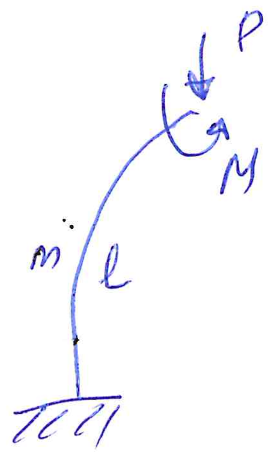
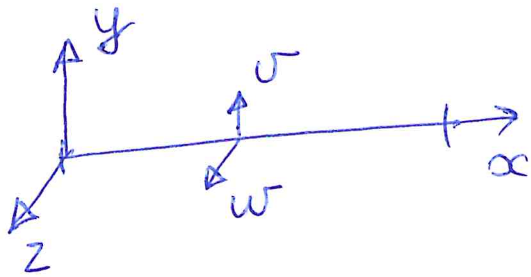
Nikolai

$EI_1,\ EI_2,\ \ell,\ m$

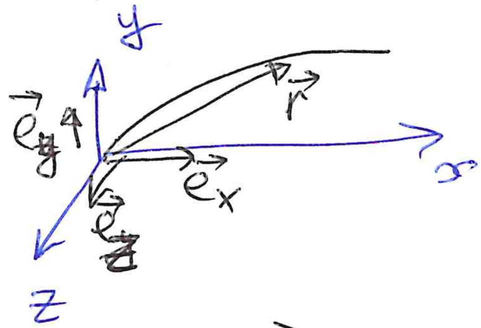


$$\begin{cases} EI_1 v^{IV} + M w''' + P v'' = 0 \,/ -m\ddot{v} \\ EI_2 w^{IV} - M v''' + P w'' = 0 \,/ -m\ddot{w} \end{cases}$$



1. Il criterio statico:

$$\begin{cases} EI_1 v^{IV} + M w''' + P v'' = 0 \\ EI_2 w^{IV} - M v''' + P w'' = 0 \end{cases}$$

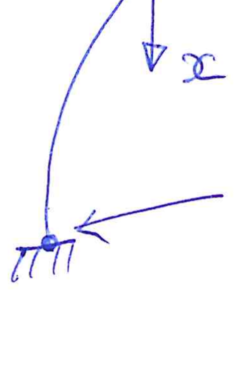


$\vec{r} = x\vec{e}_x + v\vec{e}_y + w\vec{e}_z$

invece:

$$\begin{cases} EI_1 v'' + M w' + P v = 0 \\ EI_2 w'' - M v'' + P w = 0 \end{cases}$$

$v(0) = w(0) = 0$



$v'(\ell) = w'(\ell) = 0$

$$v(x) = V e^{\lambda x}, \quad w(x) = W e^{\lambda x}$$

$\Rightarrow$ l'eq. per λ ....

$v(x) \sim \sin, \cos \ldots$

$$\begin{cases} v = c_1 \cos\alpha_1 x + c_2 \sin\alpha_1 x + c_3 \cos\alpha_2 x + c_4 \sin\alpha_2 x \\ w = c_1 \sin\alpha_1 x - c_2 \cos\alpha_2 x + c_3 \sin\alpha_2 x \cancel{+} c_4 \cos\alpha_2 x \end{cases}$$

1. $\underline{v = c_1 \cos\alpha_1 x, \quad w = c_1 \sin\alpha_1 x}$

$$-\alpha_1^2 EI_1 \underline{c_1 \cos\alpha_1 x} + M \underline{c_1}\alpha_1 \underline{\cos\alpha_1 x} + P \underline{c_1 \cos\alpha_1 x} = 0$$

$$EI_1 v'' + M w' + P v = 0$$

$$\left(-\alpha_1^2 EI_1 + M\alpha_1 + P\right) c_1 \cos\alpha_1 x = 0$$

$$-\alpha_1^2 EI_1 + M\alpha_1 + P = 0 \quad \leftarrow \alpha_1$$

$$-\alpha_1^2 EI_2 \underline{c_1 \sin\alpha_1 x} - M \alpha_1 \underline{c_1} (\underline{-\sin\alpha_1 x}) + P \underline{c_1 \sin\alpha_1 x} = 0$$

