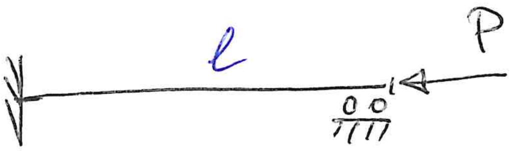
Example 3

P_c and eigen-mode ?



$\approx$ 

BCs: $v(0)=0, \quad v(l)=0;$

$v'(0)=0, \quad v'(l)=0$

$$v(x) = C_1 \cos\beta x + C_2 \sin\beta x + C_3 x + C_4$$

$$v'(x) = -C_1 \beta \sin\beta x + C_2 \beta \cos\beta x + C_3$$

BCs:
$$\boxed{\begin{aligned} & C_1 + C_4 = 0 \\ & C_2\beta + C_3 = 0 \\ & C_1 \cos\beta l + C_2 \sin\beta l + C_3 l + C_4 = 0 \\ & -C_1 \beta \sin\beta l + C_2 \beta \cos\beta l + C_3 = 0 \end{aligned}}$$

$$C_4 = -C_1, \quad C_3 = -C_2\beta$$

$$\begin{cases} \underline{C_1} \cos\beta l + \underline{\underline{C_2}} \sin\beta l - \underline{\underline{C_2}}\beta l - \underline{C_1} = 0 \\ -\underline{C_1}\beta \sin\beta l + \underline{\underline{C_2}}\beta \cos\beta l - \underline{\underline{C_2}}\beta = 0 \end{cases}$$

$$\begin{cases} c_1\left[\cos\beta l - 1\right] + c_2\left[\sin\beta l - \beta l\right] = 0 \\ c_1\left[-\beta\sin\beta l\right] + c_2\beta\left[\cos\beta l - 1\right] = 0 \end{cases}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = 0 \qquad \underline{\det A = 0}$$

$$\det A = a_{11}a_{22} - a_{12}a_{21}$$

$$\beta\left[\cos\beta l - 1\right]^2 + \beta\sin\beta l\left[\sin\beta l - \beta l\right] = 0$$

characteristic equation

$$\beta\left[\cos^2\beta l - 2\cos\beta l + 1\right] + \beta\sin^2\beta l - \beta^2 l\sin\beta l = 0$$

$$2\cancel{\beta} - 2\cancel{\beta}\cos\beta l - \beta^{\cancel{2}} l\sin\beta l = 0$$

$$2 - 2\cos\beta l - \beta l\sin\beta l = 0$$

$$\boxed{\beta l = \frac{2 - 2\cos\beta l}{\sin\beta l}} \qquad - \beta l - ?$$

$$\boxed{\beta l = 2\pi}$$

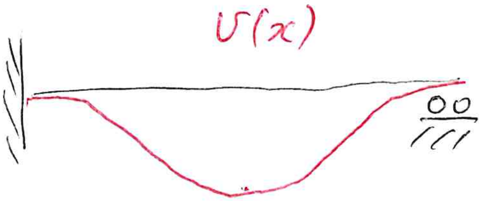
Find solution for βl.

$$2 - 2\cos\beta\ell - \beta\ell\sin\beta\ell = 0$$

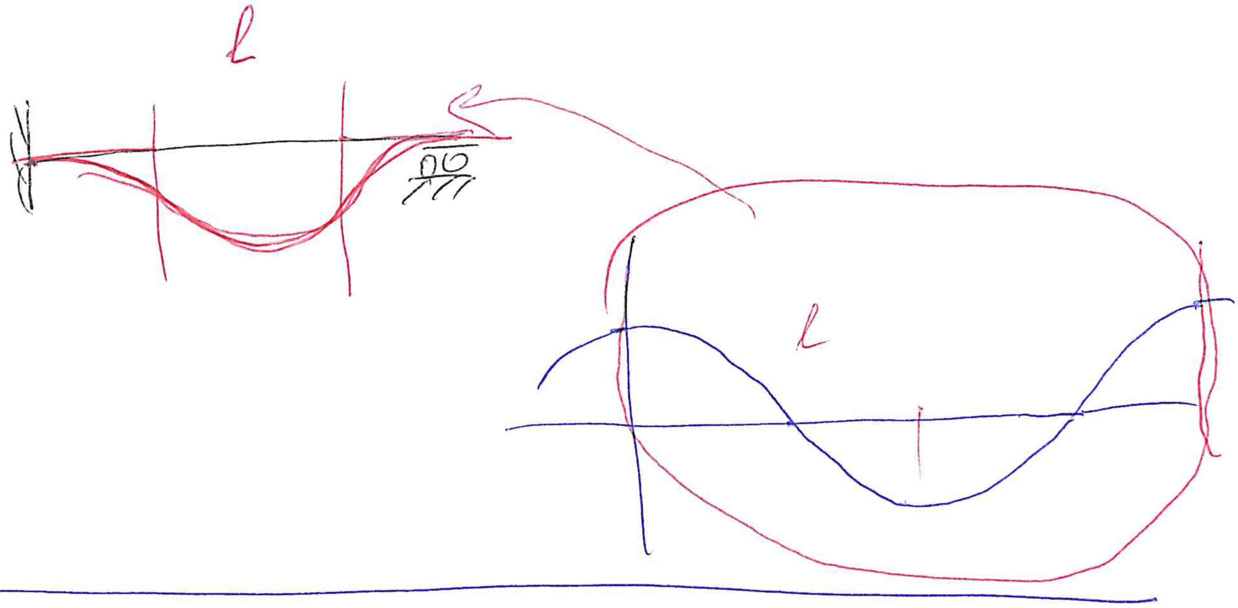
Solution is $\beta\ell = 2\pi$

So $P = \boxed{P_c = \frac{4\pi^2 EI}{\ell}}$

Shape



Shape



Another way to solve this problem :

$$v^{(4)} + \beta^2 v'' = 0 \qquad v(0) = 0, \quad v(l) = 0$$

$$v'(0) = 0, \quad v'(l) = 0$$

$$v'' \Rightarrow w \qquad \begin{cases} w'' + \beta^2 w = 0 \\ w(0) = 0, \ w(l) = 0 \end{cases}$$

$$w = c_1 \cos\beta x + c_2 \sin\beta x$$

$$w(0) = c_1 = 0$$

$$w(l) = \underline{c_2 \sin\beta l = 0} \Rightarrow \underline{\sin\beta l = 0}$$

$$\beta l = \pi \Rightarrow \boxed{P = P_c = \frac{\pi^2}{l^2} EI}$$