Dynamic stability

most general approach

Equilibrium state

Nonlinear equation. $\vec{q}$ – vector of variables

$$\vec{q} = \vec{q}(t)$$

$$F(\ddot{\vec{q}}, \dot{\vec{q}}, \vec{q}, \ldots) = 0 \qquad (*)$$

equilibrium state $\vec{q} = \vec{q}_0 = \text{const}$

Let us consider $\vec{q} = \vec{q}_0 + \vec{u}$ ← small

From $(*)$ we came to

$$M\ddot{\vec{u}} + C\dot{\vec{u}} + K\vec{u} = 0$$

after linearization

M is a matrix of mass

C is a matrix of viscosity

K is a matrix of stiffness (elastic)

"usually" they are symmetric and positive definite

$\vec{u} = \vec{u}(t)$ with $t \to \infty$

$\vec{u} = (u_1, u_2)$ Two-dimensional case

$\vec{u} \to u$ One-dimensinal case

1D : $M\ddot{u} + C\dot{u} + Ku = 0$

initial conditions: $\begin{cases} u(0) = u_0 \\ \dot{u}(0) = v_0 \end{cases}$

Ordinary differential equation (ODE)

$u \sim e^{\lambda t}$, λ is unknown

$u = Ue^{\lambda t}$

$M\lambda^2 \cancel{Ue^{\lambda t}} + C\lambda \cancel{Ue^{\lambda t}} + K\cancel{Ue^{\lambda t}} = 0$

Characteristic equation:

$$\boxed{M\lambda^2 + C\lambda + K = 0}$$

roots: λ_1, λ_2 : 1) real

$u(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$

A, B – const

2) complex roots

$$\lambda_{1,2} = a \pm i b, \qquad \underline{i^2 = -1}$$

$$u(t) = A e^{(a+ib)t} + B e^{(a+ib)t}$$

$$\rightarrow \quad u(t) = e^{at}\left[\tilde{A} \cos bt + \tilde{B} \sin bt\right]$$

