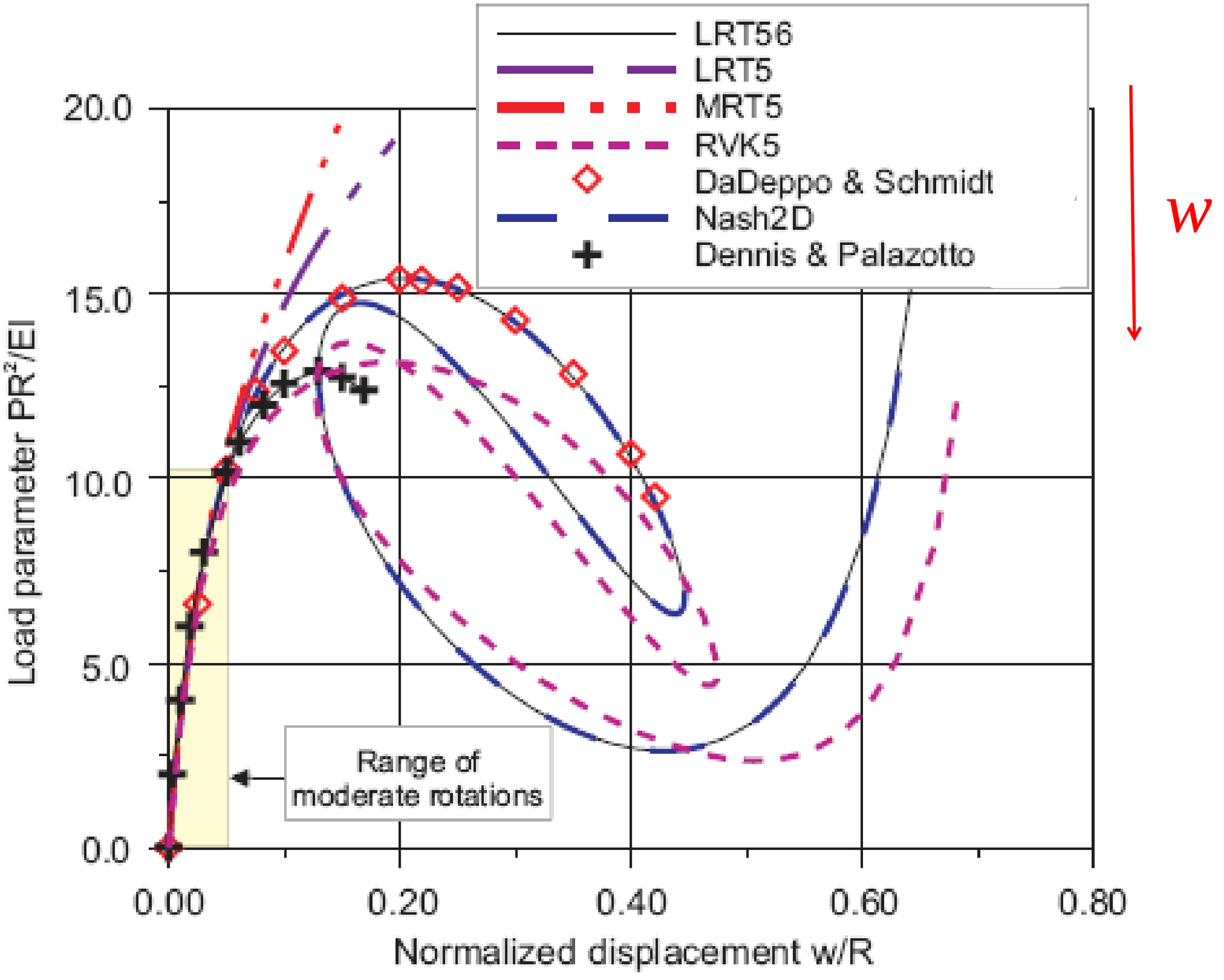
$\frac{h}{l}$	No hinges	One hinge	Two hinges	Three hinges
0.1	58.9	33	28.4	22.2
0.2	90.4	50	39.3	33.5
0.3	93.4	52	40.9	34.9