$$-\alpha_1^2 EI_2 + M\alpha_1 + P = 0$$

$$\Rightarrow \quad \boxed{EI_1 = EI_2 = EI}$$

$$\Rightarrow \quad \underline{-\alpha_1^2 EI + M\alpha_1 + P = 0}$$

α_1 e α_2 :

$$\underline{-EI\alpha^2 + M\alpha + P = 0}$$

$$\alpha^2 - \bar{M}\alpha - \bar{P} = 0 \qquad \bar{M} = \frac{M}{EI}$$

$$\boxed{\alpha_{1,2} = \frac{M \pm \sqrt{M^2 + 4P}}{2}} \qquad \bar{P} = \frac{P}{EI}$$

$c_1, c_2, c_3, c_4 - ?$

$$\begin{cases} v(x) = c_1 \cos\alpha_1 x + c_2 \sin\alpha_1 x + c_3 \cos\alpha_2 x + c_4 \sin\alpha_2 x \\ w(x) = + c_1 \sin\alpha_1 x - c_2 \cos\alpha_1 x + c_3 \sin\alpha_2 x - c_4 \cos\alpha_2 x \end{cases}$$

$v(0) = w(0) = 0$ $\qquad \boxed{c_1 + c_3 = 0}$

$$-c_2 - c_4 = 0 \rightarrow \boxed{c_2 + c_4 = 0}$$

$v'(l) = w'(l) = 0$

$$\underline{-c_1\alpha_1 \sin\alpha_1 l + c_2\alpha_1 \cos\alpha_1 l - c_3\alpha_2 \sin\alpha_2 l + c_4 \alpha_2 \cos\alpha_2 l = 0}$$

$$\underline{c_1\alpha_1 \cos\alpha_1 l + c_2\alpha_1 \sin\alpha_1 l + c_3\alpha_2 \cos\alpha_2 l + c_4\alpha_2 \sin\alpha_2 l = 0}$$

$\Rightarrow$ per $\underline{c_1}$...

$$Ac = 0 \qquad c = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -\alpha_1 \sin\alpha_1 l & \alpha_1 \cos\alpha_1 l & -\alpha_2 \sin\alpha_2 l & \alpha_2 \cos\alpha_2 l \\ \alpha_1 \cos\alpha_1 l & \alpha_1 \sin\alpha_1 l & \alpha_2 \cos\alpha_2 l & \alpha_2 \sin\alpha_2 l \end{bmatrix}$$

$$\boxed{\det A = 0}$$

$$\det A = 0 \quad \boxed{\alpha_1^2 + \alpha_2^2 = 2\alpha_1\alpha_2 \cos(\alpha_1 - \alpha_2) l}$$

α_1, α_2 :

$$\alpha^2 - \bar{M}\alpha - \bar{P} = 0 \qquad \bar{M} = \frac{M}{EI}$$

$$\alpha_{1,2} = \frac{\bar{M} \pm \sqrt{\bar{M}^2 + 4\bar{P}}}{2} \qquad \bar{P} = \frac{P}{EI}$$

$$\alpha_1\alpha_2 = -\bar{P}$$

$$\boxed{\begin{aligned}(x-x_1)(x-x_2) &= \\ = x^2 + px + q\end{aligned}}$$

$$x^2 + px + q = 0$$

$$x_1 x_2 = q$$

$$x_1 + x_2 = -p$$

$$\alpha_1 - \alpha_2 = \sqrt{\bar{M}^2 + 4\bar{P}}$$

$$\alpha_1^2 + \alpha_2^2 = \bar{M}^2 + 2\bar{P}$$

$$\bar{M}^2 + 2\bar{P} = -2\bar{P}\cos\sqrt{\bar{M}^2 + 4\bar{P}}\,l$$

$$\Rightarrow \cos\sqrt{\bar{M}^2 + 4\bar{P}}\,l = -\left(1 + \frac{\bar{M}^2}{2\bar{P}}\right)$$