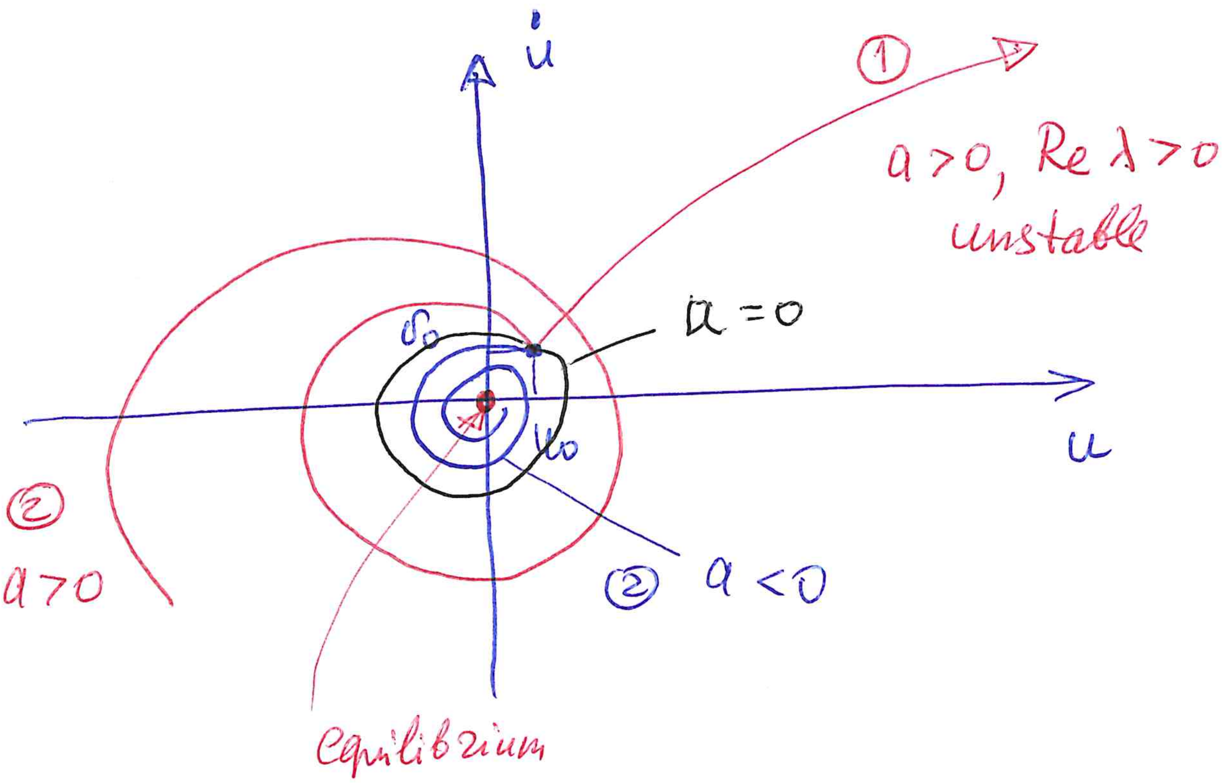
3) imaginary roots

$$\lambda_{1,2} = \pm i b$$

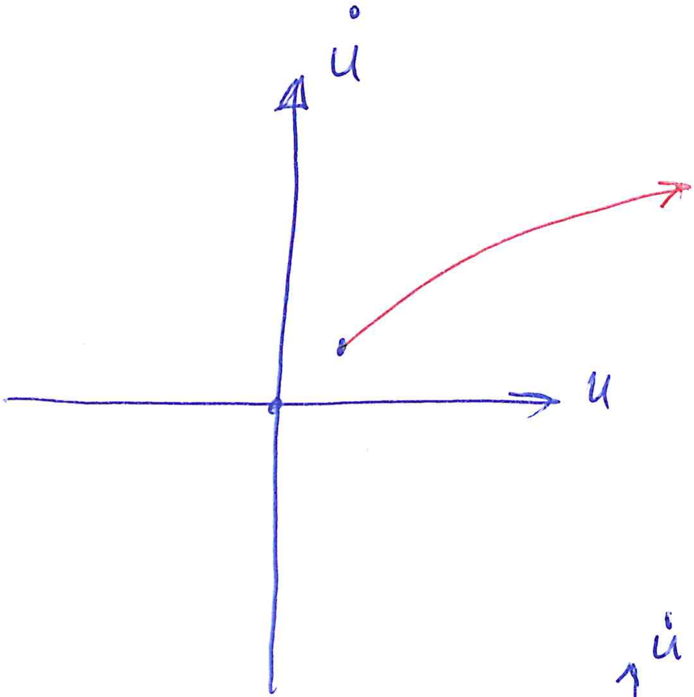
$$u(t) = A \cos bt + B \sin b b$$

Stability $|u(t)| \to 0 \quad t \to \infty$

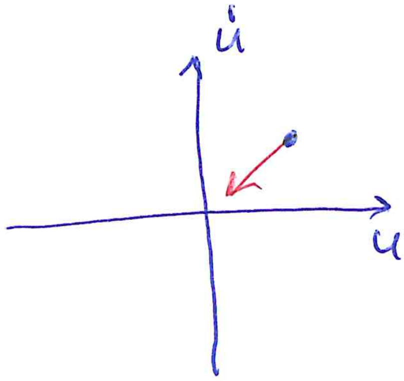
or, at least, $|u(t)| \leq Const$



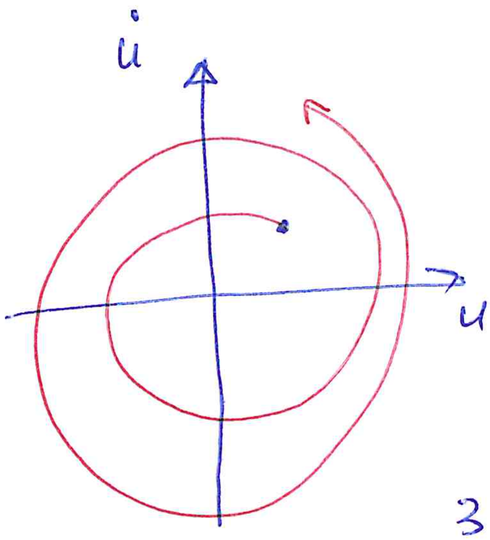
1. $a > 0$



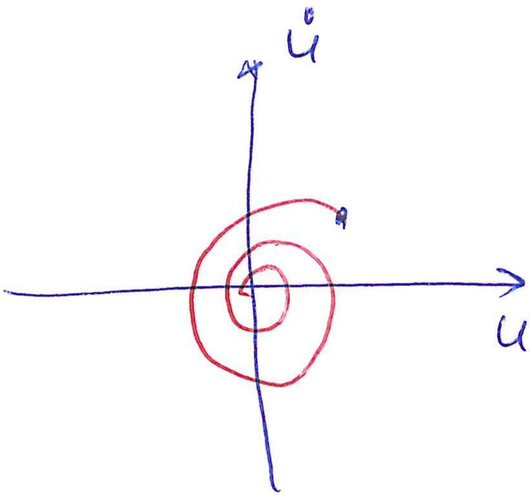
1. $a < 0$



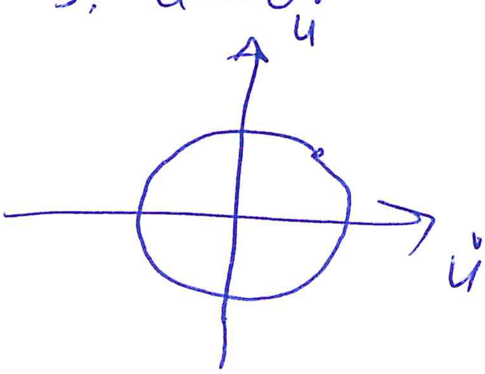
2. $a > 0$



2. $a < 0$



3. $a = 0$



Conclusion

Find λ

Find a critical parameter when $Re\,\lambda = 0$

&

$Re\,\lambda < 0$ — stability

$Re\,\lambda > 0$ — instability

asymptotic stability (instability) in sense of Lyapunov

$\Longrightarrow$ it is ~~enough~~. enough to study linear problem !

Problem with $Re\,\lambda = 0$ $\rightarrow$

linear system is stable.