0.4	80.7	46	32.8	30.2
0.5	64.0	37	24.0	24.0

Buckling e postbuckling

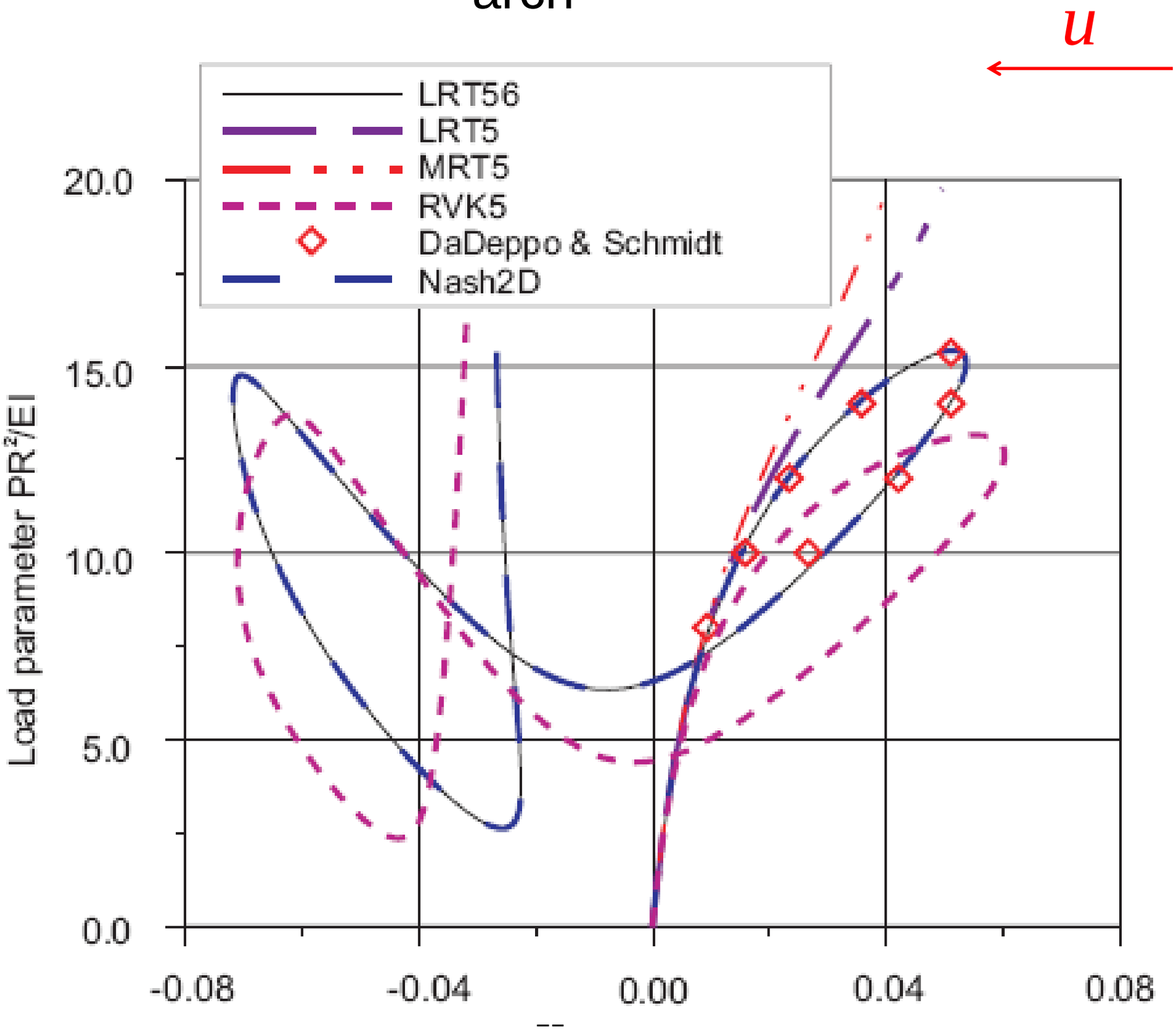
Instability of clamped-hinged circular arches