$$|\cos| \leq 1 \qquad \left|1 + \frac{\bar{M}^2}{2\bar{P}}\right| > 1$$

$$= 1 \quad \underline{\bar{M} = 0} \; !$$

$$\Rightarrow \underline{\exists \; P_c} \quad \underline{M = 0} \quad ?$$

$\Rightarrow$ Il criterio dinamico

$$\begin{cases} EI_1 v^{IV} + M w''' + P v'' + m \ddot{v} = 0 \\ EI_2 w^{IV} \cdot M v''' + P w'' + m \ddot{w} = 0 \end{cases}$$

le condizione al contorno

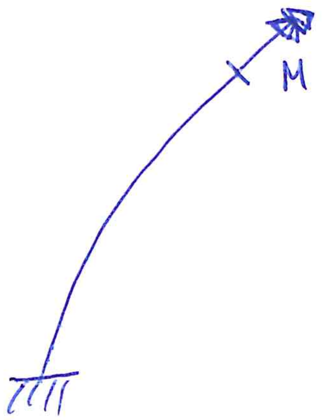
$x = 0$, $v = w = 0$

$v' = w' = 0$

$x = l$ . . . . .

$x = \ell$

a)

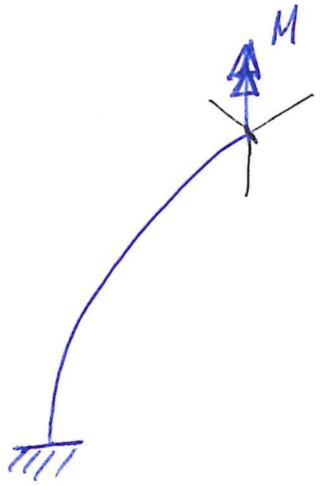


$$\begin{cases} EI_1 v''' + P v' - M w'' = 0 \\ EI_2 w'' + P w' + M v'' = 0 \end{cases}$$

$M_y = M_z = 0 \Rightarrow$

$$\begin{cases} v'' = 0, \quad w'' = 0 \end{cases}$$

b)



$$\begin{cases} = \\ = \end{cases}$$

$$\begin{cases} EI_1 v'' - M w' = 0 \\ EI_2 w'' + M v' = 0 \end{cases}$$

$$v(x,t) = \bar{V}(\bar{x})\,\ell\, e^{i\omega A t}$$

$$w(x,t) = \bar{W}(\bar{x})\,\ell\, e^{i\omega A t} \sqrt{\frac{EI_2}{EI_1}}$$

$$A = \sqrt{\frac{EI_1}{m\ell^4}}, \quad \bar{x} = x/\ell$$

$$\Rightarrow \begin{cases} V^{IV} - M_0 W''' + \beta^2 V'' - \omega^2 V = 0 \\ W^{IV} + M_0 V''' + \frac{\beta^2}{k^2} W'' - \frac{\omega^2}{k^2} W = 0 \end{cases}$$

$$M_0 = \frac{M\ell}{\sqrt{EI_1 EI_2}}, \quad \beta^2 = \frac{P\ell^2}{EI_1}, \quad k = \sqrt{\frac{I_2}{I_1}}$$

$I_2 \geq I_1$

$\Rightarrow \underline{k \geq 1}$

$$V(\bar{x}) = \sum_{n=0} V_n \bar{x}^n$$

$$W(\bar{x}) = \sum_{n=0} W_n \bar{x}^n$$

V_n, W_n – ?

$$\left.\begin{aligned} V'(0) = W'(0) = 0 \\ V(0) = W(0) = 0 \end{aligned}\right| \Rightarrow \begin{aligned} V_0, V_1 = 0 \\ W_0, W_1 = 0 \end{aligned}$$

. . .

$\underline{V_2, V_3, \; W_2, W_3}$

$$V(\bar{x}) \simeq V_2 \bar{x}^2 + V_3 \bar{x}^3 \dots$$

$$W(\bar{x}) \simeq W_2 \bar{x}^2 + W_3 \bar{x}^3 \dots$$

$$A \begin{bmatrix} V_2 \\ V_3 \\ W_2 \\ W_3 \end{bmatrix} = 0 \Rightarrow \underline{\det A = 0}$$

$$\det A = \underline{F(\underset{P}{\beta}, \underset{M}{M_0}, k, \omega)} = 0$$