Nikolai's paradox: nonconservative moment

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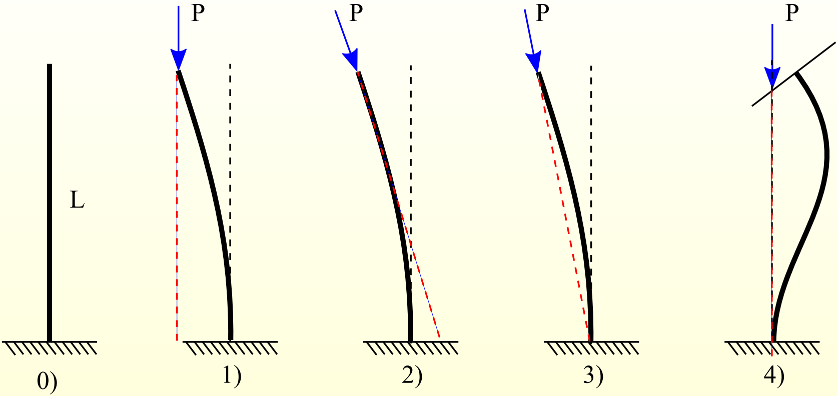
Instability of beams[1]



Critical forces: 1) $P^* = \frac{\pi^2 EI}{4L^2}$; 2) $P^* = \frac{20.5 EI}{L^2}$; 3) $P^* = \frac{\pi^2 EI}{L^2}$; 4) $P^* = \frac{20.5 EI}{L^2}$ [Reut, 1939].

[1] Feodosiev, V.I. *Advanced Stress and Stability Analysis. Worked Examples*. Springer, 2005

Follower force. Static “solution”

$$EIv^{(4)} + Pv'' = 0; \qquad v(0) = v'(0) = 0, \quad v''(L) = 0, \qquad (1)$$

Case 1.

$$v'''(L) = -Pv'(L), \qquad (2)$$

Case 2.

$$v'''(L) = 0. \qquad (3)$$

General form of solution is

$$v(x) = C_1 + C_2 x + C_3 \sin\beta x + C_4 \cos\beta x, \quad \beta^2 = \frac{P}{EI}.$$

Case 2. Solution:

$$C_1 + C_4 = 0; \quad C_2 + C_3 = 0;$$
$$C_3 \sin\beta L + C_4 \cos\beta L = 0; \quad C_3 \cos\beta L - C_4 \sin\beta L = 0.$$

Unique solution $C_k = 0$. No any other equilibrium forms!

Follower force. Dynamic solution

$$EI\frac{\partial^4 v}{\partial x^4} + P\frac{\partial^2 v}{\partial x^2} + \rho\frac{\partial^2 v}{\partial t^2} = 0; \tag{4}$$

$$v(0,t) = 0, \quad v'(0,t) = 0, \quad v''(L,t) = 0, \quad v^{'''}(L,t) = 0. \tag{5}$$

General form of solution is

$$v(x,t) = V(x)\exp i\omega t,$$

$$V(x) = C_1 \sin s_2 x + C_2 \cos s_2 x + C_3 \sinh s_1 x + C_4 \cosh s_2 x.$$

Result: $P^* = \dfrac{20.5EI}{L^2}$ when $\Re\omega = 0$.

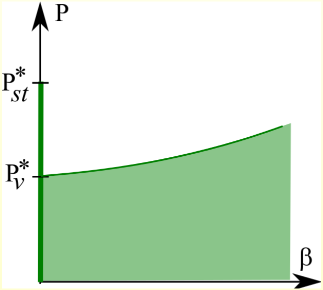
For $P < P^*$ we have real value of ω whereas for $P > P^*$ ω becomes complex.

Follower force

Consequences:

- Dynamic criterion should be used. Thus, mass distribution becomes important!
- Ziegler's paradox: destabilizing role of damping.