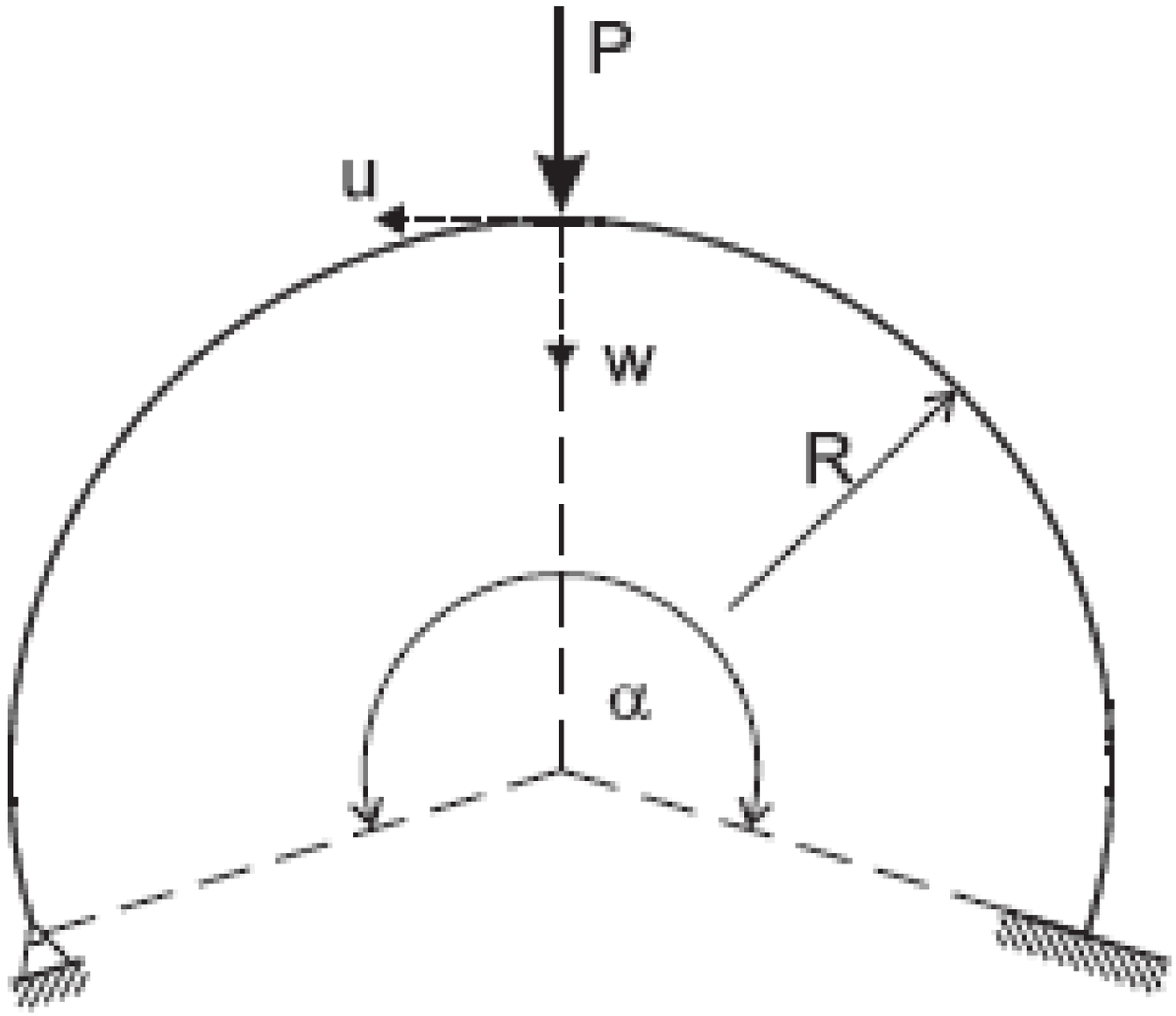
Normalized downward displacement at the crown for 100-degree arch

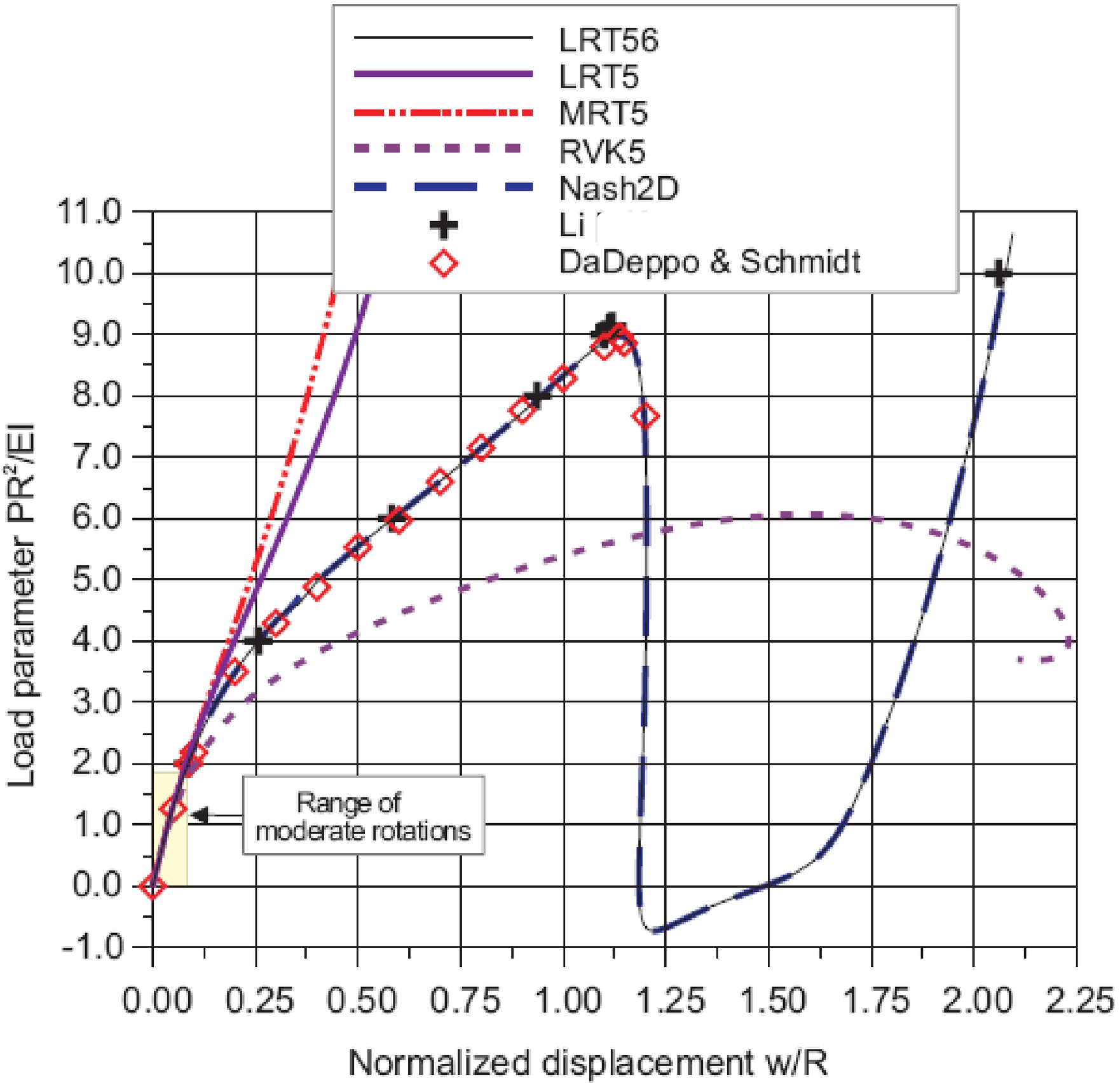


