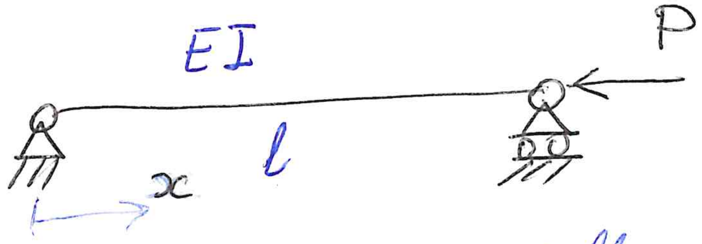
# Example 1.

Simply-supported beam

$P_c - ?$




EI - bending stiffness (rigidity)

## Equation

$$EI\, v^{(4)} + P v'' = 0$$

$$v^{(4)} + \frac{P}{EI} v'' = 0$$

Characteristic equation: $v \sim V_0 e^{\lambda x}$

$$\lambda^4 + \frac{P}{EI}\lambda^2 = 0$$

$$\lambda^2 \left(\lambda^2 + \frac{P}{EI}\right) = 0$$

$\lambda_{1,2} = 0$ – multiple solution (mult. 2)

$$\lambda_{3,4} = \pm i \sqrt{\frac{P}{EI}}$$

$$\beta = \sqrt{\frac{P}{EI}}$$

$$v(\ell) = C_2 \sin\beta\ell + C_3\ell = 0$$

$$v''(\ell) = -C_2\beta^2 \sin\beta\ell = 0$$

$$\Rightarrow \quad C_2 \neq 0 \; !$$

if $C_2 = 0$
then $C_3 = 0$
and then $v \equiv 0$!

$$\sin\beta\ell = 0 \; !$$

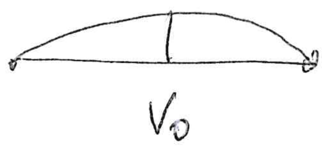
then $\boxed{C_3 = 0}$ !

$$\sin\beta\ell = 0 \quad : \quad \beta\ell = \cancel{\pm}\pi k, \quad k = 1, 2 \ldots$$

$$\beta\ell = \sqrt{\frac{P}{EI}}\,\ell = \pi k$$

$$\sqrt{\frac{P}{EI}} = \frac{\pi k}{\ell}, \quad P_k = \frac{\pi^2 k^2}{\ell^2} EI, \quad k = 1, 2 \ldots$$

$$P_c = \min_k P_k = \frac{\pi^2 EI}{\ell^2}$$




$$\boxed{P_c = \frac{\pi^2 EI}{\ell^2}}$$

Mode : $v(x) = V_0 \sin\sqrt{\frac{P_c}{EI}}\, x$

$$\begin{cases} EI\,v^{(4)} + P v^{(2)} = 0 \\ v(0) = 0 \qquad v(l) = 0 \\ v''(0) = 0 \qquad v''(l) = 0 \end{cases}$$

Boundary-value problem

let $w = v''$ $\Rightarrow$

$$\begin{cases} EI\,w'' + P w = 0, \\ w(0) = 0, \quad w(l) = 0 \end{cases}$$

$$w(x) = C_1 \cos\beta x + C_2 \sin\beta x$$

Because ch. eq. $\lambda^2 + \frac{P}{EI} = 0$

$$w(0) = C_1 = 0$$

$$w(l) = C_2 \sin\beta l = 0 \Rightarrow \beta l = \pi k,$$

$k = 1, 2 \ldots$