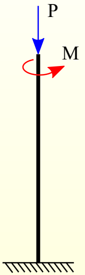
Hans Ziegler (1952)



E. Nikolai paradox

E. Nikolai (1927 – 1930)

Static "solution"



$$EIu'' + Pu = Mv', \quad EIv'' + Pv = -Mu';$$
$$u(x) = C_1 \cos a_1 x + C_2 \sin a_1 x + C_3 \cos a_2 x + C_4 \sin a_2 x,$$
$$v(x) = C_1 \sin a_1 x - C_2 \cos a_1 x + C_3 \sin a_2 x - C_4 \cos a_2 x,$$
$$a^2 + \frac{M}{EI} a - \frac{P}{EI} = 0.$$

Boundary conditions

$$u(0) = v(0) = 0, \quad u'(L) = v'(L) = 0.$$

Solvability $\det \mathbb{A} = 0$ results in

$$\cos \sqrt{\left(\frac{ML}{EI}\right)^2 + \frac{4PL^2}{EI}} = -\left(1 + \frac{M^2}{2PEI}\right)$$

E. Nikolai paradox

E. Nikolai (1927 – 1930)

Solvability condition

$$\cos\sqrt{\left(\frac{ML}{EI}\right)^2 + \frac{4PL^2}{EI}} = -\left(1 + \frac{M^2}{2PEI}\right)$$

has a solution iff $M = 0$

$$\cos\sqrt{\frac{4PL^2}{EI}} = -1, \qquad P^* = \frac{\pi^2 EI}{4L^2}.$$

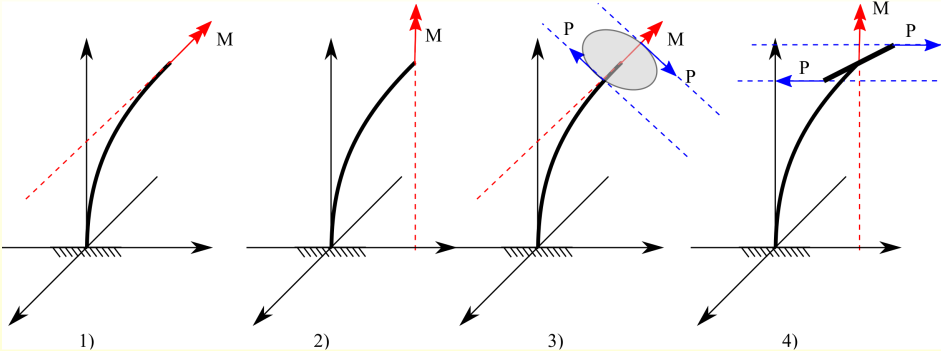
Conclusion

no equilibrium forms can exist other than the rectilinear form.

Therefore, the applied method is invalid.

Tangential and axial moments (non-conservative)

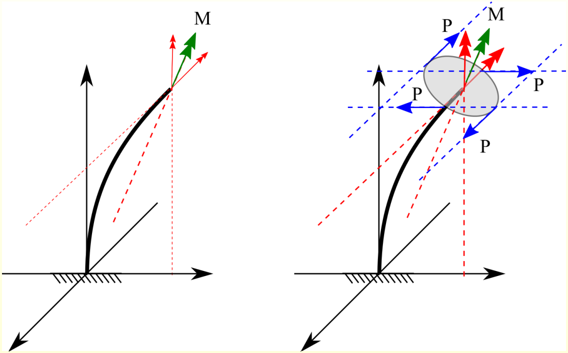
Hans Ziegler introduced: quasi-tangential, semi-tangential and pseudo-tangential moments[2]



Tangential and axial moments are given in 1) and 2) whereas their realizations are given in 3) and 4), respectively.

[2]Bolotin, V. V. *Nonconservative Problems of Theory of Elastic Stability*, Pergamon Press, 1963

Conservative moment



The moment whose vector bisects the angle between the tangent to undeformed and deformed axes of the bar is conservative.

Dynamic analysis

Equations of motion

$$EI_1\frac{\partial^4 u}{\partial x^4} - M\frac{\partial^3 v}{\partial x^3} + P\frac{\partial^2 u}{\partial x^2} + \rho\frac{\partial^2 u}{\partial t^2} = 0, \quad (6)$$

$$EI_2\frac{\partial^4 v}{\partial x^4} + M\frac{\partial^3 u}{\partial x^3} + P\frac{\partial^2 v}{\partial x^2} + \rho\frac{\partial^2 v}{\partial t^2} = 0. \quad (7)$$

Boundary conditions at $s = 0$

$$u = v = 0, \quad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0.$$

At $x = L$ we have

$$EI_1\frac{\partial^3 u}{\partial x^3} - M\frac{\partial^2 v}{\partial x^2} + P\frac{\partial u}{\partial x} = 0, \quad (8)$$

$$EI_2\frac{\partial^3 v}{\partial x^3} + M\frac{\partial^2 u}{\partial x^2} + P\frac{\partial v}{\partial x} = 0, \quad (9)$$

(transverse forces).