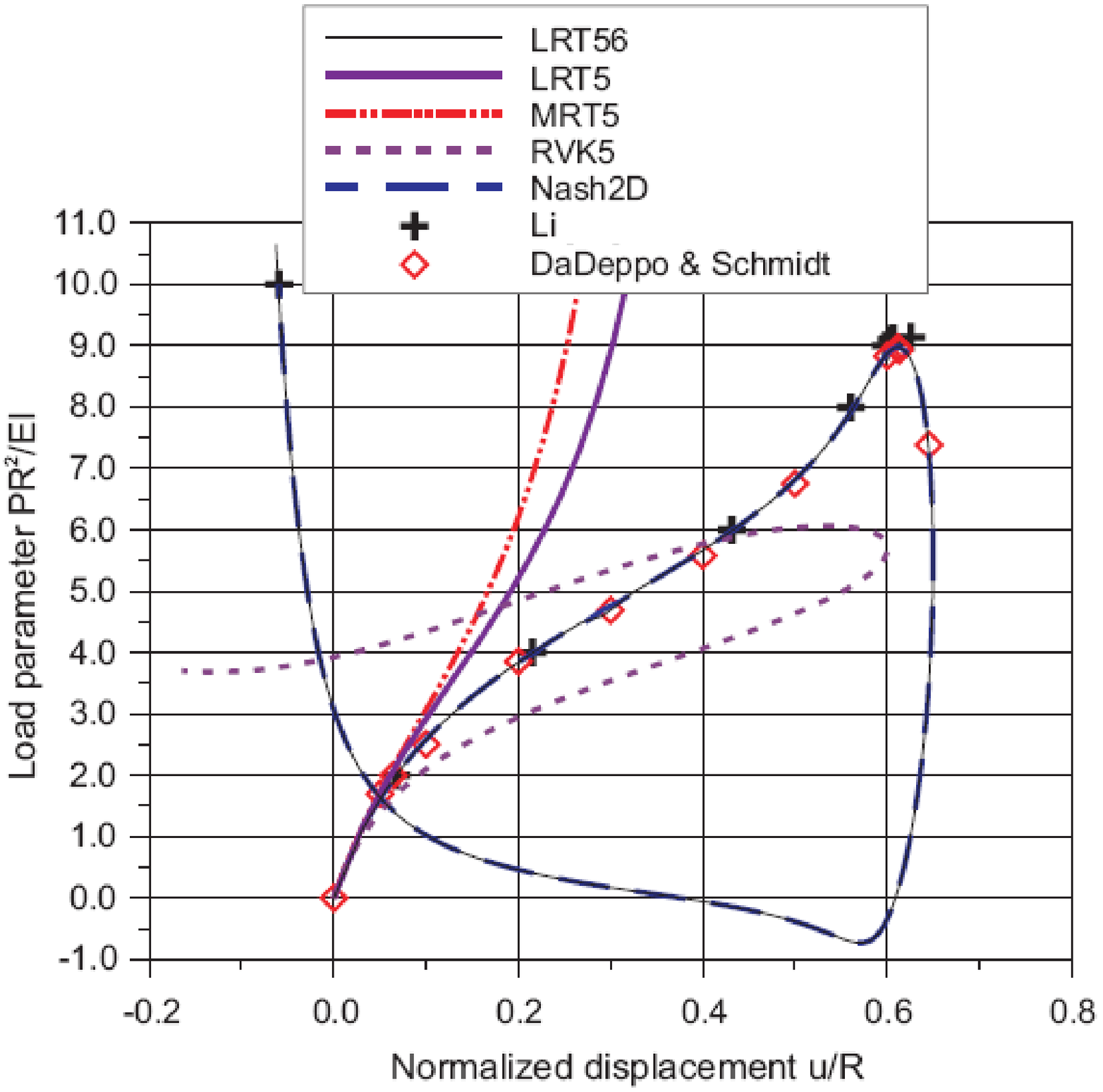
Normalized horizontal displacement at the crown for 100-degree arch



Clamped-hinged arch, α = 215 degrees

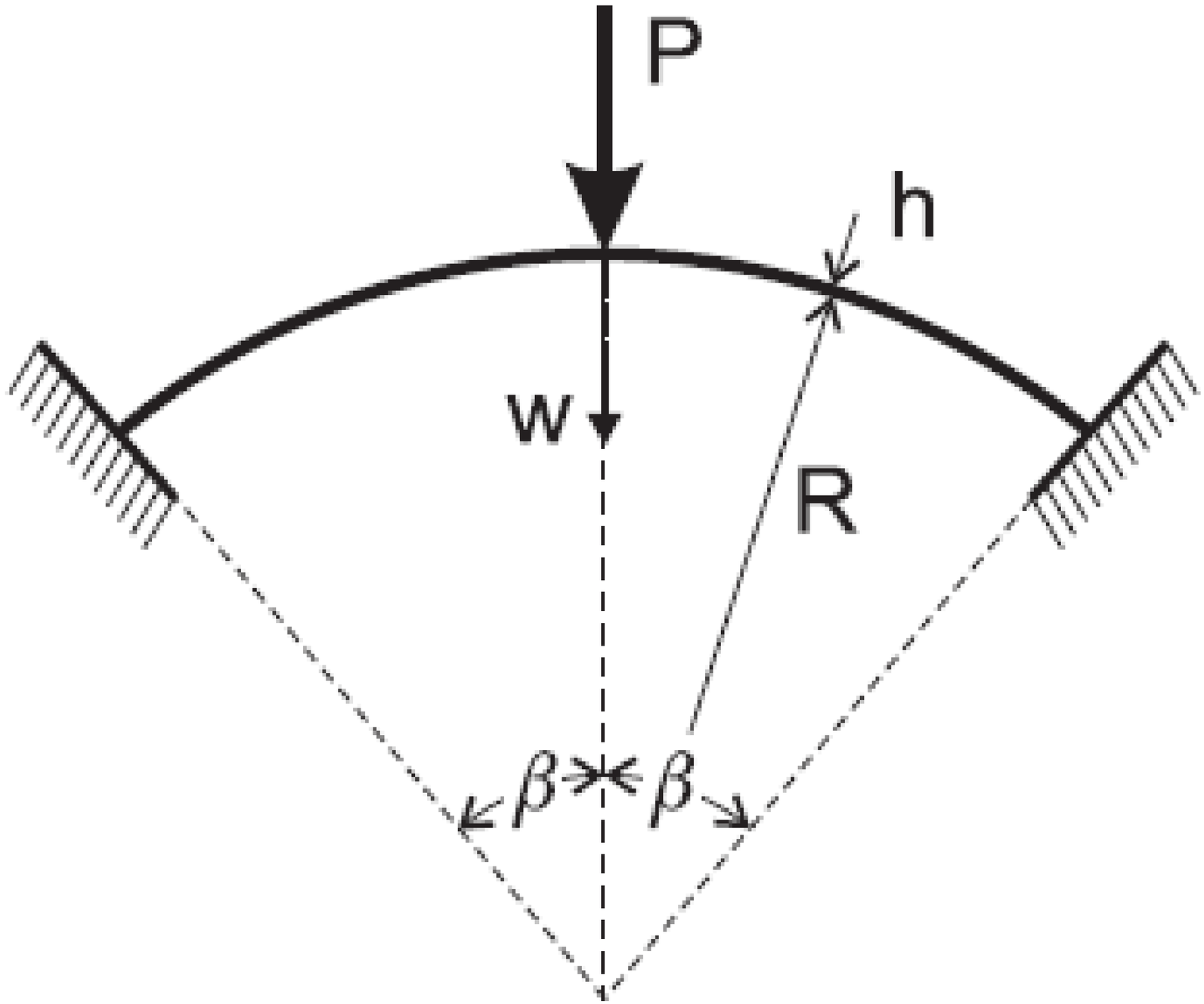


