Example 4

$$v(0)=0,\quad v'(\ell)=0$$

$$v(\ell)=0,\quad v''(\ell)=0$$

BCs:

$v(0)=0 \rightarrow$ $C_1 + C_4 = 0$

$v'(0)=0 \rightarrow$ $C_2\beta + C_3 = 0$

$v(\ell)=0 \rightarrow$ $C_1\cos\beta\ell + C_2\sin\beta\ell + C_3\ell + C_4 = 0$

$v''(\ell)=0 \rightarrow$ $-C_1\beta^2\cos\beta\ell - C_2\beta^2\sin\beta\ell = 0$

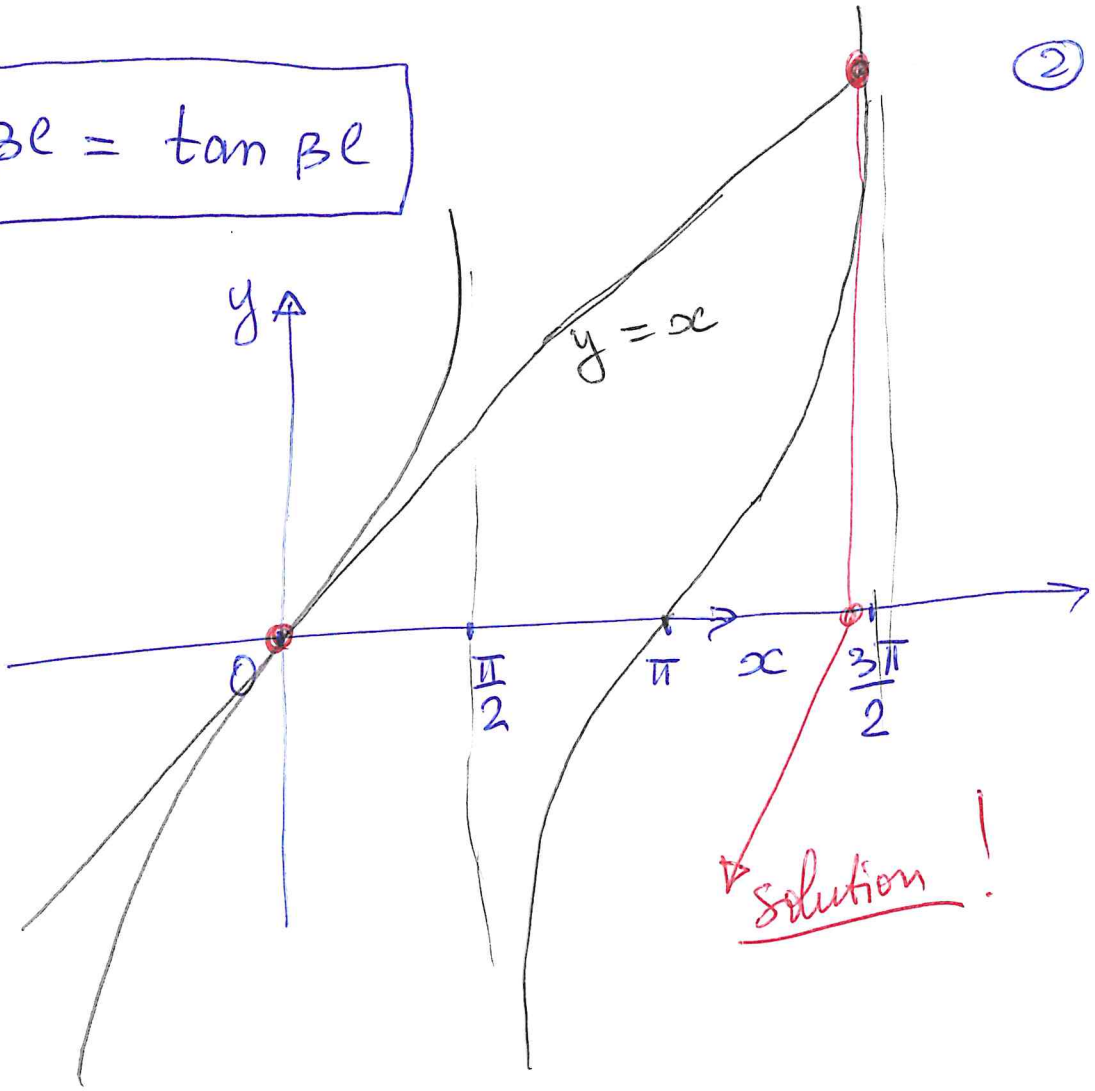
$$C_4 = -C_1,\quad C_3 = -C_2\beta$$

$$\begin{cases} C_1\left[\cos\beta\ell - 1\right] + C_2\left[\sin\beta\ell - \beta\ell\right] = 0 \\ C_1\cos\beta\ell + C_2\sin\beta\ell = 0 \end{cases}$$

$$(\cos\beta\ell - 1)\sin\beta\ell - \cos\beta\ell\,(\sin\beta\ell - \beta\ell) = 0$$

$$-\sin\beta\ell + \cos\beta\ell\,\beta\ell = 0$$

$$\beta \ell = \tan \beta \ell$$



x^* : $x = \tan x$

$$P = P_c \equiv \frac{x^{*2}}{\ell^2} EI$$