Boundary conditions for different moments

At $x = L$ depending on the loadings we have

- Tangential moment

$$\frac{\partial^2 u}{\partial x^2} = 0, \quad \frac{\partial^2 v}{\partial x^2} = 0.$$

- Axial moment

$$EI_1 \frac{\partial^2 u}{\partial x^2} - M \frac{\partial v}{\partial x} = 0,$$
$$EI_2 \frac{\partial^2 v}{\partial x^2} + M \frac{\partial u}{\partial x} = 0.$$

- Semi-follower moment (pseudo-tangential)

$$EI_1 \frac{\partial^2 u}{\partial x^2} - M \frac{\partial v}{\partial x} = 0,$$
$$\frac{\partial^2 v}{\partial x^2} = 0.$$

Harmonic-type solutions

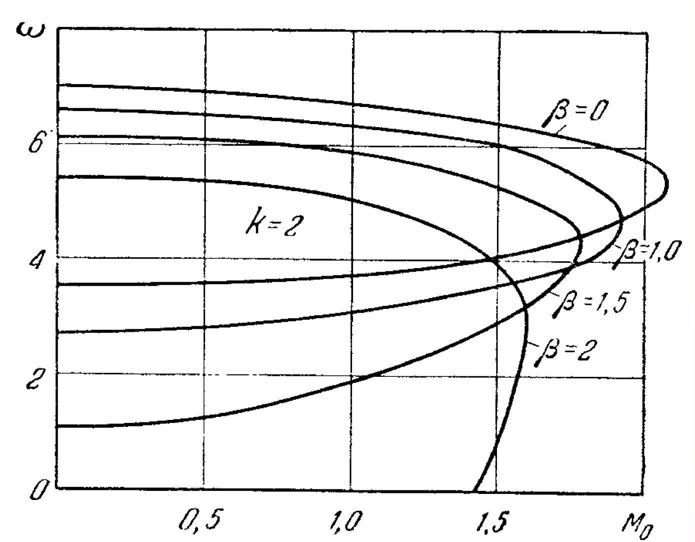
$$u(x,t) = U(x)\exp(i\omega t), \quad v(x,t) = V(x)\exp(i\omega t).$$

Dimensionless parameters

$$M_0 = \frac{ML}{E\sqrt{I_1 I_2}}, \quad \beta^2 = \frac{PL^2}{EI_1}, \quad k = \sqrt{\frac{I_2}{I_1}},$$

$$I_1 \le I_2, \quad k \ge 1.$$

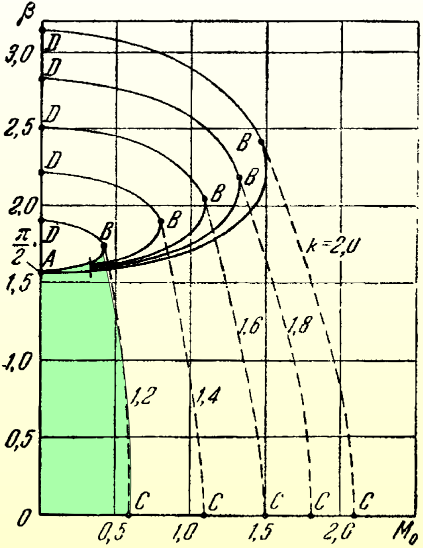
Frequency ω



Stability zones

Point A:

$$P^* = \frac{\pi^2 E I_1}{4L^2}$$



Feodosiev, V.I. *Advanced Stress and Stability Analysis. Worked Examples*. Springer, 2005

Panovko, I.G. and Gubanova, I.I. *Stability and oscillation of elastic systems: modern concepts, paradoxes and errors*. National Aeronautics and Space Administration, 1973.

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