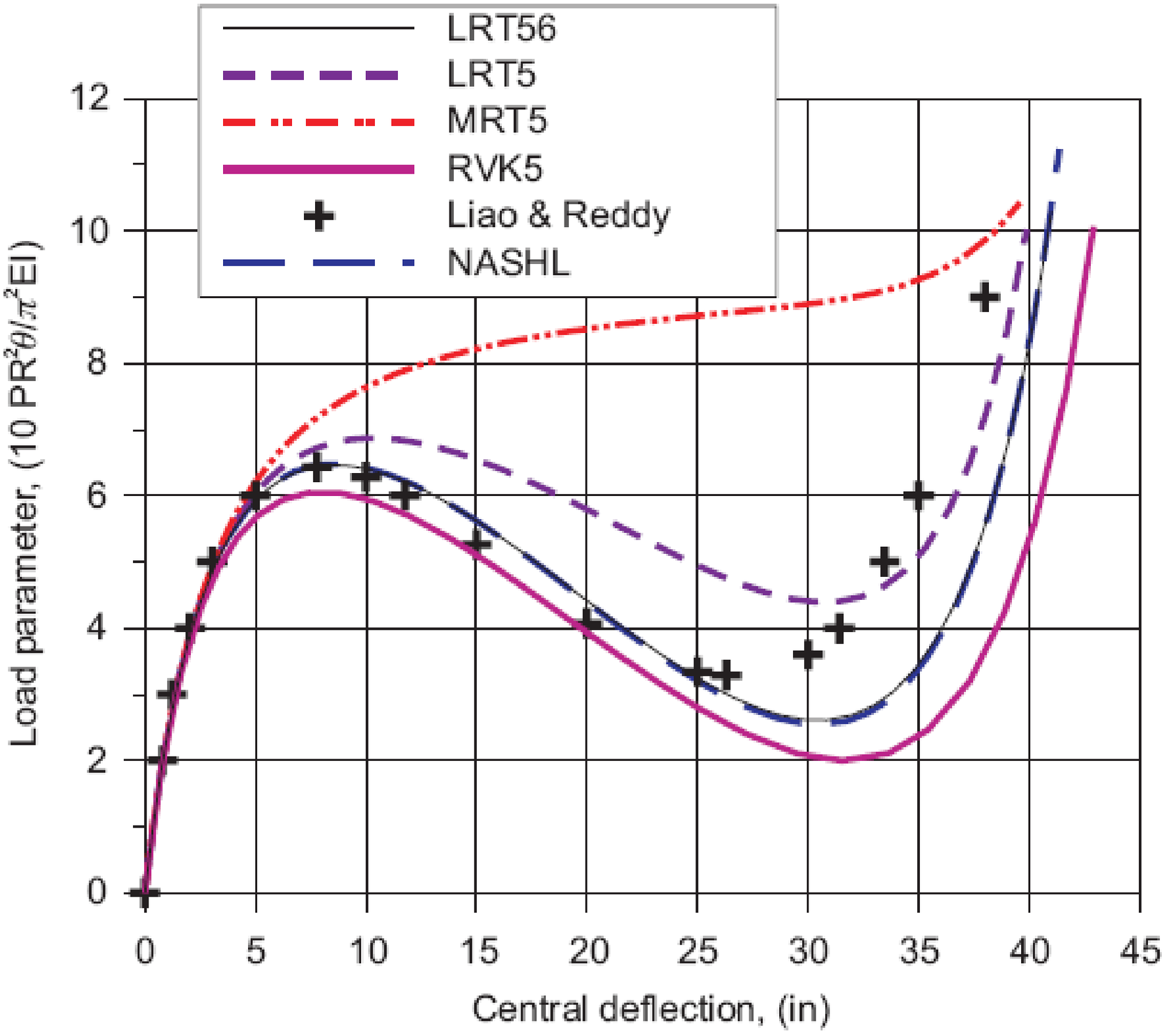

LRT56
LRT5
MRT5
RVK5
Nash2D
Li
DaDeppo & Schmidt
Load parameter PR²/EI
11.0
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0.0
-1.0
Range of moderate rotations
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
2.25
Normalized displacement w/R



Normalized horizontal displacement at the crown for 215-degree arch

Clamped laminated shallow arch under point load





Central vertical deflection for clamped laminated arch